DESIGN OF DISCRETE SLIDING MODE CONTROLLER FOR HIGHER ORDER SYSTEM

S. R. Shiledar, G. M. Malwatkar, I. S. Jadhav, G. V. Lakhekar

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Department of Instrumentation Engineering
Government College of Engineering, Jalgaon-425002, India.
shaktishiledar@gmail.com

Abstract

This paper presents a discrete sliding mode controller for higher order systems without using model order reduction techniques for single input single output systems. The proposed controller is designed with discrete form of PID as a sliding surface. To find the PID sliding surface controller tuning parameters, the traditional method of PID design or pole placement method can be used. The designed controller has flexibility in terms of range of the parameters to decide the stability and robustness of the closed loop performance and existence in terms of Lyapunov function and/or stability. Generally it is difficult to design proper controller due to inaccurate identified model of system or its parameters and external unmesaurable disturbance. The proposed controller has a simple and flexible structure having a set of tuning equations as a function of the desired performance of the systems. The discrete form of sliding surface and system states provide highly useful information to control necessary parameters of the interest for many higher or lower order systems. The systems available in real time or plant model identified by different method in the context of design of the controllers results in higher order; therefore it is necessary to direct the automation applications of systems towards higher order systems. In this paper, the examples are simulated using Mathwork's MATLAB to show and compare results proposed law with prevalent available controllers.

Keywords: Discrete sliding mode control, Higher order system, Sliding mode control, Robustness, Simulation.

I. Introduction

Generally the performance of the PID controller are not achievable properly for the higher order model unless it reduced to first or second order systems [1-3]. The main aim to design the robust controller for higher order system to minimize offset and uncertainty in the plant for that purpose design strategies for controller developed model mismatch themselves the era of the variable structure controller (VSC) and latter days it modified and is famous as the sliding mode control techniques. Sliding mode control (SMC), first introduced in the early 1950s, has been a focus to tackle system uncertainties and external disturbances with good robustness [4]. Many researchers found good results from discrete sliding mode control (DSMC) over the continuous sliding mode control [1],[4,7]. Sliding mode control recently widely used in many control engineering applications due to its robustness and simplicity of computation. It has been successfully applied to underwater vehicle, automotive transmissions engines, power systems, induction motor, robotics etc [4,6-7]. Design of Sliding mode control (SMC) mainly consists of two important phases one is the design of a sliding surface and the second is the switching law. In literature, the design methods of PI/PID controllers for higher order delayed system with model reduction method and it is required complex computation [8-9]. Many times it is observed that higher ordered model not reduced exactly to plant behavior because of model mismatch, uncertainty and disturbance occurs in the plant. Therefore it directly affects the performance and stability of the system. [3,10]

The discrete version of SMC has been used in system control whenever digitized system has good stabilization with low level acceptable sampling period [10]. It should be pointed out that DSMC is not the counterpart of the continuous sliding mode controller [4]. The literature found that state observer is usually developed to realize the DSMC, which increases the burden on controller design. An unstable control system produces due to the inappropriate design of the state observer or control law. Hence, the method was preferable to reduce the workloads of state observer design for the DSMC [11]. The delay in the system has another factor to affect the stability of the process so it is difficult to control the variables in the process. The common strategy to eliminate it required to design delay-free process pointed out in the framework of Smith's predictor and use it for controller design[12],[13]. The robustness properties of SMC are found when the system reaches the sliding surface, but it observed that during the system moves toward reaching phase system becomes undesired high-frequency oscillation (chattering) occurs, because of the discontinuous switching function, which causes to control signal oscillated around the switching surface. The chattering-effect is undesirable to the final control elements, it will be possibility to damage the control elements in the field [7,12]. To overcome this problem one way to minimize the chattering effects is to select continuous approximation for the discontinuous function signum in the controller, which is replaced by a smooth function like saturation or hyperbolic function with the appropriate binding of error within some predefined boundary [3,14]. The second approach is to introduce an adaptive switching gain, which adapts the gain as per the present conditions. Design higher order sliding mode control is another way to eliminate the chattering effect. We observed that from past increasing the computing power of electronic devices and discrete-time sampler computer-based control has become popular to design control tasks.

A key step in the design of sliding mode controllers is to introduce a proper sliding surface so that tracking errors and output deviations can be reduced to a satisfactory level. In this work a DSMC with velocity form of discrete sliding surface is used to obtain a desired set point tracking. The system used in the simulation is typically higher order with considerable time delay. The discrete time form of the continuous system is obtained using a matched pole zero method and represented in state space using controllable Canonical form. The parameters of equivalent controllers are obtained by means of traditional approach of PID control design. In this simulation both non oscillatory and moderately oscillatory but higher order with time delay systems are experimentally used to validate usefulness of the proposed control law in discrete form.

Many available SMCs are designed based on PD type sliding surface [14], [12,15]. which introduce to large steady state error due to external disturbances it overcome this drawback, SMC with PID-type sliding surface design so, that integral term introduced into the sliding surface formulation.

In overall the major contribution of this study can be summaries as follows

- The DSMC resolved limitation of CSMC in case of slow system where sampling time of digital implementation is considerably large.
- The proposed method is applicable to higher order plus dead time process with oscillatory behavior and work satisfactory under the influence of system uncertainties.
- The developed control scheme eliminates steady state error and chattering problem.

- The simulation result are presented to make qualitative comparison with traditional continues SMC and improved PID controller.

The organization of the paper is as follows section II includes short information of continuous controller and discrete sliding mode control law, section III includes simulation examples and its performance analysis while section IV remarked as the conclusion of the work .

III. Sliding Mode Controller

A. Continuous Sliding-Mode Control

In Eker's work, a continuous form of PID sliding surface with three parameters has been introduced to achieve a satisfactory closed-loop system performance which is given below

The equivalent controller given by Eker [6] is

$$u_{eq}(t) = \frac{1}{k_d C_n}\left[k_p \dot{e}(t) + k_i e(t) + k_d \ddot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)\right] \tag{1}$$

and the switching control selected is,

$$u_{sw}(t) = k_{sw} \tanh\left(\frac{s(t)}{\beta}\right) \tag{2}$$

Thus the total control is

$$u(t) = \frac{1}{k_d C_n}\left[k_p \dot{e}(t) + k_i e(t) + k_d \ddot{r}(t) + k_d A_n \dot{y}(t) + k_d B_n y(t)\right] + u_{sw}(t)$$

The switching control law used in this work is

$$u_{sw}(t) = k_{sw} \tanh\left(\frac{s(t)}{\beta}\right)$$

B. Proposed Discrete Sliding Mode Control Law

As per the design of sliding controller formulation, there are two control laws first is equivalent control and switching control law. Nowadays there are many advantages of digital control technologies due to the rapid evaluation of digital devices. Therefore it is natural to have growth of the researcher's attitude towards implementation and simulation of discrete SMC laws [12,14]. As the sampling rate is not near to infinity in practical systems, therefore, continuous term in discrete time control law introduces an unwanted phenomenon of chattering and instability. Therefore it is essential to keep discontinuous term very small to avoid instability of the system.

The representation of system in the form of discrete state space state space as

$$x(k+1) = \emptyset x(k) + Hu(k) + l(k)$$

$$y(k) = Cx(k-d) \tag{3}$$

Where $\phi \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^{n\times 1}$ and $C \in \mathbb{R}^{1\times n}$ represent discrete time state space matrices, and x(k) is state vector. The term $l(k) \in \mathbb{R}^{n\times 1}$ represents the lumped uncertainty and it is bounded. The system model (10) used to calculate the equivalent control law. The term d is used for the number of delay samples.In this paper chosen as the sliding surface is design as discrete sliding surface

$$S(k) = Kx(k) - K_t\begin{bmatrix} e(k) + kp[e(k+1) - e(k)] + kie(k) \\ +kd[e(k+1) - 2e(k) + e(k-1)] \end{bmatrix} \tag{4}$$

Where K=[K1,K2….Kn] is the gain matrix calculated through pole placement method and it used as tuning parameters for DSMC, x(k) is the state vector and Kt is the constant for controller gain. In pole placement design desired values of settling time and damping factor required to compute state feedback gain based on Ackermann's formula The error function formulated as

$$e(k) = r(k) - y(k) \tag{5}$$

Where e(k) is the error signal, r(k) is reference input and y(k) represents systems output. Formulation for the equivalent control law in discrete form when condition that S(k)=0 is the sliding surface in equation(4) the response of sliding at (k+1)th instant

$$S(k+1) = Kx(k+1) - K_t\,[u(k+1) + kp[e(k+2) - e(k+1)\,] + kie(k+1) + kd[e(k+2) - 2e(k+1) + e(k)\,]\,] \tag{6}$$

Using (3) and (6) formulated the equivalent control law as

$$S(k+1) = K\emptyset x(k) + KHu(k) + Kl(k) - K_t\,[e(k+1) + kp[e(k+2) - e(k+1)\,] + kie(k+1) + kd[e(k+2) - 2e(k+1) + e(k)\,]\,] \tag{7}$$

The equivalent control law obtained by equating equation (14) to zero given by

$$u_{eq}(k) = (KH)^{-1}\left[-K\emptyset x(k) - Kl(k) + K_t\left[e(k+1) + kp\begin{bmatrix} e(k+2) \\ -e(k+1)\end{bmatrix} + kie(k+1) + kd[e(k+2) - 2e(k+1) + e(k)]\right]\right] \tag{8}$$

The robustness with parameter variation and external disturbance is consider by introducing of high frequency discontinues function term by sgn(s(k)) function generally used. It found that the boundary layer hyperbolic function given the smooth change in the switching signal within the specified range due to which reduced the chattering

$$u_{sw} = -\tanh\left(\frac{s(k)}{\beta}\right) \tag{9}$$

The complete control law as per DSMC is

$$u(k) = u_{eq}(k) + u_{sw}(k) \tag{10}$$

III. Simulation examples

Example 1

Consider the non-oscillatory system process with open loop transfer function [2 12]

$$G(s) = \frac{1}{(s+1)(s+5)^2}e^{-0.5s}$$

The discrete time model obtained by pole placement method with sampling interval Ts=0.1 sec is

$$G(z) = \frac{0.0001473z^2 + 0.0002947z + 0.0001473}{z^3 - 2.1182z^2 + 1.466z - 0.3329}z^{-5}$$

This can be represented in the form of state space matrix in controllable canonical form as

$$\phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.3329 & -1.466 & 2.118 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0.0001473 \\ 0.0002947 \\ 0.0001473 \end{bmatrix}$$

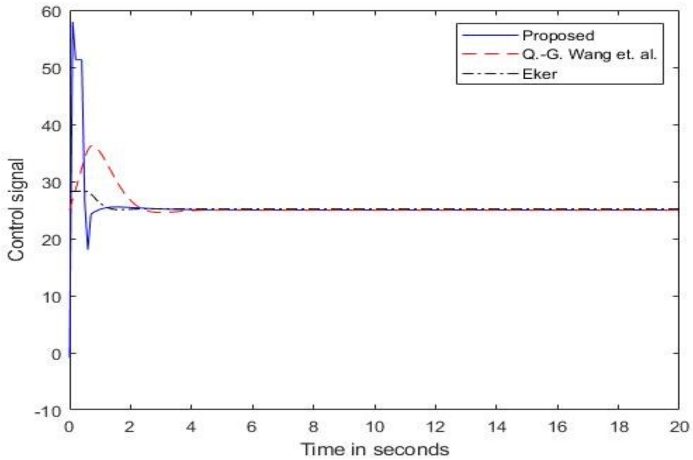

Fig. 1. Control signal generated for example 1

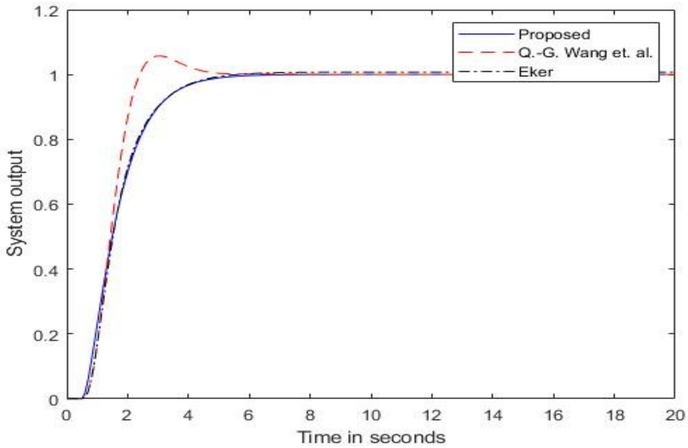

Fig. 2. System output response for example 1

For the simulated example 1 and example 2, consider the control setting values shown in Table 1.

***TABLE 1** Control setting values and*

Simulation Examples	kp	ki	kd	alpha	beta	ksw	Kt
(1)	2.3	0.01	1.65	1.999	0.09	120	17.08
(2)	2.45	0.0123	1.25	0.99	0.99	40	60

The value of pole placement with settling time (+/- 2% band) ts=2s and damping factor is 1.Gives gain matrix parameter K=[0.2493 -0.6474 0.3979].The value of Kt , alpha and beta are selected as share found from the desired performance the selected values shown in table1 , it is observed that output without chattering and obey the stability criteria. The controller parameter for Eker's SMC are kp, ki kd, and switching surface constant choosing as ksw are shown in table1.The boundary level constant β=10 chosen. For proposed study select the tuning parameters calculated with traditional method gives (12.3,-9.794, 1.032)

The Wang et al's gives the controller equation

$$G_{cwang} = 25 + \frac{18.2}{s} + 5.5s$$

As can be seen in figs(1-2),control signal response and system output compare with from proposed DSMC comparison found that large deviation in the set point tracking performance with oscillatory behavior noticed in PID response. Also less overshoot, less settling time as compare with Eker's SMC and Wang et al. In fig(1) control signal of proposed method converges faster to steady state value within 1.5sec as compare to existing method

Example 2

Consider the moderately oscillatory process with open loop transfer function [2]

$$G(s) = \frac{1}{(s^2 + 2s + 3)(s + 3)} e^{-0.3s}$$

The discrete time model obtained by pole placement method with sampling interval Ts=0.1 sec is

$$G(z) = \frac{0.0001953z^2 + 0.0003905z + 0.0001953}{z^3 - 2.1182z^2 + 1.466z - 0.3329} z^{-3}$$

This transfer function represent in the form of state space matrix in controllable canonical form as

$$\phi = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.6065 & -2.1462 & 2.5324 \end{bmatrix}, \quad H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C^T = \begin{bmatrix} 0.0001953 \\ 0.0003905 \\ 0.0001953 \end{bmatrix}$$

Obtained gain matrix for tuning parameter K=[0.5633 -1.3937 0.8304].The value of Kt, alpha and beta are selected as shown in table1 are found from the getting appropriate performance of output without chattering and follow the stability criteria. The controller parameter for Eker's SMC are kp, ki kd, Here switching surface constant choosing as ksw with boundary level constant β=10.For proposed study select the tuning parameters calculated with traditional method gives (2.109, 8.925, 0.1194).The PID controller by Wang et al's method gives

$$G_{cwang} = 5 + \frac{7.146}{s} + 3.008s$$

The comparing DSMC with output response and control signal with from Eker's SMC and Wang et al's, it is remarked as the response obtained from proposed algorithm is better than that of Eker et al's and Wang et al's technique.

In fig(3) control signal of proposed method converges faster to steady state value within 0.3sec as compare to existing method. Therefore it can be concluded that the results obtained by DSMC are better than that of the CSMC for the system with higher order dynamics

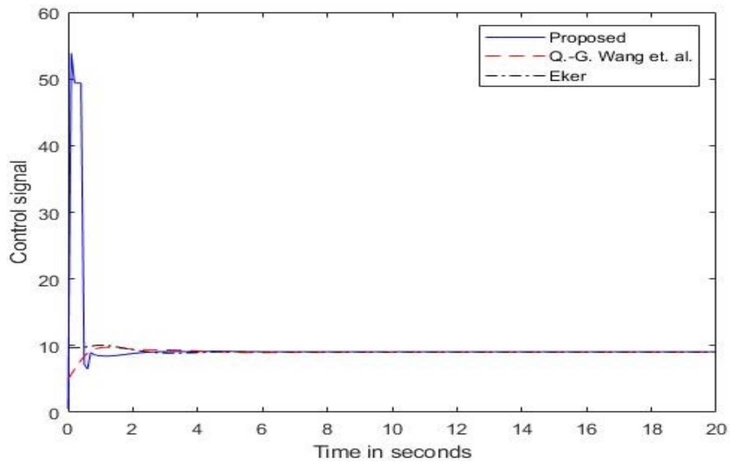

Fig. 3. Control signal genrated for example 2

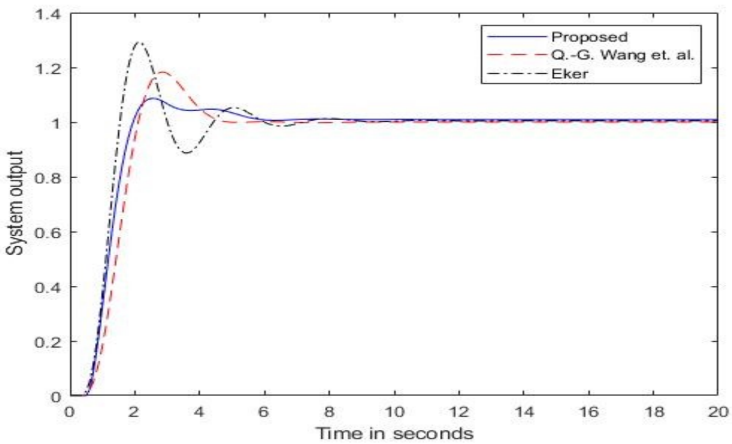

Fig. 4. System output response for example 2

IV. Conclusions

In this work, a discrete part of PID controller is used as a sliding surface to obtain the discrete sliding mode control law. The proposed discrete mode control law is applied to higher order plus delay time systems. The traditional approach of PID controller is used to obtain the tuning parameters of the discrete time sliding mode control. The design procedure given in the work looks simple and straightforward because less complexity of computations are involved in DSMC technique. Two simulation examples are included in the work are of the typical nature, the first example is monotonic with higher order plus delay time while second example is moderately oscillatory systems with higher order dynamics. The proposed DSMC is applied for set point tracking and the performance of the proposed law seem to be effective with less tracking and settling time with possibly minimum overshoot. The effort taken by the control action is also acceptable and can be useful for real time practical system. Therefore it can be concluded that the results obtained by DSMC are better than that of the CSMC for the system with higher order dynamics

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