A Two State Time-Dependent Bulk Queue Model with Intermittently Available Server

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Abstract

This paper studies the time-dependent first-in-first-out (FIFO) queuing model with a single intermittently available server and variable-sized bulk arrivals and bulk departures. The time between arrivals, servicing time, and server availability time follows an exponential distribution. The difference-differential equations are used for developing system equations, and the Laplace transforms (L.T.), and inverse transforms are used for the solution. Explicit two-state transient recursive probabilities are obtained for an exact number ofbulk arrivals and bulk departures, and in the end, few particular casesare derived.

Keywords: Bulk, Intermittently available server, two-state queuing model, Markovian

I. Introduction

The development of (i) time-dependent queueing models and (ii) continuous-time Markov chains theory and applications is feasible due to Markovian queues. Many researchers contributed in this direction, like Asmussen [1], Gross and Harris [2], Kleinrock [3], Medhi [4], Chung [5], Freedman [6], Syski [7], etc. The time-dependent analysis ofqueuing model is comparatively tedious than the corresponding steady-state analysis. Due to this, there are limited explicit expressions available in the literature, even for simple models. Also, with the above, studying queueing systems with bulk arrivals and departures make it more tedious.

The classical literature deals with queuing models, where arrivals and departures are rendered in bulk. However, there are many real-life queuing situations in which arrivals and departures are rendered in bulk. Bulk queues have been used for modeling in various cases, such as communication systems, production/manufacturing systems, centralized parallel processing computer systems, restaurants, etc., apart from theoretical structures. Erlang's M/Ek/1 [8] solution was found to be the initiation of bulk arrival queues. Gaver [9] gave the explicitsolution to the batch arrival queue. Explicit consideration of bulk arrival queues seems to have begun several years after Bailey [10] on bulk service. Later, Premchand [11] generated time-dependent probabilities of a transient queueing system, where departures occur in variable-sized batches.

Chen et al. [12] developed a Markovian bulk queues model with general state-dependent control. Ayyapann et al. [13] analyzed a single server bulk queuing model with the repairable server, heterogeneous service, multiple vacations and standby server. Nithya [14] developed a simulation model to study queues in the production system.

The assumption of the server's instantaneously availability seems plausible only in the case when the server is automatic. But, there are real-life situations where server is not available instantaneously, i.e., interruptions in taking a new unit into service just after completion of in-hand service. This type of interruption, because the server is not instantaneously available, is unpredictable, and for an improved solution, aprobability distribution may be associated. Agarwal [15] developed a model where random interruptions in service occur after completing the in-hand service. Later, Sharda and Garg [16] worked on an intermittently available server. The server can rest or work on another important task whenever the queue length ≥ 0, but before an interruption in service, the server is bound to finish the in-hand service.

Sharda [17] presented the queuing model's solution by joining the concepts of variable-sized bulk queues and intermittently available servers. Pegden and Rosenshine [18] did the pioneering work in 1982 on the two-state queuing model concept. Also, Indra and Vijay [19, 20] obtained the transient probabilities of a model with intermittently available server and variable size bulk (i) departures, and (ii) arrivals.

Chenetal. [21] discussed a bulk queue model with quasi-stationary distributions and decay properties. Banerjee etal. [22] analyzed a bulk queueing system in which service capacity is variable and service is batch size-dependent. Niranjan etal. [23] studied the state-dependent bulk arrival retrial Bernoulli feedback queues with multiple vacations, and threshold. Shanthi etal. [24] presented a computational approach for a working vacation bulk service transient queuing model.
We study the time-dependent, first-come-first-served, intermittently available single server bulk queuing model with variable-sized batch arrivals and departures in the present work.The queuing model can be applied to the data switching systems. In data switching systems, processors have Poisson streams of primary processes requiring attention. An example of such a task would be the routing of packets to an appropriate outgoing line. The processor may also be required to execute small maintenance routines whenever it is necessary. Here, the processor's primary aim is the routing of packets. Still, when maintenance is needed, the processor, after completing the packet's service, goes for maintenance by keeping the queue packets. The maintenance time is corresponding to the server's intermittently availability time.

II. The Model

The following are the assumptions of the model:

- Arrivals follows Poisson distribution and occur in variable size batches with parameter 'λ' and 'a_i' is the probability of i arrivals, where $\sum_{i=1}^{k} a_i = 1$ k represents the maximum batch capacity.
- Service times follow an exponential distribution with the parameter 'μ'. Before each service, the batch size isdetermined, either equal to the total units waiting or equal to the service channel capacity, whichever is less. 'd_j' represents the probability that server can serve j units, where $\sum_{j}^{k} d_j = 1$ k represents the maximum capacity.
- Server's availability time follows exponential distribution and 'v' is the parameter.
- Queue discipline is FIFO.

- Stochastic processes involved are: (i) units/ customer's arrivals, (ii) service times, and (iii) server availability time is statistically independent.

III. Model Definitions and Notations

'$P_{i,j,F}(t)$' represents the probability of 'i' occurrences and 'j' services by any time t and server is available, i.e., no unit is in waiting; $i \geq j \geq 0$

'$P_{i,j,B}(t)$' represents the probability of 'i' occurrences and 'j' services by any time t and server is busy, i.e., units are in waiting; $i > j > 0$

'$P_{i,j}(t)$' represents the probability of 'i' occurrences and 'j' services by any given time t; $i,j \geq 0$.

The L.T. of $P_{i,j}(t)$ is:

$$\overline{P}_{i,j}(s) = \int_0^\infty e^{-st} P_{i,j}(t)\, dt \tag{1}$$

$\sum_{\sum_{t=1}^{u} r_t = n} 1$, The sum is for all permutations of total n objects taken u (=1, 2…n) at a time, such that $\sum_{t=1}^{u} r_t = n$, $r_t > 0$.

$$\delta_{i,j} = \begin{cases} 1, & i=j \\ 0, & i \neq j \end{cases}, \qquad M_{\alpha,\beta} = \left(\sum_{\sum_{t=1}^{u} r_t = \alpha} 1 \prod_{t=1}^{u} a_{r_t} \left(\frac{\lambda}{s + \lambda + \beta} \right)^u \right)$$

IV. The solution to the Problem

Initially,

$$\left. \begin{aligned} P_{0,0,B}(0) &= 0 \\ P_{0,0,F}(0) &= 1 \end{aligned} \right\} \tag{2}$$

The system governing difference-differential equations are:

$$\frac{d}{dt} P_{i,i,F}(t) = -\lambda P_{i,i,F}(t) + \mu \sum_{\ell=1}^{i} \sum_{m=\ell}^{k} d_m P_{i,i-\ell,B}(t);\ k > i \geq 0 \tag{3}$$

$$\frac{d}{dt} P_{i,i,F}(t) = -\lambda P_{i,i,F}(t) + \mu \sum_{\ell=1}^{k} \sum_{m=\ell}^{k} d_m P_{i,i-\ell,B}(t);\ i \geq k \tag{4}$$

$$\frac{d}{dt} P_{i,j,F}(t) = -(\lambda + v) P_{i,j,F}(t) + \lambda \sum_{\ell=1}^{i-j} a_\ell P_{i-\ell,j,F}(t) + \mu \sum_{\ell=1}^{j} d_\ell P_{i,j-\ell,B}(t)(1 - \delta_{j,0});\ \ i > j \geq 0 \tag{5}$$

$$\frac{d}{dt} P_{i,j,B}(t) = -(\lambda + \mu) P_{i,j,B}(t) + \lambda \sum_{\ell=1}^{i-j-1} a_\ell P_{i-\ell,j,B}(t)(1 - \delta_{i,j+1}) + v P_{i,j,F}(t);\ i > j \geq 0 \tag{6}$$

Taking Laplace transforms of (3) to (6) along with (2) and solving recursively,

$$\bar{P}_{i,0,F}(s)=\left(\frac{1}{s+\lambda}\right)\left(\sum_{\sum_{t=1}^{u} r_t=i}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{i,0})} ;\ i\ge 0 \tag{7}$$

$$\bar{P}_{i,0,B}(s)=\left(\frac{v}{(s+\lambda)(s+\lambda+\mu)}\right)\sum_{\ell=1}^{i}\left\{\left(\sum_{\sum_{t=1}^{u} r_t=i-\ell}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+\mu}\right)^{u}\right)^{(1-\delta_{i,\ell})}\left(\sum_{\sum_{t=1}^{u} r_t=\ell}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)\right\};\ i>0 \tag{8}$$

$$\bar{P}_{i,j,F}(s)=\sum_{b_j=j}^{i}\left(\sum_{\sum_{t=1}^{u} r_t=i-b_j}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{i,b_j})}\sum_{g=1}^{j}\left(\frac{\mu(d_g)^{(1-\delta_{g,1})}}{s+\lambda}\right)^{\delta_{b_j,j}}\left(\frac{\mu d_g}{s+\lambda+v}\right)^{(1-\delta_{b_j,j})}\bar{P}_{b_j,j-g,B}(s);\ i\ge j\ge 1 \tag{9}$$

$$\bar{P}_{i,j,B}(s)=\left(\frac{v}{s+\lambda+\mu}\right)\sum_{\ell_j=j+1}^{i}\left(\sum_{\sum_{t=1}^{u} r_t=i-\ell_j}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+\mu}\right)^{u}\right)^{(1-\delta_{i,\ell_j})}\sum_{b_j=j}^{\ell_j}\left(\sum_{\sum_{t=1}^{u} r_t=\ell_j-b_j}{}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{\ell_j,b_j})}$$
$$\sum_{g=1}^{j}\left(\frac{\mu(d_g)^{(1-\delta_{g,1})}}{s+\lambda}\right)^{\delta_{b_j,j}}\left(\frac{\mu d_g}{s+\lambda+v}\right)^{(1-\delta_{b_j,j})}\bar{P}_{b_j,j-g,B}(s);\ i>j>0 \tag{10}$$

The inverse transforms of equations (7) to (10) are

$$P_{0,0,F}(t)=e^{-\lambda t} \tag{11}$$

$$P_{i,0,F}(t)=\left\{\sum_{\sum_{t=1}^{u} r_t=i}{}^{1}\prod_{t=1}^{u} a_{r_t}\left\{\left(\lambda^{u}\right)\left\{\frac{e^{-\lambda t}}{v^{u}}+\frac{e^{-(\lambda+v)t}}{(-v)}\left(\sum_{h=1}^{u}(-1)^{h+1}\left(\frac{t^{u-h}}{(u-h)!(-v)^{h-1}}\right)\right)\right\}\right\}\right\} ;i>0 \tag{12}$$

$$P_{i,0,B}(t)=\sum_{\ell=1}^{i}\left[\left[\left\{\sum_{\sum_{t=1}^{u} r_t=i-\ell}{}^{1}\prod_{t=1}^{u} a_{r_t}\left\{\left(\lambda^{u}\right)\left(\frac{e^{-(\lambda+\mu)t}t^{u-1}}{(u-1)!}\right)\right\}\right\}^{(1-\delta_{i,\ell})}\right]*\left[\left\{\sum_{\sum_{t=1}^{u} r_t=\ell}{}^{1}\prod_{t=1}^{u} a_{r_t}\left\{\left(\lambda^{u}v\right)\left\{\frac{e^{-\lambda t}}{\mu v^{u}}+\frac{e^{-(\lambda+\mu)t}}{(-\mu)(v-\mu)^{u}}\right.\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\left.+\frac{e^{-(\lambda+v)t}}{(-v)(\mu-v)}\left(\sum_{p=1}^{u}\sum_{\ell=0}^{p-1}(-1)^{p+1}\left(\frac{t^{u-p}}{(u-p)!(\mu-v)^{\ell}(-v)^{p-1-\ell}}\right)\right)\right\}\right\}\right\}\right]\right];i>0 \tag{13}$$

$$P_{i,i,F}(t)=\sum_{\ell=1}^{i}\left[\mu\left(e^{-\lambda t}\right)\left(d_{\ell}\right)^{(1-\delta_{\ell,1})}\right]*P_{i,i-\ell,B}(t)\ ;\ i\ge 1 \tag{14}$$

$$P_{i,j,F}(t)=\sum_{b_j=j}^{i}\sum_{g=1}^{j}\left[\left[\left[\left\{\sum_{\sum_{t=1}^{u}r_t=i-b_j}^{1}\prod_{t=1}^{u}a_{r_t}\left\{\left(\lambda^u\mu(d_g)^{(1-\delta_{g,1})}\right)\left\{\frac{e^{-\lambda t}}{v^u}+\frac{e^{-(\lambda+v)t}}{(-v)}\left(\sum_{h=1}^{u}(-1)^{h+1}\left(\frac{t^{u-h}}{(u-h)!(-\mu)^{h-1}}\right)\right)\right\}\right\}\right\}^{\delta_{b_j,j}}\right.\right.\right.$$

$$\left.\left.\left\{\sum_{\sum_{t=1}^{u}r_t=i-b_j}^{1}\prod_{t=1}^{u}a_{r_t}\left\{\left(\lambda^u\mu d_g\right)\left(\frac{e^{-(\lambda+v)t}t^u}{u!}\right)\right\}\right\}^{(1-\delta_{b_j,j})}\right]^{(1-\delta_{i,b_j})}\left[\left(\mu d_g\right)e^{-(\lambda+v)t}\right]^{\delta_{i,b_j}}\right]*P_{b_j,j-g,B}(t)$$

$$;i>j>0 \qquad (15)$$

$$P_{i,j,B}(t)=\sum_{\ell_j=j+1}^{i}\left[\left\{\sum_{\sum_{t=1}^{u}r_t=i-\ell_j}^{1}\prod_{t=1}^{u}a_{r_t}\left\{\left(\lambda^u v\right)\left(\frac{e^{-(\lambda+\mu)t}t^u}{u!}\right)\right\}\right\}^{(1-\delta_{i,\ell_j})}\left\{v\,e^{-(\lambda+\mu)t}\right\}^{\delta_{i,\ell_j}}\right]*\left[\sum_{b_j=j}^{\ell_j}\sum_{g=1}^{j}\right.$$

$$\left[\left[\left[\left\{\sum_{\sum_{t=1}^{u}r_t=\ell_j-b_j}^{1}\prod_{t=1}^{u}a_{r_t}\left\{\left(\lambda^u\mu(d_g)^{(1-\delta_{g,1})}\right)\left\{\frac{e^{-\lambda t}}{v^u}+\frac{e^{-(\lambda+v)t}}{(-v)}\left(\sum_{h=1}^{u}(-1)^{h+1}\left(\frac{t^{u-h}}{(u-h)!\,(-v)^{h-1}}\right)\right)\right\}\right\}\right\}^{\delta_{b_j,j}}\right.\right.\right.$$

$$\left.\left.\left.\left\{\sum_{\sum_{t=1}^{u}r_t=\ell_j-b_j}^{1}\prod_{t=1}^{u}a_{r_t}\left\{\left(\lambda^u\mu d_g\right)\left(\frac{e^{-(\lambda+v)t}t^u}{u!}\right)\right\}\right\}^{(1-\delta_{b_j,j})}\right]^{(1-\delta_{\ell_j,b_j})}\left[\left(\mu d_g\right)e^{-(\lambda+v)t}\right]^{\delta_{\ell_j,b_j}}\right]^{(1-\delta_{\ell_j,j+1})}$$

$$\left.\left[\mu\,e^{-\lambda t}\right]^{\delta_{\ell_j,j+1}}\right]*P_{b_j,j-g,B}(t)\Bigg];\ i>j>0 \qquad (16)$$

From equations (7) - (10), following is observed

$\sum_{i=0}^{\infty}\sum_{j=0}^{i}\overline{P}_{i,j}(s)=\frac{1}{s}$, therefore $\sum_{i=0}^{\infty}\sum_{j=0}^{i}P_{i,j}(t)=1$ isverified.

The L.T. $\overline{P}_{i,.}(s)$ of $P_{i,.}(t)$, i.e., occurrences of i units:

$$\overline{P}_{i,.}(s)=\sum_{j=0}^{i}\overline{P}_{i,j}(s)=\left(\frac{1}{s+\lambda}\right)\left\{M_{i,0}\right\}^{(1-\delta_{i,0})},\quad i\geq 0$$

1. Replacing $a_k=1$, for k=1; $a_k=0$, for k>1, we get

$$\overline{P}_{i,.}(s)=\sum_{j=0}^{i}P_{i,j}(s)=\left\{\frac{\lambda^i}{(s+\lambda)^{i+1}}\right\},\quad i\geq 0 \qquad (17)$$

and hence

$$P_{i,.}(t)=\frac{(\lambda t)^i}{i!}e^{-\lambda t},\quad i\geq 0 \qquad (18)$$

Equ (18) shows arrivals are distributed according to Poission distribution.

2. The L.T. of average of occurrences by any time 't' is

$$\sum_{i=0}^{\infty} i\,\overline{P}_{i,.}(s) = \left\{\frac{\lambda}{s^2}\right\} \tag{19}$$

and the inverse is

$$\sum_{i=0}^{\infty} iP_{i,.}(t) = \lambda t \tag{20}$$

Particular Cases:

I. Case 1

I. Case 1-(a): When the units are served singly, then by replacing a_k as 1 for k=1; a_k as 0 for k>1in the equations (7) to (10), we have

$$\overline{P}_{i,0,F}(s) = \left(\frac{\lambda^i}{(s+\lambda)(s+\lambda+v)^i}\right);\ i \ge 0 \tag{21}$$

$$\overline{P}_{i,0,B}(s) = \left(\frac{\lambda^i v}{(s+\lambda)\ (s+\lambda+\mu)\ (s+\lambda+v)}\right)\sum_{\ell=0}^{i-1}\left(\frac{1}{(s+\lambda+\mu)^{i-1-\ell}\ (s+\lambda+v)^{\ell}}\right); i>0 \tag{22}$$

$$\overline{P}_{i,j,F}(s) = \sum_{b_j=j}^{i}\left(\frac{\lambda}{s+\lambda+v}\right)^{i-b_j}\sum_{g=1}^{j}\left\{\frac{\mu\left((d_g)^{(1-\delta_{g,1})}\right)^{\delta_{b_j,j}}(d_g)^{(1-\delta_{b_j,j})}}{(s+\lambda)^{\delta_{b_j,j}}\ (s+\lambda+v)^{(1-\delta_{b_j,j})}}\right\}\overline{P}_{b_j,j-g,B}(s);\ i \ge j>0 \tag{23}$$

$$\overline{P}_{i,j,B}(s) = \sum_{\ell_j=j+1}^{i}\sum_{b_j=j}^{\ell_j}\left(\frac{\lambda^{i-b_j}v}{(s+\lambda+\mu)^{i+1-\ell_j}(s+\lambda+v)^{\ell_j-b_j}}\right)\sum_{g=1}^{j}\left\{\frac{\mu\left((d_g)^{(1-\delta_{g,1})}\right)^{\delta_{b_j,j}}(d_g)^{(1-\delta_{b_j,j})}}{(s+\lambda)^{\delta_{b_j,j}}(s+\lambda+v)^{(1-\delta_{b_j,j})}}\right\}.$$
$$\overline{P}_{b_j,j-g,B}(s);\ i>j>0 \tag{24}$$

Equations (21) to (24) coincide with equations (20) to (24) of Indra and Vijay [19]

II. Case 1-(b): Along with case 1-(a), when departures are also singly then by substituting d_k as 1 for k=1; d_k as 0 for k>1 in equations (21) to (24), we have

$$\overline{P}_{i,0,F}(s) = \left(\frac{\lambda^i}{(s+\lambda)(s+\lambda+v)^i}\right);\ i \ge 0 \tag{25}$$

$$\overline{P}_{i,0,B}(s) = \left(\frac{\lambda^i v}{(s+\lambda)\ (s+\lambda+\mu)\ (s+\lambda+v)}\right)\sum_{\ell=0}^{i-1}\left(\frac{1}{(s+\lambda+\mu)^{i-1-\ell}\ (s+\lambda+v)^{\ell}}\right);\ i>0 \tag{26}$$

$$\overline{P}_{i,j,F}(s) = \sum_{b_j=j}^{i}\left[\left(\frac{\lambda}{s+\lambda+v}\right)^{i-b_j}\left(\frac{\mu}{(s+\lambda)^{\delta_{b_j,j}}\ (s+\lambda+v)^{(1-\delta_{b_j,j})}}\right)\right]\overline{P}_{b_j,j-1,B}(s);\ i \ge j \ge 1 \tag{27}$$

$$\overline{P}_{i,j,B}(s) = \sum_{\ell_j=j+1}^{i}\sum_{b_j=j}^{\ell_j}\left\{\left(\frac{\lambda^{i-b_j}v}{(s+\lambda+\mu)^{i+1-\ell_j}(s+\lambda+v)^{\ell_j-b_j}}\right)\left(\frac{\mu}{(s+\lambda)^{\delta_{b_j,j}}(s+\lambda+v)^{(1-\delta_{b_j,j})}}\right)\right\}\overline{P}_{b_j,j-1,B}(s)$$
$$;i>j>0 \qquad 28)$$

Equations (25) to (28) coincide with equations (6) to (10) of Sharda and Garg [16].

III. Case 1-(c): Along with case 1-(b), when the server is instantaneously available, then by

letting
$v \to \infty$ in (25) to (28), we have

$$\bar{P}_{i,0}(s) = \left[\bar{P}_{i,0,F}(s) + \bar{P}_{i,0,B}(s)\right] = \left(\frac{\lambda^{i}}{(s+\lambda)(s+\lambda+\mu)^{i}}\right);\ i \ge 0 \tag{29}$$

$$\bar{P}_{i,i}(s) = \bar{P}_{i,i,F}(s) = \left(\frac{\mu}{s+\lambda}\right)\bar{P}_{i,i-1}(s);\ i \ge 0 \tag{30}$$

$$\bar{P}_{i,j}(s) = \left[\bar{P}_{i,j,F}(s) + \bar{P}_{i,j,B}(s)\right] = \sum_{\ell_j=j}^{i}\left[\left(\frac{\lambda}{s+\lambda+\mu}\right)^{i-\ell_j}\left(\frac{\mu}{(s+\lambda)^{\delta_{\ell_j,j}}(s+\lambda+\mu)^{(1-\delta_{\ell_j,j})}}\right)\right]\bar{P}_{\ell_j,j-1}(s)\ ;\ i > j > 0 \tag{31}$$

Equations (29) to (31) reduce to equation (5) of Pegden and Rosenshine [18].

II. Case 2

When units depart singly, then by substituting d_k as 1 for k=1; d_k as 0 for k>1 in equations (7) to (10), we have

$$\bar{P}_{i,0,F}(s) = \left(\frac{1}{s+\lambda}\right)\left(\sum_{\sum_{t=1}^{u} r_t = i}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{i,0})};\ i \ge 0 \tag{32}$$

$$\bar{P}_{i,0,B}(s) = \left(\frac{v}{(s+\lambda)(s+\lambda+\mu)}\right)\sum_{\ell=1}^{i}\left\{\left(\sum_{\sum_{t=1}^{u} r_t = i-\ell}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+\mu}\right)^{u}\right)^{(1-\delta_{i,\ell})}\left(\sum_{\sum_{t=1}^{u} r_t = \ell}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)\right\};\ i > 0 \tag{33}$$

$$\bar{P}_{i,j,F}(s) = \sum_{b_j=j}^{i}\left[\left(\sum_{\sum_{t=1}^{u} r_t = i-b_j}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{i-b_j})}\left(\frac{\mu}{(s+\lambda)^{\delta_{b_j,j}}(s+\lambda+v)^{(1-\delta_{b_j,j})}}\right)\right]\bar{P}_{b_j,j-1,B};\ i \ge j > 0 \tag{34}$$

$$\bar{P}_{i,j,B}(s) = \left(\frac{v}{s+\lambda+\mu}\right)\sum_{\ell_j=j+1}^{i}\left(\sum_{\sum_{t=1}^{u} r_t = i-\ell_j}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+\mu}\right)^{u}\right)^{(1-\delta_{i-\ell_j})}\sum_{b_j=j}^{\ell_j}\left[\left(\sum_{\sum_{t=1}^{u} r_t = \ell_j-b_j}^{1}\prod_{t=1}^{u} a_{r_t}\left(\frac{\lambda}{s+\lambda+v}\right)^{u}\right)^{(1-\delta_{\ell_j-b_j})}\left(\frac{\mu}{(s+\lambda)^{\delta_{b_j,j}}(s+\lambda+v)^{(1-\delta_{b_j,j})}}\right)\right]\bar{P}_{b_j,j-1,B}\ ;\ i > j > 0 \tag{35}$$

Equations (32) to (35) coincide with equations (6) to (9) of Indra and Vijay [20].

V. Results and Discussions

In this paper,transient probabilities are obtained using difference-differential equations. Explicit recursive probabilities of an exact number of bulk arrivals and departures are also obtained for this two-state bulk queuing model with an intermittently available server using Laplace transformsand inverse transforms. Particular cases are also derived, which shows the similarity with the already existing theoretical models. This theoretical model can be used at different businesses to utilize server time effectively.

VI. Future Scope

Future research couldbe extended by finding all the queueing system performance measures with numerical investigations using MATLAB in this direction. Also,the concept of multiple vacations with intermittently available servers can also be introduced.

VII. Conclusions

This theoretical paper has shown that the Poisson arrivals and exponential services are in variable-sized batches, with server availability as intermittent. Service times, inter-arrival times, intermittently available times are exponentially distributed. Difference-differential equations govern developed queuing systems. Laplace transforms and inverse transforms are used to get a feasible solution. Finally, recursive expressions of transient probabilities of an exact number of bulk arrivals and departures are obtained. To verify the system correctness, all the probabilitiesadded, which shows the sum equals one. The probabilities of thesemodels can be used in data switching systems, where processors have Poisson streams of primary processes requiring attention. An example of such a task would be the routing of packets to an appropriate outgoing line. The processor may also be required to execute small maintenance routines whenever it is necessary. Here, the processor's primary aim is the routing of packets. Still, when maintenance is needed, the processor, after completing the packet's service, goes for maintenance by keeping the queue packets. The maintenance time is corresponding to the server's intermittently availability time.

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