Flexible Production Inventory Model with Time Dependent Holding Cost and Reliability Process

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Abstract

The deterministic inventory model for decayed items concerns with reliability process and flexible production rate. The rate of decayed items is assumed to be a two parameters Weibull distribution. In this study, complete backlogging shortages are acceptable and holding cost is taken as linear. Further the influence of inflation has been developed and the model is established with numerically and sensitivity analysis which has been given the impact of parameters and also interpreted through graphically. The system is designed for the producer to validate the model to optimum level.

Keywords: Deterministic demand, Time dependent holding cost, Reliability process, Inflation, Shortages, Volume flexibility

I. Introduction

In manufacturing systems, the constant production rate has been explicitly assumed by numerous researchers. Such an assumption holds only if the demand of a product is known with certainty. However, with constant changes in the markets and recent trends, the demand of a product fluctuates in the long run, which may lead to shortages or high storing costs, depending upon the rise or fall. Thus, volume flexibility is an adequate tool to deal with this situation precisely.

In practicality, objects are not always absolute and their perfection depends on the reliability of manufacturing process, those are employed by the manufacturer. For making the system reliable, manufacturer has to follow several steps for the machinery as well as for the objects also. This level of optimization is only possible when the machinery of this system utilizes their energy within the best extent. For this labour force should be efficient, machinery should be extensive, no imperfect items should be there and if there is any imperfectness amongst items then, those items should be repudiated.

Schweitzer and Seidmann, (1991) were the first one to developed the model with flexibility in the machine production rate and the optimization of processing rate. Khouza (1997) extended the manufacturing model with the inconstant rate of production. Sana and Chaudhuri (2003) considered the EPQ model with flexibility for fixed deteriorating items with stock dependent demand. Bag et al. (2008) presented a mathematical structure with uncertain demand and also used the term of flexibility and reliability. In (2010) a vendor and buyer model for decaying items and volume flexible with stochastic lead time developed by Singhal et al. Singhal and Singh (2012) given the probabilistic inventory system for selling price demand with the use of volume flexible approach. Al Masud et al. (2014) worked on the reliability parameters of a production inventory model to optimize. Singhal and Singh (2014) refined a remanufacturing model with exponential demand, volume agility and probabilistic decaying items. Singhal et al. (2016) suggested a flexible production structure with time dependent demand using stochastic rate of backlogging. An analyze of the factor of reliability and time-based demand rate on inventory management developed by Mahapatra et al. (2017). Vishnoi et al. (2018) deals a vendor buyer model with expiration date, variable holding cost in an inflationary condition. Sarkar et al. (2019) proposed a mathematical multi-item model with the system reliability where the holding cost is time dependent and used conception of sustainability. Recently, manufacturing model for a deteriorating item formulated by Shaikh et al. in (2020) where the demand is price dependent and also allowing for inflation and reliability.

Numerous researchers considered that while production take place, reliability of the product can be increased by a fixed set-up cost and if producer wants to increase the reliability of the product so he has to raise the cost and set up cost can be balanced by improvement of flexibility. Many cases can be also found where holding cost vary as it depends on time and the production system. So, considering mentioned parameters we developed a mathematical deterministic model for deteriorating items where the holding cost is time dependent and inflation is also being considered.

Two parameters Weibull distribution is considered in a situation objects deteriorate over time at varying rate. In this study demand rate considered as certain and shortages are permissible with the assumption that the defective items will trade with price cut and fresh units are escalated over the unit production cost. As it's a profit, so it will be maximized. Sensitivity of important parameters is interpreted through examples.

II. Assumptions and Notations

These assumptions and notations have been assumed:

Assumptions

1. The rate of production is assumed to be variable.
2. Weibull distribution deterioration is taken in this study.
3. Holding cost is a taken as linear.
4. Unreliable items are sold at a minimum cost.
5. The permissible of shortages with complete backlogging.
6. The effect of inflation is used.
7. Demand is less than the total cost of fresh units i.e. RK>D.

Notations

I(t) Level of Inventory

K Production rate per unit time

$C_h + \gamma t$ Time dependent carrying cost per unit time, $\gamma > 0$

C_0 Set up cost/cycle

C_d Deterioration cost/unit time

C_S Shortage cost/unit time

S_1 Fresh units selling price, $S_1 = m\eta_0(K)$, m >1

S_2 Defective units selling price, $S_2 = m_1\eta_0(K)$, $0 < m_1 \leq 1$

R Rate of reliability

$Y(C_0, R) = aC_0^{-b}R^c$, Total cost of interest and depreciation and it's inversely related to set up cost and directly related to reliability with power function

r inflation rate

$\eta_0(K)$ Cost of unit production and $\eta_0(K) = N + \frac{G}{K} + HK$, where N, H and G are material cost, tool or die cost and energy and labor cost respectively.

t Product life (time to deterioration), t > 0

$\alpha\beta t^{\beta-1}$ Weibull deterioration rate, where $\alpha > 0$ and $\beta > 0$ are scale and shape parameters respectively.

III. Mathematical Formulation

This model deals with the flexibility of production under reliability process where the demand is deterministic. Initially, production starts at t=0 with zero inventory and it is further considered that production rate is always greater than the demand rate 'D' at t = T1. When inventory is 'S' i.e. maximum, production come to an end. Between the period (T1,T2), the inventory level reduces due to both demand and deterioration and at t = T2, inventory level reaches zero. Shortages starts at t = T2 and it reaches a maximum shortage level at t = T3. At T3, production began again to fulfill the shortages and the stock level end up to zero at t = T. The level of inventory with time by the following equations:

$$I'(t) + \alpha\beta t^{\beta-1} I(t) = RK - D \qquad 0 \le t \le T_1 \tag{1}$$

$$I'(t) + \alpha\beta t^{\beta-1} I(t) = -D \qquad T_1 \le t \le T_2 \tag{2}$$

$$I'(t) = -D \qquad T_2 \le t \le T_3 \tag{3}$$

$$I'(t) = RK - D \qquad T_3 \le t \le T \tag{4}$$

With these conditions $I(0) = 0$, $I(T_1) = S$, $I(T_2) = 0$, $I(T) = 0$,
Solutions from Eq. (1) to Eq. (4) are as follows Eq. (5) to Eq. (8) respectively,

$$I(t) = (RK - D)(t + \frac{\alpha t^{\beta+1}}{\beta+1})e^{-\alpha t^{\beta}} \qquad 0 \le t \le T_1 \tag{5}$$

$$I(t) = D[(T_2 - t) + \frac{\alpha}{\beta+1}(T_2^{\beta+1} - t^{\beta+1})]e^{-\alpha t^{\beta}} \qquad T_1 \le t \le T_2 \tag{6}$$

$$I(t) = D(T_2 - t) \qquad T_2 \le t \le T_3 \tag{7}$$

$$I(t) = (RK - D)(t - T) \qquad T_3 \le t \le T \tag{8}$$

At, t = T_1, $I(T_1)$ = S, from equation (6), one can get

$$S = D[(T_2 - T_1) + \frac{\alpha}{\beta+1}(T_2^{\beta+1} - T_1^{\beta+1})]e^{-\alpha T_1^{\beta}} \tag{9}$$

From equation (7) and (8), one can get

$$T_3 = \frac{(RK - D)T}{RK} + \frac{DT_2}{RK} \tag{10}$$

The present worth cost of carrying inventory during [0, T_1] and [T_1, T_2] is given by:

$$HC = [\int_0^{T_1} (C_h + \gamma t)e^{-rt} I(t)dt + \int_{T_1}^{T_2} (C_h + \gamma t)e^{-rt} I(t)dt]$$

$$= (RK - D)[C_h(\frac{T_1^2}{2} - \frac{rT_1^3}{3} - \frac{\alpha r T_1^{\beta+3}}{(\beta+1)(\beta+3)} - \frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}) + \gamma(\frac{T_1^3}{3} - \frac{rT_1^4}{4} - \frac{\alpha r T_1^{\beta+4}}{(\beta+1)(\beta+4)}$$

$$-\frac{\alpha\beta T_1^{\beta+3}}{(\beta+1)(\beta+3)})]+D[C_h(\frac{(T_2-T_1)^2}{2}-\frac{\alpha T_1 T_2^{\beta+1}}{(\beta+1)}+\frac{\alpha\beta T_2^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)}-\frac{rT_2^3}{3}$$

$$+\frac{rT_2T_1^2}{2}-\frac{rT_1^3}{3}-\frac{\alpha rT_2^{\beta+3}}{2(\beta+3)}+\frac{\alpha rT_1^2T_2^{\beta+1}}{2(\beta+1)}-\frac{\alpha rT_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha T_2T_1^{\beta+1}}{(\beta+1)})+\gamma(\frac{T_2^3}{6}-\frac{T_2T_1^2}{2}+\frac{T_1^3}{6}$$

$$+\frac{\alpha\beta T_2^{\beta+3}}{2(\beta+2)(\beta+3)}-\frac{\alpha T_1^2T_2^{\beta+1}}{2(\beta+1)}-\frac{rT_2^4}{12}+\frac{rT_2T_1^3}{3}-\frac{rT_1^4}{4}-\frac{\alpha rT_2^{\beta+4}}{3(\beta+4)}+\frac{\alpha rT_1^3T_2^{\beta+1}}{3(\beta+1)}$$

$$-\frac{\alpha rT_1^{\beta+4}}{(\beta+1)(\beta+4)}+\frac{\alpha T_1^{\beta+1}T_2}{(\beta+2)})] \tag{11}$$

The present worth of the cost of deterioration during $[0,T_1]$ and $[T_1,T_2]$ is given by:

$$DC=C_d[\int_0^{T_1}\alpha\beta t^{\beta-1}e^{-rt}I(t)dt+\int_{T_1}^{T_2}\alpha\beta t^{\beta-1}e^{-rt}I(t)dt]$$

$$=C_d[\alpha\beta(RK-D)\{\frac{T_1^{\beta+1}}{(\beta+1)}+\frac{\alpha T_1^{2\beta+1}}{(\beta+1)(2\beta+1)}-\frac{(\alpha+r)T_1^{\beta+2}}{(\beta+2)}-\frac{\alpha rT_1^{2(\beta+1)}}{2(\beta+1)^2}\}$$

$$+\alpha\beta D\{\frac{T_2^{\beta+1}}{\beta(\beta+1)}-\frac{T_2T_1^{\beta}}{\beta}+\frac{T_1^{\beta+1}}{(\beta+1)}+\frac{\alpha T_2^{2\beta+1}}{\beta(\beta+1)}-\frac{\alpha T_1^{\beta}T_2^{\beta+1}}{\beta(\beta+1)}+\frac{\alpha\beta T_2^{2\beta+1}}{(\beta+1)(2\beta+1)}$$

$$-\frac{\alpha\beta T_1^{2\beta+1}}{(\beta+1)(2\beta+1)}-\frac{rT_2^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{rT_2T_1^{\beta+1}}{(\beta+1)}-\frac{rT_1^{\beta+2}}{(\beta+2)}-\frac{\alpha rT_2^{2(\beta+1)}}{2(\beta+1)^2}+\frac{\alpha rT_1^{\beta+1}T_2^{\beta+1}}{(\beta+1)^2}$$

$$-\frac{\alpha rT_1^{2(\beta+1)}}{2(\beta+1)^2}-\frac{\alpha T_2^{2\beta+1}}{2\beta}+\frac{\alpha T_1^{2\beta}T_2}{2\beta}\}] \tag{12}$$

The present worth of the shortage cost during $[T_2, T_3]$ and $[T_3, T]$ is given by:

$$SC=C_S[\int_{T_2}^{T_3}(-I(t))e^{-rt}dt+\int_{T_3}^{T}(-I(t))e^{-rt}dt]$$

$$=C_S[D(\frac{e^{-rt_2}}{r^2}-\frac{e^{-rT}}{r^2}+\frac{T_2e^{-rt_3}}{r}-\frac{Te^{-rt_3}}{r})+RK(\frac{e^{-rT}}{r^2}-\frac{e^{-rt_3}}{r^2}-\frac{T_3e^{-rt_3}}{r}+\frac{Te^{-rt_3}}{r})] \tag{13}$$

The present worth cost of production during $[0,T_1]$ and $[T_3,T]$ is given by:

$$PC=(N+\frac{G}{K}+HK)[\int_0^{T_1}Ke^{-rt}dt+\int_{T_3}^{T}Ke^{-rt}dt]$$

$$=(NK+G+HK^2)(\frac{1}{r}-\frac{e^{-rT_1}}{r}+\frac{e^{-rT_3}}{r}-\frac{e^{-rT}}{r}) \tag{14}$$

The present worth of the selling price of the fresh units during $[0,T_2]$ and $[T_3,T]$ is given by:

$$SP=(N+\frac{G}{K}+HK)[\int_0^{T_1}RKme^{-rt}dt+\int_{T_1}^{T_2}RKme^{-rt}dt+\int_{T_3}^{T}RKme^{-rt}dt]$$

$$=(NK+G+HK^2)Rm[\frac{1}{r}-\frac{e^{-rT_2}}{r}+\frac{e^{-rT_3}}{r}-\frac{e^{-rT}}{r}] \tag{15}$$

The present worth of the selling price of the defective units during $[0,T_2]$ and $[T_3,T]$ is given by:

$$SPD=(N+\frac{G}{K}+HK)[\int_0^{T_1}(1-R)m_1Ke^{-rt}dt+\int_{T_1}^{T_2}(1-R)m_1Ke^{-rt}dt+\int_{T_3}^{T}(1-R)m_1Ke^{-rt}dt]$$

$$=(NK+G+HK^2)(1-R)m_1[\frac{1}{r}-\frac{e^{-rT_2}}{r}+\frac{e^{-rT_3}}{r}-\frac{e^{-rT}}{r}] \tag{16}$$

The present worth of the set up cost during $[0,T]$ is given by:

$$SUP = [\int_0^T C_0 e^{-rt} dt]$$

$$= \frac{C_0}{r}[1 - e^{-rT}] \tag{17}$$

The interest cost and depreciation cost is given by:

$$Y(C_0, R) = aC_0^{-b} R^{-c} \tag{18}$$

The total profit of the system is given by:

Total profit = Selling price of fresh units+ Selling price of defective units-Holding cost-
Deterioration cost-Shortage cost- Production cost- Set up cost-
Interest cost and depreciation cost

$$TP=SP+SPD-HC-DC-SC-PC-SUP-Y(C_0,R) \tag{19}$$

Where SP, SPD, HC, DC, SC, PC, SUP and $Y(C_0,R)$ are given by the equations (15), (16), (11), (12), (13), (14), (17) and (18) respectively.

IV. Solution Procedure

Total profit becomes a function of an independent variables, T_1, T_2 and T. This is the objective function. Hence, for optimal solution

$$\frac{\partial \mathrm{TP}}{\partial \mathrm{T}_1} = 0, \frac{\partial \mathrm{TP}}{\partial \mathrm{T}_2} = 0 \text{ and } \frac{\partial \mathrm{TP}}{\partial \mathrm{T}} = 0 \tag{20}$$

Provided, the values of T_1, T_2 and T satisfy the following conditions

$$\frac{\partial^2 \mathrm{TP}}{\partial {\mathrm{T}_1}^2} < 0, \frac{\partial^2 \mathrm{TP}}{\partial {\mathrm{T}_2}^2} < 0 \text{ and } \frac{\partial^2 \mathrm{TP}}{\partial \mathrm{T}^2} < 0 \tag{21}$$

To maximize the objective function for optimal solution the equation (19) is differentiate with respect to independent variables. Equation (19) gives an estimate of profit function. The equations (19) and (20) are nonlinear. These equations are solved by the software MATHEMATICA 11.3. For maximizing the profit, numerical illustration developed the optimal solutions of the model.

V. Numerical Illustrations

Adopting the parameters from the previous studies in proper units, which are as follows:
D = 50, α = 0.05, β = 2, r = 0.06, K= 80, C_h = 3, γ = 0.02, C_S = 5, a = 800, b = 0.50,
c = 0.75, R= 0.7, N =100, G = 200, H = 0.04, C_0 = 150, m = 2, m_1 = 0.6, C_d = 0.03

The optimum solutions is obtained T_1=11.6023, T_2=18.8667, T_3=25.3725, T=40.1264, Profit=204232.35

VI. Sensitivity Analysis

The analysis of sensitivity is performed by changing some parameters as Demand 'D', Deterioration parameters ('α' and 'β'), Inflation rate 'r' and Reliability parameter 'R' with the percentages of -50, -25, 25, 50 and get the variation with time and profit.

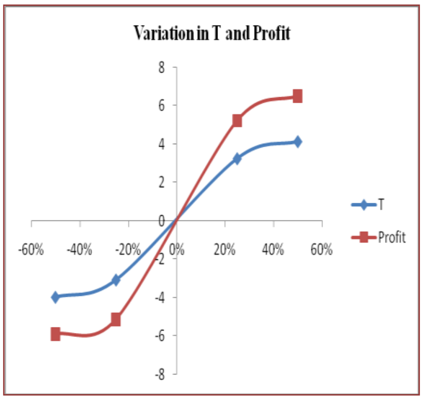
Table 1: % Change *of Optimal Solution w.r.t. time and profit*

Parameter	% Change	T_1	T_2	T_3	T	Profit
D	-50%	-8.25	-5.75	-2.08	-3.96	-5.90
	-25%	-6.61	-4.29	-1.65	-3.06	-5.12
	+25%	+5.74	+4.72	+2.19	+3.26	+5.24
	+50%	+7.82	+6.51	+2.84	+4.15	+6.53
α	-50%	-2.49	-3.87	0	-4.62	+5.33
	-25%	-1.25	-3.45	0	-3.56	+4.18
	+25%	+1.68	+3.65	0	+3.55	-4.27
	+50%	+2.46	+3.98	0	+4.34	-5.32
β	-50%	-4.26	-3.63	0	-3.06	+4.74
	-25%	-3.27	-2.84	0	-1.35	+3.90
	+25%	+3.46	+2.90	0	+1.27	-3.37
	+50%	+5.72	+3.28	0	+3.86	-4.82
r	-50%	-5.22	-3.89	0	-4.70	+7.40
	-25%	-4.72	-2.78	0	-2.63	+5.20
	+25%	+4.78	+3.24	0	+2.63	-5.24
	+50%	+5.63	+4.04	0	+4.70	-7.39
R	-50%	+6.91	+5.74	+5.02	+4.44	-6.34
	-25%	+5.37	+4.36	+4.71	+3.78	-4.69
	+25%	-5.35	-4.21	-4.73	-3.12	+4.84
	+50%	-6.92	-5.68	-5.02	-4.45	+6.26

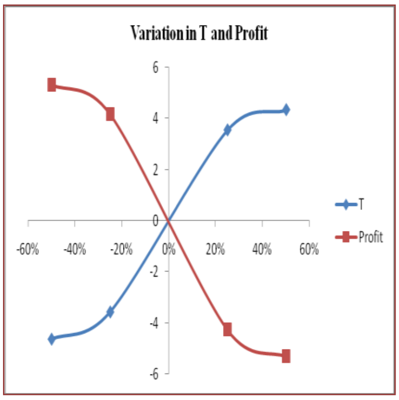
VII. Observations

The major conclusions are drawn from the numerical study. From the analysis it has been observed that:

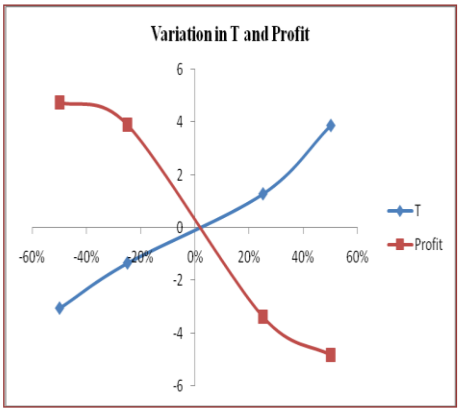
- T_1, T_2 and the profit are moderately sensitive and T_3 and T are slightly sensitive to change the parameter of demand 'D'.
- When the deterioration rates α and β increasing, then T_1, T_2, T and the profit are quite sensitive to change the scale and shape parameters of deterioration (' α 'and ' β ').
- It has been observed that when the net discount rate of inflation 'r' is increasing, the optimal profit is decreasing. T_1, T_2, T and the profit are reasonably sensitive to change the parameter of inflation 'r'.
- T_1, T_2, T_3, T and the profit are somewhat sensitive to change the parameter of reliability 'R'.
- With the increment of reliability factor, the optimal time T_1, T_2, T_3 and T are decreases and the profit is increases. With the decrement of reliability factor, the optimal time T_1, T_2, T_3 and T are increases and the profit is decreases.



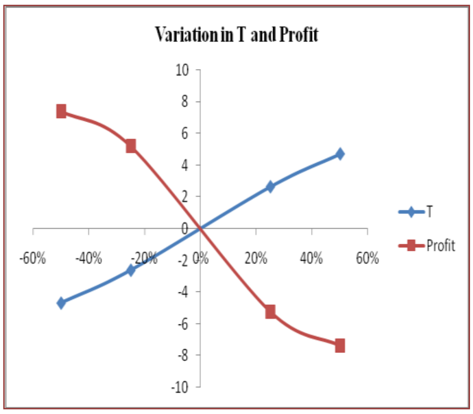
Graph 1: *Representation of the T and Profit w.r.t. 'D'*



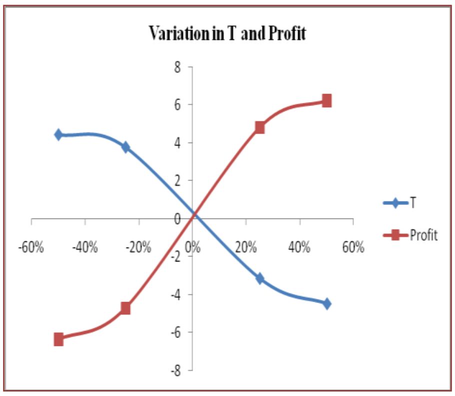
Graph 2: *Representation of the T and Profit w.r.t. 'α'*



Graph 3: *Representation of the T and Profit w.r.t. 'β'*



Graph 4: *Representation of the T and Profit w.r.t. 'r'*



Graph 5: *Representation of the T and Profit w.r.t. 'R'*

VIII. Conclusion

Flexibility and reliability for time dependent decaying items under inflation has been discussed in this deterministic inventory model. The requirement of a model with a greater level of flexibility has been highlighted. As a change has been seen throughout the globe in customer's demands across, a change has been observed in every minute and as a result the market forces like inflation change. A model which professes suppleness is sure to have a greater impact and acceptability by the organization.

The objective of this mathematical interpretation can be used to find the time dependent holding cost with the concept of volume flexibility and reliability. If a producer do not produce reliable product then, his organization will be out of contention. By dint of reliable products, producer will be able to face the upcoming challenge to survive and compete with running competitive market. Reliable of the product meant to maintain the standard quality of the object. Reliability and worth of the product have the caliber to stay in the market for long time. For instance, now-a-days (in pandemic) people have a firm believer in Swadeshi or Ayurvedic products as they have no side effects.

Sensitivity analysis is used as a mathematical technique to elucidate the results of the models. With the help of numerical illustrations, the feasibility of all the parameters has been shown. The results are found to be quite suitable and stable. This interpretation can be hold forth with fuzzy and partial backlogging.

References

[1] Al Masud, M.A., Paul, S.K. and Azeem, A. (2014). Optimisation of a production inventory model with reliability considerations, *Int. J. Logistics Systems and Management.* Vol. 17. No. 1: 22–45.

[2] Bag, S., Chakraborty, D. and Roy, A.R. (2008). A production inventory model with fuzzy random demand and with flexibility and reliability considerations, *Computers & Industrial Engineering.* 56. 1: 1-6.

[3] Khouza, M. (1997). The scheduling of economic lot size on volume flexibility production systems. *International Journal of Production Economics.* 48: 73-86.

[4] Mahapatra, G.S., Adak, S., Mandal, T.K. and Pal, S. (2017). Inventory model for deteriorating items with time and reliability dependent demand and partial backorder, *Int. J. Operational Research.* 29. 3: 344–359.

[5] Sana, S. (2003). A stochastic EOQ policy in a family of cold-drinks for a retailer, *Advanced Modeling and Optimization.* 5: 167-173.

[6] Schweitzer, P.J. and Seidmann, A. (1991). Optimizing processing rates for flexible manufacturing systems. *Management Sciences.* 37: 454-466.

[7] Shaikh,A.A., Cardenas-Barron,L.E., Manna, A.K. and Ceapedes-Mota, A. (2020). An economic production quantity (EPQ) model for a deteriorating item with partial trade credit policy for price dependent demand under inflation and reliability. *Yugoslav Journal of Operations Research (In press).*

[8] Singhal, S., Singh, S.R. and Gupta P.K. (2010). Volume Flexible supply chain system with uncertain lead time and stochastic deterioration. *International Transaction in Mathematical Sciences and Computers.* Vol. 3. No. 1: 181-193. ISSN-0974-5068.

[9] Singhal, S. and Singh, S.R. (2012). Effect of probabilistic backorder on an inventory system with selling price demand under volume flexible strategy. *International Transaction in Mathematical Sciences and Computers.* Vol. 5. No. 2: 297-304. ISSN 0974-5068.

[10] Singhal, S. and Singh, S.R. (2014). A production remanufacturing system for probabilistic decaying items with volume flexible environment. *Proceedings of 3rd International Conference on Recent Trends in Engineering & Technology (Elsevier).* 431-437: ISBN: 978-93-5107-222-5.

[11] Singhal, S., Singh, S.R. and Gupta P.K. (2016). Stochastic partial backlogging inventory models for deteriorating items with time dependent demand and volume elasticity. *International Journal of Agricultural and Statistical Sciences.* Vol. 12. No. 2: 561-567. ISSN-0973-1903.

[12] Vishnoi, M., Singh, S.R. and Singhal, S. (2018). A Supply chain inventory model with expiration date, variable holding cost in an Inflationary environment. *International Journal of pure and Applied Mathematics.* Vol.-118. No. 22: 1353-1360.

[13] Sarkar, M., Kim, S., Jemai, J., Ganguly, B. and Sarkar, B. (2019). An Application of Time-Dependent Holding Costs and System Reliability in a Multi-Item Sustainable Economic Energy Efficient Reliable Manufacturing System. *Energies.* 12. 2857: 1-19.