Two-Dimensional SRGM with Delay in Debugging by Considering the Uncertainty Factor and Predictive Analysis

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Abstract

Software is a set or group of programs and instructions, which is designed to perform a well-defined function. A software fault/error can be the cause of major productivity or economic loss. The need to determine the reliability of software led to development of the software reliability growth models (SRGMs). In research literature, exist various SRGMs which have been developed without considering fault removal time and only a few models have incorporated operating environment uncertainty. However, Most of the SRGMs are characterized under the condition that the software reliability enhancement process depends only on the testing time, which is called one-dimensional SRGM. In this article, two-dimensional SRGMs have been proposed using delay in debugging, with uncertainty in the operating environment. To represent the combined effect of used resources and testing time, the Cobb – Douglas production function has been used, which converts the one-dimensional model to two-dimensional. For better understanding the results predictive analysis is performed and to validate the proposed model results are compared with existing SRGMs using five statistical comparison criteria.

Keywords: Software, SRGMs, Predictive analysis, Reliability, Cobb-Douglas production function.

I. Introduction

Software lifecycle management through development, usage, operation and maintenance of systems, requires a systematic approach. Planning, design, coding, testing and analysis are the five phases of any software lifecycle. Software testing is the critical phase to reliably measure the functionality of the software. Any error in the software may cause considerable economic and operational damage. For successful functioning of any software, the important factor is to minimize the possibility of software failure with high reliability. "Reliability is the probability that the system will execute its indeed function under definite limit" [1]. To develop reliable software, many SRGMs have been studied but most of them are insufficient for today's complex software. The present SRGMs can be specified in two types, one-dimensional and two-dimensional models. One-dimensional SRGMs consider only testing time, whereas two-dimensional SRGMs depend on the effect of testing time as well as used resources. Kapur et al. [2] proposed a two-dimensional SRGM in perfect debugging environment for multi-release software. Singh et al. [3] considered the effect of the parameters like testing-time and testing-coverage or testing-time and testing- effort, simultaneously on fault detection process with change point. Anniprincy and Sridhar [4] discussed an S-shaped model with joint effect of testing-time and testing-coverage in imperfect debugg~~ing~~. SRGM modeling based on a CES (constant elasticity of substitution) type time function was introduced by Minamino et al. [5]. Pachauri et al. [6] studied an SRGM in imperfect debugging environment that considers the delay in fault correction.

In traditional SRGMs, it has been studied under the common belief that when a fault occurs in the software, it can be immediately removed. These models assume that the testing and operating environment of the software are the same. The first NHPP based traditional SRGM (the NHPP exponential model) was introduced by Goel and Okumoto [7]. The inflection S-shaped SRGM, depend on the failure rate of each detectable fault was discussed in Ohba [8]. Inflection S-shaped function as a fault reduction factor was given by Pachauri et al. [9].

A few models were also developed using time lag in the fault removal process. Yamada et al. [10] considered the time delay effect between the fault removal and fault detection process. An SRGM with fault dependency and various time lags, to predict the reliability, was introduced by Pradhan et al. [11]. Huang and Lin [12] proposed that in the fault removal process the correction time cannot be neglected.

In some recent publications, authors have included uncertainty of environment in the SRGMs [13 -19]. Song et al. [14] used inflection factor of error detection with vagueness in operating environment. Pham [15] introduced a new SRGM with Vtub-shaped fault detection rate. Song et al. [17] studied an SRGM with an ideal software release time and have done sensitivity analysis. Pachauri et al. [20] proposed new SRGM under the fuzzy paradigm with optimum release time and calculated total software cost.

Predictive analysis can provide advance warning of any failure, which may prove invaluable for system operators. The predictive analysis depends on historical data to make predictions about any possible future failures and errors. Recently, Song et al. [14] have introduced predictive analysis method in their SRGM.

From the literature, it is found that the factors like environmental factor, delay in debugging, effect of testing-time and used-resources, etc. are not included in an SRGM simultaneously. Therefore, motivated by Kapur et al. [2], Pachauri et al. [6] and Song et al. [14], a perfect debugging SRGM by considering uncertainty of operating environment with delay has been proposed. The predictive analysis also has been done for a better understanding of prediction. The proposed study may be useful for any software development company to check the reliability of the software based on the testing data before launching it in the market and may be used to estimate the optimum release time of ~~so~~ftware.

In this paper, two-dimensional perfect debugging SRGMs have been proposed by considering delay in fault correction and uncertainty of environment. Calculation of Mean value function (MVF) is given in Section 2 with mathematical derivation. To validate the models, the used comparison criteria, numerical results are in Section 3 and predictive analysis is shown in section 4. Finally, the conclusion is presented in Section 5.

II. Proposed model

The following are the common assumptions for these models [6, 18]:

- The software failures phenomena follow NHPP.
- Fault detection rate and residual system faults are proportional.
- The operating environment uncertainty is the multiplication of the unit failure detection rate $b(t)$ with a random variable η.
- The software system may fail during execution, due to remaining faults in the system.

- The compound effect of the used resources and testing effort is represented using Cobb-Douglas production function.

Based on these assumptions, the rate of change in MVF with uncertainty factor is given as [15]:

$$\frac{d\,m(t)}{d(t)} = \eta[b(t)][N - m(t)], \tag{1}$$

where η is a random variable (r.v.) that represents the uncertainty of operating environments with the probability density function g, $b(t)$ is the fault-finding rate function and N is the total number of faults initially [15]. The solution of the above differential equation with the initial condition $m(0) = 0$ is given as:

$$m(t) = \int_\eta N\left(1 - e^{-\eta\int_0^t b(x)dx}\right) dg(\eta), \tag{2}$$

where g has the parameters $\alpha \geq 0$, $\beta \geq 0$ and η follows the gamma distribution. After applying the r.v. η in equation (2) we get:

$$m(t) = N\left(1 - \frac{\beta}{\beta + \int_0^t b(s)ds}\right)^\alpha, \tag{3}$$

when detection rate function $b(t)$ is given as [14],

$$b(t) = \frac{b}{1 + ae^{-bt}}, a, b > 0, \tag{4}$$

then the cumulative number of faults at time t,

$$m(t) = N\left(1 - \frac{\beta}{\beta + \ln\left(\frac{a + e^{bt}}{1 + a}\right)}\right)^\alpha, \tag{5}$$

where b is the constant fault detection rate and a is the inflection factor.

After considering the testing-time and used-resources simultaneously, the modified MVF is given as [2, 6];

$$m(s,u) = N\left(1 - \frac{\beta}{\beta + \ln\left(\frac{a + e^{b(s^\gamma u^{1-\gamma})}}{1 + a}\right)}\right)^\alpha, \tag{6}$$

where s is the testing time and u is used resources. By using the delay factor which is a function of time, the new MVF $m(s,u)$ is given as;

$$m(s,u) = m(s - \varphi(s), u). \tag{7}$$

$$m(s,u) = N\left(1 - \frac{\beta}{\beta + \ln\left(\frac{a + e^{b((s-\varphi(s))^\gamma u^{1-\gamma})}}{1 + a}\right)}\right)^\alpha. \tag{8}$$

Here, two types of delay functions are considered, delayed S-shaped and inflection S-shaped curve.

Model-1

In the fault detection exercise, the delay function is modeled as an S-shaped curve. It considers the learning progress because the skills of the examiners or testers are directly proportional to time [6]. A good explanation of delayed S-shaped curve is given in [1] and mathematically it is defined as;

$$\varphi(s) = \frac{1}{b}\log(1 + bs), \tag{9}$$

after using the delay function as defined in equation (9), then the MVF is,

$$m(s,u) = N\left(1 - \frac{\beta}{\beta + \ln\left(\dfrac{a + e^{b\left(s - \frac{1}{b}\log(1+bs)\right)^{\gamma} u^{1-\gamma}}}{1 + a}\right)}\right)^{\alpha}. \tag{10}$$

Model-2

When the inflection S-shaped curve is used as the delay function which is defined as,

$$\varphi(s) = \frac{1}{b}\log\left(\frac{1 + \psi}{1 + \psi e^{-bs}}\right), \tag{11}$$

where ψ is the inflection factor of Inflected S-shaped curve [6]. Then, the MVF with delay factor is,

$$m(s,u) = N\left(1 - \frac{\beta}{\beta + \ln\left(\dfrac{a + e^{b\left(s - \frac{1}{b}\log\left(\frac{1+\psi}{1+\psi e^{-bs}}\right)\right)^{\gamma} u^{1-\gamma}}}{1 + a}\right)}\right)^{\alpha}. \tag{12}$$

III. Numerical results and discussion

For the performance validation of the proposed models, four historical data sets have been used, which are summarized in Table-1. The estimated parameter values have been obtained using the curve fitting tool in MATLAB. The performance has been obtained in terms of mean square error (MSE), sum of squared error (SSE), root mean squared error (RMSE), R-square (R^2), and adjusted R-square ($Adj\ R^2$).

Table 1: *Reference data sets*.

Data sets	Time (t)	Total testing hours	Faults	Description	References
DS1	21 (weeks)	7476	26	Data set of telecommunication system test	[1]
DS2	22(days)	93 CPU hours	86	The pattern of discovery of error	[23]

DS3	19 (weeks)	10,272 CPU hours	120	Tandem computers	[24]
DS4	12 (weeks)	5053 CPU hours	61	Tandem computers	[24]

For DS1, the estimated parameters values are given in Table-2 with comparison criteria (MSE, SSE, RMSE, R^2, $Adj\ R^2$). From Table-2, it can be seen that the obtained values of the proposed model 1 for all five criteria are, 0.537, 8.056, 0.7328, 0.9950 and 0.9933. Similarly, for the model 2, values for comparison criteria are 0.573, 8.019, 0.7568, 0.9950 and 0.9929, respectively. From the results of DS1, both the proposed models give-better results compared to the existing literature and in between model 1 & 2, model 1 gives better performance. For more clarity, the graphical representation of total number of faults with time is shown in figure-1.

Table 2: *Parameter Estimated values and Comparison results for DS1.*

No.	**Model**	**Estimated value**	**MSE**	**SSE**	**RMSE**	$\boldsymbol{R^2}$	$\boldsymbol{Adj\ R^2}$
1	GO [7]	$\hat{a} = 3923854.73$, $\hat{b} = 3.2\ \times 10^{-7}$	3.8672	73.477	1.9665	0.9582	0.9535
2	Y-DS [10]	$\hat{a} = 3{,}9.82198$, $\hat{b} = 0.1104$	1.4938	28.382	1.2222	0.9838	0.9820
3	O-IS [8]	$\hat{a} = 26.6845, \hat{b} = 0.2918$, $\hat{\beta} = 21.6856$	0.6745	12.141	0.8213	0.9931	0.9919
4	K-SRGM 3 [21]	$\hat{p} = 0.1385, \hat{b} = 0.1385$, $\hat{\alpha} = 0.1012, \hat{A} = 24.989$,	1.2295	20.902	1.1088	0.9881	0.9851
5	R-M-D NHPP [22]	$\hat{a} = 40.2018, \hat{\alpha} = 0.9319$, $\hat{\beta} = 0.1402, \hat{b} = 0.1152$	2.0059	34.101	1.4163	0.9806	0.9757
6	C-TC [16]	$\hat{b} = 2.234, \hat{\beta} = 15.2504$, $\hat{a} = 0.0043$, $\hat{N} = 26.833$, $\hat{\alpha} = 9959.1698$,	1.0939	17.502	1.0459	0.9900	0.9867
7	P-Vtub [15]	$\hat{\alpha} = 1.5176, \hat{\beta} = 11.3848$, $\hat{b} = 1.2978, \hat{N} = 25.7412$ $\hat{a} = 1.0985$,	0.7178	11.485	0.8472	0.9935	0.9913
8	S-3PFD [13]	$\hat{a} = 0.038, \hat{c} = 1488.598$, $\hat{\beta} = 0.002, \hat{b} = 0.292$, $\hat{N} = 26.889$	0.7590	12.144	0.8712	0.9931	0.9908
9	SONG -P [14]	$\hat{\alpha} = 0.2176, \hat{b} = 1.0047$, $\hat{\beta} = 155.501, \hat{N} = 47.797$ $\hat{a} = 108232.819$,	0.5864	9.3824	0.7658	0.9947	0.9929
10	Model 1	$\hat{a} = 371.1,\ \hat{\alpha} = 0.8046$, $\hat{\beta} = 1.344, \hat{\gamma} = 0.4027$, $\hat{b} = 0.2355, \hat{N} = 27.59$	**0.537**	**8.056**	**0.7328**	**0.9950**	**0.9933**
11	Model 2	$\hat{a} = 8.214,\ \hat{\alpha} = 0.6583$ $\hat{\beta} = 1.47, \hat{b} = 0.2948$, $\hat{N} = 26.73, \hat{\psi} = 243.6$, $\hat{\gamma} = 0.6273$	**0.573**	**8.019**	**0.7568**	**0.9950**	**0.9929**

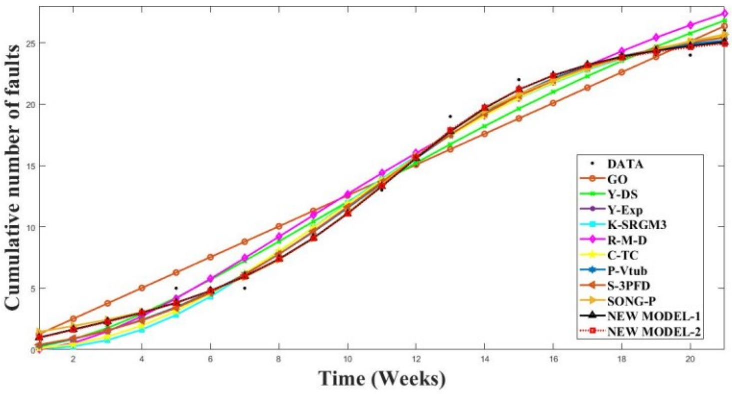

Figure 1: *Mean value function for various SRGMs for DS1.*

From Table-3, the values of the comparison criteria for DS2 are 3.3219, 53.15, 1.8226, 0.9969, 0.9959 and 3.9006, 58.51, 1.975, 0.9966, 0.9952 for model 1 & 2, respectively. These values are less than to the other existing models for the criteria MSE, SSE, RMSE and greater than for R^2, $Adj\ R^2$. Which shows the better performance for the proposed models for DS2. Again, the results of DS2 are shown in figure-2 for better understanding.

Table 3: *Parameter Estimated values and Comparison results for DS2.*

No.	Models	Estimated value	MSE	SSE	RMSE	R^2	$Adj\ R^2$
1	GO [7]	$\hat{a} = 153, \hat{b} = 0.0414$	25.190	503.818	5.019	0.9706	0.9691
2	Y-DS [10]	$\hat{a} = 94.25, \hat{b} = 0.1929$	7.645	152.883	2.765	0.9911	0.9906
3	O-IS [8]	$\hat{a} = 87.21, \hat{\beta} = 6.899,$ $\hat{b} = 0.2631$	5.909	112.317	2.431	0.9934	0.9928
4	K-SRGM 3 [21]	$\hat{A} = 10.97, \hat{p} = 1.152,$ $\hat{b} = 1.026, \hat{\alpha} = 0.890$	9.865	177.557	3.141	0.9896	0.9879
5	R-M-D NHPP [22]	$\hat{\alpha} = 1.6, \hat{b} = 0.1192$ $\hat{a} = 61.73, \hat{\beta} = 0.203,$	18.084	325.809	4.255	0.9810	0.9778
6	C-TC [16]	$\hat{a} = 0.449, \hat{\alpha} = 90.05,$ $\hat{\beta} = 1190, \hat{b} = 1.89,$ $\hat{N} = 88.34$	7.6209	129.556	2.7606	0.9924	0.9907
7	P-Vtub [15]	$\hat{a} = 15.78, \hat{b} = 1.291,$ $\hat{\beta} = 7.08, \hat{N} = 60.13,$ $\hat{\alpha} = 0.0367,$	292.20	4967.6	17.094	0.7099	0.6417
8	S-3PFD [13]	$\hat{a} = 1.117,\ \hat{c} = 136.9,$ $\hat{\beta} = 0.251, \hat{N} = 89.31$ $\hat{b} = 0.2656$	6.602	112.231	2.5694	0.9934	0.9919
9	SONG-P [14]	$\hat{a} = 58.4, \hat{\alpha} = 0.8222,$ $\hat{\beta} = 0.292, \hat{N} = 92.21$ $\hat{b} = 0.3154$	6.506	110.605	2.551	0.9935	0.9920
10	Model 1	$\hat{a} = 6.074, \hat{N} = 99.37,$ $\hat{\alpha} = 1.4E-07,$ $\hat{\beta} = 1.23, \hat{\gamma} = 0.7093,$ $\hat{b} = 0.07321$	**3.3219**	**53.15**	**1.8226**	**0.9969**	**0.9959**
11	Model 2	$\hat{a} = 4.022,\ \hat{\alpha} = 0.066,$ $\hat{\beta} = 0.919, \hat{b} = 0.066,$ $\hat{N} = 100.9 \hat{\gamma} = 0.8298$ $\hat{\psi} = 0.07275$	**3.9006**	**58.51**	**1.975**	**0.9966**	**0.9952**

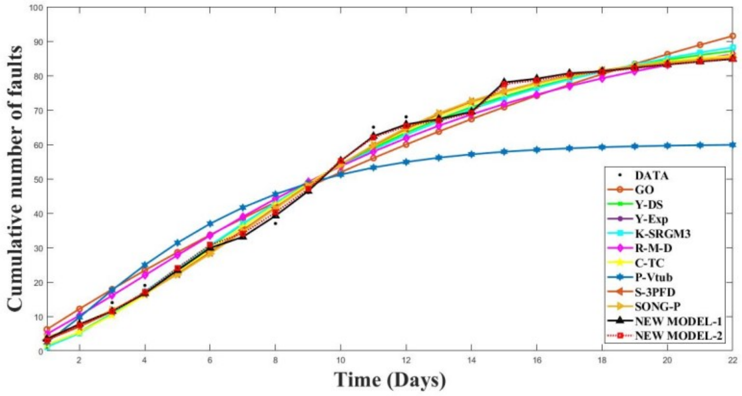

Figure 2: *Mean value function for various SRGMs for DS2.*

The results of proposed models 1, 2 for DS3 are shown in Table-4, which are 1.5055, 19.57, 1.227, 0.9992, 0.9989 and 1.5625, 18.74, 1.25, 0.9993, 0.9989, respectively. Comparison with other models in terms of MVF and time is given in figure-3. Similarly, from Table-5 the results of DS4 for both the models are 5.6644, 34.00, 2.380, 0.9932, 0.9875 and 8.8209, 44.09, 2.97, 0.9912, 0.9805, respectively. The comparative study of DS4 is shown in figure-4.

Table 4: *Parameter Estimated values and Comparison results for DS3.*

No.	Models	Estimated value	MSE	SSE	RMSE	R^2	$Adj\ R^2$
1	GO [7]	$\hat{a} = 183$, $\hat{b} = 0.0615$	26.002	442.0	5.0992	0.9824	0.9813
2	Y-DS [10]	$\hat{a} = 127.4, \hat{b} = 0.2417$	14.688	249.691	3.8325	0.9900	0.9895
3	O-IS [8]	$\hat{a} = 124.4, \hat{b} = 0.2535$ $\hat{\beta} = 3.779$	7.1268	114.0261	2.6696	0.9955	0.9895
4	K-SRGM 3 [21]	$\hat{A} = 1.858, \hat{p} = 3.791$ $\hat{b} = 2.413,$ $\hat{\alpha} = 0.9873$	18.550	278.2	4.307	0.9889	0.9867
5	R-M-D NHPP [22]	$\hat{a} = 98.13, \hat{\alpha} = 1.353,$ $\hat{b} = 0.1835,$ $\hat{\beta} = 0.215$	13.432	201.5	3.665	0.9920	0.9904
6	C-TC [16]	$\hat{\alpha} = 46.24, \hat{b} = 1.474,$ $\hat{\beta} = 26.63, \hat{N} = 125.7$ $\hat{\alpha} = 0.0787,$	13.265	185.7	3.6422	0.9926	0.9905
7	P-Vtub [15]	$\hat{a} = 60.55, \hat{\beta} = 8.282,$ $\hat{N} = 118.7, \hat{b} = 1.157,$ $\hat{\alpha} = 0.0246$	65.594	918.3	8.099	0.9634	0.9529
8	S-3PFD [13]	$\hat{a} = 0.673, \hat{c} = 39.4,$ $\hat{\beta} = 0.326, \hat{N} = 130.6$ $\hat{b} = 0.2555,$	8.2599	115.7	2.874	0.9954	0.9941
9	SONG-P [14]	$\hat{N} = 176, \hat{a} = 3181,$ $\hat{\alpha} = 0.2208,$ $\hat{\beta} = 58.16, \hat{b} = 1.096,$	2.143	30.006	1.464	0.9988	0.9984
10	Model 1	$\hat{a} = 77.35, \hat{\beta} = 45.28,$ $\hat{\alpha} = 0.659, \hat{N} = 180.9$ $\hat{\gamma} = 0.2777,$ $\hat{b} = 0.1647,$	**1.5055**	**19.57**	**1.227**	**0.9992**	**0.9989**
11	Model 2	$\hat{a} = 35.52$, $\hat{\alpha} = 0.533,$ $\hat{\beta} = 16.66, \hat{N} = 141,$ $\hat{b} = 0.2278,$ $\hat{\gamma} = 0.3356,$	**1.5625**	**18.74**	**1.25**	**0.9993**	**0.9989**

$\hat{\psi} = 72.77$

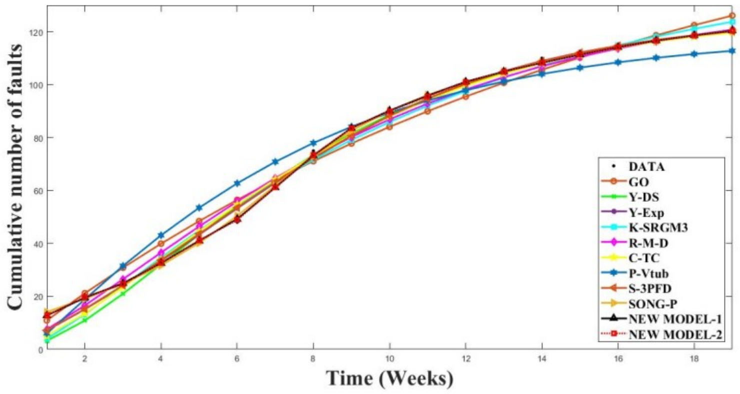

Figure 3: *Mean value function for various SRGMs for DS3.*

Table 5: *Parameter Estimated values and Comparison results for DS4.*

No.	Models	Estimated value	MSE	SSE	RMSE	R^2	$Adj\ R^2$
1	GO [7]	$\hat{a} = 244.3, \hat{b} = 0.0265$	20.894	209	4.571	0.9581	0.9539
2	Y-DS [10]	$\hat{b} = 0.2741, \hat{a} = 76.25,$	9.897	98.94	3.146	0.9802	0.9782
3	O-IS [8]	$\hat{a} = 64.4, \hat{\beta} = 11.36,$ $\hat{b} = 0.483$	6.345	57.09	2.519	0.9885	0.9860
4	K-SRGM 3 [21]	$\hat{\alpha} = 0.966, \hat{p} = 1.942,$ $\hat{A} = 3.057, \hat{b} = 2.097,$	15.296	122.4	3.911	0.9755	0.9663
5	R-M-D NHPP [22]	$\hat{a} = 59.05, \hat{b} = 0.1996,$ $\hat{\alpha} = 1.353, \hat{\beta} = 0.2526,$	20.512	145.1	4.259	0.9709	0.9600
6	C-TC [16]	$\hat{\beta} = 94.38, \hat{\alpha} = 1070,$ $\hat{b} = 1.921, \hat{a} = 0.0439$ $\hat{N} = 64.69$	10.74	75.16	3.2768	0.9849	0.9763
7	P-Vtub [15]	$\hat{\beta} = 852.4, \hat{\alpha} = 59.87,$ $\hat{b} = 0.782, \hat{a} = 1.923,$ $\hat{N} = 61.07$	6.6616	46.63	2.581	0.9906	0.9853
8	S-3PFD [13]	$\hat{a} = 29.31,\ \hat{\beta} = 2.857,$ $\hat{c} = 228.1, \hat{b} = 0.4653,$ $\hat{N} = 64.48$	9.4741	66.32	3.078	0.9867	0.9791
9	SONG-P [14]	$\hat{a} = 2262,\ \hat{\alpha} = 0.5059,$ $\hat{\beta} = 0.645, \hat{b} = 0.7314,$ $\hat{N} = 62.56$	6.287	44.01	2.507	0.9912	0.9861
10	Model 1	$\hat{a} = 48.34, \hat{\alpha} = 0.5042,$ $\hat{\beta} = 0.7441, \hat{\gamma} = 0.4634$ $\hat{b} = 0.0629, N\hat{} = 66.57$	**5.6644**	**34.00**	**2.380**	**0.9932**	**0.9875**
11	Model 2	$\hat{a} = 92.17,\ \hat{\alpha} = 0.735$ $\hat{\beta} = 0.125, \hat{b} = 0.0940$ $\hat{N} = 66.83, \hat{\gamma} = 0.8239,$ $\hat{\psi} = 0.1244$	8.8209	44.09	2.97	0.9912	0.9805

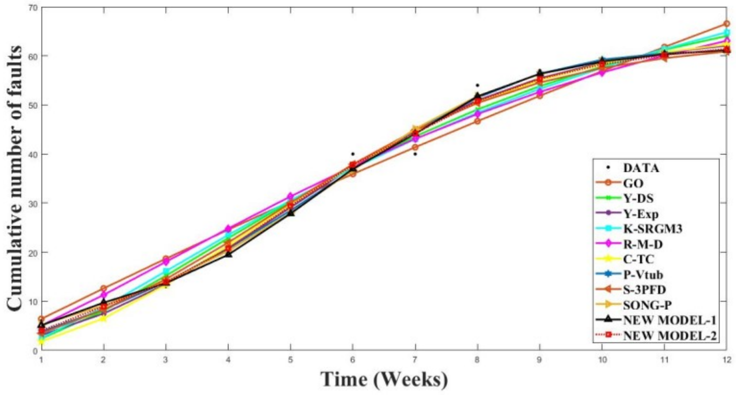

Figure 4: *Mean value function for various SRGMs for DS4*.

The relative error with time is also calculated for all the data sets, which confirms the ability to provide better accuracy. The comparison of the proposed models with other models in terms of relative error are shown in figures 5-8. Calculated values of MSE, SSE, and RMSE for proposed models are less than the existing models for all four data sets. The values of R-square and Adjusted R-square for proposed models are greater than the others. Based on these results, it can be said that these proposed models give improved results and have considerably better goodness of fit. ~~Also~~, model-1 gives ~~bet~~ter results compared to mo~~del~~-2.

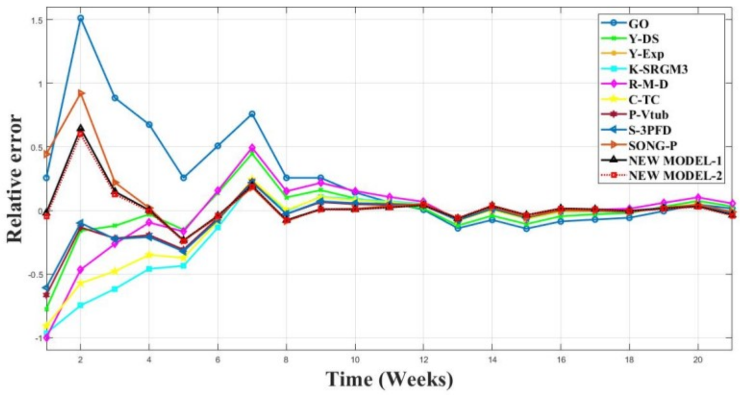

Figure 5: *Relative error curve for various SRGMs for DS1.*

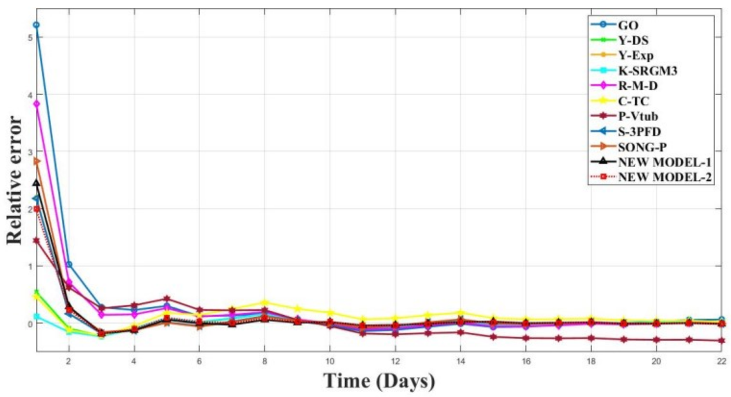

Figure 6: *Relative error curve for various SRGMs for DS2.*

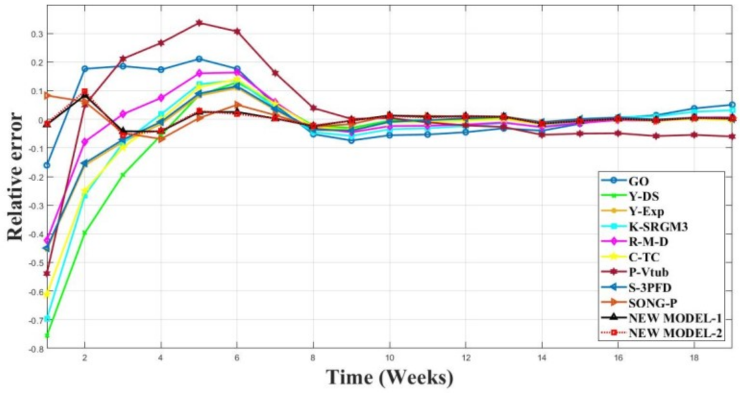

Figure 7: *Relative error curve for various SRGMs for DS3.*

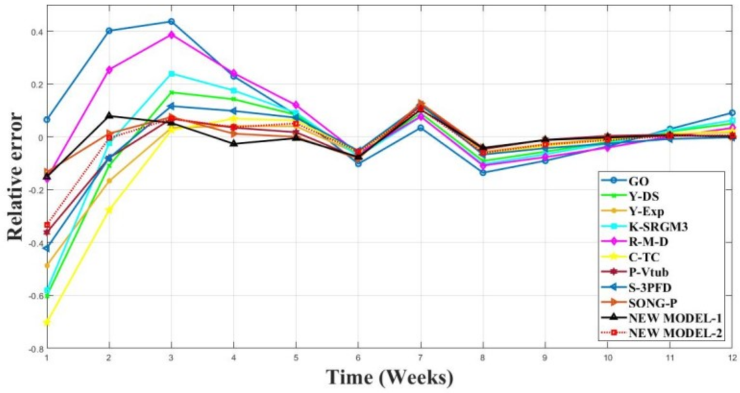

Figure 8: *Relative error curve for various SRGMs for DS4.*

IV. Predictive Analysis

In this paper, DS1 is used for the comparison of predicted values and how these differ for each model. 75% of the data set is used to estimate the parameters and the remaining 25% of the data set is used to predict the model performance. The sum of squared error of predicted value is denoted by PreSSE and it is calculated for the last 25% of the data.

The result of predictive analysis based on the six criteria (MSE, SSE, RMSE, R^2, $Adj\ R^2$ and PreSSE) is given in Table-6. From the Table-6, the values of the six criteria are 0.64, 6.417, 0.8010, 0.9924, 0.9873, 1.8684 and 0.72, 6.488, 0.8491, 0.9923, 0.9856, 1.8811 for both the models, respectively. Graphical representation of estimated MVF and predicted MVF is shown in figure-9. Based on the obtained results as shown in Table-6 and figure-9, it can be said that the proposed models give better prediction compared to the other existing models~~,~~ because the SSE and PreSSE are minimum for both the proposed models.

Table 6: *Parameter Estimated values, criteria and predictions from DS1*

No	Model	Estimated value	MSE	SSE	RMSE	R^2	$Adj\ R^2$	PreSSE
1	GO [7]	$\hat{a} = 2439.196,$ $\hat{b} = 0.00052$	4.85	8.012	2.2041	0.9199	0.9076	5.756
2	Y-DS [10]	$\hat{a} = 81.516, \hat{b} = 0.0657$	0.85	11.84	0.9195	0.9861	0.9839	128.71
3	O-IS [8]	$\hat{a} = 32.236\hat{b} = 0.2378,$ $\hat{\beta} = 16.9353$	0.67	8.690	0.8176	0.9898	0.9872	36.131
4	K-SRGM 3 [21]	$\hat{A} = 43.931, \hat{p} = 0.0265$ $\hat{b} = 1.3062, \hat{\alpha} = 2.2423$	1.02	12.24	1.0099	0.9856	0.9804	264.331

5	R-M-D NHPP [22]	$\hat{a} = 153.5748, \hat{\alpha} = 1.056, \hat{\beta} = 0.046, \hat{b} = 0.0327$	0.91	10.91	0.9535	0.9872	0.9825	188.6321
6	C-TC [16]	$\hat{a} = 0.0288, \hat{\alpha} = 193.585, \hat{\beta} = 167.613, \hat{b} = 1.6825, \hat{N} = 86.0872$	1.02	11.24	1.0107	0.9868	0.9802	160.1096
7	P-Vtub [15]	$\hat{a} = 1.2816, \hat{\alpha} = 1.1473, \hat{\beta} = 0.1982, \hat{b} = 0.9844, \hat{N} = 31.4297$	0.78	8.613	0.8849	0.9899	0.9848	32.5458
8	S-3PFD [13]	$\hat{a} = 0.1496, \hat{c} = 62.4407, \hat{\beta} = 0.1982, \hat{b} = 0.2372, \hat{N} = 36.9478$	0.82	8.989	0.9040	0.9894	0.9841	45.9267
9	SONG-P [14]	$\hat{a} = 10453.17, \hat{\alpha} = 0.4174, \hat{\beta} = 0.1175, \hat{b} = 0.53348, \hat{N} = 24.292$	0.59	6.506	0.7691	0.9923	0.9885	2.6780
10	Model 1	$\hat{a} = 26.45, \hat{\alpha} = 0.4766, \hat{\beta} = 0.4243, \hat{b} = 0.03818, \hat{N} = 26.45, \hat{\gamma} = 0.481,$	0.64	**6.417**	0.8010	**0.9924**	0.9873	**1.8684**
11	Model 2	$\hat{a} = 11.34, \hat{\alpha} = 0.7453, \hat{\beta} = 0.403, \hat{b} = 0.2607, \hat{N} = 26.18, \hat{\psi} = 86.91, \hat{\gamma} = 0.7413,$	0.72	**6.488**	0.8491	**0.9923**	0.9856	**1.8811**

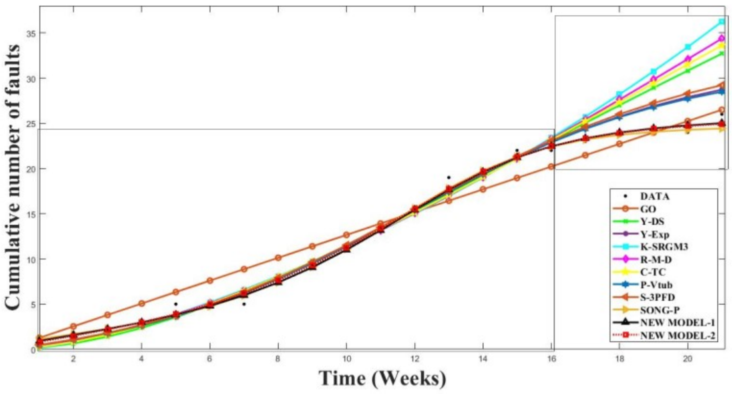

Figure 9: *Prediction of MVF for various models for DS1.*

V. Conclusion

Since-software development and testing process takes place in a controlled environment which is far away from operating environment. Therefore, uncertainty in the operating environment has been considered. To reflect the joint effect of testing time and used resources Cobb-Douglas production function has been used. Fault (if any) correction time is also affected the software costs as well as launching time. Delay in debugging has been included in two different ways, delayed S-shaped and Inflection S-shaped. In this article, two SRGMs have been proposed and these have been validated on the four real data sets. For validation, a curve fitting tool in MATLAB has been used for estimation and five statistical comparison criteria (MSE, SSE, RMSE R^2, and $Adj\ R^2$) have used to compare the results with existing literature. After validation, predictive analysis has been done to check the predictability of the proposed models using SSE and PreSSE. All the results with relative error have been figure out for better understating. On the basis of the results, it can be concluded that proposed models give significantly better performance compared to the others. In between these two models, it is found that results are improved when delayed S-shaped function has been used as delay function. In future, proposed work may include improvement of the proposed models with soft-computing techniques or introduction of additional reliability factors, i.e., change point, testing coverage, etc.

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