RT&A, Special Issue № 1 (60)
Volume 16, Janyary 2021

CHI-SQUARE STATISTICS BASED STATE ESTIMATION OF STOCHASTIC HYBRID SYSTEM WITH MISSING MEASUREMENTS

Robinson P. Paul[1], Vishvjit K. Thakar[2], Hetal Patel[3]

•

[1] Electronics & Communication department,
GTU, Gujarat, India
[2] Gokul Global University, Gujarat, India
[3] 3Electronics & Communication department, ADIT, Gujarat, India
_robinson.p.paul@gmail.com

Abstract

The objective of this paper is to estimate the successful state of Stochastic Hybrid Dynamical Systems (SHS) having a continuous and discrete dynamic state with lossy measurements without expanding the computational multifaceted nature of the calculation. Existing strategies for state estimation furnishes state estimation with computational multifaceted nature and time. Kalman and Particle Filter are the fundamental algorithms for state estimation of any Linear and Non-linear SHS. The measurements got by the sensors or any modules have limitation to provide accurate data. State estimation of any system with lossy data prone to inaccurate state estimation. In this paper, the Chi-square Statistics –Data Loss Detection (CS-DLD) algorithm based on Kalman Filter is proposed for of Stochastic Hybrid Dynamical Systems which detect the data loss and reconfigure the measurement for effective state estimation. The performance of the algorithm is checked by Linear Stochastics Hybrid System model of Aerial vehicle.

Keywords: State estimation; Dynamic Hybrid system; Lossy measurement; Chi-square statistics; Kalman filter; Particle Filter; Monte Carlo simulation; State update; Measurement update covariance matrices; Interactive Multiple Model

I. Introduction

The dominant part of the dynamic systems' approach, technique and measures have arbitrary likelihood distribution of its events known as stochastic conduct. It indicates the design which might be literally anticipated yet may not be anticipated impeccably. In a reasonable situation, a larger part of the dynamic system can be arranged as Stochastic Hybrid Systems (SHS) [1] where the system model governs by discrete and continuous dynamics. In the current stretch because of rapid advancement of integrated chips and its capacity, leading part of the dynamic systems contains computerized framework. The models range from miniature independent robots to gigantic planes. Indeed, even current development in the field of Autonomous vehicle additionally has contributed to the study of Hybrid framework improvement just as demonstrating better estimation and control of the Stochastic Hybrid

System. [2] Scientists Scholar have shown interest in the Stochastic framework's application like [3], [4], where the system model interacts with its continuous and discrete dynamics for better state estimation. In remote sensor systems, it is prominent that the remote sensor hubs, actuator hubs, and regulator hubs offer such a distinctive communication with constrained data transmission. As the quantity of hubs increases, the requirements for transmission capacity [5] exceeding the framework limit may bring about loss of data and system choke, which may to a great extent fall apart the framework execution. In the meantime, remote sensors and actuators are generally powered by batteries with constrained power resources., some of which are not even replaceable. Therefore, the effective state estimation algorithm, which handles the nondeterministic behaviour of the system, affected by the loss of measurement data and limitation of computation complexity handling is needed. Persuaded by the above essentials the for stochastic hybrid system event-triggered based approach will help to solve the problem of data loss as well as computation complexity since spearheaded by Åström and Bernhardsson [6] . In addition, it is notable that for state estimation lossy measurement is one of the most regularly happening marvels in this type of frameworks, see [7] [8][9] . In particular, The State estimation method of mixture system discussed in [10][11] unable to handle the lossy measurement with SHS modelling. The complication of the state approximation of the hybrid system is difficult than the others. State estimation procedure like AMM-ILS (adaptive multiple model iterated least squares), AMM-UKF [12] (adaptive multiple model unscented Kalman filter), IMM (Interactive multiple model) [13] and its diverse variety like IMM-KF(Interactive multiple model) – Kalman Filter), IMM-EKF(Interactive multiple model – Extended Kalman Filter) ,IMM-UKF (Interactive multiple model -unscented Kalman filter), IMM-UGHF [14] (Interactive multiple model – Unscented Gauss-Helmert model) [15] deliver good state estimation at the budget of sophisticated mathematics calculation. As Popular of the dynamic system can be effortlessly and effectively classified as the continuous time linear dynamics systems [16] but unable to incorporate discrete dynamics together. To overcome the above problem and motivated by the possibility of the fundamental Kalman Filter, the event triggered a state estimation method for the stochastic hybrid system with Chi-square statistics is proposed. The framework elements of the unmanned aerial vehicle (UAV) [17] is used to showcase the performance of our proposed Chi-square statistic - Data Loss Detection (CS-DLD) Kalman Filter algorithm for lossy measurement. The comparison of the results with standard Kalman Filter, Average and smooth filters show that our calculation gives improved estimation precision under the lossy measurement scenario with less computational complexity. The remainder of this paper is sorted out as follows: in Section II, Description of System dynamics of SHS is talked about. In Section III, we define the proposed methods for effective state estimation for lossy measurement. In Section IV, we outline the simulation results of the proposed method. The conclusion is given in Section V.

II. System Description

The state estimation of the Hybrid framework has numerous applications in the field of the biological, compound response, interchanges systems, aeroplane designing [18] , stock costs [19] and so forth. State estimation procedure introduced in [20] is a regular strategy which isn't helpful for all the hybrid system state estimation. State Estimation Problem has significant two classifications. Non-Switched Mode and Switched Mode state estimation. The Switched Mode is additionally ordered in Static and dynamic exchanged mode state estimation. Continuous-time stochastic hybrid framework model as,

RT&A, Special Issue № 1 (60)
Volume 16, Janyary 2021

$$H\colon \begin{cases} \mathrm{x}_{\mathrm{t+1}} = \mathrm{A}_{\mathrm{m}}.\mathrm{x}_{\mathrm{t}} + \varepsilon_x \\ \mathrm{Z}_{\mathrm{t}} = \alpha_m.\mathrm{x}_{\mathrm{t}} + \varepsilon_z \\ . \end{cases} \tag{1}$$

Here, x_{t} represent state equation and y_{t} represents measurement equation at a time 't'. A_{m} represent state dynamic in matrix form for mode 'm' , ε_x and ε_z are zero mean process and measurement white Gaussian noise respectively. Here we assume that they are independent. The mode transition from one state to another is governed by the guard condition α_m.

$$\alpha_{\mathrm{m}}\colon \begin{cases} \alpha_1 = 0 \quad for\ \mathrm{x}_{\mathrm{t+1}} \in \{Lossy\ measurment\} \\ \alpha_2 = 1 \quad for\ \mathrm{x}_{\mathrm{t+1}} \notin \{Actual\ measuremt\} \end{cases} \tag{1}$$

The System mode transition from actual measurement to lossy measurement are shown in below Fig. The detection of the lossy measurement provides the opportunity to update the lossy measurement for accurate state estimation.

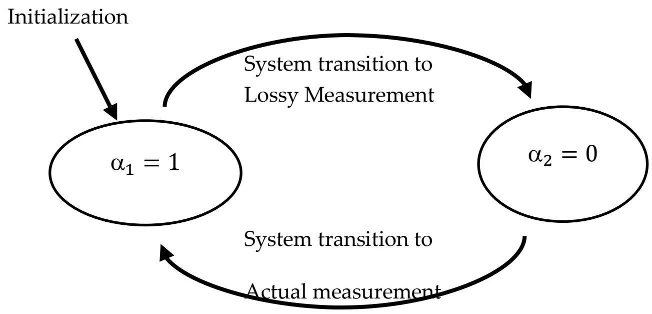

Figure. 1 *Event-triggered mode transition*

To deal with such kind of system where continuous dynamics are controlled by discrete dynamics, stochastic hybrid system modelling based on event-triggered helps to effectively predict the state with the use of Chi-square statistics with Kalman Filter for a linear system. The detail method is explained in Section 3.

III. The proposed method for state estimation

In this section, we will examine a proposed method for continuous time SHS. The performance of the proposed estimation algorithm with standard Kalman Filter (KF) [21] and Average Smoothed Filter (ASF) is investigated. The continuous-time dynamics system [22] is used with state-dependent transitions to model the UAV to detect the performance of the algorithms. This competent state estimation outcome will help us to implement the operative estimation algorithm for multiple modes hybrid system under the noisy state measurement scenario. The continuous dynamics of the system is given by

$$x_t = \begin{bmatrix} 1 & 3T \\ 0 & 1 \end{bmatrix} . x_{t-1} + \begin{bmatrix} \frac{T^2}{3} \\ 2\mathrm{T} \end{bmatrix} . \mathrm{a} + \varepsilon_x \tag{2}$$

which is known as state prediction and the measurement prediction is given by

$$z_t = \alpha_m . x_t + \varepsilon_z \tag{3}$$

Where $x_t = \begin{bmatrix} p \\ v \end{bmatrix}$; p = position, v = velocity and u_t = accelaration = a .

$$\text{Here , State Noise/Error} = \varepsilon_x = \begin{bmatrix} \sigma_p.\sigma_p. & \sigma_p\sigma_v \\ \sigma_p\sigma_v & \sigma_v.\sigma_v \end{bmatrix};$$

σ_v , σ_p = variance in velocity & position

$\varepsilon_z = \sigma_p.\ \sigma_p$ =Measurement Noise /Error

For the above structure, described in eq. (3) and eq. (4) with the distinct changeover from the lossy state to lossless state are simulated to detect the performance of the proposed algorithm for diverse % loss.

Pseudocode

Prior requirement: $\hat{x}_0$, $P_{x,0|0}$, F_k, H_k, y_k,

Set k = 1

❖ **Prediction**

Predicted state : $\hat{\mathbf{x}}_{k|k-1} = F_{k-1}\hat{\mathbf{x}}_{k-1|k-1}$

Predicted error covariance: $P_{x,k|k-1} = F_{k-1}P_{x,k-1|k-1}F'_{k-1} + Q_k$

❖ **Measurement Update**

Predicted measurement: $y_{k|k-1} = H_k\hat{\mathbf{x}}_{k|k-1}$

Innovation covariance matrix: $P_{y,k|k-1} = H_kP_{x,k|k-1}H'_k + R_k$

ek = $((y_k - \hat{y}_{k|k-1})'.P'_{y,k}.\hat{\mathbf{x}}_{k|k-1})^2$

if ek > $\chi 2$; Chi Square Test for 95% confidence interval

I Gain = 0

else

Gain: $K_k = P_{x,k|k-1}H'_kP^{-1}_{y,k|k-1}$

Updated state: $\hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + K_k(y_k - \hat{y}_{k|k-1})$

Error covariance: $P_{x,k|k} = P_{x,k|k-1} - K_kH_kP_{x,k|k-1}$

Set k = k + 1 and repeat from Prediction

Pseudocode describes the state and measurement update steps of the algorithm. The proposed method can be arranged in three principal classes. (i) System Initialization - which manages state elements where procedure and estimation condition choose the framework development. (ii) State Estimation-manages figuring of expected estimation of the state affected by arbitrary measurement with the assistance of covariance matrix with noise. (iii) Data Loss Detection- with Chi-Square Statistics of 95% confidence interval in which we abandon the lossy measurement and update it with the most appropriate Kalman gain for the compelling state estimation.

IV. Simulation results and discussion

The proposed estimation method CS-DLD Kalman Filter is compared with standard Kalman Filter (KF) [24] and Average Smoothed Filter (ASF). This productive state estimation result will assist in the effective state estimation of event-triggered SHS. In Table I, the comparison of the Average Mean Square Error for different methods is displayed for various % loss. Below enumerated parameters are used for the system simulation.

Parameter	Value	Parameter	Value
Time axis	**100**	**Measurement noise (ε_z)**	**10**
Control input (a)	**1.5**	**Process noise (ε_x)**	**0.05**
Initial state [positon;vel]	**[10;10]**	**Monte Carlo Simulation**	**100**

TABLE I:

Average MSE Simulation Results for % loss of 1 to 10 with

DLDKF filter with and without Chi-Square Test.

% Loss	Non-Smooth Filter	Smooth Filter	Kalman Filter	Proposed DLD Kalman Filter	Proposed DLDKF with CS
1	10.2980	4.9543	0.6819	0.6821	0.8275
2	12.0445	5.5013	0.5593	0.5550	0.5357
3	17.1328	7.6611	0.7299	0.7080	0.7348
4	21.5035	9.6562	0.8835	0.8490	0.8133
5	29.2830	12.9872	1.2676	1.1587	0.6178
6	36.9649	16.5515	1.8699	1.6327	0.7572
7	44.5553	23.1757	3.1364	2.6889	0.6522
8	52.2752	26.1008	4.3134	3.7425	0.6933
9	61.5342	30.9080	5.6153	4.9190	0.6596
10	70.4166	37.0281	7.3795	6.4259	0.6106
Avg	**35.6008**	**17.45242**	**2.64367**	**2.33618**	**0.6902**

As appeared in Fig. 2, The state estimation of the proposed method is better than the non-smooth filter, smooth filter and standard Kalman Filter.

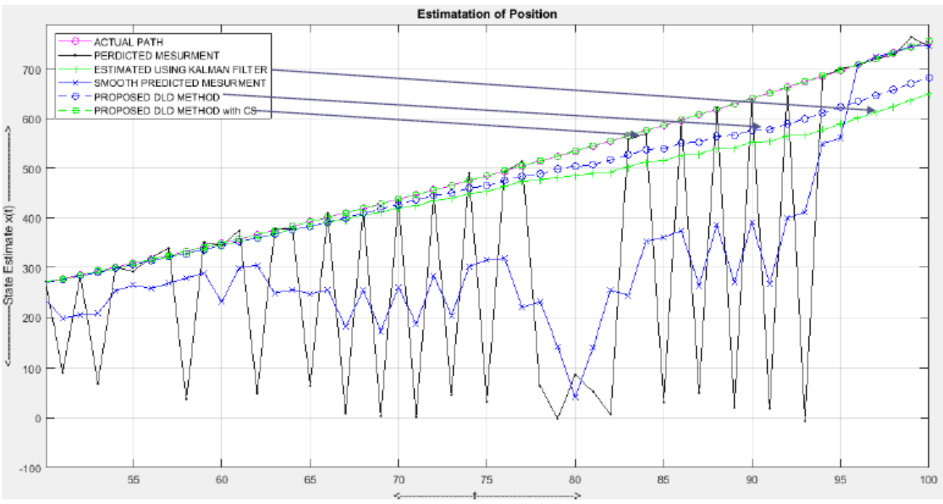

Figure 2. *Proposed Method State Simulation with standard methods*

It is plainly seen that that lossy information in the estimation, drive the state estimation calculation away from the real estimation of the state. Fig.3 shows the MSE (Mean square error) for a similar situation of Non-Smooth Filter (NSF), Average smooth Filter (ASF) and Kalman Filter (KF), where it is clear from the outcome that though Kalman Filter performs equivalent to the proposed method but unable to neglects the lossy measurement.

Fig. 4 and Table II, exhibit the hardware configuration used for the time performance calculation of our proposed algorithm. The simulation results show that with 19ms delay for 250 samples compared to standard Kalman filter, the proposed algorithm able to achieve good state estimation under the lossy measurement.

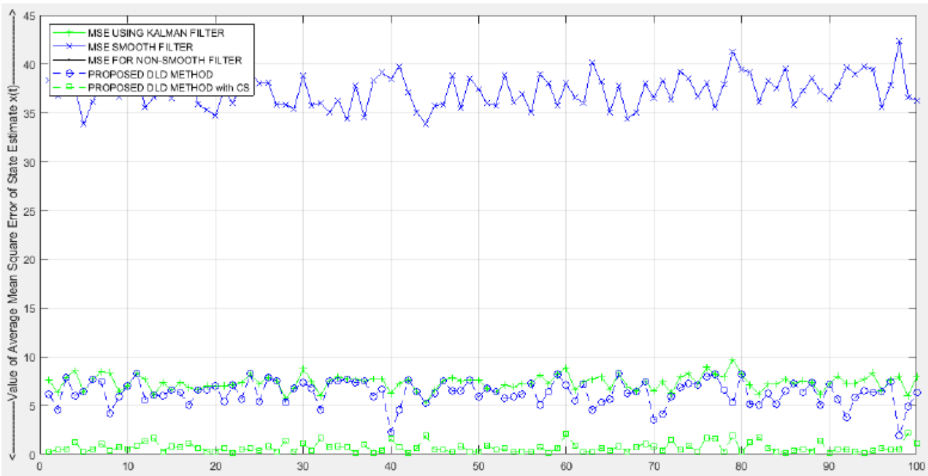

Figure. 3 *Average MSE value of state estimation at each time simulation*

For the compelling representation of the exhibition of our proposed calculation Table II demonstrates the MSE of the all technique where our proposed method gives preferable outcome over Kalman Filter (KF) and Average Smooth Filter (ASF) without expanding any computational complexity for the state estimation.

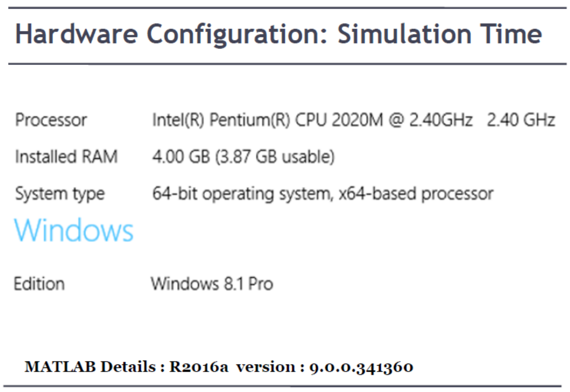

Figure 4. *Hardware configuration for the timing simulation*

TABLE II:

Proposed method computation time compared with standard Kalman filter for different samples.

Samples	State Estimation Proposed DLD Method Execution time	State Estimation Standard Kalman Filter Algorithm Execution time	Difference of Execution time In seconds
50	0.075896 seconds	0.060592 seconds	+0.015304
100	0.080629 seconds	0.062200 seconds	+0.018429
150	0.086943 seconds	0.072300 seconds	+0.014643
200	0.091026 seconds	0.074692 seconds	+0.016334
250	0.096695 seconds	0.077108 seconds	+0.019587

IV. Conclusion & future work

According to the present strategies and assessment are done, in the Event-triggered SHS framework with Chi-square statistics Kalman filter can be anticipated by the notable performance improvement in the event of the lossy measurement without expanding the complicated nature of the calculation. For linear SHS, Data Loss Detection Kalman Filter method gives 11.83% improvement over standard Kalman Filter is in terms of Average Mean Square Error while Chi-Square statics based Data Loss Detection algorithm gives 73.89 % better results in terms of Average Mean Square error compared to standard Kalman Filter for low Process Noise scenario. As the performance of the proposed algorithm depends on the process noise, the effect of process noise variation can be explored as further work.

References

[1] W. Liu and I. Hwang, "On Hybrid State Estimation for Stochastic Hybrid Systems," IEEE Trans. Automat. Contr., vol. 59, no. 10, pp. 2615–2628, 2014.

[2] W. Liu and I. Hwang, "On Hybrid State Estimation for Stochastic Hybrid Systems," IEEE Trans. Autom. Control., vol. 59, no. 10, pp. 2615–2628, 2014.

[3] C. E. Seah, H. Inseok, and I. Hwang, "Stochastic Linear Hybrid Systems: Modeling, Estimation, and Application in Air Traffic Control," Control Syst. Technol. IEEE Trans., vol. 17, no. 3, pp. 563–575, 2009.

[4] M. Dolgov, G. Kurz, and U. D. Hanebeck, "State estimation for Stochastic Hybrid Systems based on deterministic Dirac mixture approximation," in Proceedings of the American Control Conference, 2014, pp. 1408–1413.

[5] B. Yan, J. Zuo, X. Chen, and H. Zou, "Improved multiple model particle filter for maneuvering target tracking in the presence of delayed measurements," 2017 Int. Conf. Comput. Syst. Electron. Control. ICCSEC 2017, pp. 810–814, 2018.

[6] B. Bernhardsson and K. J. Åström, "Comparison of Periodic and Event Based Sampling for First-Order Stochastic Systems," 1999.

[7] R. M. Jungers and W. P. M. H. (Maurice) Heemels, "Controllability of linear systems subject

to packet losses**Raphael is supported by the Communaut_e francaise de Belgique - Actions de Recherche Concert_ees, and by the Belgian Programme on Interuniversity Attraction Poles and by the 'Actions de Recherche," Proc. 5th IFAC Conf. Anal. Des. Hybrid Syst. ADHS'15, vol. 48, no. 27, pp. 80–85, 2015.

[8] A. K. Fletcher, S. Rangan, and V. K. Goyal, "Estimation from lossy sensor data: jump linear modeling and Kalman filtering," in Information Processing in Sensor Networks, 2004. IPSN 2004. Third International Symposium on, 2004, pp. 251–258.

[9] S. Deshmukh, B. Natarajan, and A. Pahwa, "State Estimation Over a Lossy Network in Spatially Distributed Cyber-Physical Systems," IEEE Trans. Signal Process., vol. 62, no. 15, pp. 3911–3923, 2014.

[10] C. G. Cassandras, "Event-driven control, communication, and optimization," in Control Conference (CCC), 2013 32nd Chinese, 2013, pp. 1–5.

[11] I. Hwang, H. Balakrishnan, and C. Tomlin, "State Estimation for Hybrid Systems: Applications to Aircraft Tracking *."

[12] R. Yang, Y. Bar-Shalom, and G. W. Ng, "Altitude Estimation Using Multipath with a Two-Dimensional Radar over Spherical Earth," IEEE Trans. Aerosp. Electron. Syst., vol. 54, no. 2, pp. 770–782, Apr. 2018.

[13] Y. Ruan and L. Hong, "Use of the interacting multiple model algorithm with multiple sensors," Math. Comput. Model., vol. 44, no. 3–4, pp. 332–341, Aug. 2006.

[14] X. Fu, Y. Jia, F. Yu, and J. Du, "New interacting multiple model algorithms for the tracking of the manoeuvring target," IET Control Theory Appl., vol. 4, no. 10, pp. 2184–2194, Oct. 2010.

[15] Z. Chenyang and X. Linbo, "Fault Detection for Stochastic Switched System Based on Fuzzy Adaptive Unscented Kalman Filter," 2018 IEEE 4th Int. Conf. Control Sci. Syst. Eng. ICCSSE 2018, vol. 1, no. 4, pp. 360–363, 2018.

[16] J. Ru and X. R. Li, "Variable-structure multiple-model approach to fault detection, identification, and estimation," IEEE Trans. Control Syst. Technol., vol. 16, no. 5, pp. 1029–1038, 2008.

[17] F. Morbidi, R. A. Freeman, and K. M. Lynch, "Estimation and control of UAV swarms for distributed monitoring tasks," in American Control Conference (ACC), 2011, 2011, pp. 1069–1075.

[18] Chze Eng Seah and Inseok Hwang, "State Estimation for Stochastic Linear Hybrid Systems with Continuous-State-Dependent Transitions: An IMM Approach," IEEE Trans. Aerosp. Electron. Syst., vol. 45, no. 1, pp. 376–392, Jan. 2009.

[19] F. P. Dipaolo, J. H. Drew, A. G. Glen, and M. Lawrence, "Cumulative Istributionfunction : Stock Price Forecasting," no. Icimia, pp. 394–398, 2017.

[20] Z. H. E. CHEN, "Bayesian Filtering: From Kalman Filters to Particle Filters, and Beyond," Statistics (Ber)., vol. 182, no. 1, pp. 1–69, 2003.

[21] Q. Li, R. Li, K. Ji, and W. Dai, "Kalman Filter and Its Application," in 2015 8th International Conference on Intelligent Networks and Intelligent Systems (ICINIS) ,pp.74-77, 2015.

[22] C. Lei, C. Qian, W. Tian, and R. Jia, "State estimation in computer virus epidemic dynamical systems using hybrid extended Kalman filter," in 2017 IEEE International Conference on Information and Automation (ICIA),pp.349-354, 2017.