An EOQ Policy for Decaying Items with Constant Rate of Demand and Decreased Willingness of Buying over the Life Cycle of Item

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Abstract

In present study an EOQ model is proposed for decaying items with constant demand in which the process of demand is related to the willingness to receive the good quality items by giving the power to the costumers of checking the quality of items before reaching the effective purchasing. The cost of disposing is also considered for unsold units alongside the regular costs for storage and acquirement. This study presents a mathematical analysis to acquire the EOQ policy under certain conditions. To minimize the total expected inventory cost, a linear decrement is assumed in purchasing during the cycle of product life. Then the optimal parameters are found for the model with the help of a numerical example. In the last a numerical sensitivity analysis is presented to prove that the traditional EOQ model for decaying items is approximated by the proposed study during the sufficiently large life cycle.

Key words: Inventory, decaying items, demand, willingness, life cycle, costs.

I. Introduction and Literature review

In real situations it can be observed that many products like beverages, medicines, unpreserved drinks and foods etc. have a short life cycle. Due to the quick deterioration of products, there is a decrement in the willingness of customers to purchase such kind of products and so a big issue is there to manage all related management costs. This makes a situation of elaborating an inventory policy considering all related limitations that come from this situation. For case, it must be discussed here that the product quality over time, or the effect of deterioration which results the interest of customers to purchase the products. The deterioration of products results waste material which has to be getting rid of and it requires some cost which has an important role in market business or production industry. Thus, concept of deterioration of products is always required in the study of inventory control theory.

During the last few decades numerous researchers have elaborated inventory policies (models) for decaying products such as food items, blood banks, volatile liquids, medicines, volatile liquids, dairy products, electronic equipments etc..In [1] Ghare and Schrader were the first promoters for elaborating an inventory model with exponential decay. After it Covert and Philip [2] extended study [1] by taking constant rate of deterioration. In [3] an inventory model is considered for decaying products with time related demand without shortages. In [4] the study [3] is extended by allowing the shortages. Later, in [5] the demand pattern is generalized to any logconcave function. Further in studies [6] and [7] the demand function is generalized to include any continuous function (non-negative) which changes with time. Lately, in [8] an important survey is presented on the latest patterns in the study of decaying items. The conventional EOQ scheme [9] recommends that the rate of demand is always constant over the time which is responsible for inventory depletion. In [10] a detailed discussion is presented on depletion in inventory by taking an exponential decrement of demand. In [11] the decaying products are classified in two distinct parts, one with fixed decrement and second with finite life services. Authors such as in [12] and [13] studied an updated review on decaying products. In study [14] an EOQ policy is elaborated by taking cycle time related demand with holding and ordering costs. In articles [15], [16], [17]and [18] the EOQ models are developed with inventory reduction. Such models are identified by the direct modeling of inventory as a derivational relation and including only the holding and ordering costs. In study [19] a safety stock placement is presented. In study [20] a pricing factor is discussed in two separate studies and a policy developed for a system of decisions to optimize it. Instead of product deterioration level in many cases, customers want to buy any other product and the interest to buy will depend on the quality of product only. Besides to decay, price has a big impact on demand of products. Practically, a decrement in price of a product results to significant demand and sales volume. Thus, the price scheme is a basic factor that customers and sellers use to optimize gain and therefore studies with price-related demands have a great importance in the literature of inventory theory. In article [21] a deteriorating inventory policy is elaborated with price-related demand. In study [22] a pricing and replenishment scheme is studied for decaying items with a price related rate of demand that decreased with time. In article [23] the dynamic pricing and problem of lot size for decaying items with partial backlogging are presented.

Some other articles like [24] and [25] represent that there are various ways to control inventories in warehouses related to the business firms. In article [26] a single-item policy is elaborated with stock-level related demand. The researchers also present a price depletion model later the decay of the items which enhance more number of the customers. In article [27], the researchers studied a more realistic pricing model in a supply chain system. In study [28] researchers presented a perishable model that manages to a Weibull distribution. It is more realistic and practical work that contains costs due to the shortage and deterioration of the items. In [29] a pricing model is proposed for products with short life and variable demand. In study [30] a pricing model is proposed for products with shot life in a closed-loop system of supply with arbitrary orders.

In this study the charge of dumping of deteriorated or unsold items is an important factor. In some studies, the researchers presented a pricing model related to the evaluation of dynamic quality and check the effects of timings of occurrence and concession. This work presents two different ideas for an EOQ schemes: first one, the units or amounts that persist later every cycle bring out a disposing cost and ,other one, the request for a spoilable accords to an event, that is distinct to the sales process. The demand is always related to the interest of a client to get an item, whereas the sale correlates to the real effectual buying willingness of the client. Such situations occur only for decaying products; it is known that during the lifetime cycle of an item the willingness of buying will be decreased and once the customer observed it, there is a lack in interest to buy.

In this study, it is assumed that the readiness to buy a product reduces linearly across the lifetime of a decaying item. The demand is assumed as a constant.. The study is modeled for the cost of disposal for deteriorated items. Thus an EOQ model for decaying items is presented where the buying capacity from the customer side declines slowly during the life phase of product. Further the simulation results are there which present a high level correctness in forecasted results by the preferred mathematical equations.

The next part of this work is arranged as follows: The required assumptions and notations are presented in Section-**2**.The description of model and its solution method given in Section-**3**. The numerical example and its solution presented in in Section-4. The managerial implications of the study are presented in section-5. In the last, the conclusion and future scope of the study are presented in Section 6.

II. Used Assumptions and Notations

Assumptions

Following are the required and used assumptions throughout the whole study.

- The readiness to buy from the customer side declines linearly to the deterioration of an item and becoming zero at the end.
- Shortages of products are not permitted.
- Rate of demand is constant.
- Deteriorated or unsold products are disposed of at the end of their expiry.
- Instantaneous replacement.
- However this study is modeled to show the nature for a single product, but the sale and demand of an item is unconstrained to that of any other item so that the presented equations hold.

Notations

Following are the required and used assumptions throughout the whole study.

d : Per unit time demand.
Q : Quantity of order.
A : Quantity of demand (per year).
n :Annual number of orders.
Z : Life cycle of product.
T : Time between two consecutive buying orders.
K(j) : Damage or deterioration at the end of cycle j.
$I_1(t, j)$: Level of inventory for cycle j at time t.
$I_2(j)$: Level of average inventory for cycle j.
X(t, j) : Selling for cycle j at time t .
Y(t, j) : Collected selling for cycle t at time t.
P(t, i) : Probability of selling of demand for cycle j at time t.
H : Per unit holding cost (annual).
O : Cost of ordering.
D : Cost of disposing the product (per unit).

III. Model Description and Solution method

With the help of above notations and assumptions it is obvious that for the phase $j \le n, t \le T$,the probability of selling of one unit (at time t of phase j) is:

$$P(t,j) = 1 - t/Z \tag{1}$$

If d is the willingness to buy an item at time t of phase j then to determine the counting of sold units of products at that time

Let $x_1, x_2, x_3, \ldots\ldots, x_d$ be the random variables which denote the sales for all the d units. If these sales are independent then selling at time t for the cycle j is given by

$$X(t, j) = x_1 + x_2 + x_3, \ldots\ldots, + x_d$$

The Expected value of X(t, j) is given by

$$E(X(t, j)) = E(x_1 + x_2 + x_3, \ldots\ldots, + x_d)$$
$$= E(x_1)+E(x_2)+E(x_3), \ldots\ldots, +E(x_d)$$
$$= (1-t/Z) + (1-t/Z)+ (1-t/Z)+\ldots\ldots+ (1-t/Z) = d(1-t/Z) \text{ i.e}$$
$$E(X(t, j)) = d(1-t/Z) \quad .(2)$$

This expected value of X(t, j) is required to find the total cost equation detailed later .

To find the per cycle expected average inventory

For a cycle j at time t , the sales are given by X(t, j). If the process is continuous over time then the collective selling Y(t, j) cannot exceed the integral of X(t, j):

$$Y(t, j) = \int_0^t X(t, j)dt \quad (3)$$

In real situations, the level of inventory is the gap between the sold and ordered units:

$$I_1(t, j) = Q - Y(t, j) = Q - \int_0^t X(t, j)dt \quad (4)$$

The level of average inventory for cycle j:

$$I_2(j) = \frac{\int_0^T I_1(t, j)dt}{T} = \frac{\int_0^T [Q - Y(t, j)]dt}{T}$$
$$= \frac{\int_0^T Qdt - \int_0^T Y(t, j)dt}{T} = \frac{\int_0^T Qdt - \int_0^T \left[\int_0^t X(t, j)dt\right]dt}{T} \quad (5)$$

The expected value of $I_2(j)$ is given as

$$E(I_2(j)) = E\left(\frac{\int_0^T Qdt - \int_0^T \left[\int_0^t X(t, j)dt\right]dt}{T}\right) = \frac{E\left(\int_0^T Qdt\right) - E\left(\int_0^T \left[\int_0^t X(t, j)dt\right]\right)dt}{T}$$
$$= \frac{\int_0^T Qdt - \int_0^T \left[\int_0^t E(X(t, j))dt\right]dt}{T} = \frac{\int_0^T Qdt - \int_0^T \left[\int_0^t d(1-t/Z)dt\right]dt}{T}$$
$$= \frac{\int_0^T Qdt - d\int_0^T (t - t^2/2Z)dt}{T} = \frac{QT - d\left(\frac{T^2}{2} - \frac{T^3}{6Z}\right)}{T} \quad (6)$$
$$= Q - d\left(\frac{1}{2} - \frac{T}{6Z}\right) = Q - Q\left(\frac{1}{2} - \frac{Q}{6dZ}\right) = Q\left(\frac{1}{2} + \frac{Q}{6dZ}\right), \forall T < Z$$

If the order quantity, $Q \geq dZ$, the solution of equation (6) can be evaluated up to Z, provided that in the extra range of time T−Z the inventory will be zero.

$$E(I_2(j)) = \frac{QZ - d\left(\frac{Z^2}{2} - \frac{Z^3}{6Z}\right)}{T} = \frac{QZ}{T} - \frac{dZ^2}{3T} = dZ - \frac{d^2Z^2}{3Q}, \forall T \geq Z \quad (7)$$

This result shows that every cycle has the nature that the average. If the time of cycle is smaller than the life of product then average inventory will be $Q\left(\frac{1}{2}+\frac{Q}{6dZ}\right)$. Therefore, there is a classic average inventory Q/2 plus a factor $Q^2/6dZ$. The second factor is less than Q/6 in magnitude and approaches to zero as Z grows.

To find per cycle expected number of damaged or spoiled units

The number of damaged units will be difference between ordered and sold quantities after the ending of cycle T i.e

$$K(j) = Q - Y(T, j) = Q - \int_0^T X(t, j)dt \tag{8}$$

The expected value of K(j) is given as

$$\begin{aligned} E(K(j)) &= E\left(Q - \int_0^T X(t, j)dt\right) = E(Q) - E\left(\int_0^T X(t, j)dt\right) \\ &= Q - \int_0^T E(X(t, j))dt = Q - \int_0^T d(1 - t/Z)dt \end{aligned} \tag{9}$$

There are cases to solve the integral in equation (9)

Case-I: If Q <dZ then integral in (9) will be until T i.e

$$\begin{aligned} E(K(j)) &= Q - d(T - T^2/2Z) = Q - dT(1 - T/2Z) \\ &= Q - Q(1 - Q/2dZ) = Q^2/2dZ, \forall T < Z \end{aligned} \tag{10}$$

Case-II: If $Q \geq dZ$ then integral in (9) will be until Z i.e

$$E(K(j)) = Q - d(Z - Z^2/2Z) = Q - dZ/2, \forall T \geq Z \tag{11}$$

If the result in equation (10) is analyzed then it can be noticed that Q/2 is greater than $Q^2/2dZ$ (Q <dZ) and approaches to zero as the life of product rises .

To find the EOQ that depletes the total annual expected total cost

The total inventory cost involved in this model is given as the sum of the stock holding cost, total cost of ordering the materials, and the total cost of damage or spoilage per unit:

$$TIC = O\frac{A}{Q} + HI_2 + D\sum_{i=1}^{n} K(i) \tag{12}$$

The expected total inventory cost is given as

$$\begin{aligned} E(TIC) &= E(O\frac{A}{Q}) + E(HI_2) + E(D\sum_{i=1}^{n} K(i)) \\ &= O\frac{A}{Q} + HI_2 + D\sum_{i=1}^{n} E(K(i)) \end{aligned} \tag{13}$$

If Q <dZ then from equations (6), (10) and (13):

$$\begin{aligned} E(TIC) &= O\frac{A}{Q} + HQ\left(\frac{1}{2}+\frac{Q}{6dZ}\right) + D\sum_{i=1}^{n}\frac{Q^2}{2dZ} = O\frac{A}{Q} + HQ\left(\frac{1}{2}+\frac{Q}{6dZ}\right) + D\frac{Q^2}{2dZ}n \\ &= O\frac{A}{Q} + HQ\left(\frac{1}{2}+\frac{Q}{6dZ}\right) + D\frac{Q^2}{2dZ}\frac{A}{Q} = O\frac{A}{Q} + HQ\left(\frac{1}{2}+\frac{Q}{6dZ}\right) + D\frac{QA}{2dZ} \end{aligned} \tag{14}$$

Differentiating (14) with respect Q and putting equal to zero

$$\frac{dE(TIC)}{dQ} = -O\frac{A}{Q^2} + \frac{H}{2} + \frac{HQ}{3dZ} + \frac{DA}{2dZ} = 0$$

$$\Rightarrow 2HQ^3 + 3dZHQ^2 + 3DAQ^2 - 6dZOA = 0 \Rightarrow 2HQ^3 + 3(AD + dHZ)Q^2 - 6dZOA = 0$$

$$\Rightarrow Q^3 + \frac{3}{2}.\frac{(AD + dZH)}{H}Q^2 - \frac{3dZOA}{H} = 0 \qquad (15)$$

If $Q \geq dZ$ then from equations (7), (11) and (13):

$$E(TIC) = O\frac{A}{Q} + H\left(dZ - \frac{2d^2Z^2}{3Q}\right) + D\sum_{i=1}^{n}\left(Q - \frac{dZ}{2}\right) = O\frac{A}{Q} + H\left(dZ - \frac{2d^2Z^2}{3Q}\right) + \frac{D(Q - dZ/2)A}{Q}$$

After solving

$$E(TIC) = HdZ + DA + \frac{(6AO - 4Hd^2Z^2 - 3DAdZ)}{6Q} \qquad (16)$$

If $6AO - 4Hd^2Z^2 - 3DAdZ$ = W then equation (16) will be decreasing if W > 0. Since *Q* may have a biggest value/ year as *A*, it will be the best ordering value for the developed conditions. If W < 0 then equation (16) will be increasing and Q will have the minimum value dZ as an optimal value. Although equations (14) and (16) have the similar value in Q = dZ that proposes, the probability of a quantity Q <dZ may persist that the cost decreases in equation (14). Definitely it is unclear that decaying items get single order only per year. so the total optimal cost will be assumed from the solution of equation (15). It is convenient to determine Q and if the base equation is considered for T < Z ,the stock holding cost approaches to be one of the mentioned costs in the model(if the complete time of life cycle approaches to be very big). The cost related with disposed units approaches to be decreased when the life of item approaches to grow.

Solution Method

Consider equation (15) and put

$$g(Q) = Q^3 + \frac{3}{2}.\frac{(AD + dZH)}{H}Q^2 - \frac{3dZOA}{H}$$

$$\Rightarrow g'(Q) = 3Q^2 + \frac{3Q(AD + dZH)}{H}, \text{which is positive for every value of } Q \geq 0.$$

Thus g(Q) is an increasing function in the intervals $]-\infty, -(AD + dZH)/H]$ and $[0, \infty[$ and it is decreasing in $]-(AD + dZH)/H, 0[$ .Definitely , Q=0 is a comparative lowest , known that $g(0) < 0$. This statement assurances the occurrence of one real and positive root of $g(Q)$.

From equation (14), it can be observed that total expected annual cost increases with the increment in D or H. Although this cost decreases with the increment in d and Z. Generally the results of total expected annual cost are careful to the parameter Z which is involved in two components of cost. The limit of cost function as Z tends to infinity, given as below:

$$\lim_{Z\to\infty} E(TIC) = \lim_{Z\to\infty}\left[O\frac{A}{Q} + HQ\left(\frac{1}{2} + \frac{Q}{6dZ}\right) + D\frac{QA}{2dZ}\right] = O\frac{A}{Q} + \frac{HQ}{2}$$

This result shows that the model tends to the normal EOQ for the products with large life cycle. To find the positive root of cubic equation g(Q) = 0, the Cardano approximation method is used. For the cubic equation $ax^3 + bx^2 + cx + d = 0$ this method is given as below:

$$x_1 = L_1 + L_2 - \frac{b}{3a},\ x_2 = -\frac{L_1 + L_2}{2} - \frac{b}{3a} + \frac{i\sqrt{3}}{2},\ x_3 = -\frac{L_1 + L_2}{2} - \frac{b}{3a} - \frac{i\sqrt{3}}{2}$$

$$L_1 = \sqrt[3]{R + \sqrt{S^3 + R^2}},\ L_2 = \sqrt[3]{R - \sqrt{S^3 + R^2}} \text{ and in turn}$$

$$R = \frac{3ac - b^2}{9a^2},\ S = \frac{9abc - 27a^2d - 2b^3}{54a^3}$$

If Q_1, Q_2 and Q_3 be the roots of g(Q) = 0, then these roots can be obtained by using the above formula.

IV. Numerical Example and Solution

Consider the following values of the various parameters:
A=25,000 units, D=$520/unit, H=$110/unit, Z=30 days,O=$100.000/Order. By considering 360 functioning days in a year and applying the method of Cardano approximation the order quantity Q is obtained:

*Q** ≈ 812.36 *E(TIC(Q*))* ≈ $4'974.164.6.

To study the probabilistic nature of the interest to buy, a case of the study is simulated by software with the related parameters. The calculation is done with 20 random cases. In every case, the optimal expected cost is determined and the gap percentage is calculated with the help of optimal cost determined by the simulation. For every simulated instance, The optimal order quantity for each simulated instance is calculated theoretically as per the solution of equation (15) by the method of Cardano approximation.

The results of the study are shown in Table-1. The maximum gap is observed as 3.42%, and it is less than 1% usually. This shows that there is an accuracy of high level in the derived equations even under the random situations. The values of *Q** are estimated to the closest integer.

Table: 1

A	O	D	H	Z	Q*	Predicted optimal cost	Simulated optimal cost	Percentage gap
1,000,000	200,500	150	20	12	2782	$12.2304	$12.2182	.09
21,000	45,000	1200	450	25	302	$ 5.4412	$5.4642	.36
62,000	350,000	22,000	2550	65	522	$66.4202	$66.5638	.22
500,000	150,000	250	65	40	9510	$15.7846	$15.7583	.17
1250	5.200,000	100,00	35,00	100	180	$69.8632	$69.9896	.18
500	32,000	55,000	22,000	52	10	$ 3.5142	$ 3.5250	.30
2100	32,000	1000	550	17	72	$ 1.7195	$ 1.7305	.62
2600	250	5	2	25	121	$ 8.7284	$ 8.8152	.98
24,500	5500	45	14	70	1072	$ 230.1542	$234.4342	1.82
85,600	10,000	2100	360	50	342	$ 5.2726	$ 5.2961	.44
100	250	25	12	22	6	$ 5.8326	$ 5.6326	3.42
12,500	450	30	5	12	97	$ 103.0564	$ 102.4120	.62
500	100	5	1	30	45	$ 2.5732	$ 2.5611	.47
7600	150	3	3	4	88	$ 21.2346	$ 20.9710	.77
36,000	240	7	5	6	192	$ 82.8815	$ 83.3654	.59
9600	1000	100	10	48	165	$ 125.0456	$ 122.7650	1.82
300	2500	90	32	75	424	$ 22.8758	$ 22.7086	.73
66,000	140	3	1	12	54	$ 38.6456	$ 38.8786	.60

33,000	700	45	27	65	402	$ 105.1172	$ 105.3960	.264
25,000	11,000	250	12	100	782	$ 633.7036	$ 638.5234	.75

V. Managerial Implications

This study offers a more suitable and realistic structure for inventory modeling of decaying products, which creates domain-relevant results achieving the acceptability of research to the industry activities. In particular, the over time deterioration effect for such products may relent in scrap which has to be get rid of, thus a cost of disposal is required and bad effect on functioning is there. Normally, it is decided by the managers that what to do with the products that could decay. Hence, firms are counseled to recognize and determine the suitable levels of inventory for decaying, which may suggest their operations for supply chain to settle and thereby ignore partly consequences. Earlier study represents many ideas in which the inventory for decaying products is analyzed by managers. In addition, few of the studies propose the models with gradual deterioration of products. Although, here it is argued that the managers now have to focus to the interest of customers to buy, because of the deteriorated quality of the products. The executives should include the welfares coming from a more appropriate inventory model for decaying products, while they should be in position to do effort to good recognition the nature of a client whose interest to buy a product could deplete with the deteriorating quality of the products.

VI. Conclusion and Future Scope

In this study, an EOQ model is elaborated for decaying products with constant demand while the probability of buying from the side of customers depletes linearly over the life phase of product. The study`s aim is to find the quantity of ordering that optimizes the expected total annual cost. With the help of considered assumptions an equation of third order is constructed to determine the value of Q which is used to lessen the entire cost and this cost decreases with the increment in the life product. An unique solution is proved to this equation and this equation has a positive root which is determined by the method of Cardano approximation. In this study the resulting cost is very tactful to the product life phase.

In situations where (i) time of replenishment is small relatively, (ii) the units are to be disposed in the end of every cycle and (iii) the loss of the product quality is linear throughout its life phase, then this study could be applied well.

The results are validated through the simulation by taking the random behavior in the interest of customers to buy. The simulated cases presented a perfect exactness in the projection of the generated equations, specified that the forecasted cost is deflected upwards by 3.42 % as per the simulated optimal cost.

Further, the model can be elongated for future research by including new alternatives of this study, such as probabilistic demand, non-linear natures for the quality loss in decaying products, penalization due to unsatisfied demand and other things that which are helpful to the related studies.

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