Ranking of Software Reliability Growth Models: A Entropy-ELECTRE Hybrid Approach

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Abstract

Software reliability is estimated using software reliability growth models. In the last few decades, numerous software reliability growth models (SRGMs) have been established. Some models are developed with the consideration of perfect debugging, imperfect debugging, testing coverage, testing effort, and fault reduction factor. Generally, SRGMs are dependent on dataset and thus it is a challenging task to select SRGM appropriately based on the need of software user. As the inappropriate selection of SRGM can lead to inaccurate results and consequently delay in software release. To address this issue, we have combined the two multi-criteria decision-making (MCDM) approaches namely Entropy and Elimination et Choice Translating Reality (ELECTRE) methods. The proposed approach identifies the criterion importance as to select suitable SRGM, comparison of criteria is important. The outcomes are based on the aggregate value of dominant matrix that is being used for SRGMs ranking. The working of proposed Entropy-ELECTRE method is demonstrated on a real time data set on which ten SRGMs are compared against six evaluation criteria. The findings play an important role in determining the SRGM's appropriateness for decision-maker.

Keywords: SRGM, MCDM, Entropy, ELECTRE, software reliability

I. Introduction

Computer systems have become an integral part of our life. Software systems are now involved in almost every aspect of human life, due to this their importance and demands are increasing. To build such computer systems, we will need increasingly complex and large-scale software systems. The software applications have marked their presence from critical applications of mission like military, defense to safety application including medical process [1]. A software failure of such systems may result in a financial loss, loss of human life, or the collapse of a critical operation. Also, it's important to determine if a software system will meet consumer expectations without failing before releasing it. Furthermore, with the expansion of the existing market, the complexity and size of software has continued to grow. Since, humans have created software systems, a large number of software flaws must be incorporated into the software product during the process of development. Thus, application of relevant technologies is important in the development of a highly reliable software system. Hence, software reliability is considered as an essential quality factor. "Software reliability refers to the software's failure-free operations in a given time period and specific environment". It measures the failure free services rendered by software to its authorized consumers. In the software industries, prediction and estimation of software reliability enables to meet complexities of software development. It is becoming more challenging for software managers to efficiently develop highly reliable software systems. In order to obtain fault-free software, there is a requirement of process to track fault content and reliability. Hence, a mathematical relationship termed as a software reliability growth model (SRGM) describing the process of finding and removing errors to increase software reliability is introduced.

A software system is tested during the software testing phase to find and rectify the remaining software defects (or errors) and hence software reliability improves. Such a failure detection process is represented by a software reliability growth model (SRGM). SRGMs are considered as a mathematical model that statistically examine software reliability and hence provide a measure of the software product's quality. SRGM establishes mathematical relations between time of testing and the failure occurrence rate at the testing time to quantify software reliability. These mathematical relations are made using statistical formulas, stochastic processes and probability. During the testing phase, SRGM make use of failure data to forecast the reliability of software over the span of its operational life. SRGM consider failure data as input to generate the prediction for reliability as an output in the form of mathematical functions. Thus, SRGM is referred as a parametric model with the parameters based on availability of software failure data. For a given data set, different SRGMs must be selected for comparison and then identification of best suited model must be done. Also comparison criteria for selecting best SRGM should be chosen wisely. The outcome for selecting best SRGM may be numerous models or there may be none at all. If there is no outcome, a model should be created based on the technique and surroundings. Further, the models have to undergo process again if the outcomes of model selection results into two or more. Sometimes, it is possible that the comparison outcome for two models may be similar to each other. Thus, the outcome of such comparison may not be accurate. Therefore, selection of best SRGM is considered as a multi- criteria decision making (MCDM) problem.

In operation research and management science, multi-criteria decision-making (MCDM) is the most prominent areas of multidisciplinary research. According to Belton and Stewart [2], MCDM provides a method for making justifiable, understandable, and rational decisions. As selecting best software reliability growth model among the given SRGMs is considered as MCDM problem so there is necessity to develop a system to resolve the MCDM problem effectively. Since SRGMs are data-dependent, therefore there is always a need to rank the SRGM. Traditional approaches such AHP, VIKOR, COPRAS, PROMETHEE, DEMATEL, MOORA, and many more have certain complexities and limitations. We tried to resolve the situation where software engineers have to choose a model for the software testing among the availability of various models. The objective of this study is to find responses to the questionnaire that have been collected based on the data gathered from research. Table 1 depicts the research questions based on software reliability growth models.

Table 1: *Research Questions*

S.No	Research Questions
1	Identification of SRGM for software engineers.
2	Selection of alternative SRGM for comparison.
3	Determination of ranking criteria.
4	Calculation of weights using Entropy method.
5	Ranking of SRGMs based on ELECTRE method.

In this paper, we have integrated two MCDM approaches namely Entropy and ELECTRE to find the best software reliability growth model for a dataset. We have considered ten well-known Non-homogeneous Poisson process model (NHPP) based software reliability growth models. These ten models were examined based on six evaluation criteria. We have used Entropy approach to find the weights of evaluation criteria and have used these weights in ELECTRE approach for the ranking of SRGMs. The primary contributions of the study are as follows:

- Proposed Entropy-ELECTRE as a combined approach.
- Application of Entropy approach to calculate weights of evaluation criteria.
- ELECTRE approach is used to rank alternative SRGMs.
- A real time data set is used to illustrate the proposed approach.

Further, remaining sections of the paper are organized as follows: Section II describes the related research on software reliability growth models (SRGMs) and multi criteria decision making (MCDM). Section III explains the methodology of combined Entropy-ELECTRE approach. Section IV specify the selection of ranking criteria and software reliability growth models for comparison. Section V illustrates proposed approach numerically on a real time data set. Lastly, section VI concludes the paper with future work.

II. Literature Review

Numerous software reliability growth models (SRGMs) has been developed for estimating the reliability of a software system. Chang et al. [3] presented a novel testing coverage software reliability model that takes into account operating environment uncertainty. Zhang et al. [4] developed a Fault removal model that incorporates efficiency of fault removal into software reliability model. Goel and Okumoto [5] developed a model that described failure detection as an NHPP and assumed that the hazard rate is proportional to the number of defects remaining in the software. Kapur et al. [6] proposed software reliability models in the presence of error generation and imperfect debugging. Dhavakumar and Gopalan [7] proposed chaotic grey wolf optimization algorithm (CGWO) as a new technique to quantify attributes of software reliability growth models. CGWO is a heuristic system that depicts execution by achieving complicated parameter optimization and solving application design challenges. Gao [8] proposed simulation approach to model Fault detection process (FDP), Fault correction process (FCP), and Fault introduction process (FIP) together and considered debuggers featuring contribution differently to FDP, FCP and FIP. Li et al. [9] proposed testability growth models on the basis of NHPP that takes into account the testability growth effort while simultaneously rectifying delay and imperfect correction.

Kaur et al. [10] proposed a mathematical model for firms that provide patching services with the assumption that sometimes corrective steps may implement infected patches. Erto et al. [11] proposed new generalized inflection S-shaped software reliability growth model. It is unique, very flexible finite failure Poisson process that covers the commonly used Goel-Okumoto model, inflection S-shaped model, and Goel generalized Nonhomogeneous Poisson process as special cases. Li and Pham [12] proposed a generalized model based on a non-homogeneous Poisson process (NHPP) that covers the uncertainty of the operating environment and imperfect debugging and its impact on fault detection rate in the evaluation of software. Zeephongsekul et al. [13] introduced a variation of EM algorithm, the expectation conditional maximization (ECM) algorithm and provided a viable option to estimate the parameters of nonhomogeneous Poisson (NHPP) software reliability growth models (SRGM). Kumar et al. [14] proposed a model to allocate resources in an effective way to reduce costs with fault correction process (FCP) and fault detection process (FDP) in a dynamic environment. Vizarreta et al. [15] focuses on several applications of SRGM framework that are critical for the adequate management based on SDN networks. Raghuvanshi et al. [16] proposed the time-variant fault detection Software Reliability Model that analyses numerous well-known algorithms on several performance measures and contains different software properties for model development.

Lee et al. [17] proposed a novel SRGM considering software failures that are interdependent. Authors used numerous evaluation criteria to assess the proposed model's goodness-of-fit with the previous results of nonhomogeneous Poisson process SRGMs on real-world datasets. Zhu and Pham [18] designed a martingale-based generalized multiple-environmental-factors software reliability growth model with related unpredictability. On the basis of randomness authors included a stochastic software fault detection procedure in the model. Kumar and Ram [19] highlighted the current advances and applications of artificial intelligence, data mining, and many other approaches in the predictive modelling and analytics in software reliability engineering. Kumar and Sahni [20] described the estimation of testing efforts in a dynamic environment with the assumption that debugging costs associated with each release follow a learning curve.

Garg and Ram [21] explained how to deal with uncertainty in reliability optimization using maintenance scheduling, soft computing, uncertainty, and fuzzy optimization scheduling strategies. Cai et al. [22] proposed a mechanism to minimize the disparities between adjacent trace files and incorporated certain unique mutation/crossover strategies into the genetic algorithm (GA). Kumar and Sahni [23] used FCP and FDP in a dynamic environment to assign testing resources in a way to reduce costs throughout the testing process. Kumar et al. [24] developed a reliability growth model based on software patching to enable software systems more cost-effective and reliable, minimizing software release time and testing cost. Kumar et al. [25] developed a model considering two stage process of fault detection and removal incorporating the effects of resources and testing time.

Over the last few decades, Multi-Criteria Decision Analysis has been used widely. Its importance in several application fields has grown dramatically, particularly new approaches have developed and existing ones have improved. Amirghodsi et al. [26] introduced Decision Making Trial and Evaluation Laboratory (DEMATEL) and ELECTRE decision-making approach on grey numbers from both quantitative and qualitative methods to address the technology provider selection problem systematically. Anser et al. [27] used analytical hierarchy process (AHP) and Fuzzy-VIKOR methods used to resolve the problem of selection of the optimal site for the installation of solar projects. Ho et al. [28] provides review work for supplier evaluation and selection on multi criteria decision making.

Sevkli [29] proposed a new approach known as fuzzy technique for ELimination Et Choix Traduisant la REalite´ (ELECTRE) for supplier selection problem by considering it multi criteria decision problem. Aruldoss et al. [30] addressed the problem in fuzzy multi criteria decision making techniques. Lin et al. [31] proposed an approach for evaluation of eutrophication based on Monte Carlo simulation (MCS) and technique for order preference by similarity to an ideal solution (TOPSIS). Rani et al. [32] proposed a new divergence measure based on fuzzy TOPSIS for evaluating and selecting renewable energy sources in multi-criteria decision-making challenges. Torlak et al. [33] used VIseKriterijumska Optimizacija I Kompromisno Resenje (VIKOR) - structural equation modeling (SEM) to develop service provider benchmarks and to analyze a multi methodology approach in the internet sector.

Mohammed [34] applied the techniques and concept of multi-criteria decision-making in a fuzzy environment to project prioritizing and selection in portfolio management. Yazdani et al. [35] introduced a combined compromise decision-making algorithm with the use of several aggregation strategies. Deveci et al. [36] proposed a technique to prioritize the benefits of different methods of real-time traffic management using fuzzy multi-criteria decision making (MCDM). Kumar et al. [37] presented a novel hybrid entropy weight based multi-criteria decision-making (MCDM) method and Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) approach to select suitable SRGM and used it for identify and rank SRGMs in the most efficient manner. Ikram at al. [38] aimed to find a way on the development of an integrated management system (IMS) using AHP-Fuzzy VIKOR approach. Arabsheybani et al. [39] applied a fuzzy multi-objective optimization model based on the ratio analysis (MOORA) to analyze the overall performance of supplier's. Kumari and Mishra [40] extended traditional complex proportional assessment (COPRAS) approach to resolve the multi criteria decision making (MCDM) problem of green supplier selection with intuitionistic fuzzy sets (IFSs).

III. The Proposed Approach

The proposed methodology is developed using the MCDM techniques. This methodology is based on the combination of Entropy method and ELECTRE method by considering ten models and six evaluation criteria. The techniques of Entropy and ELECTRE are appropriate to determine weight of evaluation criteria, rank the SRGMs, and to choose the best SRGM in the decision matrix. The weights of each evaluation criterion is obtained by using Entropy method and alternative SRGMs are ranked by using ELECTRE method to find the best SRGM. A hierarchical model is used to explain the process to determine the best SRGM. The objective is to choose the best SRGM among a given set of SRGMs. The model is divided into stages to make the process easier. The initial stage of the model includes the identification of criteria for evaluation. The second stage includes identification of alternative SRGMs followed by the calculation of SRGMs parameter using SPSS 20. In the next stage weights of each evaluation criterion are calculated by using the methodology of Entropy followed by the application of ELECTRE method to find best SRGM and hence SRGMs are ranked. The process of hierarchical model is explained by the flowchart shown in Figure 1. The methodology of Entropy and ELECTRE are discussed below.

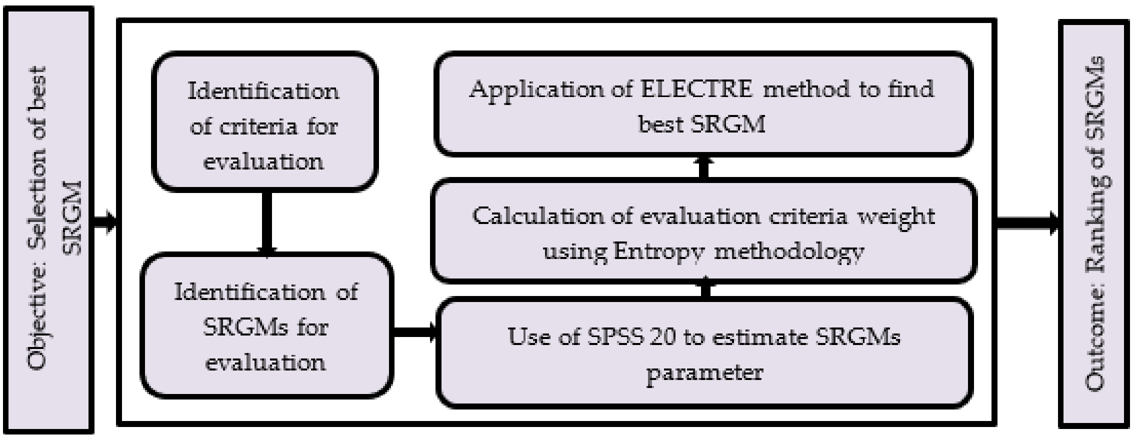

Figure 1*: Hierarchical model for best SRGM*

Shannon proposed the concept of entropy in 1948. "The Entropy method is a generic form of Monte Carlo simulation which is applied in complicated estimation and optimization problems for minimizing the error". It's a decision-making tool for determining the selection criteria weights in MCDM application. The Entropy approach should be employed for quantitative data volume measurement as well as for computing proportionate weight information [37]. Moreover, in information theory, entropy may be used to compute the predicted value of information in a given message. In this study, entropy is used to find the weight of each criterion and the steps involved in the process of Entropy method are explained in Figure 2.

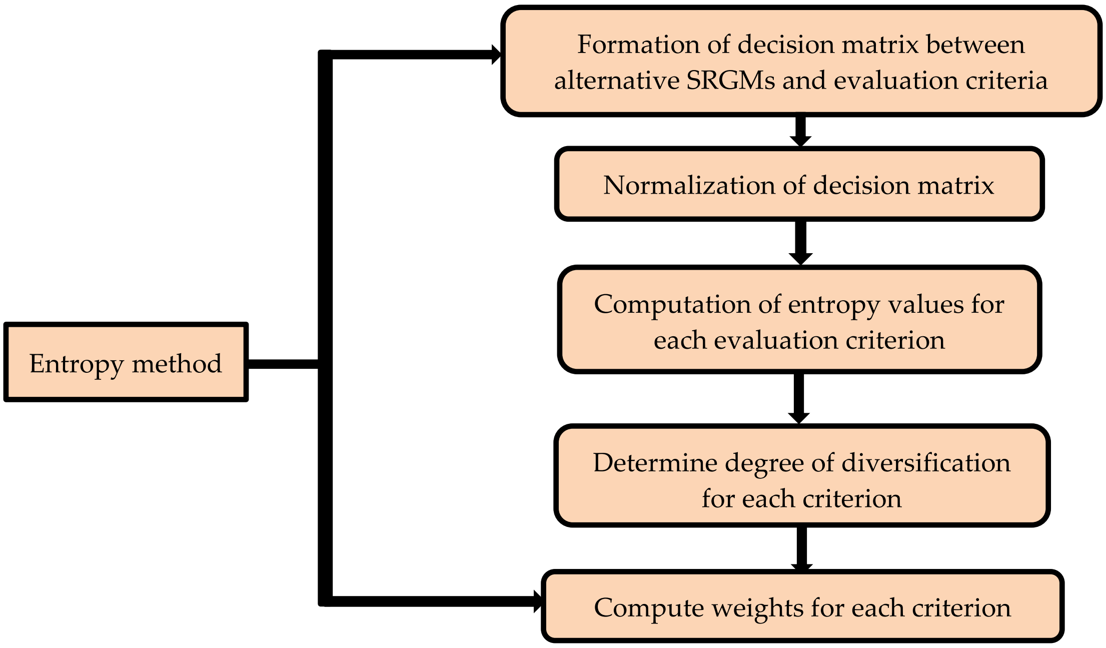

Figure 2: *Flowchart of Entropy method*

Let us assume SRGMs consisting of 1,2,3,...,p SRGMs and 1,2,3,...,q selection criterion for each alternative SRGM, where z_{pq} denotes the value of estimated parameter for p^{th} SRGM based on the q^{th} evaluation criterion and matrix A describes the collection of p^{th} SRGM and q^{th} evaluation criterion.

$$A = \begin{array}{c} \\ T_1 \\ T_2 \\ \cdots \\ \cdots \\ T_p \end{array} \begin{array}{c} \begin{array}{ccccc} D_1 & D_2 & \cdots & \cdots & D_q \end{array} \\ \begin{bmatrix} z_{11} & z_{12} & \cdots & \cdots & z_{1q} \\ z_{21} & z_{22} & \cdots & \cdots & z_{2q} \\ \cdots & \cdots & \cdots & \cdot\cdot & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ z_{p1} & z_{p2} & \cdots & \cdots & z_{pq} \end{bmatrix} \end{array}$$

where " $T_1, T_2,\ldots,T_p$ " signifies the alternative SRGM and " $D_1, D_3,\ldots,D_q$ " signifies the evaluation criteria.

The normalized decision matrix (S_{mn}) of evaluation criteria for each alternative SRGM using entropy method is represented by Equation (1) given below

$$S_{mn} = \frac{z_{mn}}{\sum_{m=1}^{p} z_{mn}}, m = 1,2,\ldots,p; n = 1,2,\ldots,q \tag{1}$$

The normalized decision matrix (S_{mn}) of evaluation criteria for each alternative SRGM given in Equation (1) is used to determine the value of entropy (e_n). The value of entropy of each evaluation criterion is calculated by using Equation (2) given below

$$e_n = \frac{-\sum_{m=1}^{p} S_{mn} \ln S_{mn}}{\ln p}; m = 1,2,\ldots,p; n = 1,2,\ldots,q \tag{2}$$

The value of entropy (e_n) described in Equation (2) is used to find the value of degree of diversification (d_n). The degree of diversification (d_n) of each evaluation criterion is obtained by using Equation (3) given below

$$d_n = 1 - e_n; n = 1,2,\ldots,q \tag{3}$$

The weights (w_n) of each evaluation criterion is measured by using value of degree of diversification (d_n) represented by Equation (3). The weights (w_n) of each evaluation criterion is calculated by using Equation (4) given below

$$w_n = \frac{d_n}{\sum_{n=1}^{q} d_n}; n = 1,2,\ldots,q \tag{4}$$

ELECTRE (Elimination et Choice Translating Reality) is a technique used to find solutions of problem for the situations involving in Multi Criteria Decision Making. The methodology of ELECTRE is based on the study of ranking relations. It analyses the relations of ranking among alternatives using indexes of concordance and discordance. The best alternative that a decision-maker chooses over the other alternative is measured using concordance and discordance indexes and eliminate the alternatives that are not suitable so the best solution or alternative can be obtained. The steps involved in the process of ELECTRE method is explained in the Figure 3 using flowchart and calculation for ELECTRE method can be obtained by equations given below involved in the process of ELECTRE method [41].

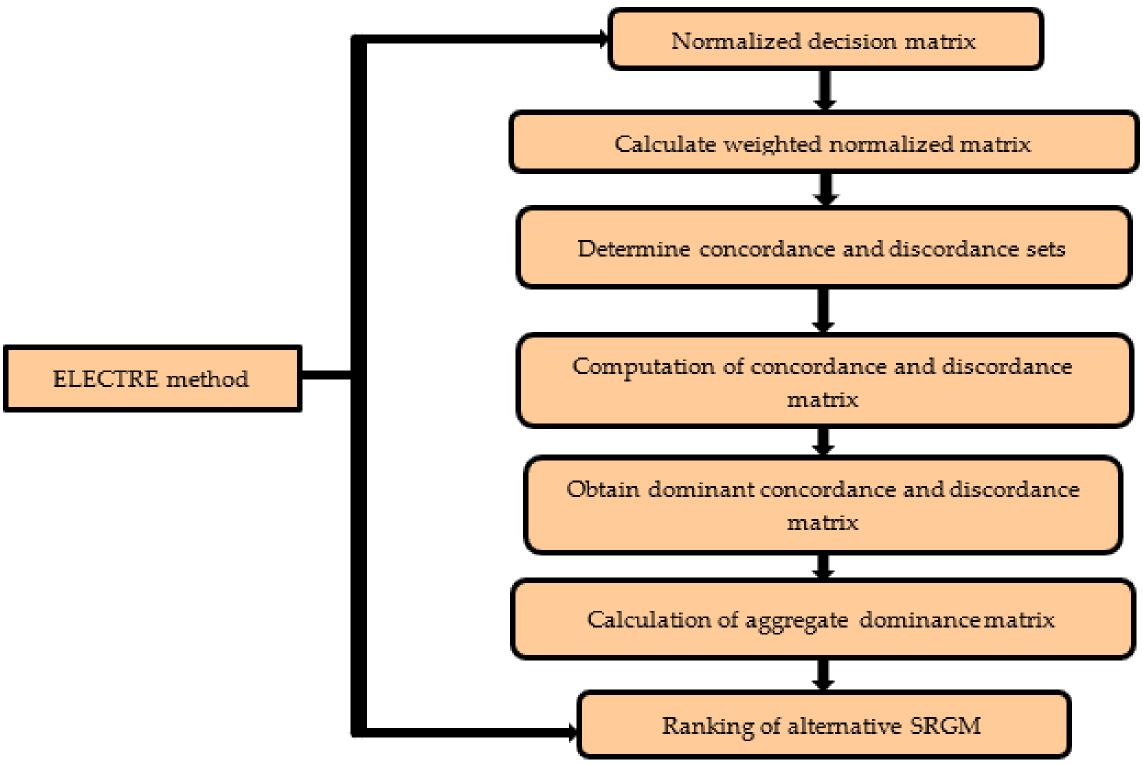

Figure 3: *Flowchart of ELECTRE method*

Equation (5) gives the normalized value (r_{mn}) of decision matrix based on ELECTRE method. Results of the normalized decision matrix of evaluation criteria for each alternative SRGM using Equation (5) will be a matrix R, which is described by equation (6).

$$r_{mn} = \frac{z_{mn}}{\sqrt{\sum_{m=1}^{p} z_{mn}^2}}; m = 1,2,\ldots,p; n = 1,2,\ldots,q \tag{5}$$

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1q} \\ r_{21} & r_{22} & \cdots & r_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ r_{p1} & r_{p2} & \cdots & r_{pq} \end{bmatrix} \tag{6}$$

The normalized decision matrix R in Equation (6) is used to find the weighted normalized matrix X of evaluation criteria for each alternative SRGM. The weights (w_n) of each criterion are predetermined by using Equation (4) and is used in Equation (7) to find the weighted normalized matrix X as shown below.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1q} \\ x_{21} & x_{22} & \cdots & x_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ x_{p1} & x_{p2} & \cdots & x_{pq} \end{bmatrix} = \begin{bmatrix} w_1 r_{11} & w_2 r_{12} & \cdots & w_q r_{1q} \\ w_2 r_{21} & w_2 r_{22} & \cdots & w_q r_{2q} \\ \cdots & \cdots & \cdots & \cdots \\ w_1 r_{p1} & w_2 r_{p2} & \cdots & w_q r_{pq} \end{bmatrix} \tag{7}$$

Concordance and discordance sets will be calculated by comparing the data of matrix X with every pair and their results are obtained as shown below. Concordance sets can be obtained by using Equation (8) and discordance sets can be obtained by using Equation (9).

$$\{c_{ab}\} = \{n \mid x_{an} \geq x_{bn}\} for n = 1,2,..q; a,b = 1,2,\ldots,p and a \neq b \tag{8}$$

$$\{d_{ab}\} = \{n \mid x_{an} < x_{bn}\} for n = 1,2,\ldots,q; a,b = 1,2,\ldots,p and a \neq b \tag{9}$$

Matrix $C = [C_{ab}]_{p \times p}$ for concordance consisting of alternative SRGM corresponding to each SRGM is calculated by adding weight values of the elements of concordance sets by using Equation (8) as shown in Equation (10). Discordance matrix $D = [D_{ab}]_{p \times p}$ consisting of alternative SRGM corresponding to each SRGM is calculated by using Equation (9) as shown below in Equation (11).

$$C_{ab} = \sum_{n \in c_{ab}} w_n; n = 1,2,\ldots,q \tag{10}$$

$$D_{ab} = \frac{\max[|x_{an} - x_{bn}|] n \in d_{ab}}{\max[|x_{an} - x_{bn}|] \forall n} \tag{11}$$

To calculate the dominant concordance matrix G consisting of alternative SRGM corresponding to each SRGM, there is need to find threshold value $\overline{C}$ by using Equation (12). Now, threshold value $\overline{C}$ is used to find dominant concordance matrix $G = [g_{ab}]_{p \times p}$ as shown below in Equation (13).

$$\overline{C} = \frac{\sum_{a=1}^{p}\sum_{b=1}^{p} c_{ab}}{p(p-1)} \tag{12}$$

$$g_{ab} = \begin{Bmatrix} 1, c_{ab} \geq \overline{C} \\ 0, c_{ab} < \overline{C} \end{Bmatrix} \tag{13}$$

To calculate the dominant discordance matrix H consisting of alternative SRGM corresponding to each SRGM, there is need to find threshold value $\overline{D}$ by using Equation (14). Now, threshold value $\overline{D}$ is used to find dominant discordance matrix $H = [h_{ab}]_{p\times p}$ as shown below in Equation (15).

$$\overline{D} = \frac{\sum_{a=1}^{p}\sum_{b=1}^{p} d_{ab}}{p(p-1)} \tag{14}$$

$$h_{ab} = \begin{Bmatrix} 1, h_{ab} \geq \overline{D} \\ 0, h_{ab} < \overline{D} \end{Bmatrix} \tag{15}$$

The value of aggregate dominance matrix $F = \left[f_{ab}\right]_{p\times p}$ consisting of alternative SRGM corresponding to each SRGM is obtained by the multiplication of matrices G and H as shown below in Equation (16).

$$f_{ab} = g_{ab} \times h_{ab} \tag{16}$$

The ranking of SRGMs is based on ascending or descending order of the sum of rows of matrix F.

IV. Selection of ranking criteria and SRGM

There are a variety of SRGMs available at present. Thus, it is required to analyze and verify the reliability of SRGM. This section describes the evaluation of ranking criteria and selection of alternative SRGM. The section I explains the evaluation of ranking criteria and section II explains the selection of alternative SRGM.

I. Evaluation of ranking criteria for SRGM

There is no model that satisfies all conditions among the SRGM. In contrast, different models anticipate very different outcomes. Sometimes a model gives good result for a given data set but the same model does not work well for other data set. Therefore, evaluation of model should be done based on specific data set and hence there is need to select criteria for evaluation. We have used the following criteria for evaluation.

- Mean Square Error (MSE) is defined as the distance between estimated and actual data and can be calculated by using the equation given below [42]

$$MSE = \frac{1}{n-N}\sum_{i=1}^{n}(y_i - \hat{m}(t_i))^2$$

n → number of observations

y_i →total number of faults detected upto to time t_i in terms of the testing data.

$m(t_i)$→estimated value of cumulative fault number up to t_i based on the mean value function, i=1,2,…,n.

N →number of parameters in the model

- R^2 is the second criteria for evaluation of SRGM and is defined as correlation index of the regression curve equation [43]. It is calculated by the formula given below

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{m}(t_i))^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2}$$

$$where \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

- Adjusted R^2 is used as a third criteria for evaluation of SRGM and is obtained by the equation shown below [43]

$$Adjusted R^2 = 1 - \frac{(1-R)(n-1)}{n-P-1}$$

where R→value of R^2

P→number of predictors in the fitted model

- The predictive-ratio risk (PRR) calculates the distance between an estimated data and actual data and is measured by the following equation [44]

$$PRR = \sum_{i=1}^{n}\left(\frac{\hat{m}(t_i) - y_i}{\hat{m}(t_i)}\right)^2$$

- The fifth criterion used for evaluation of SRGM is Predictive power (PP) [44]. It measures the distance between the model's estimation and the actual data. It can be measured by the equation given below.

$$PP = \sum_{i=1}^{n}\left(\frac{\hat{m}(t_i) - y_i}{y_i}\right)^2$$

- The Akaike information criterion (AIC) [45] defines a model's capacity to maximise the likelihood function that is directly proportional to the degrees of freedom and is calculated as

$$AIC = -2LogL + 2N$$

II. Selection of SRGM

In order to make experimental analysis of proposed technique, ten NHPP SRGM are selected to determine the accuracy of proposed Entropy-ELECTRE technique. The selected ten SRGM with their mean value function are described in Table 2.

V. Numerical Example

The objective of this example is to check the performance of the proposed integrated Entropy-

ELECTRE method so that the systematic ranking of alternative SRGM could be made with the association of relevant ranking criteria for evaluation. A software failure data set is used to check the numerical applicability of proposed method. In this study, we used a real time data set to calculate the parameters of model for ten SRGMs. The data set is taken from [46]. Table 3 represents the failure data set wherein 14 weeks 38 faults were observed. The parameter values are estimated by using SPSS 20.0 for ten NHPP based SRGM. The estimated parameter values of SRGMs for data set is presented in Table 4 and estimated values of ranking criteria for each alternative SRGM is shown in Table 5.

Table 2: *Selected SRGMs and their Mean value functions*

No	Model Name	Model Type	Mean Value Function
1	**Li-Pham model [1]**	S-Shaped	$m(t)=a(1+\alpha' t)-a\left(\frac{\beta}{\beta+pt^r}\right)^{\alpha}-a\alpha'\left(\frac{\beta}{\beta+pt^r}\right)^{\alpha}\left[t+\sum_{n=1}^{\infty}\left(\frac{p}{\beta+pt^r}\right)^{n}\frac{t^{nr+1}}{n(nr+1)}\frac{1}{beta(n,\alpha)}\right]$
2	**Chang et al. [3]**	S-Shaped	$m(t)=N\left\{1-\left(\frac{\beta}{\beta+(at)^b}\right)^{\alpha}\right\}$
3	**Fault removal model [4]**	S-Shaped	$m(t)=\frac{a}{p-\beta}\left\{1-\left(\frac{(1+\alpha)e^{-bt}}{1+\alpha e^{-bt}}\right)^{\frac{c}{b}(p-\beta)}\right\}$
4	**Goel and Okumoto model [5]**	Concave	$m(t)=a(1-e^{-bt})$
5	**Kapur et al. model [6]**	S-Shaped	$m(t)=\frac{A}{1-\alpha}\left[1-\left(\left(1+bt+\frac{b^2t^2}{2}\right)e^{-bt}\right)^{p(1-\alpha)}\right]$
6	**HD/GO model [47]**	Concave	$m(t)=\log[(e^{a}-c)/(e^{ae^{-bt}}-c)]$
7	**Roy et al. model [48]**	Concave	$m(t)=a\alpha(1-e^{-bt})-\frac{ab}{b-\beta}(e^{-\beta t}-e^{-bt})$
8	**Teng-Pham model [49]**	S-Shaped	$m(t)=\frac{a}{p-q}\left\{1-\left(\frac{\beta}{\beta+(p-q)\ln\left(\frac{c+e^{bt}}{c+1}\right)}\right)^{\alpha}\right\}$
9	**Yamada exponential model [50]**	Concave	$m(t)=a(1-e^{-\gamma\alpha(1-e^{-\beta t})})$
10	**Yamada Rayleigh model [50]**	S-Shaped	$m(t)=a(1-e^{-\gamma\alpha(1-e^{-\beta t^2/2})})$

Table 3: *Failure Data Set*

Week No.	Faults	Cumulative detected faults
1	2	2
2	11	13
3	2	15
4	4	19
5	3	22
6	1	23
7	1	24
8	2	26
9	4	30
10	0	30
11	4	34
12	1	35
13	3	38
14	0	38

Table 4: *Estimated Parameter values of SRGMs*

No	Model Name	Estimated Parameter
1	**Li-Pham model [1]**	$\alpha^{'} = 0.1904, \hat{\beta} = 9.451, \hat{a} = 10.67,$ $\hat{\alpha} = 59, \hat{p} = 0.04222, \hat{r} = 3$
2	**Chang et al model [3]**	$\hat{N} = 584, \hat{a} = 0.3318, \hat{\alpha} = 0.03498, \hat{b} = 1.11, \hat{\beta} = 0.9627$
3	**Fault removal model [4]**	$\hat{\beta} = 0.3616, \hat{a} = 25.24, \hat{\alpha} = 2.701e-06$ $\hat{b} = 7.944e-06, \hat{c} = 0.2162, \hat{p} = 0.9085$
4	**Goel and Okumoto model [5]**	$\hat{a} = 46.14, \hat{b} = 0.1182$
5	**Kapur et al. model [6]**	$\hat{A} = 24.25, \hat{\alpha} = 0.3188, \hat{b} = 1,$ $\hat{p} = 0.6313$
6	**HD/GO model [47]**	$\hat{a} = 46.14, \hat{b} = 0.1182, \hat{c} = 1$
7	**Roy et al. model [48]**	$\hat{a} = 473.9, \hat{\alpha} = 1.038, \hat{b} = 0.3889, \hat{\beta} = 0.003857$
8	**Teng-Pham model [49]**	$\hat{a} = 78.53, \hat{\alpha} = 0.06219, \hat{b} = 0.3843, \hat{\beta} = 0.291,$ $\hat{c} = 3.186e-08, \hat{p} = 0.853, \hat{q} = 0.6711$
9	**Yamada exponential model [50]**	$\hat{a} = 68.09, \hat{\beta} = 0.05826, \hat{\gamma} = 0.8222, \hat{\alpha} = 1.741$
10	**Yamada Rayleigh model [50]**	$\hat{a} = 38.09, \hat{\beta} = 0.03646, \hat{\gamma} = 1.656, \hat{\alpha} = 1.4$

Table 5: *Estimated values of ranking criteria for each alternative SRGM*

S.No	SRGM	MSE	R^2	Adjusted R^2	PRR	PP	AIC
1	**Li -Pham model [1]**	1.46	0.9916	0.9864	0.1071	0.1933	62.7092
2	**Chang et al. [3]**	4.0544	0.9738	0.9622	0.4961	2.9906	355.24
3	**Fault removal model [4]**	5.4512	0.9687	0.9491	0.528	2.5785	70.8309
4	**G-O model [5]**	3.6342	0.9687	0.9661	0.5283	2.5742	62.8309
5	**Kapur et al. model [6]**	14.52	0.8958	0.8768	2.4295	0.6419	76.6596
6	**HD/GO model [47]**	3.9645	0.9687	0.963	0.5283	2.5742	64.831
7	**Roy et al. model [48]**	3.287	0.9764	0.9693	0.5115	4.2796	66.7588
8	**Teng-Pham model [49]**	5.2514	0.9736	0.951	0.5125	3.5656	72.6973
9	**Yamada exponential model [50]**	4.116	0.9704	0.9616	0.5188	2.8112	66.7085
10	**Yamada Rayleigh model [50]**	16.1	0.8844	0.8498	1.9072	0.5447	82.0585

To determine the best SRGM among alternative SRGM, ranking of ten SRGM is done based on the proposed method as discussed in section III. The proposed methodology is applied on ten SRGM and the results are shown in following Tables. Initially we will use Entropy method to find weights of each criterion. Equation (2) is used to compute the values of normalized entropy (e_n) for each criterion. The result of the calculated entropy is shown in Table 6. The weights (w_n) of each criterion is calculated by using Equation (4) and the results obtained is shown in Table 7.

Table 6: *Normalized* (e_n) *entropy for each criterion*

Criteria(D_n, n=1 to 6)	Normalized entropy (e_n, n=1 to 6)
D_1	0.89474
D_2	0.99972
D_3	0.99956677
D_4	0.86562
D_5	0.91207
D_6	0.89199

Table 7: *Weights* (w_n) *of each criterion*

Criteria(D_n, n=1 to 6)	Weights (w_n, n=1 to 6)
w_1	0.24126
w_2	0.00065
w_3	0.000992966
w_4	0.308
w_5	0.20154
w_6	0.24756

Table 8: *Normalized decision (R) matrix of ranking criteria for each alternative SRGM using ELECTRE method*

S.No	SRGM	MSE	R^2	Adjusted R^2	PRR	PP	AIC
1	Chang et al. [3]	0.669650496	0.313079277	0.309992524	0.072804014	1.08043627	305.980859
2	Fault removal model [4]	1.210540268	0.309808538	0.301609114	0.082467858	0.80318718	12.164564
3	G-O model [5]	0.538037637	0.309808538	0.312510547	0.082561598	0.80051056	9.57188754
4	HD/GO model [47]	0.640282825	0.309808538	0.310508211	0.082561598	0.80051056	10.1909922
5	Kapur et al. model [6]	8.588715974	0.264933513	0.257407737	1.746025885	0.04977569	14.2489914
6	Li -Pham model [1]	0.086836182	0.324629377	0.325781668	0.003393093	0.00451384	9.53484301
7	Roy et al. model [48]	0.440143626	0.314753323	0.314584225	0.077394152	2.2125284	10.8060773
8	Teng-Pham model [49]	1.123427907	0.312950689	0.302817904	0.077697064	1.53584624	12.8140847
9	Yamada exponential model [50]	0.690153573	0.310896876	0.30960604	0.079619016	0.95469794	10.7897996
10	Yamada Rayleigh model [50]	10.55958281	0.258233303	0.241798704	1.075992755	0.03584241	16.326691

Table 9: *Weighted Normalized Matrix (X) of ranking criteria for each alternative SRGM*

S.No	w_n	**0.241256715**	**0.000648498**	**0.000992966**	**0.30799854**	**0.20154373**	**0.24755955**
	SRGM	MSE	R^2	Adjusted R^2	PRR	PP	AIC
1	Chang et al. [3]	0.161557679	0.000203031	0.000307812	0.02242353	0.21775516	75.7484832
2	Fault removal model [4]	0.292050968	0.00020091	0.000299488	0.02539998	0.16187734	3.01145396
3	G-O model [5]	0.129805193	0.00020091	0.000310312	0.025428852	0.16133789	2.36961215
4	HD/GO model [47]	0.154472531	0.00020091	0.000308324	0.025428852	0.16133789	2.52287741
5	Kapur et al. model [6]	2.072085401	0.000171809	0.000255597	0.537773424	0.01003198	3.52747387
6	Li -Pham model [1]	0.020949812	0.000210522	0.00032349	0.001045068	0.00090974	2.36044143
7	Roy et al. model [48]	0.106187605	0.000204117	0.000312371	0.023837286	0.44592123	2.67514761
8	Teng-Pham model [49]	0.271034526	0.000202948	0.000300688	0.023930582	0.30954018	3.172249
9	Yamada exponential model [50]	0.166504184	0.000201616	0.000307428	0.024522541	0.19241339	2.6711179
10	Yamada Rayleigh model [50]	2.547570259	0.000167464	0.000240098	0.331404198	0.00722381	4.04182825

Now ELECTRE method is used to rank alternate SRGM so that best SRGM can be determined. Normalized decision matrix is obtained by using Equation (5) and the obtained result is shown in Table 8. Equation (7) is used to compute the weighted normalized matrix. The results for weighted normalized matrix is shown in Table 9. Concordance and discordance matrices consisting of alternative SRGM corresponding to each SRGM are calculated by using Equation (10) and Equation (11). The results of the concordance and discordance matrices are shown in Table 10 and Table 11. Equation (13) and Equation (15) are used to find the values of dominant concordance and discordance matrices. The obtained results for dominant concordance and discordance matrices consisting alternative SRGM corresponding to each SRGM are shown in Table 12 and Table 13.

Table 10: *Concordance matrix (C) consisting alternative SRGM corresponding to each SRGM*

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
C_1	0.0000	0.7578	0.6910	0.6910	0.4507	0.9984	0.4888	0.2492	0.4507	0.4507
C_2	0.5493	0.0000	0.6910	0.6910	0.2032	0.9984	0.7968	0.5493	0.7968	0.2032
C_3	0.3090	0.3096	0.0000	0.5112	0.2032	0.9984	0.5493	0.3090	0.3090	0.2032
C_4	0.3090	0.3096	0.9990	0.0000	0.2032	0.9984	0.5493	0.3090	0.3090	0.2032
C_5	0.5493	0.5493	0.7968	0.7968	0.0000	0.9984	0.7968	0.7968	0.7968	0.5112
C_6	0.0016	0.0016	0.0016	0.0016	0.0016	0.0000	0.0016	0.0016	0.0016	0.0016
C_7	0.5112	0.2032	0.4507	0.4507	0.2032	0.9984	0.0000	0.2032	0.4507	0.2032
C_8	0.7508	0.4507	0.6910	0.6910	0.2032	0.9984	0.7968	0.0000	0.6910	0.2032
C_9	0.5493	0.2032	0.6910	0.6910	0.2032	0.9984	0.5493	0.3090	0.0000	0.2032
C_{10}	0.5493	0.7968	0.7968	0.7968	0.4888	0.9984	0.7968	0.7968	0.7968	0.0000

Table 11: *Discordance matrix (D) consisting alternative SRGM corresponding to each SRGM*

	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8	D_9	D_{10}
D_1	0.0000	0.0018	0.0000	0.0000	0.0265	0.0000	0.0031	0.0015	0.0001	0.0333
D_2	0.0018	0.0000	0.0000	0.0000	1.0000	0.0000	0.0031	1.0000	0.0897	1.0000
D_3	0.0000	1.0000	0.0000	1.0000	1.0000	0.0001	0.3055	1.0000	1.0000	1.0000
D_4	1.0000	1.0000	0.0000	0.0000	1.0000	0.0001	1.0000	1.0000	1.0000	1.0000
D_5	1.0000	0.2899	0.0779	0.0789	0.0000	0.0000	0.2217	0.1663	0.0957	1.0000
D_6	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	1.0000
D_7	1.0000	1.0000	0.0773	0.1697	1.0000	0.0000	0.0000	1.0000	0.2379	1.0000
D_8	1.0000	0.1307	0.0019	0.0023	1.0000	0.0000	0.2744	0.0000	0.0012	1.0000
D_9	1.0000	1.0000	0.0030	0.0061	1.0000	0.0001	1.0000	1.0000	0.0000	1.0000
D_{10}	1.0000	0.0686	0.0637	0.0644	0.4012	0.0000	0.1797	0.1328	0.0778	0.0000

Table 12: *Dominant concordance matrix (G) consisting alternative SRGM corresponding to each SRGM*

	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8	G_9	G_{10}
G_1	0	1	1	1	0	1	0	0	0	0
G_2	1	0	1	1	0	1	1	1	1	0
G_3	0	0	0	1	0	1	1	0	0	0
G_4	0	0	1	0	0	1	1	0	0	0
G_5	1	1	1	1	0	1	1	1	1	1
G_6	0	0	0	0	0	0	0	0	0	0
G_7	1	0	0	0	0	1	0	0	0	0
G_8	1	0	1	1	0	1	1	0	1	0
G_9	1	0	1	1	0	1	1	0	0	0
G_{10}	1	1	1	1	0	1	1	1	1	0

Table 13: *Dominant discordance matrix (H) consisting alternative SRGM corresponding to each SRGM*

	H_1	H_2	H_3	H_4	H_5	H_6	H_7	H_8	H_9	H_{10}
H_1	1	1	1	1	1	1	1	1	1	1
H_2	1	1	1	1	0	1	1	0	1	0
H_3	1	0	1	0	0	1	1	0	0	0
H_4	0	0	1	1	0	1	0	0	0	0
H_5	0	1	1	1	1	1	1	1	1	0
H_6	0	0	0	0	0	1	0	0	0	0
H_7	0	0	1	1	0	1	1	0	1	0
H_8	0	1	1	1	0	1	1	1	1	0
H_9	0	0	1	1	0	1	0	0	1	0
H_{10}	0	1	1	1	1	1	1	1	1	1

Table 14: *Aggregate dominance matrix (F) of alternative SRGM*

	SRGM											Sum	Rank
1	Chang et al. [3]	0	1	1	1	0	1	0	0	0	0	4	5
2	Fault removal model [4]	1	0	1	1	0	1	1	0	1	0	6	3
3	G-O model [5]	0	0	0	0	0	1	1	0	0	0	2	7
4	HD/GO model [47]	0	0	1	0	0	1	0	0	0	0	2	7
5	Kapur et al. model [6]	0	1	1	1	0	1	1	1	1	0	7	1
6	Li -Pham model [1]	0	0	0	0	0	0	0	0	0	0	0	10
7	Roy et al. model [48]	0	0	0	0	0	1	0	0	0	0	1	9
8	Teng-Pham model [49]	0	0	1	1	0	1	1	0	1	0	5	4
9	Yamada exponential model [50]	0	0	1	1	0	1	0	0	0	0	3	6
10	Yamada Rayleigh model [50]	0	1	1	1	0	1	1	1	1	0	7	1

The final step is to determine the value of aggregate dominance matrix based on the Equation (16). The obtained result is shown in Table 14. The SRGM that has maximum number of 1 in its row will be ranked one. Based on the result Kapur et al. [6] and Yamada Rayleigh model [50] is ranked one as it has maximum number of 1 in a row. The graphical representation of rank of models is shown in Figure 4.

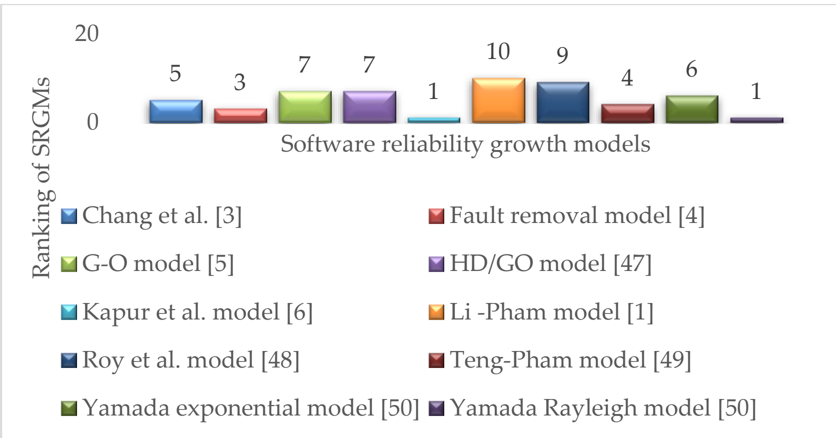

Figure 4: *Graphical representation of ranking of models*

VI. Conclusion

The objective of this study was the ranking of various software reliability growth models (SRGMs) to select suitable SRGM. In this paper, we have combined the two well-known MCDMs approach to rank and select the suitable SRGM. The renowned Shannon entropy approach is combined with Elimination et Choice Translating Reality (ELECTRE) approach to find the appropriate result. The Shannon entropy approach is applied to determine the weights of each criteria and ELECTRE is applied to rank the SRGMs so as to find suitable SRGM. The application of proposed approach shows that Kapur at al. [6] and Yamada Rayleigh model [50] are the most suitable software reliability growth models. The results obtained shows that the proposed method is suitable to rank SRGMs considering multiple criteria to determine suitable SRGM. The present study can be extended by comparing large number of criteria for comparison as it will give more accurate results. Further, proposed approach can be compared with the existing multi criteria decision making approaches.

References

[1] Li, Q. and Pham, H., (2017). NHPP software reliability model considering the uncertainty of operating environments with imperfect debugging and testing coverage. *Applied Mathematical Modelling*, 51:68-85.

[2] Belton, V and Stewart, T. Multiple criteria decision analysis: an integrated approach, *Springer Science & Business Media*, 2002.

[3] Chang, I.H., Pham, H., Lee, S.W. and Song, K.Y., (2014). A testing-coverage software reliability model with the uncertainty of operating environments. *International Journal of Systems Science: Operations & Logistics*, 1:220-227.

[4] Zhang, X., Teng, X. and Pham, H., (2003). Considering fault removal efficiency in software reliability assessment. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 33:114-120.

[5] Goel, A.L. and Okumoto, K., (1979). Time-dependent error-detection rate model for software reliability and other performance measures. *IEEE transactions on Reliability*, 28:206-211.

[6] Kapur, P.K., Pham, H., Anand, S. and Yadav, K., (2011). A unified approach for developing software reliability growth models in the presence of imperfect debugging and error generation. *IEEE Transactions on Reliability*, 60:331-340.

[7] Dhavakumar, P. and Gopalan, N.P., 2021. An efficient parameter optimization of software reliability growth model by using chaotic grey wolf optimization algorithm. *Journal of Ambient Intelligence and Humanized Computing*, 12:3177-3188.

[8] Gao, K., (2021). Simulated software testing process and its optimization considering heterogeneous debuggers and release time. *IEEE Access*, 9:38649-38659.

[9] Li, T., Si, X., Yang, Z., Pei, H. and Ma, Y., (2019). NHPP Testability Growth Model Considering Testability Growth Effort, Rectifying Delay, and Imperfect Correction. *IEEE Access*, 8:9072-9083.

[10] Kaur, J., Anand, A., Singh, O. and Kumar, V., (2021). Measuring Software Reliability under the Influence of an Infected Patch. *Yugoslav Journal of Operations Research*, 31:249-264.

[11] Erto, P., Giorgio, M. and Lepore, A., 2018. The generalized inflection S-shaped software reliability growth model. *IEEE Transactions on Reliability*, 69:228-244.

[12] Li, Q. and Pham, H., 2019. A generalized software reliability growth model with consideration of the uncertainty of operating environments. *IEEE Access,* 7:84253-84267.

[13] Zeephongsekul, P., Jayasinghe, C.L., Fiondella, L. and Nagaraju, V., (2016). Maximum-likelihood estimation of parameters of NHPP software reliability models using expectation conditional maximization algorithm. *IEEE Transactions on Reliability*, 65:1571-1583.

[14] Kumar, V., Khatri, S.K., Dua, H., Sharma, M. and Mathur, P., (2014). An assessment of

testing cost with effort-dependent fdp and fcp under learning effect: a genetic algorithm approach. *International Journal of Reliability, Quality and Safety Engineering,* 21:1450027.

[15] Vizarreta, P., Trivedi, K., Helvik, B., Heegaard, P., Blenk, A., Kellerer, W. and Machuca, C.M., (2018). Assessing the maturity of sdn controllers with software reliability growth models. *IEEE Transactions on Network and Service Management*, 15:1090-1104.

[16] Raghuvanshi, K.K., Agarwal, A., Jain, K. and Singh, V.B., (2021). A time-variant fault detection software reliability model. *SN Applied Sciences*, 3:1-10.

[17] Lee, D.H., Chang, I.H. and Pham, H., (2020). Software reliability model with dependent failures and SPRT. *Mathematics*, 8:1366.

[18] Zhu, M. and Pham, H., (2020). A generalized multiple environmental factors software reliability model with stochastic fault detection process. *Annals of Operations Research*, 1-22.

[19] Kumar, V. and Ram, M. eds. Predictive Analytics: Modeling and Optimization. CRC Press, 2021.

[20] Kumar, V. and Sahni, R., (2020). Dynamic testing resource allocation modeling for multi-release software using optimal control theory and genetic algorithm. *International Journal of Quality & Reliability Management.*

[21] Garg, H. and Ram, M. eds. Reliability management and engineering: challenges and future trends. CRC Press, 2020.

[22] Cai, Y., Qasem, S.N., Garg, H., Parvin, H., Pho, K.H. and Mansor, Z., (2021). Optimal Reordering Trace Files for Improving Software Testing Suitcase. *CMC-COMPUTERS MATERIALS & CONTINUA*, 67:1225-1239.

[23] Kumar, V. and Sahni, R., (2016). An effort allocation model considering different budgetary constraint on fault detection process and fault correction process. *Decision Science Letters*, 5:143-156.

[24] Kumar, V., Singh, V.B., Dhamija, A. and Srivastav, S., (2018). Cost-reliability-optimal release time of software with patching considered. *International Journal of Reliability, Quality and Safety Engineering*, 25:1850018.

[25] Kumar, V., Mathur, P., Sahni, R. and Anand, M., (2016). Two-dimensional multi-release software reliability modeling for fault detection and fault correction processes. *International Journal of Reliability, Quality and Safety Engineering*, 23:1640002.

[26] Amirghodsi, S., Naeini, A.B. and Makui, A., (2020). An integrated Delphi-DEMATEL-ELECTRE method on gray numbers to rank technology providers. *IEEE Transactions on Engineering Management,* 1-17.

[27] Anser, M.K., Mohsin, M., Abbas, Q. and Chaudhry, I.S., (2020). Assessing the integration of solar power projects: SWOT-based AHP–F-TOPSIS case study of Turkey. *Environmental Science and Pollution Research*, 27:31737-31749.

[28] Ho, W., Xu, X. and Dey, P.K., (2010). Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of operational research*, 202:16-24.

[29] Sevkli, M., (2010). An application of the fuzzy ELECTRE method for supplier selection. *International Journal of Production Research*, 48:3393-3405.

[30] Aruldoss, M., Lakshmi, T.M. and Venkatesan, V.P., (2013). A survey on multi criteria decision making methods and its applications. *American Journal of Information Systems*, 1:31-43.

[31] Lin, S.S., Shen, S.L., Zhou, A. and Xu, Y.S., (2020). Approach based on TOPSIS and Monte Carlo simulation methods to evaluate lake eutrophication levels. *Water Research*, 187:116437.

[32] Rani, P., Mishra, A.R., Mardani, A., Cavallaro, F., Alrasheedi, M. and Alrashidi, A., (2020). A novel approach to extended fuzzy TOPSIS based on new divergence measures for renewable energy sources selection. *Journal of Cleaner Production*, 257:120352.

[33] Torlak, N.G., Demir, A. and Budur, T., (2021). Using VIKOR with structural equation modeling for constructing benchmarks in the Internet industry. *Benchmarking: An International*

Journal. https://doi.org/10.1108/BIJ-09-2020-0465

[34] Mohammed, H.J., (2021). The optimal project selection in portfolio management using fuzzy multi-criteria decision-making methodology. *Journal of Sustainable Finance & Investment*, 1-17.

[35] Yazdani, M., Zarate, P., Zavadskas, E.K. and Turskis, Z., (2019). A Combined Compromise Solution (CoCoSo) method for multi-criteria decision-making problems. *Management Decision*, 57: 2501-2519.

[36] Deveci, M., Pamucar, D. and Gokasar, I., (2021). Fuzzy Power Heronian function based CoCoSo method for the advantage prioritization of autonomous vehicles in real-time traffic management. *Sustainable Cities and Society*, 69:102846.

[37] Kumar, V., Saxena, P. and Garg, H., (2021). Selection of optimal software reliability growth models using an integrated entropy–Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) approach. *Mathematical Methods in the Applied Sciences*. https://doi.org/10.1002/mma.7445

[38] Ikram, M., Zhang, Q. and Sroufe, R., (2020). Developing integrated management systems using an AHP-Fuzzy VIKOR approach. *Business Strategy and the Environment*, 29:2265-2283.

[39] Arabsheybani, A., Paydar, M.M. and Safaei, A.S., (2018). An integrated fuzzy MOORA method and FMEA technique for sustainable supplier selection considering quantity discounts and supplier's risk. *Journal of cleaner production*, 190:577-591.

[40] Kumari, R. and Mishra, A.R., (2020). Multi-criteria COPRAS method based on parametric measures for intuitionistic fuzzy sets: application of green supplier selection. *Iranian Journal of Science and Technology, Transactions of Electrical Engineering*, 44:1645-1662.

[41] Zongzhou Yang, Qi Xu, Xilan Qiu and Huijin Wang (2008). An applied study on the method for supplier selection with PCA and ELECTRE. *IEEE International Conference on Service Operations and Logistics, and Informatics,* 2151-2156, doi: 10.1109/SOLI.2008.4682890.

[42] Hwang, S. and Pham, H., (2008). Quasi-renewal time-delay fault-removal consideration in software reliability modeling. *IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans*, 39:200-209.

[43] Chiu, K.C., Huang, Y.S. and Lee, T.Z., (2008). A study of software reliability growth from the perspective of learning effects. *Reliability Engineering & System Safety*, 93:1410-1421.

[44] Pham, H., (2003). Software reliability and cost models: Perspectives, comparison, and practice. *European Journal of operational research*, 149:475-489.

[45] Pham, H. and Zhang, X., (2003). NHPP software reliability and cost models with testing coverage. *European Journal of Operational Research*, 145:443-454.

[46] J.D. Musa, A. Iannino, K. Okumoto. Software reliability: measurement, prediction, Application, McGraw-Hill, New York, 1987.

[47] Hossain, S.A. and Dahiya, R.C., (1993). Estimating the parameters of a non-homogeneous Poisson-process model for software reliability. *IEEE Transactions on Reliability*, 42:604-612.

[48] Roy, P., Mahapatra, G.S. and Dey, K.N., (2014). An NHPP software reliability growth model with imperfect debugging and error generation. *International Journal of Reliability, Quality and Safety Engineering*, 21:1450008.

[49] Teng, X. and Pham, H., (2006). A new methodology for predicting software reliability in the random field environments. *IEEE Transactions on Reliability*, 55:458-468.

[50] Yamada, S., Ohba, M. and Osaki, S., (1983). S-shaped reliability growth modeling for software error detection. *IEEE Transactions on reliability*, 32:475-484.