An Order Level EOQ Inventory Model for Perishable Products with Price Reliant Demand, Time Reliant Holding Cost and Shortages

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Abstract

In present climate of competition, each prime company wants to increase the planning of pricing in order to get more profit; controlling the cost of inventories is one of the finest policies of it. The present research study elaborates an order level inventory policy for perishable items where the rate of decay varies with time directly. The holding and ordering costs are taken the functions of time. The policies of optimality and decision rules are developed to optimize the total inventory cost,. The shortages are totally backlogged and allowed. A numerical example is given to explain and verify the study. In order to examine the effect of major parameters on decision-making, a sensitivity analysis is performed. Eventually, conclusions are presented along with some managerial perceptions.

Keywords: Perishable products, demand, deterioration, holding cost, shortages.

I. Introduction

In the present era mathematical modelling is an urgent need in different areas of research and development. The study of inventory control theory is not possible without mathematical ideas and modelling. To minimize the inventory related total cost, it is important for the management to agree how much and when to produce or order and it is much important when there is deterioration or decay in the inventory. The meaning of deterioration or decay is the damage or loss of the real value of the inventory, which results a big loss and this loss cannot be ignored. Many products such as flowers, medicines, vegetables, cosmetic items, electronic equipments facing the problem of deterioration and lost their values in the period of normal storage. In this study of inventory controls, the optimal inventory policies are determined for such type of perishable items and the total loss is to be minimized. In the study of inventory theory demand is the main factor, demand may be dependent and independent. If the demand of an inventory of a product depends to another product then it is dependent otherwise independent. In many inventory systems the rate of demand is supposed as constant, but in case of present solid goods it not feasible to consider constant demand always. The different rate of demand may be considered such as stock dependent, cost dependent, time dependent, selling price dependent etc. In our study an order level inventory policy for decaying products is developed with selling price dependent rate of demand as there is a big role of selling price demand in present business environment.

II. Literature Review

In [25], authors presented a research article on economic lot size inventory policy with selling price related demand by considering freight and quantity discounts. In [28], authors proposed a research article for ameliorating items with selling price related rate of demand. In [29], authors studied an inventory policy with time and list price related demand. In maximum models, the cost of holding is well-known and considered as constant but it is not always constant; many studies are there where cost of holding is taken as variable. In the study of EOQ models, many researchers such as in [1] and [2]. The researchers in [11] and [22] used various functions for holding cost. In article [13], researchers developed some refined policies for linear type of demands. In articles [19] and [21] EOQ policies for perishable products, presented with time- related demand.

In the present study an order level inventory policy for perishable products is presented for a system of sole warehouse. Rate of demand is considered as a function of list price (selling). Here the shortages are completely backlogged and allowed. The purpose of the current study is to present an inventory policy for deteriorating items with selling price related demand and time related holding cost with shortages. There are two types of selling price related demand one is linear and other is exponential. If there are uniform changes in the rate of demand of any item per unit selling price, then it is linear selling price dependent demand. But in general it is not noticed in the present era market. Other is exponentially selling price dependent demand which is also unrealistic as the rate of exponential change is really on top level and rarely such high exponential rate of change exists in the market. Therefore researchers have an alternate approach of more realistic selling price dependent demand. A brief literature review working with selling price dependent demands is given as follow.

In the study of inventory models, researchers take interest mainly on two factors first one is the rate of deterioration and the second one is variation in rate of demand with time In [3], authors presented as cheme of approximate solution in all cases of deterministic and time related pattern of demand. In [7], authors presented an inventory model for a demand of linear trend without shortages but the calculation of the solution was very complicate. Researchers in [12], extended the model [7] and presented an analytical study for positive linear pattern demand. In [9] , the authors also worked on the model of [7] and determined an exact formula for EOQ model with demand which increases linearly.

In [8],an easy solution is provided to adjust the EOQ for both the cases of linear demand pattern, either it is increasing or decreasing. But till now the consideration of shortages and deterioration were not included properly. In [10], authors studied an inventory procedure for decaying items where the demand was varying with time. Later, Sachan extended the same and covered the option of backlogging. In [17], a heuristic inventory model is developed to determine economic order quantities with constant rate of inventory deterioration and time proportional demand over time. In study [14], an inventory model is presented for replenishment policies for the items with deterministic demand nature, positive linear trend and shortages. In [15], the same work extended by Murdesh war to minimize the total inventory cost and given an analytic study to find the decision policy for the selection of times and with finite time-horizon. The same contribution was given by many researchers such as [16], [18], [20] and [23].In [4], a model is presented by using two parameters Weibull and relaxed the assumption of constant rate of deterioration. Further in [5], the author extended the same model by using Weibull distribution of three-parameters. Later in [6], authors also used a Weibull distribution of two parameters for decay and presented an inventory scheme with fixed rate of replenishment. Recently many researchers are following these investigations and working on these. In [39],a model is developed to control the best capital within a working environment. In articles [24] and [27], models are presented for perishable items with time-related demand and shortages. In article [26], the structural properties of an inventory system is elaborated with various demands. In article [30], an extension of inventory policies with discrete holding costs. In articles [31], [32], [33] and [34}, researchers discussed various inventory methods for decaying items with price and stock- level related demands. In articles [35], [36], [37] and [38], researchers discussed various inventory schemes for decaying items with different demand rates and shortages. In articles [40] and [41], researchers studied inventory and rating strategy for defective products with sales returns exploration inaccuracies and incomplete reorders under inflationary environment. In article [42], optimal manufacturing transportation schemes are discussed for vendor and producer in a conditional closed-loop SPM for commutable delivery wrapping by usingmetaheuristics policy.

III. Useful Assumptions and Notations

Assumptions

To elaborate the present mathematical study, the following assumptions and limitation are used

1. A single warehouse is considered.
2. Shortages are taken backlogged and allowed.
3. The period of schedule is constant without supplying lead time.
4. The primary level of stock is raised to order level at the starting of every period.
5. When deterioration of the units has been received into the inventory then it is considered.
6. The rate of demand depends on selling price and given in the form of $M(x) = ux^{-v}, u, v > 0$,where x denotes the selling price.
7. At any time $t > 0$, the deterioration rate heed the Weibull distribution with two-parameters, given as $\phi = \mu\delta t^{(\delta-1)}$,where $\mu(0 < \mu < 1)$ and $\delta(\delta > 0)$ which is a shape parameter.

Notations

To elaborate the present mathematical study, the following notations and limitation are used through the study.

$M(x)$: Demand rate function.
ϕ : Rate of deterioration.
$I(t)$: The level of inventory at time t.
T : Time period of a schedule
S_m : Level of maximum Shortage
S_i : Level of primary stock at the starting of each inventory.
P_1 : Cost of shortage
P_2 : Per unit cost of shortage
H_c : Holding Cost
H_i : Per unit holding cost
D_c : Per unit cost of deterioration
M_ϕ : Total-deteriorating units

IV. Mathematical formulation and Analysis

There are two mainly factors, responsible for decreasing the inventory level. The first one is demand and the second one is deterioration. At time t = 0, the initial stock is S_i, the level of stock approaches to 0 at $t = t_1$. Next the shortages arise and reach to $t\,S_m$ level at t = T. In the interval (0, T) the generated differential equations for the inventory level I(t) at time t, are given as below

$$\frac{dI(t)}{dt} + \mu\delta t^{(\delta-1)} I(t) = -ux^{-v}, \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI(t)}{dt} = -ux^{-v}, \qquad t_1 \le t \le T \tag{2}$$

Using the boundary conditions $I(0) = S_i, I(t_1) = 0$ and $I(T)=S_m$, to solve the equations (1) and (2), We have

$$I(t) = \left[S_i - \mu x^{-\delta} \left(t + \frac{\mu}{\delta+1} t^{\delta+1} \right) \right] e^{-\mu t^{\delta}} \tag{3}$$

$$I(t) = u(t_1 - t)x^{-v} \tag{4}$$

Substitute $t = t_1$ in equation (4), we have

$$S_i = ux^{-v} \left(t_1 + \frac{\mu}{\delta+1} t^{\delta+1} \right) \tag{5}$$

Now, substitute t =T in equation (5), we have

$$S_m = u(T - t_1)x^{-v} \tag{6}$$

Now we find the total deteriorating units in the interval (0,T)

$$\begin{aligned} M_\phi = \int_0^{t_1} \phi I(t)dt \; &= S_i \left(\mu t_1^{\delta} - \frac{\mu^2}{2} t_1^{\delta} \right) \\ &- ux^{-v} \left[\frac{\mu\delta}{(\delta+1)} t_1^{\delta+1} - \frac{\mu^2\delta}{(\delta+1)(2\delta+1)} t_1^{2\delta+1} - \frac{\mu^3\delta}{(\delta+1)(3\delta+1)} t_1^{3\delta+1} \right] \end{aligned} \tag{7}$$

Now the deteriorating cost DC= $D_c M_\phi$

$$\begin{aligned} &= D_c S_i \left(\mu t_1^{\delta} - \frac{\mu^2}{2} t_1^{\delta} \right) \\ &- ux^{-v} D_c \left[\frac{\mu\delta}{(\delta+1)} t_1^{\delta+1} - \frac{\mu^2\delta}{(\delta+1)(2\delta+1)} t_1^{2\delta+1} - \frac{\mu^3\delta}{(\delta+1)(3\delta+1)} t_1^{3\delta+1} \right] \end{aligned} \quad .(8)$$

During the period (0,T), the holding cost is given as

$$\begin{aligned} H_c = \int_0^{t_1} I(t)dt \; &= H_i S_i \left(t_1 - \frac{\mu}{(\delta+1)} t_1^{\delta+1} \right) \\ &- uH_i x^{-v} D_c \left[\frac{t_1}{2} - \frac{\mu\delta}{(\delta+1)(\delta+1)} t_1^{\delta+2} - \frac{\mu^2}{(\delta+1)(2\delta+2)} t_1^{2\delta+2} \right] \end{aligned} \tag{9}$$

Now the cost of shortages is given as

$$P_1 = P_2 \left(-\int_{t_1}^{T} I(t)dt \right) = uP_2 x^{-v} \left(\frac{T^2 + t_1^2}{2} - Tt_1 \right) \tag{10}$$

Now the total inventory cost is given as

$$TIC = DC + H_c + P_1$$

$$
\begin{aligned}
&= D_c S_i\left(\mu t_1^{\delta} - \frac{\mu^2}{2} t_1^{\delta}\right) \\
&- u x^{-v} D_c\left[\frac{\mu\delta}{(\delta+1)} t_1^{\delta+1} - \frac{\mu^2\delta}{(\delta+1)(2\delta+1)} t_1^{2\delta+1} - \frac{\mu^3\delta}{(\delta+1)(3\delta+1)} t_1^{3\delta+1}\right] \\
&+ H_i S_i\left(t_1 - \frac{\mu}{(\delta+1)} t_1^{\delta+1}\right) - u H_i x^{-v} D_c\left[\frac{t_1}{2} - \frac{\mu\delta}{(\delta+1)(\delta+1)} t_1^{\delta+2} - \frac{\mu^2}{(\delta+1)(2\delta+2)} t_1^{2\delta+2}\right] \\
&+ u P_2 x^{-v}\left(\frac{T^2 + t_1^2}{2} - T t_1\right)
\end{aligned}
\tag{11}
$$

Equation (11) gives the total inventory cost associated with model.

V. Numerical Verification and Sensitivity Analysis

In this section, the present study is illustrated by using the following numerical example
Consider an order level inventory scheme for decaying products with the following parameter values
u=10, v =1, μ =0.0052, δ =0.41, x =6.00, H_i =5.00, P_2 =4.00, D_c =2.00 Substitute these parameters values, we get, S_i =29.9879 = 30units (units approximately) and the total inventory cost TIC=2967.97

Table: 1*Sensitivity analysis of the Model*

Change in parameter Values		S_i	t_1	TIC
H_i	2	44.86	26.60	1783.65
	3	38.47	22.88	2291.52
	4	33.64	19.99	2671.21
	5	29.90	17.82	2968.38
	6	26.87	15.92	3205.12
P_2	1	11.16	6.76	1112.54
	2	19.20	11.41	1907.22
	3	25.21	14.87	2503.21
	4	29.82	17.77	2967.25
	5	33.45	20.00	3204.25
D_c	0.5	29.87	17.72	2967.91
	1	29.87	17.72	2968.05
	1.5	29.87	17.72	2968.12
	2	29.87	17.72	2968.34
	2.5	29.87	17.72	2968.46
T	25	18.65	11.10	1159.12
	30	22.40	13.39	1669.42
	35	26.15	15.51	2272.55
	40	29.86	17.70	2968.25
	45	33.59	19.92	3757.56
X	3	59.57	17.74	5936.72
	4	44.83	17.74	4453.44
	5	35.86	17.74	3562.10
	6	29.66	17.74	2968.32
	7	25.62	17.74	2544.32

The following graphs are showing the impact of different parameters on total inventory cost.

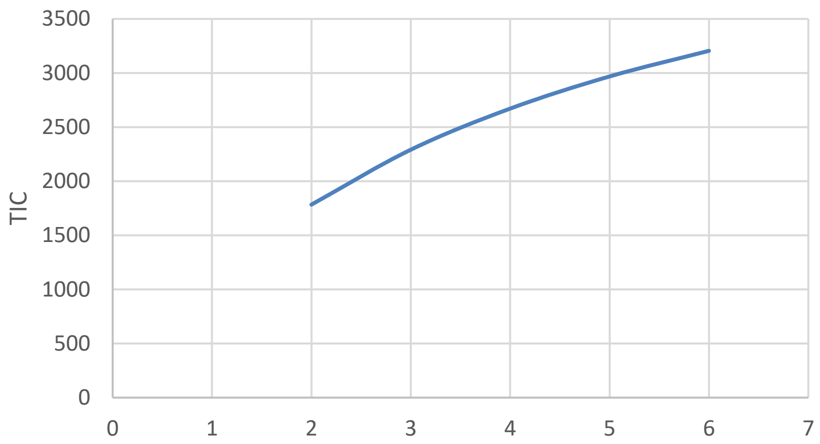

Figure-01:Change in parameter Hi

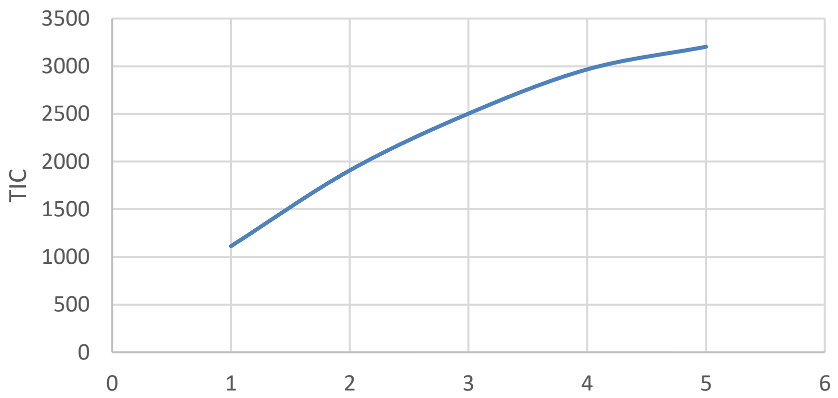

Figure-2: Change in parameter P2

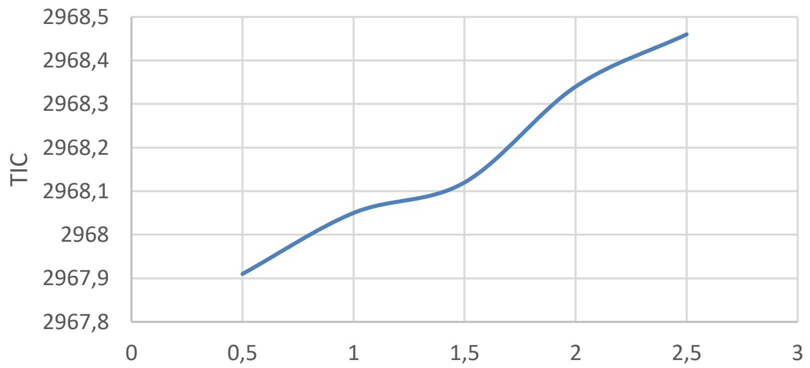

Figure-03: Change in parameter Dc

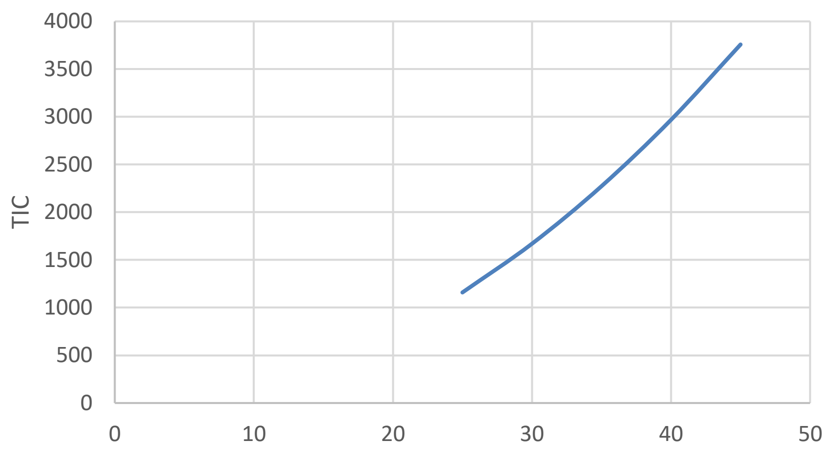

Figure-04: Change in parameter T

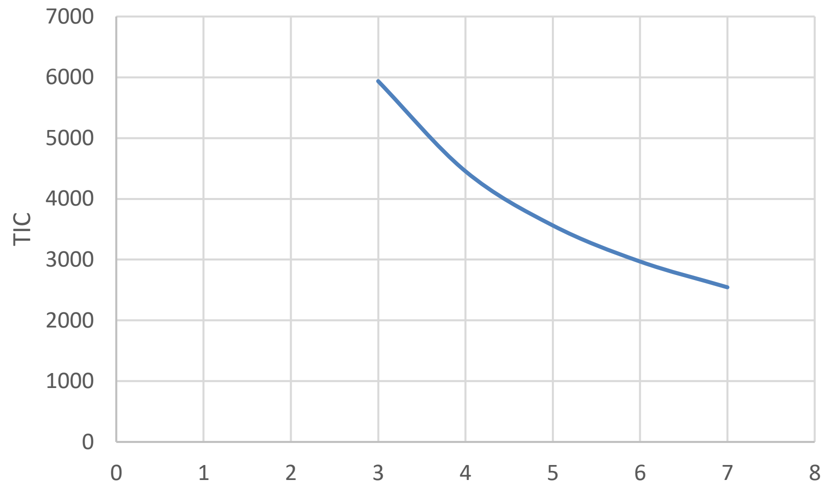

Figure-5: Change in parameter X

Observations

The observations of the above table are as follows.

1. The TIC i.e cost of total inventory increases as the shortage cost (P_2) per unit increases.
2. The (TIC) increases with the holding cost (H_i) (per unit).
3. The (TIC) increases with the time period (T).
4. The (TIC) decreases as the price (x) changes.
5. After increases in deteriorating cost there is a little change in TIC.

V. Conclusion and Future Scope

This study presents an order level inventory EOQ policy for deteriorating items. The rate of demand which is deterministic is taken as a function of price (selling). Here the shortages are considered fully backlogged and allowed in this current study. Also the cost of holding is taken as the function of time. In many conditions the stock out cannot be ignored as many stocked items give a high value of profit as comparison to its backorder cost. Decay is a natural process in any system of inventories and it is always necessary to consider it, so it also considered in the present study. The study is verified by a numerical example. Sensitivity is performed to observe the behaviour of the decision variables.

This study can be extended by inculcating other constraints and different preservation technologies. It can be extended for inventory models with two warehouses system in which PT can be used in any of the warehouses. Further, it can be extended for order quantity of production and to study the models for perishable items by taking different demand rates and costs.

References

[1]. Naddor, E. (1966). Inventory Systems Wiley, New York.

[2]. Van Der Veen, B. (1967). Introduction to the Theory of Operational Research. Philip Technical Library, Springer-Verlag, New York.

[3]. Silver, E. A. and Meal, H. C., (1969). A simple modification of the EOQ for the case of a varying demand rate. Production of Inventory Management, 10(4), 52-65.

[4]. Covert, R. P. and Philip, G. C., (1973). An EOQ model for items with Weibull distribution deterioration. AIIE Transactions, 5, 323-326.

[5]. Philip, G. C.(1974). A generalized EOQ model for items with Weibull distribution deterioration. AIIE Transactions, 6, 159-162.

[6]. Misra, R. B. (1975). Optimum production lot-size model for a system with deteriorating inventory. International Journal of Productions Research, 13, 495-505.

[7]. Shah, Y. K. and Jaiswal, M. C. (1977). An order-level inventory model for a system with constant rate of deterioration. Opsearch, , 14, 174-18 w ww.ajer.org Page 16

[8]. Silver, E. A. (1979). A simple inventory replenishment decision rule for a linear trend in demand. Journal of Operational Research Societ, 30, 71-75.

[9]. Ritchie, E. (1980). Practical inventory replenishment policies for a linear trend in demand followed by a period of steady demand. Journal of Operational Research Society, 31, 605-613.

[10]. Dave, U. and Patel, L. K.. (1981). (T, Si) policy inventory model for deteriorating items with time proportional demand. Journal of Operational Research Society, 32, 137-142.

[11]. Weiss, H.J., (1982), Economic Order Quantity models with nonlinear holding cost, European Journal of Operational Research, 9, 56-60.

[12]. Mitra, A. Cox, J. F. and Jesse, R.R. (1984). A note on determining order quantities with a linear trend in demand. Journal of Operational Research Society, 35, 141-144.

[13]. Goyal, S. K. (1986) . On improving replenishment policies f or linear trend in demand. Engineering Costs and Production Economics,, 10, 73-76

[14]. Deb, M. and Chaudhuri, K. (1987) A note on the heuristic for replenishment of trended inventories considering shortages. Journal of Operational Research Society, 38, 459-463.

[15]. Murdeshwar, T. M., (1988). Inventory replenishment policies for linearly increasing demand considering shortages: Journal of Operational Research Society, 39, 687-692.

[16]. Goyal, S. K. (1988). A heuristic for replenishment of trended inventories considering shortages. Journal of Operational Research Society, 39, 885-887.

[17]. Bahari-Kashani, H. (1989). Replenishment schedule for deteriorating items with time-proportional demand. Journal of Operational Research Society, 40, 75-81.

[18]. Goswami, A. and Chaudhuri, K. S. (1991) An EOQ model for deteriorating items with a linear trend in demand. Journal of Operational Research Society, 42(12), 1105-1110.

[19]. Xu, H. and Wang, H. (1991), An economic ordering policy model for deteriorating items with time-proportional demand. European Journal of Operational Research,, 24, 21-27.

[20]. Chung, K. J., and Ting, P. S.(1993) A heuristic for replenishment of deteriorating items with a linear trend in demand. Journal of Operational Research Society, 1993, 44(12), 1235-1241.

[21]. Chung, K. J. and Ting, P. S. (1994) On replenishment schedule for deteriorating items with time proportional demand. Production Planning and Control, 5, 392-396.

[22]. Goh, M. (1994), EOQ models with general demand and holding cost functions. European Journal of Operational Research. 73, 50-54.

[23]. Kim, D. H., (1995). A heuristic for replenishment of deteriorating items with linear trend in demand. International Journal of Production Economics, 39, 265-270.

[24]. Jalan, A. K., Giri, R. R. and Chaudhuri, K. S., (1996) EOQ model for items with Weibull distribution deterioration, shortages and trended demand. International Journal of System Science, 1996, 27(9), 851-855.

[25]. Burwell T.H., Dave D.S., Fitzpatrick K.E., Roy M.R., (1997). Economic lot size model for price-dependent demand under quantity and freight discounts, International Journal of Production Economics, 48(2), 141-155.

[26]. Jalan, A. K. and Chaudhuri, K. S., (1999). Structural properties of an inventory system with deterioration and trended demand. International Journal of System Science, 30(6), 627-633.

[27]. Lin, C., Tan, B. and Lee, W. C., (2000). An EOQ model f or deteriorating items with time-varying demand and shortages. International Journal of System Science, 31(3), 39-400.

[28]. Mondal, B., Bhunia, A.K., Maiti, M.,(2003), An inventory system of ameliorating items for price dependent demand rate, Computers and Industrial Engineering, 45(3), 443-456.

[29]. You, S.P., (2005), Inventory policy for products with price and time-dependent demands, Journal of the Operational Research Society, 56, 870-873.

[30].Timothy L. Urban. (2008) An extension of inventory models with discretely variable holding costs. International Journal of Production Economics 114:1, 399-403.

[31]. Suresh K. G., Chun-Tao Chang. (2009) Optimal ordering and transfer policy for an inventory with stock dependent demand. European Journal of Operational Research 196:1, 177-185.

[32]. Chun-Tao Chang, Shuo-Jye Wu, Li-Ching Chen. (2009) Optimal payment time with deteriorating items under inflation and permissible delay in payments. International Journal of Systems Science 40:10, 985-993.

[33]. Chun-Tao Chang, Yi-Ju Chen, Tzong-Ru Tsai, Wu Shuo-Jye. (2010) Inventory models with stock- and price-dependent demand for deteriorating items based on limited shelf space. Yugoslav Journal of Operations Research20:1, 55-69

[34].Sana, S.S., (2012) The EOQ model – A dynamical system. Applied Mathematics and Computation218:17, 8736-8749.

[35]. Guchhait. P, Manas. Kand Maiti. M (2013) Production-inventory models for a damageable item with variable demands and inventory costs in an imperfect production process. International Journal of Production Economics 144:1, 180-188.

[36]. Julia. P and Stefan.V(2014) Integrating deterioration and lifetime constraints in production and supply chain planning: A survey. European Journal of Operational Research 238:3, 654-674.

[37]. Chun-Tao Chang, Mei-Chuan Cheng, Liang-Yuh Ouyang. (2015) Optimal pricing and ordering policies for non-instantaneously deteriorating items under order-size-dependent delay in payments. Applied Mathematical Modelling 39:2, 747-763.

[38] Sarkar B, Guchhait. R, Cárdenas-BarrónL. E. (2019) How does an industry manage the optimum cash flow within a smart production system with the carbon footprint and carbon emission under logistics framework? International Journal of Production Economics,(SCIE), Vol 213, pp. 243-257.

[39] Sarkar. B,Guchhait R Sarkar B and Kim . N (2019) Impact of Safety factors and setup time reduction in a two-echelon supply chain management, Robotics and Computer-Integrated Manufacturing, (SCIE), Vol. 55(B), pp. 250 – 258.

[40].Khanna, A, Kishore. A, Sarkar. B and Jaggi C.K (2020) Integrated vendor-buyer strategies for imperfect production systems with maintenance and warranty policy, RAIRO – Operations Research, (SCIE), Vol. 54(2), pp. 435 – 450.

[41]. Khanna, A, Kishore. A ,Sarkar. B and Jaggi C.K (2020) Inventory and pricing decisions for imperfect quality items with inspection errors, sales returns, and partial backorders under inflation, RAIRO – Operations Research, (SCIE), Vol 54(1), pp. 287 – 306.

[42] Sarkar, B, Tayyab, M, Kim, N and Habib, M.S (2019) Optimal production delivery policies for supplier and manufacturer in a constrained closed-loop supply chain for returnable transport packaging through metaheuristic approach, Computers & Industrial Engineering, (SCIE), Vol 135, pp.987 – 1003.