Signature Analysis of Bleaching and 2-out-of-4 Systems

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Abstract

A real-life bleaching system is proposed, and its reliability function is estimated using universal generating function technique. The units are connected to each other in series and parallel arrangements. Further, a 2-out-of-4 system is described and taken as an example where out of 4 units, if 2 are working the system will perform its task. Universal Generating Function technique is applied to both the systems and by using Owen's and Boland method signature, tail signature, Barlow-Proschan Index and expected time of the proposed complex system is calculated.

Keywords: Universal generating function; signature; tail signature; 2-out-of-4 system; independent identically distributed; Barlow-Proschan index.

I. Introduction

In past few decades, many efforts had been done in the field of reliability to evaluate the reliability characteristics of various systems and models. Engineering and mathematics connect with reliability theory perfectly. When any system is allowed to work in definite mode and under specified environment, the representation of the characteristics of a working of the system is reliability engineering. Reliability engineering tells us regarding the performance of the framework and therefore the operating time period of it. Reliability theory have helped to establish finest designs or models for various systems. When any system performs a given task in a specified time limit with different efficiency and performance rates, called multi-state systems (MSS). Eryilmaz et al. [16] had thought about a k-out-of-n structure with n independent units having three states, where state "2" was assumed to be the perfectly working state, state "1" is assumed to be partially working state and state "0" is failed state. This research creates formulae for the endurance capacities comparing to the two diverse framework's states depicted previously. For representation purposes, mathematical model which expects that the debasement happens as per a Markov cycle is introduced. Eryilmaz et al. [19] created formulae for the endurance capacities relating to the two distinctive framework's states depicted previously. For representation purposes, a mathematical model which accepts that the degradation happens as per a Markov process is introduced. There are many techniques to estimate the reliability of MSS and UGF technique is one of them. In the past, many researchers had shown interest in UGF technique and also to evaluate the signature of the complex system [1,12,23]. Levitin [25] used the UGF technique to seek out the reliability of a consecutive *k*-out-of-*r*-from-*n*:F system in multi-state case. Author had taken a linear multi-state sliding window system in which system had *n* elements and condition for failure of the system was the failure of k elements out of r consecutive elements. Levitin [24] considered a framework comprises of *n* straightly requested multi-state components. Every component can have different states: from complete-disappointment up to consummate working. A performance rate is related with each state. The sliding window framework fails if the amount of the performance rates of any *r* sequential multi-state components is lower than a base permissible level. Author proposes another model that sums up the sequential *k*-out-of-*r*-from-*n*:F framework to the multi-state case. Levitin [26] describes the use of UGF technique in different type of systems i.e., if the system is a parallel then what would be the algorithms, if the system is a *k*-out-of-*n* type then the algorithms will change etc. Levitin and Ben-Haim [27] that proposed the linear consecutive sliding window system and the model had *n* multi state elements which were linearly ordered. The condition for failure of the system was when in out of *m* consecutive groups of *r* consecutive elements the sum of the performance rates of units was lesser then demand *W*. Levitin and Dai [28] in which the system consists of *n* linearly ordered elements and the condition for failure was defined when at least *k* elements failed in the *m* groups which were consecutively overlapped and had *r* units in each group where the units in the group are also consecutive. Levitin and Dai [29] proposed a model in which there were m independent linearly ordered multistate elements in the system, and the conditions for failure were if the sum of performance rates was lower than the minimum allowable rates in at least *k* number of groups in which each group consists of *r* elements arranged consecutively. Jafary and Fiondella [2] had developed a model which was discrete and continuous for a multistate system and based on series and parallel UGF where the elements, consists of multistate units, were identical but correlated.

da Costa Bueno [3] discussed how to calculate the importance of every unit for a given system by using its signature representation. The definition of signature was given in sense of compensator transform and in the context of system signature. Eryilmaz [23] deliberate the reliability characteristics of *m*-consecutive-*k*-out-of-*n*:F system with overlapping runs through signature and used it to evaluate different reliability characteristics of the system. Harish et al. [4] assessment the reliability with the help of vague lambda-tau methodology for industrial system in which the collected information about the units of the system was uncertain and the nature of the information was also inaccurate. Also, rather than fuzzy set theory author had used intuitionistic fuzzy set theory to control the uncertainty in the data. Da and Hu [5] given the definition of the bivariate signature in terms of order statistics of lifetime elements and then establish the formula, for 3-state system and calculating the bivariate variables. Kumar and Singh [6, 7] estimated the signature reliability characteristics of the complex and binary sliding window coherent system, with the assistance of UGF, reliability function, signature and minimal signature of the sliding window system were also assessed. Kumar and Singh [8] had proposed the A-within-B-from-D/G sliding window coherent system, by considering the case of multiple failures. Authors used UGF technique and Owen's method to obtain the signature of the system. Kumar and Ram [9] considered a 2-out-of-5-linear consecutive system and discussed the signature of the system. Some other parameters for example tail signature, expected lifetime and expected cost from reliability function were also evaluated. Kumar and Ram [10] had proved that in spite of having number of techniques and methods for evaluation such as fuzzy, supplementary and Markov chain technique, UGF technique gave more amended results in case of interval-valued. Kumar and Singh [11] had revisited the sliding window system and estimated the interval-valued reliability using UGF technique.

In the above discussion about signature and UGF, a bleaching system has been considered which the mixture of series and parallel arrangements and reliability is defined with the assistance of state performance. A 2-out-of-4 system is also considered in which out of 4 elements at least 2 elements should work for the whole system to be in working condition. The elements of the system are independent identically distributed. A brief discussion of UGF technique and signature is given in Section 2. The description of the considered complex model is given in Section 3. Then a k-out-of-n system is considered and solved with the help of UGF technique. The description of the model is given in Section 4. In Section 5, algorithms and used formulas are discussed for the complex system and in Section 6 procedures for calculating the reliability of the k-out-of-n arrangement is given. An example of complex system is considered and solved according to the algorithms and formulas in Section 7 and in Section 8 an example of 2-out-of-4 system is also considered and solved with the help of algorithms given in the 5 and 6 Sections. At last, in Section 9 the conclusion is drawn and the results are discussed.

II. Universal Generating Function and Signature

UGF was first introduced by Ushakov in 1986, this technique is said to be effective within the analysis of function of reliability for the system. With this technique one can optimize the various kind of complex and multi-state devices. UGF technique includes the contribution of the performance of all the units existing in an entire system and then calculate the performance distribution of the system based on the performance distribution of its units. To reduce the problem, it joins different units of the device by using configuration operators.

The UGF of an independent discrete random variable Y is expressed in a polynomial form as

$$U(Z) = \sum_{m=1}^{M} q_m z^{y_m},$$

where, the possible values of the variable y are m and the possibility of the system in working state is q_m.

In 1985 Samaniego first introduced the concept of signature. To characterize the system reliability, signature is used and it is a very useful tool. Signature tells us mainly about the probability of the failure time of the system's unit, i.e., at what time the particular unit of the system will fail and which is the last unit whose failure will cause the failure of the whole system. The system signature is given by the n-tuples whose kth coordinate s_k is the probability that the kth unit failure causes the system to fail. That is,

$$S_k = P_r(T_s = T_{k:n}),$$

where, $T_{k:n}$ denotes the kth smallest lifetime, i.e., the k^{th} order statistics computed by rearranging the variables $T_1, \dots, T_n$ in ascending order of magnitude.

III. Description of Bleaching System

A bleaching system is considered which comprises of both series and parallel structure. The proposed system is a type of complex system that can't be simplified as pure series structure or pure parallel structure, this is the combination of both. The system is said to be in series configuration when on the failure of one element the whole system fails and the system is said to be in parallel configuration, when till the last working element of the system will work i.e.; the system fails when the last working element fails.

In the proposed model, the system is simplified and then the reliability function is evaluated with the help of UGF technique. In the given case, 2 and 3 is in parallel and 4 and 5 are also in parallel with each other. 1 is in series with both of the above.

Hence UGF of the bleaching system is defined as follows

$U_6(z) = \max\,(U_2(z), U_3(z))$ (1)

$U_7(z) = \max\,(U_4(z), U_5(z))$ (2)

$U_8(z) = \min\big(U_1(z), U_6(z), U_7(z)\big)$ (3)

Reliability of the system will be calculated (Levitin, 2003) as

$R = U'(z)$ at $(z=1)$.

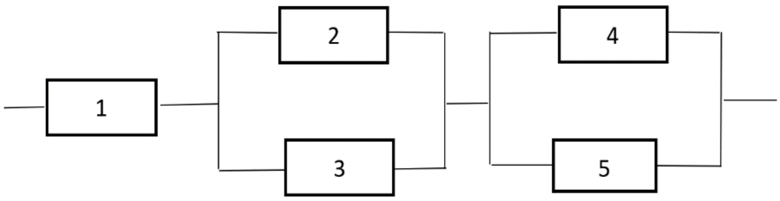

Figure 1. *Block diagram of bleaching system*

IV. Description of k-out-of-n System

Considered a k-out-of-n arrangement system where total n number of identical units are connected with each other and the system will perform its desired task if at least k units out of n will work perfectly. These type of systems are called k-out-of-n:G systems. If k elements of the system fail and that lead to the failure of the whole system i.e., the system will fail if at least k elements fail, that kind of systems are known as k-out-of-n:F system.

Pure series and parallel system are also be the special cases of k-out-of-n systems. As it is known that in pure series system, the failure of any one unit will lead to the failure of the whole system. So, this case can be taken as 1-out-of-n systems where out of n identical units if even one unit fails the whole system will go to fail state, and in pure parallel system, when all the units fail then the system is considered to be fail i.e., the failure of the last working unit will lead to the failure of the whole system. So, this case can be considered as n-out-of-n systems, where out of n identical units the system will go to the failed state only when all n units will fail. The k-out-of-n system also have a wide application in the technical field.

V. Algorithm for Computing Various Measures

1. Algorithm for computing the signature with the assistance of reliability function.

Calculate signature of the considered system from reliability function in the following manner. Firstly, by using Boland's formula [13] the signature of the system will be evaluated. The formula for the estimation of signature is

$$B_a = \frac{1}{\binom{s}{s-a+1}} \sum_{\substack{k\subseteq[s] \\ |k|=s-a+1}} \varphi(K) - \frac{1}{\binom{s}{s-1}} \sum_{\substack{K\subseteq[s] \\ |K|=s-1}} \varphi(K) \quad (4)$$

Compute the reliability polynomial for above complex structure using

$$K(P) = \sum_{j=1}^{s} e_j \binom{s}{j} P^j q^{n-j} \quad (5)$$

where, $e_i = \sum_{i-s-j+1}^{s} w_i,\ j = 1,2,\dots s$.

The reliability function of the system is to be obtained with the help of Taylor evolution at $w = 1$ from the polynomial form, which we get from the above algorithm for the calculation of reliability function. So, the formula for reliability function is

$$P(w) = w^s K\left(\frac{1}{w}\right). \tag{6}$$

Then the estimation of the values of Tail signature for the complex system with $(p+1)$ -tuple $W = (W_0,...,W_s)$ is done with the following formula [20,21,22].

$$W_a = \sum_{i=a+1}^{s} w_i = \frac{1}{\binom{s}{s-a}} \sum_{|H|=s-a} \varphi(K). \tag{7}$$

Now by using Marichal and Mathonet [14] and equation (7), the Tail signature of the system is calculated by using the formula given below.

$$W_a = \frac{(s-1)!}{s!} d^a P(1),\ a = 0,1,\dots,s. \tag{8}$$

With the help of Tail signature of the system, finally the signature of the system is calculated as follows

$$w = W_{a-1} - W_a,\ a = 1,2,\dots,s. \tag{9}$$

2. The Algorithm to estimate the expected lifetime of the system by using minimum signature.

Calculation of the expected lifetime for the system is done using minimal signature. The elements of the system are considered to be independent and identically distributed and the mean time to failure (MTTF) is calculated for the elements which have exponentially distributed element with mean $\otimes$.

Then by using the formula given in [15] estimate expected lifetime $E(T)$ of the system with the help of given formula.

$$E(T) = \mu \sum_{i=1}^{n} \frac{e_i}{i}, \tag{10}$$

where, $e = (e_1, e_2, \dots, e_n)$ is a vector coefficient obtained with the help of minimal signature.

3. Algorithm for obtaining the Barlow-Proschan index for system.

Using [16,17,18] the Barlow-Proschan Index will be calculated from reliability function of the complex system where the elements of the system are independent and identically distributed. So, the following formula is used to estimate the Barlow-Proschan Index

$$I_{BP}^{(a)} = \int_0^1 (\partial_a K)(w)dw,\ a = 1,2,\dots,n \tag{11}$$

where, k are reliability functions of system.

4. Algorithm to determine the expected value of the system [23].

Next the expected value is to be calculated for the elements of the proposed system. The formula used for the expected value for the elements is given below.

$$E(X) = \sum_{i=1}^{n} iw_i,\ i = 1,2,\dots,n\ . \tag{12}$$

Then two measures are to be calculated at last. One $E(X)$ and second is $\frac{E(X)}{E(T)}$ for the proposed bleaching system.

VI. Algorithm for Computing Various Measures

First it is to note that the operator $\otimes_+$contains the associative property and the following procedure can be defined by using the structure function mathematically. With this procedure one can get the reliability function of k-out-of-n system as follows:
First, determine the u-function of each element in the following form
$u_i(z) = p_i z^1 + (1 - p_i)z^0$.
Assign $U_1(z) = u_1(z)$.
Obtain $U_i(z) = U_{i-1}(z)\otimes_+ u_i(z)$
For all $i = 2,3, \dots, n$.
Probability mass function (p.m.f.) of the random variable X is represented by the last u-function $U_n(z)$ calculated for the system.
Calculate the u-function $U(z)$ which represents the p.m.f of structure function

$$\emptyset(X_1, \dots, X_n) = 1(\sum_{i=1}^{n} X_i \geq k)$$

as $U(z) = U_n(z) \otimes$ k, where $\varphi(X, k) = 1(X \geq k)$.
Calculate the system's reliability with the help of the following formula
$E(\varphi(X, k)) = U'(1)$ (differentiate the final function at $z = 1$).

VII. Example of the Bleaching Structure

Consider a complex bleaching structure as shown in Figure 1, in which unit 1 is in series with the combination of unit (2, 3) and (4, 5). Unit 2, 3 and 4, 5 are connected in parallel with each other. The reliability function of corresponding system using UGF can be calculated as follows:
UGF of all elements is defined as

$$U_1(\mathrm{z}) = P_1\, z^1 + (1 - P_1)z^0 \tag{13}$$
$$U_2(\mathrm{z}) = P_2 z^1 \, + (1 - P_2)z^0 \tag{14}$$
$$U_3(\mathrm{z}) = P_3 z^1 + (1 - P_3)z^0 \tag{15}$$
$$U_4(\mathrm{z}) = P_4 z^1 + (1 - P_4)z^0 \tag{16}$$
$$U_5(\mathrm{z}) = P_5 z^1 + (1 - P_5)z^0 \tag{17}$$

Hence in Figure 1, $U_2(z), U_3(z)$ and $U_4(z), U_5(z)$ are in parallel with each other, the reliability of the units in the UGF form is as follows:

$$U_6(z) = \max\,(U_2(z), U_3(z)) \tag{18}$$
$$U_7(z) = \max\,(U_4(z), U_5(z)) \tag{19}$$

Now,$U_1(z)$ is in series with $U_6(z)$ and $U_7(z)$. So, the reliability function of the unit in UGF form is as follows:

$$U_8(z) = \min\,(U_1(z), U_6(z), U_7(z)) \tag{20}$$

After simplifying the proposed complex system, the reliability function of the system can be estimated from$U_8(z)$ as
Now evaluate UGF of the above
$U_6(z) = \max\,(U_2(z), U_3(z))$
$\Rightarrow P_2 z^1 + (1 - P_2)z^0 \,\otimes\, P_3 z^1 + (1 - P_3)z^0$
$\Longrightarrow (P_2 + P_3 - P_2P_3)z^1 + (1 - P_2)(1 - P_3)z^0$.
$U_7(z) = \max\,(U_4(z), U_5(z))$
$\Longrightarrow P_4 z^1 + (1 - P_4)z^0 \,\otimes\, P_5 z^1 + (1 - P_5)z^0$
$\Longrightarrow (P_4 + P_5 - P_4P_5)z^1 + (1 - P_4)(1 - P_5)z^0$.
$U_8(z) = \min\,(U_1(z), U_6(z), U_7(z))$
$\Longrightarrow P_1 z^1 + (1 - P_1)z^0 \,\otimes (P_2 + P_3 - P_2P_3)z^1 + (1 - P_2)(1 - P_3)z^0 \,\otimes (P_4 + P_5 - P_4P_5)z^1 + (1 - P_4)(1 - P_5)z^0$
$\Longrightarrow (P_1P_2 + P_1P_3 - P_1P_2P_3)(P_4 + P_5 - P_4P_5)z^1 + [(P_2 + P_3 - P_2P_3 - P_1P_2 - P_1P_3 + P_1P_2P_3)(P_4 + P_5 - P_4P_5) + (P_1 - P_1P_3 - P_1P_2 + P_1P_2P_3)(P_4 + P_5 - P_4P_5) + (1 - P_3 - P_2 + P_2P_3 - P_1 + P_1P_3 + P_1P_2 -$

$P_1P_2P_3)(P_4+P_5-P_4P_5)+(P_1P_2+P_1P_3-P_1P_2P_3)(1-P_4-P_5+P_4P_5)+(P_2+P_3-P_2P_3-P_1P_2-P_1P_3+P_1P_2P_3)(1-P_4-P_5+P_4P_5)+(P_1-P_1P_3-P_1P_2+P_1P_2P_3)(1-P_4-P_5+P_4P_5)+(1-P_3-P_1+P_1P_3-P_2+P_2P_3+P_1P_2-P_1P_2P_3)(1-P_4-P_5+P_4P_5)]z^0.$

Hence, reliability of the system is (Levitin, [25]) as

$$R = P_1P_2P_4+P_1P_2P_5-P_1P_2P_4P_5+P_1P_3P_4+P_1P_3P_5-P_1P_3P_4P_5-P_1P_2P_3P_4-P_1P_2P_3P_5+P_1P_2P_3P_4P_5.$$

Now, if all the elements are independent and identically distributed i.e.

$P_1=P_2=P_3=P_4=P_5=P$, then the reliability function of the complex system is

$$R = 4P^3-4P^4+P^5 \tag{21}$$

1. Signature of the Bleaching Structure

Using Owen's method, the reliability function of the bleaching system is obtained in the form of p as follows

$H(p)=4P^3-4P^4+P^5.$

Now from equation (6), the polynomial function is

$P(v)=P^5H\left(\frac{1}{P}\right)=1-4P+4P^2.$

To obtain the tail signature P of the complex bleaching system, procedure given in section 5.1 is followed

$P=\left(1,\frac{4}{5},\frac{2}{5},0,0,0\right).$

Now, to estimate the signature of the considered bleaching system equation (9) from section 5.1 is used

$P=\left(\frac{1}{5},\frac{2}{5},\frac{2}{5},0,0\right).$

2. Barlow-Proschan Index of Bleaching System

Now with the help of equation (11) calculate the Barlow-Proschan index for the considered bleaching system as follows

$I_{BP}^{(1)}=\int_0^1(4P^2-4P^3+P^4)dp=\frac{8}{15}.$

Similarly, Barlow-Proschan index $I_{BP}^{(K)}$ for $K=(2,\dots,5)$ of all elements is

$I_{BP}=(\frac{8}{15},\frac{7}{60},\frac{7}{60},\frac{7}{60},\frac{7}{60}).$

3. Expected Lifetime of Bleaching System

Using Equation (5) from above the minimal signature M of the bleaching system is determined as Minimal signature $(0,0,4,-4,1)$

Using minimal signature, expected $E(t)$ is obtained as

$E(t)=0.534.$

4. Expected Cost Rate

Using equation (12), the expected value of the considered bleaching system is

$E(X)=2.2.$

Expected cost rate $=E(X)/E(t)$,

$=4.1199$

VIII. Example of the 2-out-of-4 Structure

The reliability of the considered 2-out-of-4 system can be calculated by using the algorithms as discussed above in section 6 such as

$$U_1(z)=P_1z^1+(1-P_1)z^0 \tag{22}$$
$$U_2(z)=P_2z^1+(1-P_2)z^0 \tag{23}$$
$$U_3(z)=P_3z^1+(1-P_3)z^0 \tag{24}$$

$U_4(z) = P_4 z^1 + (1 - P_4) z^0$ (25)

Now following the second step from the above algorithm

$U_1(z) = u_1(z) = P_1 z^1 + (1 - P_1) z^0,$

Following the third step from the algorithm and obtaining the other u-functions,

$U_2(z) = (P_1 z^1 + (1 - P_1) z^0)(P_2 z^1 + (1 - P_2) z^0)$

$U_2(z) = P_1P_2 z^2 + (P_1 + P_2 - 1P_1P_2) z^1 + (1 - P_1 - P_2 + P_1P_2) z^0,$

$U_3(z) = P_1P_2 z^2 + (P_1 + P_2 - 1P_1P_2) z^1 + (1 - P_1 - P_2 + P_1P_2) z^0 (P_3 z^1 + (1 - P_3) z^0),$

$U_3(z) = P_1P_2P_3 z^3 + (P_1P_2 + P_1P_3 + P_2P_3 - 3P_1P_2P_3) z^2 + (P_1 + P_2 + P_3 - 2P_1P_2 - 2P_1P_3 - 2P_2P_3 + 3P_1P_2P_3) z^1 + (1 - P_1 - P_2 - P_3 + P_1P_2 + P_2P_3 + P_1P_3 - P_1P_2P_3) z^0,$

$U_4(z) = P_1P_2P_3 z^3 + (P_1P_2 + P_1P_3 + P_2P_3 - 3P_1P_2P_3) z^2 + (P_1 + P_2 + P_3 - 2P_1P_2 - 2P_1P_3 - 2P_2P_3 + 3P_1P_2P_3) z^1 + (1 - P_1 - P_2 - P_3 + P_1P_2 + P_2P_3 + P_1P_3 - P_1P_2P_3) z^0 (P_4 z^1 + (1 - P_4) z^0,$

$U_4(z) = P_1P_2P_3P_4 z^4 + (P_1P_2P_4 + P_1P_3P_4 + P_2P_3P_4 + P_1P_2P_3 - 4P_1P_2P_3P_4) z^3 + (P_1P_2 + P_1P_3 + P_1P_4 + P_2P_3 + P_2P_4 + P_3P_4 - 3P_1P_2P_3 - 3P_1P_2P_4 - 3P_1P_3P_4 - 3P_2P_3P_4 + 6P_1P_2P_3P_4) z^2 + (P_1 + P_2 + P_3 + P_4 - 2P_1P_4 - 2P_2P_4 - 2P_3P_4 - 2P_1P_2 - 2P_1P_3 - 2P_2P_3 + 3P_1P_2P_4 + 3P_2P_3P_4 + 3P_1P_3P_4 + 3P_1P_2P_3 - 4P_1P_2P_3P_4) z^1 + (1 - P_1 - P_2 - P_3 - P_4 + P_1P_2 + P_2P_3 + P_1P_3 + P_1P_4 + P_2P_4 + P_3P_4 - P_1P_2P_3 - P_1P_2P_4 - P_2P_3P_4 - P_1P_3P_4 + P_1P_2P_3P_4) z^0.$

Hence, from the above u-function, the expression for the reliability function of the considered 2-out-of-4 system is

$R = 3P_1P_2P_3P_4 - 2P_1P_2P_4 - 2P_1P_3P_4 - 2P_1P_2P_3 - 2P_2P_3P_4 + P_1P_2 + P_1P_3 + P_1P_4 + P_2P_3 + P_2P_4 + P_3P_4.$

Let all the elements of 2-out-of-4 system are independent and identically distributed i.e.,

$$P_1 = P_2 = P_3 = P_4 = P$$

After, applying above condition, the reliability function of the 2-out-of-4 system will become

$R = 3P^4 - 8P^3 + 6P^2$ (26)

1. Signature of the 2-out-of-4 System

Now, the reliability function of the considered system in the form of p by using Owen's method is as follows

$H(p) = 6P^2 - 8P^3 + 3P^4.$

Now from Equations (6) (calculation of signature) polynomial function is

$P(v) = P^5 H\left(\frac{1}{P}\right) = 3 - 8P + 6P^2.$

With the help of equation (8) in section 5.1 obtain the tail signature P of 2-out-of-4 structure

$P = (1, 1,1,0,0).$

Now, using equation (9) estimate the signature of the considered 2-out-of-4 structure,

$P = (0,0,1,0).$

2. Barlow-Proschan index of 2-out-of-4 structure system

Now with the help of equation (11) calculate the Barlow-Proschan index of the 2-out-of-4 structure as

$I_{BP}^{(1)} = \int_0^1 (3P^3 - 6P^2 + 3P) dp = \frac{1}{4}.$

Similarly, for $K = (2, \dots, 4)$ the Barlow-Proschan index $I_{BP}^{(K)}$ for all elements of the considered 2-out-of-4 system is as follows

$I_{BP} = (\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4},).$

3. Expected Lifetime of the System

Now, using Equation (5) from above we get the minimal signature M of the 2-out-of-4 system as Minimal signature $(0,6,-8,3)$.

Using minimal signature, we obtain expected $E(t)$ such as

$E(t) = 0.4167.$

4. Expected Cost Rate

Using equation (12) of algorithm 5.4, the expected value of the 2-out-of-4 structure is determined as $E(X) = 3$,
Calculating the expected cost rate for the system using the formula
Expected cost rate = $E(X)/E(t)$,
= 7.19943.

IX. Conclusion

The signature reliability characteristics of bleaching system have been studied. This study mainly aims to evaluate the reliability function, signature, minimal signature, Barlow-Proschan Index, expected cost and expected lifetime of the considered system with UGF technique and also system have equal reliability with independent and identically distributed elements. This research discusses about the series parallel arrangement of the system and the failure of the system as a whole due to the failure of units. Some fundamental results concluded in this paper are as follows: Signature $P = \frac{1}{5}, \frac{2}{5}, \frac{2}{5}, 0, 0$, Barlow-Proschan index $I_{BP} = \frac{8}{15}, \frac{7}{60}, \frac{7}{60}, \frac{7}{60}, \frac{7}{60}$, and expected cost rate of the system by using Owen's method is 4.1199. One can extend his/her work in the area of complex system with the help of this paper.
Also, 2-out-of-4 system is considered, where out of 4 units in the system at least 2 elements should work for the whole system to work properly. The following system is described in brief and is solved to get the reliability of the system. The system is solved with the help of the algorithm discussed above and results are drawn.

Signature $P = 0,0,1,0$ Barlow-Proschan index $I_{BP} = \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}$, and expected cost rate of the system by using Owen's method is 7.19943. The area of any analysis within the field of reliability and its characteristics is very beneficial for designing more reliable with low-cost systems. This study is abundantly helpful for the engineers, designers, researchers and students of various fields. Also, signature of the designed systems can be calculated with the help of order statistics for the future research purpose.

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