# Inventory Policy for Deteriorating Items with Two-Warehouse and Effect of Carbon Emission

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**Abstract**

*In today's era, humans have shifted from the stage of fulfilling their needs to the stage of freedom where they indiscriminately use all the resources. This has increased the concern for protecting our nature and developing sustainable practices. So, an EOQ inventory model is formulated for decaying items with capacity constraint policy where demand is stock-dependent. Shortages are allowed with condition of partial backlogging. The impact of carbon emission is taken on holding cost and deterioration cost. Due to different conditions of storage, the deterioration rate is also different in different warehouses. The main focus of this study is to optimize total cost with carbon emission and total cycle time. An algorithm for the inventory model to find the best output is formulated here. Numerical example with sensitivity is also discussed to show the impact of carbon emission.*

**Keywords:** Deterioration, Carbon-Emission, Two-Warehouse, EOQ Model, Shortages.

## I. Introduction

Carbon emission is one among many greenhouse gas emissions that takes place after human activities or other processes. It refers to the release of carbon dioxide gas in the atmosphere from vehicles, industries, etc. Carbon dioxide gas traps the incoming solar radiations which results in global warming. When the heat does not escape, it leads to inflation in the earth's temperature. This increase in the temperature in not beneficial for all the species living on earth. It leads to change in climate of regions affecting agriculture, monsoon patterns, leads to melting of icebergs, glaciers etc. It also increases respiratory diseases in humans. It is very important to reduce carbon emissions. It can be reduced by taking measures to reduce the production of $CO_2$ by using filters, using public transportation or carpooling, adopting the ideas of composting for bio-degradable wastes and cycling or walking over small distances is a very healthy option to decrease carbon emission.

Due to environmental conditions, the quality of an item changes that is referred as deterioration phenomena. Deterioration occurs because of change in temperature, transportation and also the storage standard play an important role. Some of items deteriorate with time and lose their quality. Some items have specific lifetime because of some specific things which they are made of. After the completion of the time period, the item turns to waste and with nearing of time its quality decreases.

Warehousing is necessary for continuous, seasonal demand. The limited space and high rental costs in large market places such as malls, supermarkets etc., has made it difficult to have large showrooms there. The management decides to buy large number of items either when there is an impressive discount in cost for bulk order or when there are some issues in often acquirement and also if the item demand is inevitably high. The available space then falls small for the recently purchased items, to solve the problem of storage a rental warehouse is hired. The RW (Rented Warehouse) could be located in close proximity to the owned warehouse or at some distance with enough space for the items. The holding cost in RW increases than the one in OW (Owned Warehouse). To decrease the total holding cost demands are first fulfilled from RW. The demand rate in supermarkets is influenced by volume of stock-level. Over the past many years, a lot of interest has been shown to the situations in which the demand rate depends on the on-hand stock level. It is nearly not possible to stock high quantities in store because of increased holding costs and during high demands this issue leads to storage in the inventory systems. Thus, we assume inventory system for decaying goods with stock-based demand rate which has different rate of decaying in various warehouses, where shortages are allowed.

## II. Literature Review

Many researchers have done work on two-warehouse inventory model. Hartley [1] firstly formulated the two-warehouse inventory structure. He considers the rented warehouse causing the higher total carrying cost rather than owned warehouse. Sharma [2] recommended two-warehouse system for decaying items. Mandal & Phujdar [3] offered model for two-warehouse with stock-based demand for decaying items. Pakkala and Achary [4] established model for two-warehouse with stock-based demand for decaying items under the condition of shortages. Shah [5] offered a survey of literature on structure of inventory. Zhou and Yang [6] preferred model for two-warehouse facilities with stock-based demand for decaying items. Alfares [7] initiated work on model for two-warehouse for decaying items with variable carrying cost and the rate of demand is stock-based. Singh et al. [8] suggested a system with two-warehouse under the condition of partially backlogged for decline goods. Yang et al. [9] explored a system for two-warehouse with the impact of inflation and shortages. The rate of demand is stock-based. Singh et al. [10] considered system for two-warehouse with the impact of trade credit and shortages. Here, the rate of demand is based on presently stock. Bhunia et al. [11] proposed structure for two-warehouse with shortages and the rate of demand is constant and known. Rastogi and Rathore [12] projected system for two-warehouse with the impact of preservation technology and shortages. Here, the rate of demand to be stock-based.

Sarkar et al. [13] developed policy related to inventory which gave an effect on environment with the condition of partial backlogging. An offer of multi-trade-credit period is given to the dealer to increase the business. Panda et al. [14] proposed policy related to inventory with the effect on trade credit under the condition of partial backlogging. An offer of multi-trade-credit period is given to the dealer to increase the trade. Demand is the combination of advertisement, cost and stock. Harit et al. [15] developed policy related to inventory with the effect on trade credit and inflation. Chauhan and Yadav [16] discussed the policy of inventory for decline goods with capacity constraint using genetic algorithm and stock -dependent demand. Yadav et al. [17] initiated model with stock-based and ramp type demand function for decline items, with reverse money and shows the impact of carbon emission. Gautam et al. [18] offered a sustainable production policy under the effect of volume agility and the technology of preservation with price-reliant demand. Sarkar et al. [19] explored the policy of inventory for deteriorating items with carbon emission. Mishra et al. [20] formulated model with controllable carbon emission for deteriorating items.

Sepehri [21] initiated model with for deteriorating Item with maximum lifetime and carbon emissions under trade credit policy. Al Arjani [22] offered a sustainable online-to-offline model for a supply chain management where lead time is controllable and function of demand is a variable.

The next part of this work is arranged as follows: Introduction with literature review is presented in Section 1, Section 2 includes assumptions & notations used in the paper, Section 3 deals with mathematical formulation of model, the process of solution with numerical example is presented in Section 4 and Section 5 contains sensitivity analysis and in the last, the conclusion and future scope of the study are presented in section 6.

## III. Assumptions and Notations

3.1 Assumptions:

Following are the required and used assumptions throughout the whole study.

(i) The rate of demand D(t) is a function of immediate inventory level and taken as: $D(t) = \begin{Bmatrix} p+q(t), 0 \le t \le t_1 \\ p, t_1 \le t \le T \end{Bmatrix}$ where $p, q > 0$.Shortages are allowed and unfulfilled demand is partially backlogged. The fraction of backordered is $\frac{1}{1+\lambda(T-t)}$ where $\lambda$ is a positive constant.

(ii) Time limit is infinite.

(iii) Size of OW is limited and RW has unlimited size.

(iv) Firstly, consumed the RW goods and then consumed the OW goods.

(v) Lead time is zero.

(vi) For the best result, we consider maximum decaying amount for times in OW, $bI_1 < D$.

(vii) Carbon emission is assumed for the best result of keeping the decaying goods and warehousing the goods.

3.2 Notations:

$Q_r(t)$: At time t level of positive stock in RW
$Q_o(t)$: At time t level of positive stock in OW
$C_o$: ordering cost
$C_d$: Cost of deterioration
$I_1$: Capacity of OW
$I_2$: Capacity of RW
$C_{ho}$: Holding cost in OW
$C_{hr}$: Holding cost in RW
$C_{re}$: Carbon emission cost for holding goods in RW
$C_{oe}$: Carbon emission cost for holding goods in OW
$C_s$: Shortage cost
$C_l$: Lost sale cost
a: deterioration cost in RW, $0 \le a<1$
b: deterioration cost in OW, $0 \le b<1$
$C_{ae}$: Carbon emission cost form deteriorating goods in RW
$C_{be}$: Carbon emission cost form deteriorating goods in OW
$t_1$: Time of zero stock keeping unit in RW
$t_2$ : Time of zero stock keeping unit in OW
TC: Total average cost

p: Initial rate of demand
q: Positive rate of demand

## IV. Formulation of Mathematical Model

In formulation of mathematical model assume the intervals [0, $t_1$], [$t_1$, $t_2$] and [$t_2$, T]. The level of RW decline due to decay and demand in [0, $t_1$] and reaches to zero at $t=t_1$. After this demand of customer is fulfilled by the stock available in OW in [$t_1$, $t_2$]. After this shortage takes place that are partially backlogged. The level of OW decline due to decay and demand and reaches to zero at $t=t_2$ as shown in Figure1.

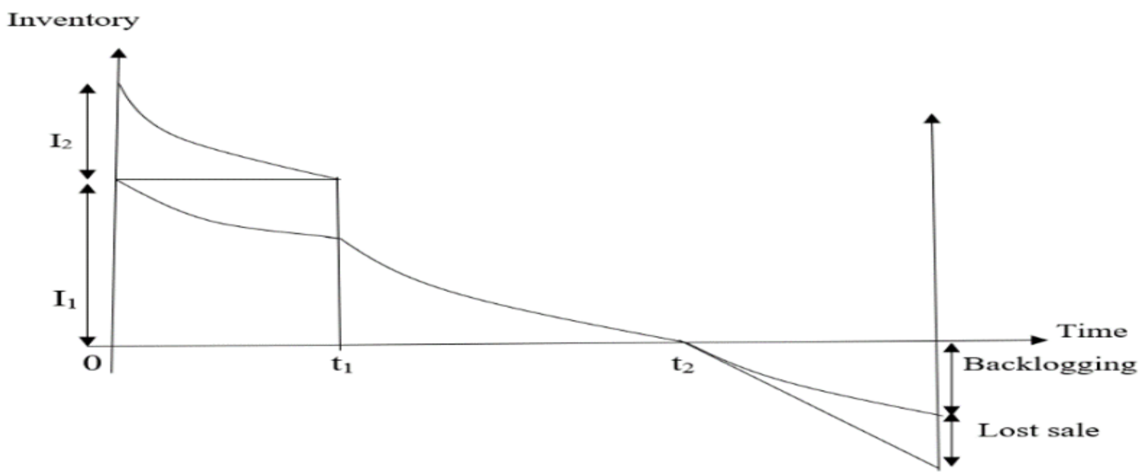


**Figure 1:** *Two-warehouse Inventory System*

The level of stock between $0 \le t \le T$ is represented by the following differential equations:

$$\frac{dQ_r(t)}{dt} = -(p + qQ_r(t)) - aQ_r(T),\ \ 0 \le t \le t_1 \ \ \text{With } Q_r(t_1) = 0 \tag{1}$$

$$\frac{dQ_o(t)}{dt} = -bQ_o(t),\ \ 0 \le t \le t_1 \ \text{With } Q_o(0) = I_1 \tag{2}$$

$$\frac{dQ_o(t)}{dt} = -(p + qQ_o(t)) - bQ_o(t),\ \ t_1 \le t \le t_2 \ \ \text{With } Q_o(t_2) = 0 \tag{3}$$

Using equations (1), (2) and (3),

$$Q_r(t) = \frac{p}{(q+a)}[e^{(q+a)(t_1-t)} - 1],\ 0 \le t \le t_1 \tag{4}$$

$$Q_o(t), = I_1 e^{-bt}, 0 \le t \le t_1 \tag{5}$$

$$Q_o(t) = \frac{p}{(q+b)}[e^{(q+b)(t_2-t)} - 1], t_1 \le t \le t_2 \tag{6}$$

At $t = t_1$, by equations (5) and (6),

$$I_1 e^{-bt_1} = \frac{p}{(q+b)}[e^{(q+b)(t_2-t_1)} - 1] \tag{7}$$

Now $$t_2 = t_1 + \frac{1}{(q+b)}\log(1 + \frac{(q+b)I_1 e^{-bt_1}}{p}) \tag{8}$$

Here $t_2$ is a function of $t_1$. In $[t_2, T]$ shortages starts and level of stock based on demand and some of the demand is lost. So,

$$\frac{dQ_o(t)}{dt} = -\frac{p}{1+\lambda(T-t)}, \; t_2 \le t \le T \quad With\, Q_o(t_2) = 0 \tag{9}$$

$$Q_o(t) = -\frac{p}{\lambda}\left[\{\log(1+\lambda(T-t_2)\} - \{\log(1+\lambda(T-t)\}\right] \tag{10}$$

Now we proceed to find the various cost associated with stock as follows:
Ordering cost (OC)= $C_o$
Holding cost (HC)is the combination of holding cost and carbon emission cost. So, (from equation (4))

$$\text{HC}_{\text{RW}} = (C_{hr} + C_{re})\int_0^{t_1} Q_r(t)dt$$

$$HC_{RW} = \frac{(C_{hr} + C_{re})p}{(q+a)^2}[e^{(q+a)t_1} - (q+a)t_1 - 1] \tag{11}$$

$$HC_{OW} = (C_{ho} + C_{oe})\left(\int_0^{t_1} Q_o(t)dt + \int_{t_1}^{t_2} Q_o(t)dt\right)$$

$$HC_{OW} = (C_{ho} + C_{oe})\left[\frac{I_1}{b}(1-e^{-bt_1}) + \frac{p}{(q+b)^2}\left(e^{(q+b)(t_2-t_1)} - (q+b)(t_2-t_1) - 1\right)\right] \tag{12}$$

Shortage cost (SC)= $C_s \int_{t_2}^{T} -Q_0(t)dt$

$$SC = \frac{pC_s}{\lambda^2}\left(\lambda(T-t_2) - \{\log(1+\lambda(T-t_2)\}\right) \tag{13}$$

Lost Sale Cost (LSC) = $C_l p \int_{t_2}^{T}\left(1 - \frac{1}{1+\lambda(T-t)}\right)dt$

$$LSC = \frac{pC_l}{\lambda}\left(\lambda(T-t_2) - \{\log(1+\lambda(T-t_2)\}\right) \tag{14}$$

Cost of deterioration in RW: $D_R = Q_r(0) - \int_0^{t_1} D(t)dt$

$$D_R = \frac{pa}{(q+a)^2}[e^{(q+a)t_1} - (q+a)t_1 - 1]$$

Cost of deterioration in OW: $D_O = Q_o(0) - \int_{t_1}^{t_2} D(t)dt$

$$D_O = I_1 - p(t_2 - t_1) - \frac{pb}{(q+b)^2}[e^{(q+b)(t_2-t_1)} - (q+b)(t_2-t_1) - 1]$$

Deterioration cost (DC) is the combination of deterioration cost and carbon emission cost. So,

$$D_c = (C_d + C_{ae})D_R + (C_d + C_{be})D_O$$

$$D_c = (C_d + C_{ae})\{D_R + D_O\}$$

$$D_C = \begin{bmatrix} \left(C_d + C_{ae}\right)\left(\frac{pa}{(q+a)^2}[e^{(q+a)t_1} - (q+a)t_1 - 1]\right) + \\ \left(C_d + C_{be}\right)\begin{pmatrix} I_1 - p(t_2 - t_1) - \frac{pb}{(q+b)^2}[e^{(q+b)(t_2-t_1)} - \\ (q+b)(t_2-t_1) - 1] \end{pmatrix} \end{bmatrix} \tag{15}$$

Total average cost = [OC+ HC+ SC+ LSC+DC]

$$TC(t_1,T) = \frac{1}{T}\begin{Bmatrix} C_o + \frac{(C_{hr} + C_{re})p}{(q+a)^2}[e^{(q+a)t_1} - (q+a)t_1 - 1] + \\ (C_{ho} + C_{oe})\left[\frac{I_1}{b}(1 - e^{-bt_1}) + \frac{p}{(q+b)^2}\begin{pmatrix} e^{(q+b)(t_2-t_1)} \\ -(q+b)(t_2-t_1) \\ -1 \end{pmatrix}\right] + \\ \frac{p(C_s + \lambda C_l)}{\lambda^2}\left(\lambda(T - t_2) - \{\log(1 + \lambda(T - t_2)\}\right) + \\ \left(C_d + C_{ae}\right)\left(\frac{pa}{(q+a)^2}[e^{(q+a)t_1} - (q+a)t_1 - 1]\right) + \\ \left(C_d + C_{be}\right)\begin{pmatrix} I_1 - p(t_2 - t_1) - \\ \frac{pb}{(q+b)^2}[e^{(q+b)(t_2-t_1)} - (q+b)(t_2-t_1) - 1] \end{pmatrix} \end{Bmatrix} \tag{16}$$

To optimize the total cost function, the necessary conditions are $\frac{\delta TC(t_1,T)}{\delta t_1} = 0 \,\&\, \frac{\delta TC(t_1,T)}{\delta T} = 0$

and sufficient are
$$\frac{\partial^2 TC(t_1,T)}{\partial t_1^2} > 0 \,\&\, \frac{\partial^2 TC(t_1,T)}{\partial T^2} > 0 \; and$$
$$\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(t_1,T)}{\partial t_1 T}\right) > 0$$

## V. Numerical Example

Algorithm:

Step 1.

(i) Begin with $t_1(1) = t_2$ .

(ii) Put the value of $t_1$ in to equation $\frac{\delta TC(t_1, T)}{\delta t_1} = 0$ and obtain $T_{(1)}$ .

(iii) By using $T_{(1)}$ , obtain $T_{1(2)}$ from equation $\frac{\delta TC(t_1, T)}{\delta T} = 0$ .

(iv) Reoccur (ii) and (iii) until changeless $t_1$ & $T$ comes.

Step 2: - (i) If $t_1 \le t_2$, then use step (3).

(ii) If $t_2 < t_1$, then set $t_1 = t_2$, find T from equation $\frac{\delta TC(t_1, T)}{\delta T} = 0$ and then use step (3).

Step 3: - Calculate Total Cost.

*Numerical Example:* To find the optimal result of total cost, suppose the following data has been taken in Table 1:

**Table 1:** *Value of Different Parameter*

| $C_o$ | $C_{hr}$ | $C_{re}$ | $C_{ho}$ | $C_{oe}$ | $C_s$ | $C_l$ |
|---|---|---|---|---|---|---|
| 500 | 4 | 0.03 | 6 | 0.02 | 20 | 10 |
| b | $I_1$ | $C_d$ | p | q | $C_{ae}$ | $C_{be}$ |
| 0.06 | 150 | 4 | 1000 | 15 | 0.04 | 0.02 |
| $C_d$ | $t_1$ | a | $\lambda$ | b | | |
| 4 | 0.1250 | 0.05 | 0.2 | 0.06 | | |

From this value, we find $t_2$ = 0.9801, T= 95.622, TC= 113603.5551

## VI. Sensitivity Analysis

We explore that the acuteness for the optimal solution of the system by using the distinct values of the variables. The following results are observed.

**Table 2:** *Impact of Demand and Deterioration on Policy of Inventory*

| Parameter | Change in cost in % | $t_2$ | T | Total Cost |
|---|---|---|---|---|
| p | -20 | 1.018 | 161.69 | 96278.24 |
| | -10 | 0.997 | 122.49 | 103592.72 |
| | 10 | 0.966 | 78.31 | 123753.14 |
| | 20 | 0.954 | 66.29 | 133216.94 |
| $C_d$ | -20 | 0.838 | 14.86 | 188657.78 |
| | -10 | 0.911 | 28.46 | 135562.90 |
| | 10 | 1.057 | 173.13 | 117136.17 |
| | 20 | 1.131 | 884.44 | 109339.43 |

From Table-2, We noticed that when demand function rise, T & $t_2$ also rise & Total cost fall with concerning demand function. As demand function falls, T & $t_2$ also changed and Total cost with concerning demand function. While demand function & $t_2$ reduce then total cost also rises. We see that the effect of deterioration cost on $t_2$, T& total cost. As effect of deterioration cost the reduce, Total cost increase while $t_2$& T fall and as the impact of deterioration cost increase, Total cost fall while $t_2$& T rise.

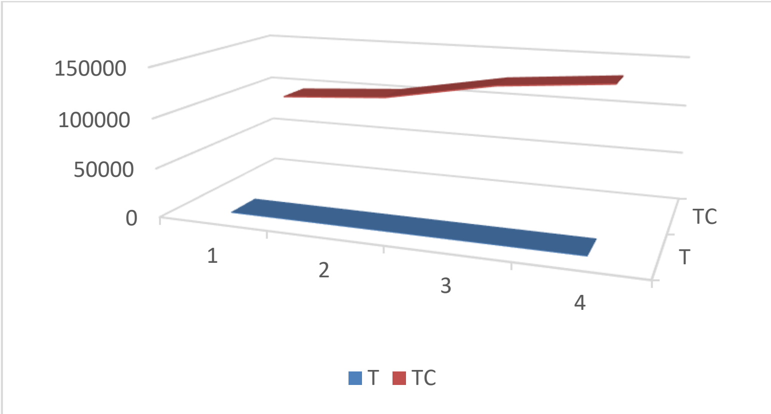


**Figure 2:** *Impact of demand function on Total cost*

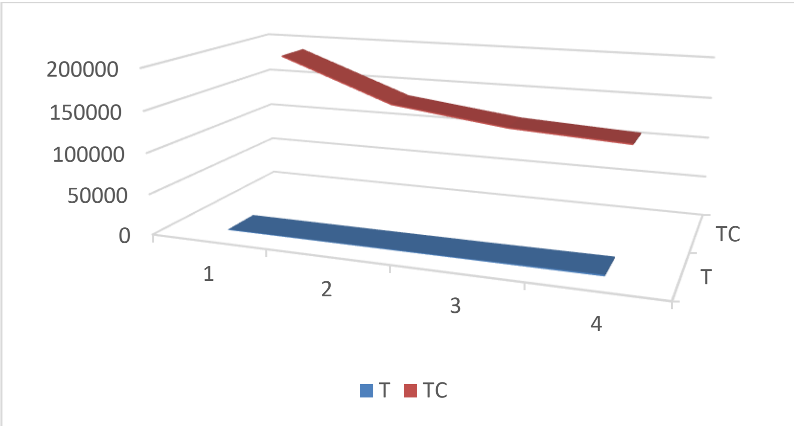


**Figure 3:** *Impact of Deterioration cost on Total Cost*

## VII. Conclusion

In the present study, a two-warehouse inventory model for deteriorating items is formulated with the effect of carbon emissions where demand is a function of stock available in warehouses which also permits shortages. There are different conditions of storage in different warehouses. Therefore, the rate of deterioration is also different for each warehouse. The impact of sustainability is taken with holding cost and deterioration cost. To demonstrate the work a numerical example with analysis of sensitivity regarding various variables is taken and it is observed that total average cost is favorably accessible for the different the value of demand function, deterioration cost and cycle time. It is favorably receptive for the deteriorating parameter. $t_2$ is favorably receptive for the different value of deteriorating parameter and highly receptive for the cycle time. The future extension for this model is different trade credit period, effect of inflation and time-dependent ordering cost etc.

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