Stocks' Data Mathematical Modeling using Differential Equations: The Case of Healthcare Companies in Athens Stock Exchange

Miltiadis Chalikias [1], Ioannis Triantafyllou[2]*, Michalis Skordoulis [3], Dimitris Kallivokas[4] Panagiota Lalou[5]

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[1]Department of Accounting and Finance, School of Business and Economics, University of West Attica, 250 Thivon and Petrou Ralli, GR12244, Egaleo, Greece; mchalikias@hotmail.com (M.C.);
[2]Department of Computer Science & Biomedical Informatics
School of Science, University of Thessaly, 2-4 Papasiopoulou str.35131 Lamia (Greece), itriantafyllou@uth.gr
[3]Department of Forestry and Management of the Environment and Natural Resources, School of Agricultural and Forestry Sciences, Democritus University of Thrace, 193 Pantazidou, GR68200, Orestiada, Greece; mskordoulis@gmail.com
[4]Department of Business Administration, School of Business and Economics, University of West Attica, 250 Thivon and Petrou Ralli, GR12244, Egaleo, Greece; dkaliv@uniwa.gr
[5]Aspropyrgos Merchant Marine Academy, Paralia Aspropyrgou, 19300, Attica, Greece, glalou@hotmail.com
*Corresponding author

Abstract

Stock prices' prediction is fundamental for investment decision-making. In this research, a differential equations model is developed for stock prices prediction. More specifically, a 7×7 differential equations system based on Lanchester's combat models will be used. Data concerning the short-term stock's prices of healthcare firms listed in Athens Stock Exchange will be analyzed in order to develop and evaluate the stocks' prices predictive model. The obtained results revealed the differential equations model potential for stock prices' prediction in the short-term.

Keywords: stock price prediction, forecasting, differential equations, Lanchester's combat model

I. Introduction

Stock markets are formal, organized and regulated markets for securities whose prices are determined by the law of supply and demand. In these markets the opposite expectations of investors are met for the formation of stock prices at a given time. More specifically, there are always some investors who believe that the price of a stock is going to fall and others who believe that price of the same stock is going to rise. The former are trying to sell their stocks pushing their price to fall, while the latter are trying to buy these stocks, pushing their prices to rise. Investors see the stock markets as an alternative form of investing their capitals, in order to gain a satisfactory return, higher than that these other investments such as bank deposits or government bonds.

Stock markets are also parts of the financial systems and, like banks, they provide the means and services to transfer funds from investors' savings to firms. Based on stock markets, investors expect positive returns, which are achieved through the growth of the firms leading to a rise in the stock prices.

Several studies suggest a correlation between many factors such as political and financial stability and stock prices [1]. Macroeconomic and psychological factors can affect stock prices [2]. Invest decisions in stocks are found to be affected by factors such as optimism and pessimism as well [3]. Furthermore, stock prices can be affected by factors such as macroeconomic data, market circles, trade balance, firms' profits, technology and globalization [4].

Predicting stock prices would be really crucial for investment decision-making [5–6]. Despite the many factors that can affect stock prices, their prediction can be achieved even it is a difficult process. This is the main reason why stock prices prediction is in the spotlight of most of the investors and professional analysts [7].

Chang and Liu [7], propose a model to predict stocks' future prices using a first order Takagi–Sugeno model. Their model was tested on Taiwan Stock Exchange stocks and the model's output outperformed other approaches such regression analysis. Schöneburg [8], used neural networks to predict German stock prices by the aid of temporary and not-long lasting framework. The proposed model achieved a degree of accuracy up to 90%. Neural networks were used by Kohara *et al*. [9] as well. In their research, they used data from 330 days to estimate their model's coefficients. Adebiyi *et al*. [10], used the following ARIMA model to predict the future prices of stock prices:

$$Y_t = \varphi_0 + \varphi_1 Y_{t-1} + +\varphi_2 Y_{t-2} + \ldots + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \ldots - \theta_q \varepsilon_{t-q} \quad (1)$$

where Y_t is the real value and ε_t is the random error at t, φ_i and θ_j are the coefficients, p and q are the integers called autoregressive and moving average respectively. The same authors [11] are comparing the accuracy of ARIMA and neural networks models in predicting stock prices. Their results show that both the models achieve effective forecast for stock prices. Hafezi *et al*. [12], proposed a bat-neural network model based on a multi-agent framework to predict DAX stock prices in quarterly periods of eight years.

Katsouleas *et al*. [13] proposed a generalized differential equations model by the aid of the so-called Lanchester's combat approach to predict the healthcare firms of Athens Stock Exchange (ASE) stocks' prices. The model was based on the prior work of Chalikias and Skordoulis on Lanchester's combat approach concerning the case of a duopolistic market [14] as there is evidence that warfare models can be applied in business cases [14–17]. The primary differential equations model was the following one:

$$\begin{aligned} \frac{dx}{dt} &= -ay + f(t) \\ \frac{dy}{dt} &= -bx + g(t) \end{aligned} \quad (2)$$

where x(t) and y(t) refer to the amount of ready-for-use product items for sale of firm A and B correspondingly, f(t) and g(t) refer to their respective increase and decrease rates, while ay(t), bx(t) correspond to the handy product items' rates.

The principal objective of the present manuscript is to develop a differential equations model by the aid of Lanchester's combat approach for stock price prediction.

II. Methods

The data used in this research concern healthcare firms listed in ASE. In Greece, citizens receive health care from both public and private providers. The increasing problems on public health care system is the main factor which is responsible for the growth of private sector [18]. The private healthcare sector represents the 32.9% of health care market in Greece [19]. Greek private health services contain diagnostics centers and clinics as primary and secondary health care units respectively. The 5 largest groups of the private healthcare sector correspond to 53% all of the market's stocks [20]. This market contains seven firms in Athens stock exchange, Axon (AXON), Euromedica (EUROM), Iaso (IASO), Iatriko Athinon (IATR), Lavipharm (LAVI), Medicon Hellas (MENTI) and Hygeia (YGEIA).

The Athens Stock Exchange constitutes the only authorized stock market in Greece. Before 2002 comes to an end, almost three hundred seventy-five firms had been included, while their overall capitalization equal to € 85.5 billion. Only ASE affiliates may carry out purchase and sale requisitions for shares via the so-called Integrated Automatic Electronic Trading System (OASIS) of the market. The ASE is actually an order-driven market, since its affiliates can continually commence offer requisitions in the system from 11:00 a.m. to 4:00 p.m. [21].

The study used historical stock prices of ASE healthcare firms during a 12-month period. More specifically, the stock data were picked out daily data files of ASE containing for all of the months the closing stock prices from the first day of each month.

For developing the deferential equations approach, we utilized the random variables T, U, V, W, X, Y and Z which correspond to the stock prices of the market's 7 firms. Thus, the next 7 × 7 differential equations system was primarily concluded:

$$\begin{cases} \frac{dT}{dt} = \frac{b}{a}U + \frac{c}{a}V + \frac{d}{a}W + \frac{e}{a}X + \frac{f}{a}Y + \frac{g}{a}Z \\ \frac{dU}{dt} = \frac{a}{b}T + \frac{c}{b}V + \frac{d}{b}W + \frac{e}{b}X + \frac{f}{b}Y + \frac{g}{b}Z \\ \frac{dV}{dt} = \frac{a}{c}T + \frac{b}{c}U + \frac{d}{c}W + \frac{e}{c}X + \frac{f}{c}Y + \frac{g}{c}Z \\ \frac{dW}{dt} = \frac{a}{d}T + \frac{b}{d}U + \frac{c}{d}V + \frac{e}{d}X + \frac{f}{d}Y + \frac{g}{d}Z \\ \frac{dX}{dt} = \frac{a}{e}T + \frac{b}{e}U + \frac{c}{e}V + \frac{d}{e}W + \frac{f}{e}Y + \frac{g}{e}Z \\ \frac{dY}{dt} = \frac{a}{f}T + \frac{b}{f}U + \frac{c}{f}V + \frac{d}{f}W + \frac{e}{f}X + \frac{g}{f}Z \\ \frac{dZ}{dt} = \frac{a}{g}T + \frac{b}{g}U + \frac{c}{g}V + \frac{d}{g}W + \frac{e}{g}X + \frac{f}{g}Y \end{cases} \quad (3)$$

III. Results

Let us consider the following n × n system of differential equations:

$$\frac{dx}{dt} = Ax \quad (4)$$

where $x(t) = (x_1(t), \dots, x_n(t))^T$ is an n × 1 vector of functions of the variable t, the coefficient matrix $A \in \mathbb{R}^{n \times n}$ takes the following form:

$$A = \begin{bmatrix} 0 & \frac{a_2}{a_1} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ \frac{a_1}{a_2} & 0 & \frac{a_3}{a_2} & \cdots & \frac{a_{n-1}}{a_2} & \frac{a_n}{a_2} \\ \vdots & & \ddots & \ddots & & \\ \frac{a_1}{a_{n-1}} & \frac{a_2}{a_{n-1}} & \frac{a_3}{a_{n-1}} & \cdots & 0 & \frac{a_n}{a_{n-1}} \\ \frac{a_1}{a_n} & \frac{a_2}{a_n} & \frac{a_3}{a_n} & \cdots & \frac{a_{n-1}}{a_n} & 0 \end{bmatrix} \quad (5)$$

and $[a_j]_{j=1}^n \subset \mathbb{R}$ in its definition (2) satisfy $\sum_{j=1}^n a_j = 1$.

An interesting and somewhat surprising result is that the spectrum $\sigma(A)$ is in fact independent of these parameters $[a_j]_{j=1}^n$. More precisely, it may be shown that $\sigma(A)$ is intimately related to the order n of the matrix A, including simply the eigenvalue pair $\lambda_1 = n - 1$ with algebraic multiplicity (n-1) and $\lambda_2 = n - 1$. In this direction, we will prove its characteristic polynomial may be factored as follows:

$$\chi_A(\lambda) = \det(A - \lambda I_n) = (-1)^n(\lambda + 1)^{n-1}(\lambda - n + 1) \quad (6)$$

by induction on n. Indeed, this assertion is readily verified for $n = 2$ and $n = 3$.

To simplify our analysis for larger n, for an arbitrary $n \times n$ matrix X we introduce the notation $X_{i,j}$ for its $(n - 1) \times (n - 1)$ submatrix deduced by erasing its i-th row and j-th column. Our recursive assumption may then be stated as follows:

$$\chi_{A_{n,n}}(\lambda) = \det\left(A_{n,n} - \lambda I_{n-1}\right) = (-1)^{n-1}(\lambda + 1)^{n-2}(\lambda - n + 2) \quad (7)$$

since $A_{n,n}$ is simply the leading $(n - 1) \times (n - 1)$ submatrix of A. Laplace expansion along the last row of $A - \lambda I_n$ yields to:

$$\chi_A(\lambda) = \det(A - \lambda I_n) = \sum_{j=1}^{n-1} \frac{a_j}{a_n}(-1)^{n+j}\det\left((A - \lambda I_n)_{n,j}\right) + (-\lambda)(-1)^{n+n}\det\left((A - \lambda I_n)_{n,n}\right) = \sum_{j=1}^{n-1} \frac{a_j}{a_n}(-1)^{n+j}\det\left((A - \lambda I_n)_{n,j}\right) + (-1)^n(\lambda + 1)^{n-2}(\lambda - n + 2) \quad (8)$$

To continue, we turn our attention to $\left[\det\left((A - \lambda I_n)_{n,j}\right)\right]_{j=1}^{n-1}$ and note the following properties:

Lemma 1. (a) $\det\left((A - \lambda I_n)_{n,1}\right) = \frac{a_n}{a_1}(\lambda + 1)^{n-2}$. (b) $\det\left((A - \lambda I_n)_{n,j}\right) = (-1)^{j+1}\frac{a_n}{a_j}(\lambda + 1)^{n-2}$, for $j = 2, \dots, n - 1$.

Proof. (a) By induction on n. Indeed, denoting:

$$B \equiv (A - \lambda I_n)_{n,1} = \begin{bmatrix} \frac{a_2}{a_1} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ -\lambda & \frac{a_3}{a_2} & \cdots & \frac{a_{n-1}}{a_2} & \frac{a_n}{a_2} \\ \vdots & \ddots & & \ddots & \\ \frac{a_2}{a_{n-1}} & \frac{a_3}{a_{n-1}} & \cdots & -\lambda & \frac{a_n}{a_{n-1}} \end{bmatrix} \in \mathbb{C}^{(n-1)\times(n-1)} \quad (9)$$

A direct computation verifies the statement for $n = 3$, since $\begin{vmatrix} \frac{a_2}{a_1} & \frac{a_3}{a_1} \\ -\lambda & \frac{a_3}{a_2} \end{vmatrix} = \frac{a_3}{a_1}(\lambda + 1)$.

Proceeding further, we make the recursive assumption:

$$\left(\det\left(B_{n-1,n-1}\right)=\right)\begin{vmatrix} \frac{a_2}{a_1} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-2}}{a_1} & \frac{a_{n-1}}{a_1} \\ -\lambda & \frac{a_3}{a_2} & \cdots & \frac{a_{n-2}}{a_2} & \frac{a_{n-1}}{a_2} \\ \vdots & \ddots & & \ddots & \\ \frac{a_2}{a_{n-2}} & \frac{a_3}{a_{n-2}} & \cdots & -\lambda & \frac{a_{n-1}}{a_{n-2}} \end{vmatrix} = \frac{a_{n-1}}{a_1}(\lambda+1)^{n-3}, \quad (10)$$

whereby this determinant is independent of $\left[a_j\right]_{j=2}^{n-2}$ and involves only a_{n-1} i.e. the nominator of the last column entries in (10), and a_1, the denominator of the first row in (10).

Since $B_{n-1,n-2}=\begin{bmatrix} \frac{a_2}{a_1} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-2}}{a_1} & \frac{a_n}{a_1} \\ -\lambda & \frac{a_3}{a_2} & \cdots & \frac{a_{n-2}}{a_2} & \frac{a_n}{a_2} \\ \vdots & \ddots & & \ddots & \\ \frac{a_2}{a_{n-2}} & \frac{a_3}{a_{n-2}} & \cdots & -\lambda & \frac{a_n}{a_{n-2}} \end{bmatrix}$ has the same formulation as in (10), but with a_n instead of a_{n-1} in its last column, we conclude that:

$$\det\left(B_{n-1,n-2}\right)=\frac{a_n}{a_1}(\lambda+1)^{n-3} \quad (11)$$

On the other hand, it is immediately revealed that:

$$\det\left(B_{n-1,j}\right)=0, \text{ for j=1,2, . . . , n-3} \quad (12)$$

since each of these minors includes a pair of linearly dependent rows; namely, rows 1 and j+1. Hence, Laplace expansion of (9) along its last row verifies the assertion:

$$\begin{aligned}\det(B) &= \sum_{j=1}^{n-3}\frac{a_{j+1}}{a_{n-1}}(-1)^{(n-1)+j}\det\left(B_{n-1,j}\right)+(-\lambda)(-1)^{(n-1)+(n-2)}\det\left(B_{n-1,n-2}\right) \\ &+\frac{a_n}{a_{n-1}}(-1)^{(n-1)+(n-1)}\det\left(B_{n-1,n-1}\right) \overset{(9)}{=} \ 0+\lambda\det\left(B_{n-1,n-2}\right)+\frac{a_n}{a_{n-1}}\det\left(B_{n-1,n-1}\right) \\ &=\lambda\frac{a_n}{a_1}(\lambda+1)^{n-3}+\frac{a_n}{a_{n-1}}\frac{a_{n-1}}{a_1}(\lambda+1)^{n-3}=\frac{a_n}{a_1}(\lambda+1)^{n-2}.\end{aligned}$$

(b) For $j = 2$ we have:

$$\begin{aligned}\text{et}\left((A-\lambda I_n)_{n,2}\right) &= \begin{vmatrix} -\lambda & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ \frac{a_1}{a_2} & \frac{a_3}{a_2} & \cdots & \frac{a_{n-1}}{a_2} & \frac{a_n}{a_2} \\ \vdots & \ddots & & \ddots & \\ \frac{a_1}{a_{n-1}} & \frac{a_3}{a_{n-1}} & \cdots & -\lambda & \frac{a_n}{a_{n-1}} \end{vmatrix} = (-1)\begin{vmatrix} \frac{a_1}{a_2} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ -\lambda & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ \vdots & \ddots & & \ddots & \\ \frac{a_1}{a_{n-1}} & \frac{a_3}{a_{n-1}} & \cdots & -\lambda & \frac{a_n}{a_{n-1}} \end{vmatrix} \\ &= (-1)\frac{a_1}{a_2}\begin{vmatrix} \frac{a_1}{a_1} & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ -\lambda & \frac{a_3}{a_1} & \cdots & \frac{a_{n-1}}{a_1} & \frac{a_n}{a_1} \\ \vdots & \ddots & & \ddots & \\ \frac{a_1}{a_{n-1}} & \frac{a_3}{a_{n-1}} & \cdots & -\lambda & \frac{a_n}{a_{n-1}} \end{vmatrix}\end{aligned} \quad (13)$$

where we have used elementary properties to bring the matrix under consideration in the general form (9). Since the previous statement ensures the determinant of (13) is dependent only on the parameters a_n (appearing as nominator of the last column entries) and a_1 (denominator of first row entries), we obtain:

$$\det\left((A-\lambda I_n)_{n,2}\right) = (-1)\frac{a_1}{a_2}\frac{a_n}{a_1}(\lambda+1)^{n-2} = (-1)\frac{a_n}{a_2}(\lambda+1)^{n-2} \tag{14}$$

The remaining assertions for $2 < j \leq n-1$ are proved similarly.

The statements in Lemma 1 may be summarized together as:

$$\det\left((A-\lambda I_n)_{n,j}\right) = (-1)^{j+1}\frac{a_n}{a_j}(\lambda+1)^{n-2}, \text{ for } j = 1, \dots, n-1.$$

Hence, plugging (14) in (8) and after some algebraic manipulations, we reach:

$$\begin{aligned}\chi_A(\lambda) &= (-1)^n(\lambda+1)^{n-2}\left[\sum_{j=1}^{n-1}(-1)^{2j+1} + \lambda(\lambda-n+2)\right] \\ &= (-1)^n(\lambda+1)^{n-2}\left(\lambda^2-(n-2)\lambda-(n-1)\right) \\ &= (-1)^n(\lambda+1)^{n-2}(\lambda+1)(\lambda-n+1)\end{aligned}$$

whereby (6) and $\sigma(A) = \left[\underbrace{-1, \dots -1}_{n-1}, n-1\right]$ are immediate.

Denoting $\left[e_j\right]_{j=1}^{n} \subset \mathbb{R}^n$ the standard basis vectors, i.e.$e_j = \left(\underbrace{0,\dots 0}_{j-1}, 1, \underbrace{0,\dots 0}_{n-j}\right)^T$ it is straightforward to check that span $\left[-\frac{a_j}{a_1}e_1 + e_j\right]_{j=2}^{n}$ is the $(n-1)-$dimensional eigenspace associated to $\lambda_1 = -1$, while $\left(\frac{a_n}{a_1}, \frac{a_n}{a_2}, \dots, \frac{a_n}{a_{n-1}}, 1\right)^T$ is an eigenvector corresponding to $\lambda_2 = n-1$.

Hence, the general solution to (4) takes the form:

$$x(t) = e^{-t}\begin{bmatrix} -\frac{a_n}{a_1} & -\frac{a_{n-1}}{a_1} & -\frac{a_{n-2}}{a_1} & \cdots & -\frac{a_2}{a_1} \\ 0 & 0 & \cdots & 0 & 1 \\ \vdots & & & \cdots & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & \cdots & & \end{bmatrix}\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_{n-1}\end{pmatrix} + c_n e^{(n-1)t}\begin{pmatrix}\frac{a_n}{a_1} \\ \frac{a_n}{a_2} \\ \vdots \\ \frac{a_n}{a_{n-1}} \\ 1\end{pmatrix} \tag{15}$$

with arbitrary $\left[c_j\right]_{j=1}^{n}$ or, equivalently,

$$x_1(t) = -\left(\sum_{j=1}^{n-1} c_j\frac{a_{n-j+1}}{a_1}\right)e^{-t} + c_n\frac{a_n}{a_1}e^{(n-1)t}$$

$$x_j(t) = c_{n-j+1}e^{-t} + c_n\frac{a_n}{a_j}e^{(n-1)t} \quad (\text{for } j = 2, \dots, n-1),$$

$$x_n(t) = c_1e^{-t} + c_ne^{(n-1)t} \quad \blacksquare$$

We note that the case of (4) for $n = 4$ has been previously studied by Chalikias *et al*. [22] in relation to the issue of banking industry in Greece.

For the set of the examined stocks the following data were extracted. The function of time has been estimated in order to fit the above solution to the real data [22]. Because of the different monotony of every stock, different functions of time for every stock were used. More specifically the time functions of the following table were used.

Table 1. *Stocks' time functions.*

Stock symbol	Variable i	Time function
AXON	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i^3
EUROM	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i^4
IASO	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i
IATR	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i
LAVI	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i^4
MENTI	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i^3
YGEIA	$i = -0.05$ where $j = 0, 0.083, 0.166, 0.025, 0.033, \dots, 1$	i^4

If we change a_i coefficients with the stock percentages we take the c_i coefficients of the model.

Table 2. *Model's c_i coefficients.*

Coefficient	Value
C_1	–177.324
C_2	–1.93013
C_3	–1064.27
C_3	0.643501
C_5	0.258205
C_6	–151.673

In order to evaluate the good fit of the experimental results, the real stocks' data and the model's predicted data were compared with Wilcoxon Test as the precaution of normality weren't satisfied.

Table 3. *Wilcoxon Test results.*

	Real data – predicted data
Z	–1.574
Asymp. Sig. (2-tailed)	0.116

Based on the above table, we conclude that the real data and model's predicted have the equal distribution with the same median as Wilcoxon Test's null hypothesis is accepted ($\text{Asymp. Sig.} (2 - \text{tailed}) = 0.116$).

Furthermore, the same results are drawn by Sign Test as shown in the following table ($\text{Asymp. Sig.} (2 - \text{tailed}) = 0.909$).

Table 4. *Sign Test results.*

	Real data – predicted data
Z	–0.115
Asymp. Sig. (2-tailed)	0.909

The following figure shows the real data and the model's predicted values.

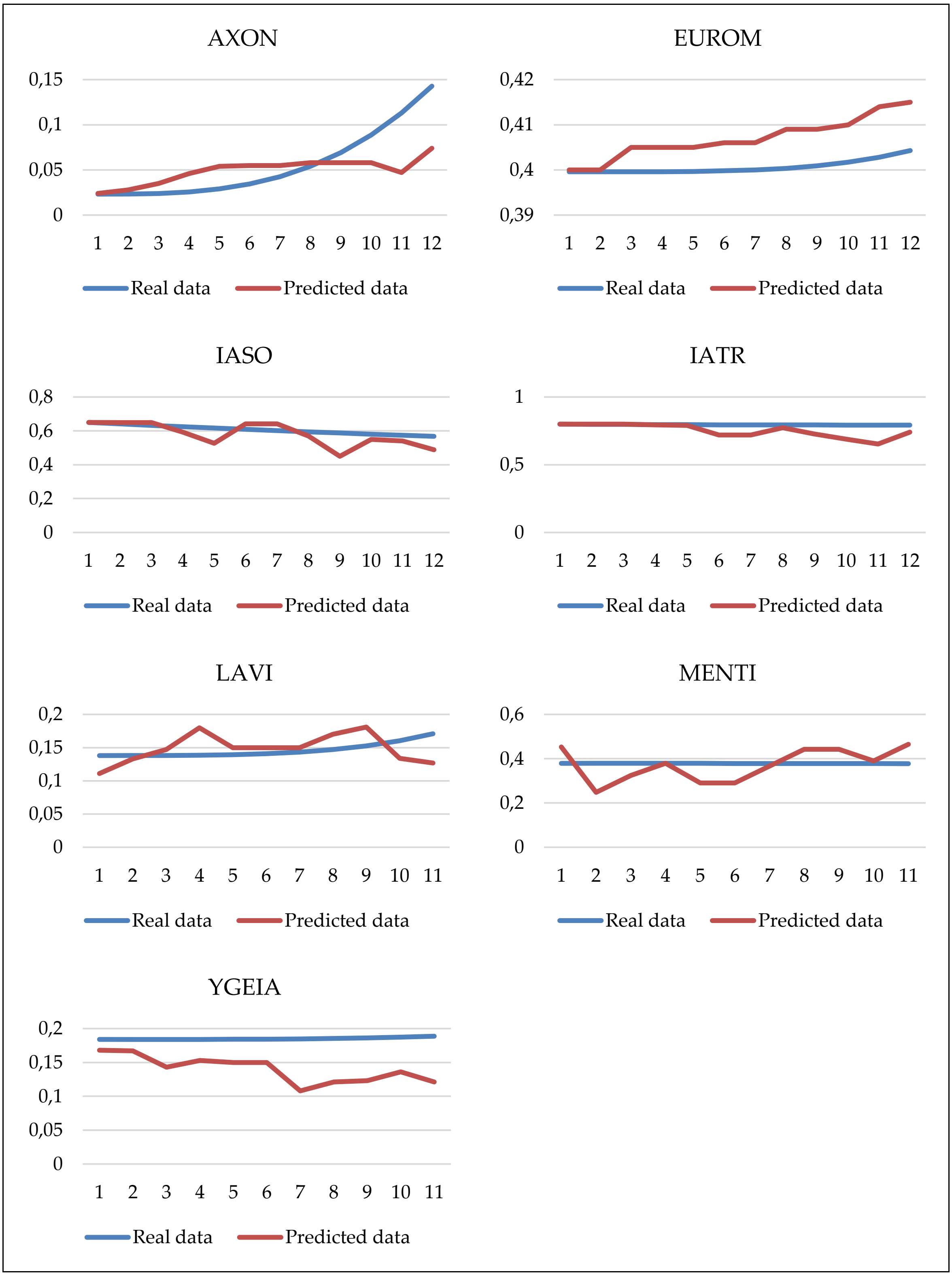

Figure 1. *Real data and the model's predicted values.*

IV. Discussion

In this paper we proposed a differential equations approach by the aid of Lanchester's combat model which can predict stock prices. Input variables concerned the 7 private healthcare firms listed in ASE. The data analysis was based on 7×7differential equations model. The experimental results were found to have equal distribution and same median as the real data. This demonstrated the model's good fit. Thus, the method used can be applied in cases of stock price prediction. Furthermore, another scope of Lanchester's combat models is found, as there is no other application of these models in such a case.

As already mentioned, the data used in this study concern a 12-month period. Similar models are used in various cases of long-term data. In these cases, the models' predictive capability is high [14-16]. However, such models have not be used in stock prices prediction cases. Thus, a long-term model could be analyzed in a future research. Furthermore, in future models, stock prices prediction could take into consideration various factors such as the economy of a country [23-24], the political structure of a country [22,25], or psychological factors [26]. Several modifications and extensions of the proposed methodological approach seem to be of some research interest for future work, since the specific topic is really contemporary and meets a variety of real data applications.

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