Reliability Analysis of Periodically Inspected Systems under Imperfect Preventive Maintenance

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Abstract

In this study a periodically inspected system is analyzed which is subject to imperfect maintenance policy. The considered system is inspected and maintained periodically and passes through a fixed number of imperfect repairs before being replaced. The imperfect effect of the preventive maintenance is modelled on the basis of the increasing failure rate of the system. Since the preventive maintenance is imperfect, it brings the considered system to an operating state which lies between two states, namely the "as bad as old state" and "as good as new state". The distribution of the failure time of the system and that of the repair times are assumed to be arbitrary. The times required for the preventive maintenance of the system and its replacement are further assumed to be negligible. Incorporating the given facts, the reliability of the periodically inspected system is evaluated. Also, various reliability measures like availability, steady-state availability are estimated. An optimal interval of inspection is estimated so that the total cost in minimized. With the help of a case study on Nuclear Power Plant System (NPPS) the derived results are illustrated.

Keywords: reliability, imperfect preventive maintenance, steady-state availability.

I. Introduction

Maintenance systems are of utmost importance for various equipments to operate as desired, to facilitate them to function as required etc. Maintenance has a major impact on delivery, quality and cost of various systems. If maintenance is optimized, it can play a very significant role to improve the ability of an organization to fulfil its objectives. Maintenance includes preventive and corrective actions so as to reduce the chance of failure which not only further enhances reliability of the system but also reduces operation cost. There are four major categories of maintenance namely, perfect, minimal, imperfect, worst repair. Last few decades have seen a considerable amount of attention being paid to the area of maintenance modelling in various fields of reliability. For instance, Levitin and Lisnianski [10] considered a multi-state system and generalized a preventive maintenance policy to optimize it. In order to estimate the reliability of the system many techniques including universal generating function have been incorporated. Also, a genetic algorithm has been used for the optimization purpose. El-Ferik and Ben-Daya [7] investigated an age-based model (hybrid) subject to imperfect preventive maintenance. The model involves two types of failure modes namely, maintainable and non-maintainable. Also, both the hazard rate and the age (effective) have been incorporated in the study. The optimal number of maintenance actions have been found out so that the expected cost is minimized. Castro [2] investigated a system which can fail due to two possible failure modes namely, maintainable mode and non-maintainable mode. Also, at the instance of failure a minimal repair is carried out. Preventive maintenances are conducted at regular intervals with the system being replaced after a fixed number of maintenance cycles. The basic problem was to find a feasible length between consecutive maintenance cycles so that cost rate is minimized. Bartholomew et al. [1] in their study dealt with the notion of scheduling the imperfect preventive maintenance of the system. The model was based on Kijima where each application of maintenance reduces the age of the system. Also, a numerical study illustrates the optimum schedule for different measures of the age-based model. Soro et al. [17] in their paper examined a multi-state system which is subject to preventive maintenance (imperfect) and estimated its availability, reliability etc. The minimal repair brought back the system to the previous state without impacting the rate of failure. The system has been modeled as a Markov process to evaluate the various performance measures. Liu et al. [11] in their paper proposed a model for the maintenance of mission-oriented systems which are exposed to continuous shocks and degradation. The degradation of the system has been modelled as a Weiner process. Also, an optimal policy for the maintenance of the system has been developed. Mercier and Castro [13] considered a degrading system and compared two models subject to imperfect repair. The level of degradation has been modelled by a Gamma process (non-homogeneous). A comparison has been made between both the models based on stochastic property. Furthermore, optimal strategies for maintenance have been explored.

All the aforementioned models have been studied in one or the other way to improve the availability and reliability of the systems. There has been an extensive study on the reliability models in the past. Liu and Kapur [12] studied a multistate system and developed various reliability measures. It was assumed that the considered system follows Markov process. Also, it was focused on evaluating considered systems for system designing. Singh [16] considered a system which consists of two subsystems. Subsystem-1 is a k-out of n subsystem while subsystem-2 consists of two units arranged in a parallel configuration. The failure rates have been assumed to be constant while the repair is governed by general and exponential distributions. Various performance measures like reliability, cost have been evaluated with the technique of supplementary variable, Laplace and copula methodology. Fan et al. [9] developed a model for estimating the reliability dependent of a system subject to failure processes which often results in the shock dependence. The method of Monte Carlo simulation has been used to calculate the various reliability measures. A real-time application has been demonstrated to study the dependent behavior of the failures. Song et al. [18] in their study considered a multi-state system whose components are dependent. In the proposed study stochastic multi valued models have been presented and a comparative analysis has been done to illustrate the model. Cha et al. [4] studied a model in consideration to the systems which operate in a given environment. The system is subject to a Poisson process of shocks where each of the shocks has a double effect on the system. Hence, the considered system is bivariate and the study is illustrated with numerical examples. Dong et al. [6] investigated a parallel system with redundance comprising of various components which are non-identical. Also, a self-healing mechanism is developed with respect to the damage load and numbers of shock arrivals. In this study reliability model is proposed to evaluate the closed form expressions for reliability and preventive maintenance. Furthermore, a Nelder-Mead simplex method has been applied to find the feasible age of replacement of the system. Nautiyal et al. [14] in their study considered a k out of n network and focused on evaluating reliability and its other measures. The method of Gumbel- Hougaard has been applied to obtain the various performance measures. Eloy and Dawabsha [8] considered a multistate system with warm standby. There are two major states in which the system has been partitioned namely, internal degradation and shocks. The process of repair comprises of more than one repairperson. The proposed model is presented and built in an algorithm form which makes its implementation to be easier. Cao et al. [3] examined a multi-state system having multiple components and explored how the process of ageing affects the process of failure in the considered system. The analysis of reliability and its various measures is done on the basis of a semi-Markov chain. With the help of an example of a transformer the proposed method has been demonstrated. Pan et al. [15] in their study desired to model a method for the evaluation of reliability. To model the parameter for obtaining the reliability the Wiener process has been incorporated. Then after Copula function has been effectively used to demonstrate the proposed method. Chen et al. [5] investigated about multi-layer systems and studied the behavior of the failure in the system. Further, a simulation method has been proposed in order to estimate the reliability of the system along with a maintenance cost model.

In this paper, we have restricted our study to imperfect preventive maintenance in which the health of the considered system lies between "as good as new" and "as bad as old." Also, we have considered a system which is inspected at regular constant intervals and is subject to a fixed number of maintenance cycles before being replaced. The motivation for performing inspections periodically arises from the fact that it may reduce the cost which is involved in continuous monitoring of the system. Some real-life instances where systems are inspected periodically are gas detectors, safety valves etc. Also, a perfect repair might require high costs, thus various imperfect repairs are done before the system is actually replaced. The failure rate model has been used to model the effect of imperfect maintenance system. Also, corrective maintenance is carried out whenever there is a failure in the system. Hence, the proposed model extends the contribution to the existing literature since it allows the effect of imperfect maintenance on the periodically inspected system which is followed by a corrective repair at the occurrence of every failure.

II. Model description and notations

II.I System Description

In this section we discuss the failure rate model for a periodically inspected system which has been used as a modelling framework of imperfect maintenance. For this we assume that the lifetime of the considered system follows general distribution with the corresponding failure rate function being denoted by $\sigma(t)$. Based on these assumptions, following observations are made:

- The failure rate $\sigma(t)$ of the system increases strictly after every mth imperfect preventive maintenance and becomes $l_m\sigma(t)$ when it was $\sigma(t)$ before the preventive maintenance took place.
- Thus, $L_m\,\sigma(t)$ is the failure rate in the mth preventive maintenance where $L_m = l_0\,l_1{*}l_2{*}\ldots\ldots{*}l_{m-1}$ and $1 = l_0 < l_1 < l_2 < \ldots\ldots\ldots < l_{Q-1}$

II.II Assumptions

1. At time $t = 0$ the system is operational.
2. The system is inspected at regular intervals and is preventively maintained.
3. The failure rate $\sigma_m(t)$ is strictly increasing.
4. The failure is detected only at the time of inspection and the system is subject to corrective repair at the instance of a failure.
5. The times required for preventive maintenance and replacement of the considered system is assumed to be negligible.
6. The system becomes as good as new after every replacement.
7. The probability density function of service times θ_m corresponding to the mth imperfect maintenance obeys general distribution $Y(t)$.
8. The cost rate associated with every inspection, preventive maintenance is denoted by I_{FC} and C_{CPM}

respectively. The cost associated with every system replacement is given by C_{REPL}. The cost rate during the system downtime is denoted by D_C. The CM expense (cost) rate corresponding to the failure of the system is represented by C_{RC}.

II.III Notations

$A(t)$	System's instantaneous availability
$R(t)$	System's reliability
CM	Corrective Maintenance
$\sigma(t)$	Failure Rate of the system.
i.i.d.	Identically and independently distributed
Q	Number of Preventive maintenance cycles
η_m	Failure times corresponding to the mth preventive maintenance
$B_m(t)$	Distribution function of η_m, m =1, 2, ..., Q-1.
$b_m(t)$	Probability density function of η_m, m =1, 2, 3…., Q-1.
θ_m	Repair times during the mth preventive maintenance, m = 1, 2…, Q-1.
$Y(t)$	cdf of repair times.
$y(t)$	Probability density function for repair time
H	Periodicity of inspection
N_T	Number of inspections performed in every system restoration
LC	Total length of cycle requires for restoration of the system
I_{FC}	Fixed cost of the inspection
C_{CPM}	Preventive maintenance cost
C_{REPL}	Cost associated with the replacement of the system
D_C	Cost rate(penalty) while system is in downtimes
TC_R	Total cost in a cycle (renewal) when the system is restored

III. Reliability of the periodically inspected system

The current section develops expressions to evaluate the reliability and availability of the periodically inspected system with imperfect maintenance modelled in the previous section. Thus, the function for the estimation of the reliability is derived as follows:

$$R(t)=\prod_{m=1}^{Q-1}P(\Omega_m>t)=\prod_{m=1}^{Q-1}R_m(t)$$

Thus, proposition-1 which is given below helps to calculate the instantaneous/point availability of the periodically inspected system based on the renewal theorem.

III.1 Instantaneous Availability Analysis

Proposition-1: A periodically inspected system is maintained such that the failure rate of the system is modelled by an imperfect preventive maintenance policy. The failure times η_m are governed by a general distribution function $B_m(t)$ in the mth preventive maintenance (m =1, 2, …., Q-1) respectively. Service times θ_m are assumed to follow general distribution$Y(t)$. Thus, the expression for the instantaneous availability of the considered periodically inspected system can be derived as follows:

$$A(t)=\prod_{m=1}^{Q-1}R_m(t)+\sum_{j=0}^{[t/H]-1}\sum_{m=1}^{Q-1}\int_{jH}^{(j+1)H}l_m\sigma(t)R(t)\int_{0}^{t-(j+1)H}A(t-(j+1)H-z)y(z)dz$$

Proof. There can be two possible states for the considered periodically inspected system, namely the down-state, which corresponds to $\Omega(t) = 0$, or in the up-state, which implies $\Omega(t) = 1$.
Based on the assumption that the system is operable at $t = 0$ and no inspection and preventive maintenance has been performed in the first interval, i.e. $[0, H)$.
Thus, from the fundamental definition of availability we have the following:
Since the system is replaced at Qth cycle, hence, $Q-1 = J$ (say) will be the total number of inspections

$$A(t) = P \text{ (the system is available at an instant } t) = P(\Omega(t) = 1) \quad (1)$$

performed till the time system gets replaced. Furthermore, the time of failure for the periodically inspected system Ω are related as, $(J-1)H < \Omega < JH$. Consequently, the probability mass function of J is given as

$$P(J = j) = R(jH) - R((j+1)H) \quad (2)$$

Therefore, equation (1) becomes

$$A(t) = \sum_{j=0}^{[t/H]-1} P(\Omega(t) = 1, J = j) + \sum_{j=[t/H]}^{\infty} P(\Omega(t) = 1, J = j) \quad (3)$$

The probability that the first failure of the system happens to be in $[(j-1)H, jH]$ is expressed by the first half of the above equation and the system gets repaired/restored at time jH, where $j = 2, 3, \ldots$ while the second half of (3) directs toward the event that system survives before time t and is represented as given below:

$$P(\Omega(t) = 1, J \geq [t/H]) = P(\Omega > t) \quad (4)$$

Since there are Q maintainable cycles, the failure rate in mth preventive maintenance cycle shall be $\sigma_m(t)$, $m = 1, 2, \ldots, (Q-1)$, hence the first half of equation (3) becomes

$$\sum_{j=0}^{[t/H]-1} P(\Omega(t) = 1, J = j) = \sum_{j=0}^{[t/H]-1} \sum_{m=1}^{Q-1} P(\Omega(t) = 1, J = j, M = m) P(J = j, M = m) \quad (5)$$

The event $\{J = j, M = m\}$ implies no failure has occurred in $[0, jH]$ and it is only in the consequent interval, i.e. $[jH, (j+1)H]$ there is a probability of the failure of the considered system.
Thus, we have

$$P(J = j, M = m) = P\{jH < \Omega < (j+1)H, \Omega_m = \min(\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_{Q-1})\}$$

$$= \int_{jH}^{(j+1)H} R(t) l_m \sigma(t) dt \quad (6)$$

In the considered model, it is assumed that the failure rate of the periodically inspected system becomes $l_m\sigma(t)$ after the mth preventive maintenance when it was $\sigma(t)$ before the mth preventive maintenance. The repair rate of the system remains unchanged. Thus, the probability associated with the failure of the considered system in the mth imperfect preventive maintenance cycle can be obtained from equation (6) as

$$P(\Omega_m = \min(\Omega_1, \Omega_2, \ldots\ldots\ldots \Omega_{Q-1})) = \int_0^{\infty} R(t) l_m \sigma(t) dt \quad (7)$$

The total sojourn repair time θ_m elapsed in the restoration of the system due to system failure in the mth preventive maintenance is governed by the distribution function $Y(z)$. After the sojourn time to repair is elapsed the operating unit is functional again and the process repeats itself. Furthermore, the same process is repeated every time. Then, we have,

$$P(\Omega(t)=1 \mid J=j, M=m) = \int_{0}^{t-(j+1)V} P(\Omega(t)=1 \mid J=j, M=m, \theta_m = z) y(z) dz$$

$$= \int_{0}^{t-(j+1)H} A(t-(j+1)H-z) y(z) dz \tag{8}$$

Using the equations (7) and (8), the expression (5) becomes

$$\sum_{j=0}^{[t/H]-1} P(\Omega(t)=1), J=j) = \sum_{j=0}^{[t/H]-1} \sum_{m=1}^{Q-1} \int_{jH}^{(j+1)H} R(t) l_m \sigma(t) \int_{0}^{t-(j+1)H} A(t-(j+1)H-z) y(z) dz \tag{9}$$

Substituting the equations (8) and (9) in (3), the instantaneous availability of the periodically inspected system is obtained as

$$A(t) = \prod_{m=1}^{Q-1} R_m(t) + \sum_{j=0}^{[t/H]-1} \sum_{m=1}^{Q-1} \int_{jH}^{(j+1)H} l_m \sigma(t) R(t) \int_{0}^{t-(j+1)H} A(t-(j+1)H-z) y(z) dz \tag{10}$$

III.II Long-run availability

The following proposition gives the expression for estimating the steady-state availability of the periodically inspected system subject to imperfect maintenance.

Proposition-2: The availability of the proposed periodically inspected system in the steady state is obtained as follows:

$$A = \frac{\int_0^\infty R(t) dt}{\sum_{j=0}^{\infty} (j+1)H(R(jH) - R(j+1)H) + \sum_{m=1}^{Q-1} E(\theta_m) \int_0^\infty R(t) l_m \sigma(t) dt}$$

Proof: With the help of the renewal theorem the long-run availability of the considered periodically inspected system is derived as the ratio of expectation of the uptime of the system and the sum of the expectations of the system uptimes and downtimes.

Thus, based on Proposition 1, the expectation of the uptimes of the considered periodically inspected system is obtained as follows:

$$\text{E (Up times)} = \text{E}(\Omega) \tag{11}$$

the length(expected) of each restoration cycle (E(*LC*)) is calculated on the basis of the probability decomposition method which is obtained as given below:

$$E(LC) = \sum_{j=0}^{\infty} (j+1)H(R(jH) - R(j+1)H) + \sum_{m=1}^{Q-1} E(\theta_m) \int_0^\infty R(t) l_m \sigma(t) dt \tag{12}$$

The first half of equation (12) corresponds to the expected time between any unexpected/sudden failures to the time it has been restored again. The second half of equation (12) denotes the expected time for restoration.

On combining equations (11) and (12) the expression for the availability (in the steady state) of the considered periodically inspected system is derived as

$$A = \frac{\int_0^\infty R(t)dt}{\sum_{j=0}^{\infty}(j+1)H(R(jH)-R(j+1)H)+\sum_{m=1}^{Q-1}\theta_m\int_0^\infty R(t)l_m\sigma(t)dt} \tag{13}$$

Corollary 2.1: Here the bounds for the steady state availability, i.e., the upper bound and lower bound availability denoted by A_U and A_L respectively for the risk system are obtained as follows:

$$A_U = \frac{\int_0^\infty R(t)dt}{E(\Omega)+\sum_{m=1}^{Q-1}E(\theta_m)\int_0^\infty l_m\sigma(t)R(t)dt} \tag{14}$$

$$A_L = \frac{\int_0^\infty R(t)dt}{E(\Omega)+L+\sum_{m=1}^{Q-1}E(\theta_m)\int_0^\infty l_m\sigma(t)R(t)dt} \tag{15}$$

Clearly, equations (15) and (14) signify that the lower bound of availability is less than the availability of the considered system which is further less than the upper bound of availability.

III. III Maintenance Analysis

As per the maintenance policy implemented in the current study, the system may experience *Q* preventive maintenance cycles. Corrective maintenance is done whenever there is a system failure while preventive maintenance is carried out whenever an inspection is performed. Towards the end of every *Q*th cycle, the considered system is replaced with a new one. The expected duration of system replacement is assumed to be negligible. The optimal maintenance policy consists on finding a feasible value of the interval of inspection and the replacement period. Thus, the average maintenance cost rate is then a function of the both variables *H* and *Q*, and hereafter denoted by *W* (*H*, *Q*) and is defined as:

$$W(H,Q) = \frac{E(TC_R)}{E(LC)} \tag{16}$$

where $E(TC_R)$ is the expected total cost.

Proposition 3: For a periodically inspected system which is inspected and preventively maintained at regular intervals, the average rate of cost of maintenance is obtained as given below:

$$W(H,Q) = \frac{I_{FC}\sum_{j=0}^{\infty}(j+1)(R(jH)-R(j+1)H)+D_C\left(\sum_{j=0}^{\infty}(j+1)(R(jH)-R(j+1)H)-\int_0^\infty R(t)dt\right)+\sum_{m=1}^{Q-1}C_{RCm}E(\theta_m)\int_0^\infty R(t)l_m\sigma(t)dt+(Q-1)C_{CPM}+C_{REPL}}{\sum_{j=0}^{\infty}(j+1)(R(jH)-R(j+1)H)+\sum_{m=1}^{Q-1}E(\theta_m)\int_0^\infty R(t)l_m\sigma(t)dt}$$

Proof: In every renewal cycle when the considered system is restored then the expected(total) maintenance cost rate involved is given as follows:

$$E(TCR) = IFC\ E(NT) + E(C_{RC}) + DC\ E(Downtimes) + CCPM\ E(Q-1) + E(CREPL) \tag{17}$$

Hence, the mean cost involved in the corrective maintenance (CM) in every restoration cycle is given as

$$E(C_{RC}) = \sum_{m=1}^{Q-1} C_{RCm} \int_0^{\infty} l_m \sigma(t) R(t) dt \tag{18}$$

where $N_T = J+1$ is the total count of inspections.
Also, the expectation of the system downtime is obtained as:

$$E(Downtimes) = \sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) - \int_0^{\infty} R(t) dt \tag{19}$$

Therefore, the expectation of the cost (total) for every cycle when the periodically inspected system is restored is

$$E(TC_R) = I_{FC} \sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) + D_C \left(\sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) - \int_0^{\infty} R(t) dt \right) + \sum_{m=1}^{Q-1} C_{RCm} E(\theta_m) \int_0^{\infty} R(t) l_m \sigma(t) dt + (Q-1) C_{CPM} + C_{REPL}$$

Combining the results of equation (20) and equation (11) into equation (16), the mean cost rate for maintenance with respect to the imperfect preventive maintenance is obtained as given below:

$$W(H,Q) = \frac{I_{FC} \sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) + D_C \left(\sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) - \int_0^{\infty} R(t) dt \right) + \sum_{m=1}^{Q-1} C_{RCm} E(\theta_m) \int_0^{\infty} R(t) l_m \sigma(t) dt + (Q-1) C_{CPM} + C_{REPL}}{\sum_{j=0}^{\infty} (j+1)(R(jH) - R(j+1)H) + \sum_{m=1}^{Q-1} E(\theta_m) \int_0^{\infty} R(t) l_m \sigma(t) dt}$$

IV. Evaluation of Reliability Characteristics

Numerical Example: An Application of Nuclear Power Plant System

Background

Nuclear power plant systems (NPPS) need unremitting and careful monitoring from time to time. Whenever there is a chance of unsafe conditions in the NPPS corrective measures must be taken to ensure proper functioning of the system. In order to control such circumstances appropriate knowledge of the prevailing conditions and the ones to be changed should be known at prior. The diagnosis of any fault in the plant should be known very soon. Thus, preventive maintenance plays a major role in this direction. Through such maintenance policies the plant operators and engineers can perform the required preventive and corrective actions, if needed. The reactor coolant pump is an integral part of the NPPS. It prevents and safeguards the vibrations which may lead to trippage of the reactor. To illustrate the various obtained results, we assume that there is a fault in the reactor coolant of the NPPS. The failure time of the reactor coolant is governed by the distribution $B_m(t)$ = 1-exp($-mt$), $m > 0$. Thus, as per the assumptions since l_0 =1, then the distribution for the failure time $B_1(t)$ =1-exp ($-t$) and the failure rate remains 1 in the interval [1,2]. Let us now suppose that l_1 = 2. Hence the rate of failure becomes 2 in the interval [2,3]. Also, let the distribution for repair, i.e., $Y(z)$ = 1- exp($-3z$). We further assume that the system is replaced at 3rd cycle. Furthermore, the values of the relevant system parameters used to evaluate the desired results are listed below in Table 1.

Table 1: *System Parameter Values*

Parameter	
Parameter	Values

H	1 time unit
I_{FC}	1 cost unit
$C_{RC1}=C_{RC2}$	2 cost unit
C_{REPL}	1 cost unit
D_C	10 cost unit

IV.1. Reliability and Availability Evaluation

With respect to the imperfect maintenance policy, the results of Section 4 are well illustrated through the graphs plotted in Fig. 1 and Fig. 2. It is clear from the Fig. 1 that the reliability of the considered system decreases with the increase in time and approaches to zero over a long period of time. The Fig. 2 corresponds to the instantaneous availability of the system with respect to the inspection period H =1. It can be seen from the Fig. 2 that instantaneous availability is same as the system's reliability in the time [0,1). In the consequent interval which is followed by a preventive repair at H =1 the system availability increases till the time t = 1.6 and then further decreases till t = 2. The failure rate now further increases in the interval [2,3] as the system is subject to imperfect maintenance. Thus, in the interval [2,3] failure is detected and repaired which increases the availability of the system. From this figure the steady state availability is found to be 0.2405 %. Finally, the system is replaced at the end of the interval [2,3] after the preventive maintenance. Fig. 3 shows the sensitivity of the considered periodically inspected system with respect to the inspection period (H). It illustrates the availability of the system when inspection period varies for H = 1, H = 1.5, H = 2. As anticipated, Fig. 3 validates that when there is a longer gap between two inspection periods i.e., the value of H increases the system availability decreases.

IV.II. Optimal Inspection Policy

Fig. 4 reflects the steady state availability of the system with respect to different periodicity of inspections. As seen from the figure that it is a strictly decreasing curve, i.e., larger the periodicity lesser is the steady-state availability of the system. The current study seeks, however, to obtain an optimal inspection period which will minimize the average cost rate. Fig. 5 represents the average long run maintenance cost rate when periodicity of inspection is H =1. It is quite evident from the graph that the maintenance cost rate decreases initially and eventually becomes constant. From this figure the minimum cost rate is attained when H = 0.8 and the cost rate is found to be 9.5327. Fig. 6 shows the effect of varying replacement periods on the long run maintenance cost rate of the system. It demonstrates that as the value of Q increases the cost rate also increases. It is quite intuitive also since a larger gap between consecutive replacement periods will result in performing inspections more frequently thereby increasing the frequency of preventive maintenances. Hence, the cost rate of the system increases substantially.

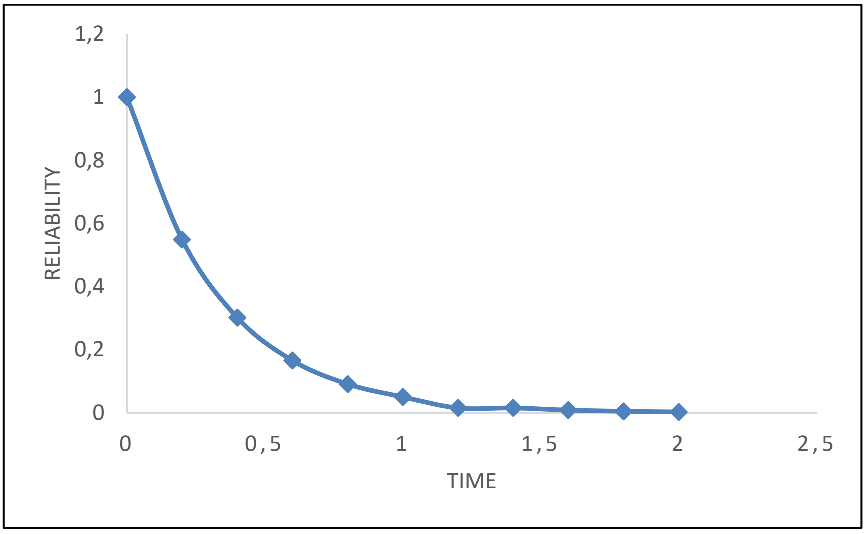

Fig. 1: *Reliability of the system.*

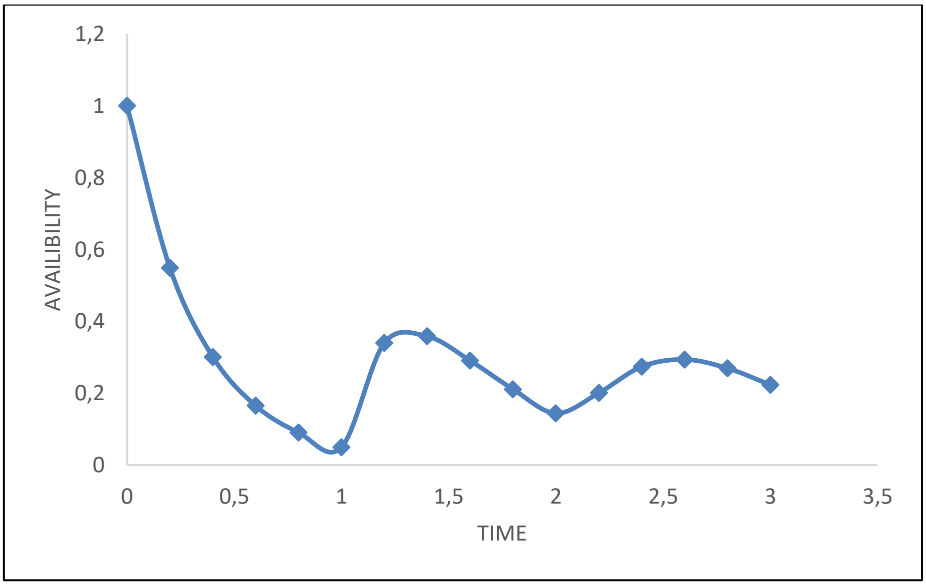

Fig. 2: *Availability when two imperfect repairs are followed by a replacement: H =1.*

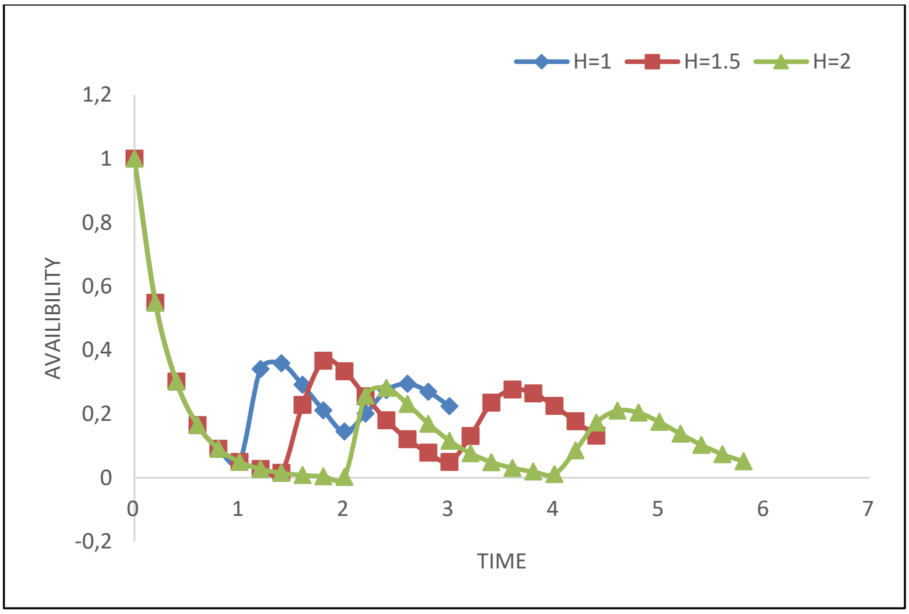

Fig. 3: *Sensitivity of the NPPS system*

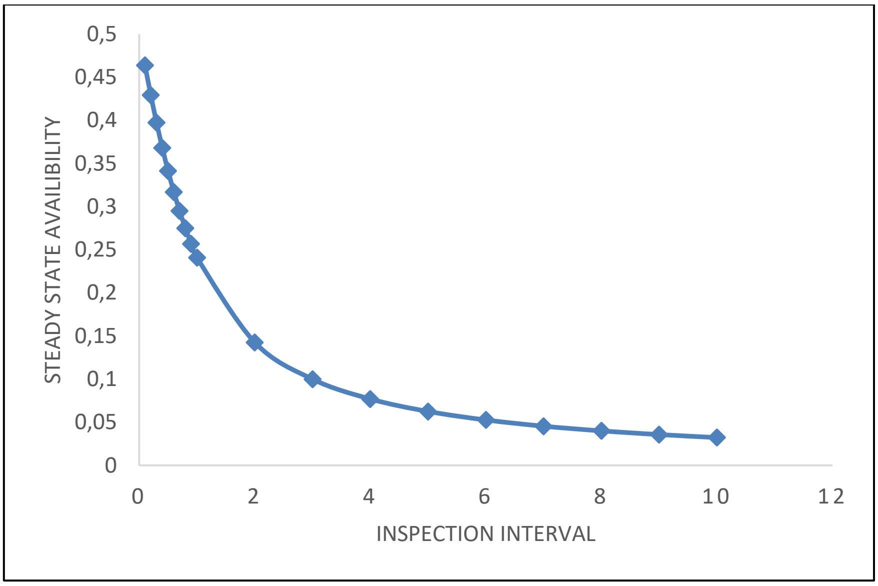

Fig. 4: *Steady-state Availability versus H.*

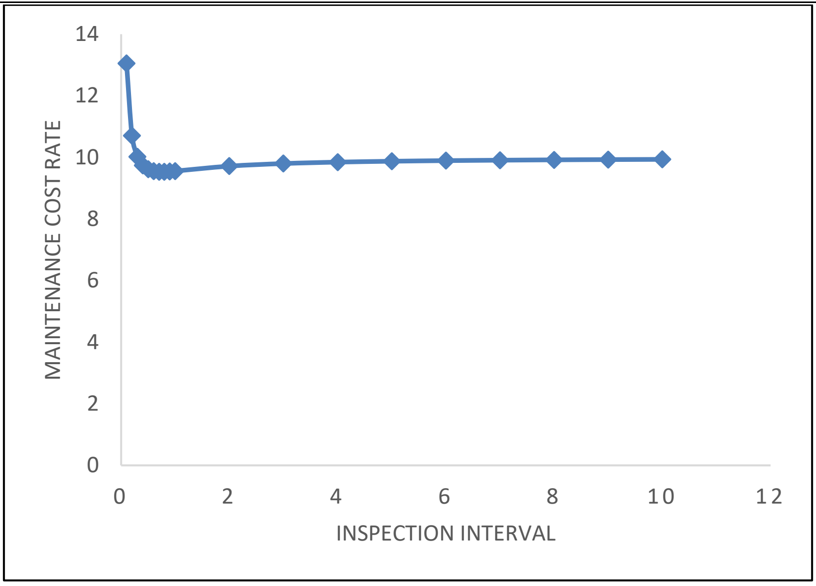

Fig. 5: *NPPS's maintenance cost rate.*

V. Discussion

The current study developed an imperfect preventive maintenance model for a periodically inspected system. The considered system is assumed to be inspected periodically for the detection of any possible failure. Imperfect preventive maintenance actions are modelled on the basis of a failure rate. The strictly increasing failure rate ensures and allows to represent the occurring failures in the system and the imperfect effect of the preventive maintenance. Furthermore, a case study on NPPS is then proposed and the results are illustrated with times of failure following exponential distribution. Also, the times to repair have been assumed to be exponentially distributed with the periodicity of inspection as one.

Conflicts of interest: Authors hereby state that there are no conflicts of interest in this manuscript. This is soul work of authors which has not submitted in any other journal. The manuscript follows ethically guidelines for the journal.

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