

An Economical Order Quantity Inventory Model for Time-Dependent Deterioration Rate with Price Dependent Demand under Permitted Delay

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Abstract

In the current study, we develop an inventory model for the deteriorating items under the variable holding cost. The decline (or deterioration) rate depends on time; also, the demand rate depends on the price of the item. The shortage cost is linear in nature. The interest rate is a component of selling prices and shortages. Supplier offers a trade-credit period to the customer during which there is no interest charged, but upon the prescribed time limit expiry, the supplier will charge some interest. This study validates with a numerical example and explains the sensitivity analysis, and the optimal solution not only exists but also feasible.

Keywords: Inventory, permitted delay, deterioration and time varying holding cost.

I. Introduction

Generally, an inventory is typical of idle resources in organizations for upcoming use. Manufacturing organizations naturally have lists of raw materials, apparatuses, tools, equipment, semi-finished items, finished items, etc. In service organizations such as banks, financial institutions, hospitals, etc., the inventory consists of various items to be used in the multiple types of forms (for various banking operations), brochures, and pamphlets (for details of different banking policies, schemes, and instruments), etc. Banks also have inventories of currency notes and coins. Hospitals have medical equipment stocks such as a syringe, thermometer, drip, various one-time instruments used by medical professionals, etc. other accessories such as bandages, cotton, spirit, etc., to multiple medicines. Thus, no organization works without inventory.

Due to the effects of deterioration, this assumption is not always applicable to certain commonly used physical commodities such as wheat, rice, or some other sort of grains, vegetables, organic products, and so on. A few parts of these products have been damaged, decayed, gasifier, or influenced by various elements. These degraded parts are not so much lost to the stock management office. The decrease in items is one of the severe variables in any stock and creation framework. The acknowledgment of this factor provoked modellers to consider the corruption factor as one of the displaying angles. The weakening stock issue has been widely concentrated by various analysts now and then. Covert and Philip [4] proposed a model of things under a consistent rate of weakening; Deb and Chaudhuri [5] finished the absolute most recent work here, Aggarwal et al. [2] broadened

the Goyal model when the circumstance deteriorated. Goyal and Giri [9] projected a typical model affected by item weakening. Roy [17] proposed an inventory model for deteriorating items with price dependent demand and time-varying holding cost. Kumar et al. [14] built up a stock model that decayed after some time; Kingsman [11] developed the effect of payment rules on ordering and stockholding in purchasing. Aarya and Kumar [1] proposed a stock model with various restrictions at a constant decaying rate.

An EOQ model accepts that the retailer must compensate the provider following getting the merchandise in practical situations. Providers may regularly permit retailers to direct advance financing to expand requests or diminish stock. This implies the vendor will support the purchaser with a credit period to settle the sum owed, and during this period, the sum owed won't acquire any intrigue. Goyal [8] built up a stock model affected by professional credit; additionally, Goh [7] presented an EOQ model with general demand and holding cost functions; Teng et al. [18] established an optimum pricing and assembling strategy under permitted delay, Mondal et al. [16] demonstrated a model of upgrading objects under demand rate depends on price, Kumar et al. [13] built up a model for the diverse interest rates under exchange credit and in [12] a stock model of the incremental holding cost with the admissible instalment delay proposed, Weiss [19] presented an economic order quantity model with non-linear holding cost. Giri and Chaudhuri [6] presented a heuristic model for deteriorating items with shortages and time-varying demand and costs, and Yu [20] proposed an inventory policy for products with price and time-dependent demands.

In the field of inventory management, many authors use various other types of needs and factors. Some researchers believe that the demand rate is static, linear, price-related, and inventory-related. The actual target demand may be related to time, inventory, and price. Haley [10] offered Inventory policy and trade credit financing, Chapman et al. [3] developed Credit policy and inventory control, Kumar et al. [15] presented a model on preservation technology with trade credits under demand rate dependent on an advertisement, time and selling price.

In view of the above writing survey, we set up a degradation model. When the corruption rate is relative to time, and the holding cost is variable, the interest rate is an element of selling price and all-out deficiency, bringing about lack and total accumulation. On account of postponed instalment authorization, the interest rate is an element of the business cost, and we utilize a numerical example to check the model.

Assumptions and Notations:

Assumptions:

- Deterioration rate changes with time.
- Shortage is permitted and 'completely backlogged'.
- Demand function is a component of trade cost.
- Cost of ownership is linear.
- Renewal or 'replacement' rate is immediate.
- Leading- time is zero.
- Trade credit is permitted.

Notations:

- 1) C_1 : Shortage cost per unit time.
- 2) C_2 : Cost of an item per unit.
- 3) $\theta(t) = \theta t$: Deterioration rate.
- 4) $f(p) = (\alpha p^{-\beta}) > 0$ is the demand rate, where $\alpha, \beta > 0$.
- 5) $H(t) = (h + at)$, where $a > 0$, $h > 0$ is holding cost per unit time.
- 6) q : Order quantity per cycle.
- 7) p : Marketing charge per unit item.
- 8) M is the trade credit period.
- 9) A is the ordering cost.

- 10) T is the time period.
- 11) In period $0 < t < T_1$ the inventory is positive.
- 12) In time (T_1) the stock is drained because of the crumbling and request of the thing. At time (T_1) the stock goes to zero and shortage starts.
- 13) I_e : Interest received for each unit time.
- 14) I_p : Interest paid for each unit time with $I_p > I_e$.

II. Model Formulation and Solution

Modeling, and Solution of Proposed Model

$$\frac{dI(t)}{dt} + \theta(t) I(t) = -f(p), \quad 0 \leq t \leq T_1 \quad (1)$$

$$\frac{dI(t)}{dt} = -f(p), \quad T_1 \leq t \leq T \quad (2)$$

with initial condition $I(T_1) = 0$

Solution of equations (1) and (2) are given as follows:

$$I(t) = (\alpha p^{-\beta}) \left[(T_1 - t) + \theta \left(\frac{T_1^3}{6} - \frac{T^3}{3} - \frac{T_1 t^2}{2} \right) + \theta^2 \left(\frac{T_1^5}{40} - \frac{t^5}{15} - \frac{t^2 t_1^3}{12} + \frac{t_1 t^4}{8} \right) \right], \quad 0 \leq t \leq T_1 \quad (3)$$

$$\text{and } I(t) = -(\alpha p^{-\beta})(t - T_1) = (\alpha p^{-\beta})(T_1 - t), \quad T_1 \leq t \leq T \quad (4)$$

Holding cost

$$HC = \int_0^{T_1} H(t) I(t) dt \\ = (\alpha p^{-\beta}) h \cdot \left[\frac{T_1^2}{2} + \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90} \right] + a(\alpha p^{-\beta}) \left[\frac{T_1^3}{6} + \frac{\theta T_1^5}{40} + \frac{\theta^2 T_1^7}{336} \right] \quad (5)$$

Shortage cost

$$SC = -C_1 \int_{T_1}^T [-(t - T_1)f(p)] dt \\ = C_1 \frac{(\alpha p^{-\beta})}{2} (T - T_1)^2 \quad (6)$$

Stock loss due to deterioration

$$D = (\alpha p^{-\beta}) \int_0^{T_1} e^{-\frac{\theta t^2}{2}} dt - (\alpha p^{-\beta}) \int_0^{T_1} dt \\ = (\alpha p^{-\beta}) \left[\frac{\theta T_1^3}{6} + \frac{\theta^2 T_1^5}{40} \right] \quad (7)$$

Order quantity

$$q = C_2 \left[D + \int_0^T (\alpha p^{-\beta}) dt \right] \\ = C_2 (\alpha p^{-\beta}) \left[\frac{\theta T_1^3}{6} + \frac{\theta^2 T_1^5}{40} \right] + (\alpha p^{-\beta}) T \quad (8)$$

Presently, there are two prospects with respect to the delay period M of allowable deferral in installments.

Case I: $M \leq T_1$,

Case II: $M > T_1$

Case I: $M \leq T_1$: In this case, the interest payable for each period of unsold stock after the maturity date (M) is

$$IP_1 = C_2 I_p \int_0^{T_1} I(t) dt \\ = C_2 (\alpha p^{-\beta}) I_p \left[\left(\frac{T_1^2}{2} - \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90} \right) - \left(MT_1 - \frac{M^2}{2} \right) - \theta \left(\frac{MT_1^3}{6} - \frac{M^4}{12} - \frac{M^3 T_1}{6} \right) - \theta^2 \left(\frac{MT_1^5}{40} - \frac{M^6}{90} - \frac{M^3 T_1^3}{36} + \frac{M^5 T_1}{40} \right) \right] \quad (9)$$

In the calculation, the interest earned (IE_1) in each period is found as

$$IE_1 = C_2 I_e \int_0^{T_1} t \cdot f(p) dt$$

$$= C_2 I_e (\alpha p^{-\beta}) \frac{T_1^2}{2} \tag{10}$$

Total profit function is

$$\begin{aligned} Z(T, T_1, p) &= p.f(p) - \frac{1}{T} [\text{OrderingCost} + \text{ShortageCost} + \text{HoldingCost} + \text{OrderQuantity} + IP_1 - IE_1] \\ &= (\alpha p^{1-\beta}) - \frac{1}{T} \left[A + \frac{C_1(\alpha p^{-\beta})}{2} (T - T_1)^2 + (\alpha p^{-\beta})h \left\{ \frac{T_1^2}{2} + \frac{\theta T_1^4}{12} \right. \right. \\ &\quad \left. \left. + \frac{\theta^2 T_1^6}{90} \right\} + a(\alpha p^{-\beta}) \left\{ \frac{T_1^3}{6} + \frac{\theta T_1^5}{40} + \frac{\theta^2 T_1^7}{336} \right\} \right. \\ &\quad \left. + C_2(\alpha p^{-\beta}) \left\{ T + \frac{\theta T_1^3}{6} + \frac{\theta^2 T_1^5}{40} \right\} + C_2(\alpha p^{-\beta}) I_p \left\{ \left(\frac{T_1^2}{2} - \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90} \right) \right. \right. \\ &\quad \left. \left. - \left(MT_1 - \frac{M^2}{2} \right) - \theta \left(\frac{MT_1^3}{6} - \frac{M^4}{12} - \frac{M^3 T_1}{6} \right) \right. \right. \\ &\quad \left. \left. - \theta^2 \left(\frac{MT_1^5}{40} - \frac{M^6}{90} - \frac{M^3 T_1^3}{36} + \frac{M^5 T_1}{40} \right) \right\} - \frac{C_2 I_e (\alpha p^{-\beta}) T_1^2}{2} \right] \end{aligned}$$

Let $T_1 = aT$; $0 < a < 1$

Thus, total profit is

$$\begin{aligned} Z(T, p) &= (\alpha p^{1-\beta}) - \frac{1}{T} \left[A + \frac{C_1(\alpha p^{-\beta})}{2} (1-a)^2 T^2 + h(\alpha p^{-\beta}) \left\{ \frac{a^2 T^2}{2} + \frac{\theta a^4 T^4}{12} \right. \right. \\ &\quad \left. \left. + \frac{\theta^2 a^6 T^6}{90} \right\} + a(\alpha p^{-\beta}) \left\{ \frac{a^3 T^3}{6} + \frac{\theta a^5 T^5}{40} + \frac{\theta^2 a^7 T^7}{336} \right\} + C_2(\alpha p^{-\beta}) \left\{ T + \frac{\theta a^3 T^3}{6} + \frac{\theta^2 a^5 T^5}{40} \right\} \right. \\ &\quad \left. + C_2(\alpha p^{-\beta}) I_p \left\{ \left(\frac{a^2 T^2}{2} - \frac{\theta a^4 T^4}{12} + \frac{\theta^2 a^6 T^6}{90} \right) - \left(MaT - \frac{M^2}{2} \right) - \theta \left(\frac{Ma^3 T^3}{6} - \frac{M^4}{12} - \frac{M^3 aT}{6} \right) \right. \right. \\ &\quad \left. \left. - \theta^2 \left(\frac{Ma^5 T^5}{40} - \frac{M^6}{90} - \frac{M^3 a^3 T^3}{36} + \frac{M^5 aT}{40} \right) \right\} - \frac{C_2 I_e (\alpha p^{-\beta}) a^2 T^2}{2} \right]. \end{aligned}$$

Presently, our goal is to optimize $Z(T, p)$. The essential situations for expanding the profit are

$$\frac{\partial Z(T, p)}{\partial T} = 0 \text{ and } \frac{\partial Z(T, p)}{\partial p} = 0$$

We obtain

$$\begin{aligned} &\left[-\frac{A}{T^2} + \frac{C_1(\alpha p^{-\beta})(1-a)^2}{2} + h(\alpha p^{-\beta}) \left\{ \frac{a^2}{2} + \frac{\theta a^4 T^2}{4} + \frac{\theta^2 a^6 T^4}{18} \right\} \right. \\ &\quad \left. + a(\alpha p^{-\beta}) \left\{ \frac{a^3 T}{3} + \frac{\theta a^5 T^3}{10} + \frac{\theta^2 a^7 T^5}{56} \right\} \right. \\ &\quad \left. + C_2(\alpha p^{-\beta}) \left\{ \frac{\theta a^3 T}{3} + \frac{\theta^2 a^5 T^3}{10} \right\} \right. \\ &\quad \left. + C_2(\alpha p^{-\beta}) I_p \left\{ \left(\frac{a^2}{2} - \frac{\theta a^4 T^2}{4} + \frac{5\theta^2 a^6 T^4}{4} \right) - \left(\frac{M^2}{2T^2} \right) - \theta \left(\frac{Ma^3 T}{3} + \frac{M^4}{12T^2} \right) \right. \right. \\ &\quad \left. \left. - \theta^2 \left(\frac{Ma^5 T^3}{10} + \frac{M^6}{90T^2} - \frac{M^3 a^3 T}{18} \right) - \frac{C_2 I_e (\alpha p^{-\beta}) a^2}{2} \right\} = 0 \right. \tag{11} \end{aligned}$$

and

$$\begin{aligned} &\alpha(1-\beta)p^{-\beta} - \frac{1}{T} \left[-\frac{C_1(\alpha\beta p^{-\beta-1})(1-a)^2}{2} T^2 - h(\alpha\beta p^{-\beta-1}) \left\{ \frac{a^2 T^2}{2} + \frac{\theta a^4 T^4}{12} + \frac{\theta^2 a^6 T^6}{90} \right\} \right. \\ &\quad \left. - a(\alpha\beta p^{-\beta-1}) \left\{ \frac{a^3 T^3}{6} + \frac{\theta a^5 T^5}{40} + \frac{\theta^2 a^7 T^7}{336} \right\} - C_2(\alpha\beta p^{-\beta-1}) \left\{ T + \frac{\theta a^3 T^3}{6} + \frac{\theta^2 a^5 T^5}{40} \right\} \right. \\ &\quad \left. - C_2 I_p (\alpha\beta p^{-\beta-1}) \left\{ \left(\frac{a^2 T^2}{2} - \frac{\theta a^4 T^4}{12} + \frac{5\theta^2 a^6 T^6}{4} \right) - \left(MaT - \frac{M^2}{2} \right) \right. \right. \\ &\quad \left. \left. - \theta \left(\frac{Ma^3 T^3}{6} - \frac{M^4}{12} - \frac{aTM^3}{6} \right) - \theta^2 \left(\frac{Ma^5 T^5}{40} - \frac{M^6}{90} - \frac{M^3 a^3 T^3}{36} + \frac{M^5 aT}{40} \right) \right\} + \frac{C_2 I_e (\alpha\beta p^{-\beta-1}) a^2 T^2}{2} \right] = 0 \tag{12} \end{aligned}$$

Case II: $M > T_1$

For this situation, the interest unpaid for each cycle is zero, when $T_1 < M \leq T$ on the grounds that the provider can be forked over the required funds at the time M , the allowable delay. In this manner, the premium received in each cycle is the premium received during the great stock time frame in addition to the premium earned from the money contributed during the timeframe (T_1, M) after the

stock is depleted at time T_1 , and it is given by

$$\begin{aligned}
 IE_2 &= C_2 I_e \int_0^{T_1} f(p).tdt + C_2 I_e (M - T_1) \int_0^{T_1} f(p) dt \\
 &= C_2 I_e f(p) \frac{T_1^2}{2} + C_2 I_e (M - T_1) f(p). T_1 \\
 IE_2 &= C_2 I_e (\alpha p^{-\beta}) T_1 \left(M - \frac{T_1}{2} \right)
 \end{aligned} \tag{13}$$

Total profit is

$$\begin{aligned}
 Z(T, T_1, p) &= p.f(p) - \frac{1}{T} [A + \text{ShortageCost} + \text{HoldingCost} + q + IP_2 - IE_2] \\
 &= (\alpha p^{1-\beta}) - \frac{1}{T} \left[A + \frac{C_1(\alpha p^{-\beta})}{2} (T - T_1)^2 + (\alpha p^{-\beta}) h \left\{ \frac{T_1^2}{2} + \frac{\theta T_1^4}{12} + \frac{\theta^2 T_1^6}{90} \right\} \right. \\
 &\quad \left. + a(\alpha p^{-\beta}) \left\{ \frac{T_1^3}{6} + \frac{\theta T_1^5}{40} + \frac{\theta^2 T_1^7}{336} \right\} - C_2 I_e (\alpha p^{-\beta}) T_1 \left(M_1 - \frac{T_1}{2} \right) \right]
 \end{aligned} \tag{14}$$

Let $T_1 = aT$; $0 < a < 1$.

Thus, the profit function is

$$\begin{aligned}
 Z(T, p) &= (\alpha p^{1-\beta}) - \frac{1}{T} \left[A + \frac{C_1(\alpha p^{-\beta})}{2} (1 - a)^2 T^2 + h(\alpha p^{-\beta}) \left\{ \frac{a^2 T^2}{2} + \frac{\theta a^4 T^4}{12} + \frac{\theta^2 a^6 T^6}{90} \right\} \right. \\
 &\quad \left. + a(\alpha p^{-\beta}) \left\{ \frac{a^3 T^3}{6} + \frac{\theta a^5 T^5}{40} + \frac{\theta^2 a^7 T^7}{336} \right\} - C_2 I_e a T (\alpha p^{-\beta}) \left(M_1 - \frac{aT}{2} \right) \right]
 \end{aligned} \tag{15}$$

Presently, our goal is to optimize $Z(T, p)$. The essential conditions for expanding the profit are

$$\frac{\partial Z(T,p)}{\partial T} = 0 \text{ and } \frac{\partial Z(T,p)}{\partial p} = 0.$$

$$\begin{aligned}
 \Rightarrow &\left[-\frac{A}{T^2} + \frac{C_1(\alpha p^{-\beta})(1-a)^2}{2} + h(\alpha p^{-\beta}) \left\{ \frac{a^2}{2} + \frac{\theta a^4 T^2}{4} + \frac{\theta^2 a^6 T^4}{18} \right\} \right. \\
 &\quad \left. + a(\alpha p^{-\beta}) \left\{ \frac{a^3 T}{3} + \frac{\theta a^5 T^3}{10} + \frac{\theta^2 a^7 T^5}{56} \right\} - C_2 a (M - aT) (\alpha p^{-\beta}) \right] = 0
 \end{aligned} \tag{16}$$

$$\begin{aligned}
 \text{And } \alpha(1-\beta)p^{-\beta} - \frac{1}{T} &\left[-\frac{C_1(1-a)^2(\alpha\beta p^{-\beta-1})}{2} T^2 - h(\alpha\beta p^{-\beta-1}) \left\{ \frac{a^2 T^2}{2} + \frac{\theta a^4 T^4}{12} + \frac{\theta^2 a^6 T^6}{90} \right\} \right. \\
 &\quad \left. - a(\alpha\beta p^{-\beta-1}) \left\{ \frac{a^3 T^3}{6} + \frac{\theta a^5 T^5}{40} + \frac{\theta^2 a^7 T^7}{336} \right\} + C_2 I_p a T (\alpha\beta p^{-\beta-1}) \left(M - \frac{aT}{2} \right) \right] = 0
 \end{aligned} \tag{17}$$

Solving equations (11) to (17), we find the T^* , and p^* . Also, optimize the function $Z^*(T, p)$ using the essential conditions for maximizing of $Z(T, p)$ are

$$\frac{\partial^2 Z(T,p)}{\partial T^2} < 0, \quad \frac{\partial^2 Z(T,p)}{\partial p^2} < 0 \text{ and } \frac{\partial^2 Z(T,p)}{\partial T^2} \cdot \frac{\partial^2 Z(T,p)}{\partial p^2} - \frac{\partial^2 Z(T,p)}{\partial T \partial p} > 0 \text{ at } (T^*, p^*).$$

III. Illustrative Example

Table-1: Sensitivity analysis table

Parameter	Parameter Changing (%)	Value of Parameter	T	p	Z	Changing in Z*(%)
A	-50%	125	54	13759	413	-8%
	-25%	187.5	61	16842	426	-5%
	0%	250	69	24434	450	0%
	25%	312.5	72	24444	449	-0.2%
	50%	375	74	25061	450.77	0.17%
α	-50%	50	72	14394	205.91	-54%
	-25%	75	69	17424	319.85	-29%
	0%	100	69	24434	450	0%
	25%	125	64	20659	549.54	22.1%
	50%	150	68	35908	683.68	51.93%
β	-50%	0.425	30	182990	106120	23482%
	-25%	0.6375	60	151030	7534	1574%

	0%	0.85	69	24434	450	0%
	25%	1.0625	58.8	1546.4	56	-87.6%
	50%	1.275	62.48	580.2293	11.84	-97.37%
M	-50%	0.0325	67	20658	438.97	-2%
	-25%	0.04875	69	24434	450.5349	0%
	0%	0.065	69	24434	450	0%
	25%	0.08125	69	24434	450.5357	0.1%
	50%	0.0975	69	24434	450.5361	0.12%
H	-50%	0.25	69	24493	450.71	0%
	-25%	0.375	72	32079	469.84	4%
	0%	0.5	69	24434	450.5353	0%
	25%	0.625	67	20659	438.97	-2.6%
	50%	0.75	67	20658	438.96	-2.57%
C ₂	-50%	10	70	15247	419.29	-7%
	-25%	15	67	46841	497.62	11%
	0%	20	69	24434	450	0%
	25%	25	66	24444	450.33	0.1%
	50%	30	61	16914	425.0958	-5.53%
C ₁	-50%	0.65	71	28270	461.04	2%
	-25%	0.975	69	24443	450.69	0%
	0%	1.3	69	24434	450	0%
	25%	1.625	66	20685	438.87	-2.5%
	50%	1.95	69	28271	460.48	2.33%
a	-50%	0.1	56.2	2230.5	306	-32%
	-25%	0.15	67.8	6162.7	363.28	-19%
	0%	0.2	69	24434	450	0%
	25%	0.25	53	17454	426.89	-5.1%
	50%	0.3	44	176669	606.61	34.80%
θ	-50%	0.01	0.75	12227	608.86	35%
	-25%	0.015	69	1.5294	419.18	-7%
	0%	0.02	69	24434	450	0%
	25%	0.025	62	20659	4388.72	875.3%
	50%	0.03	62	28875	461.83	2.63%
I _p	-50%	0.08	70	15279	419.14	-7%
	-25%	0.12	67	16844	425.39	-5%
	0%	0.16	69	24434	450	0%
	25%	0.2	69	3.2079	469.711	4.4%
	50%	0.24	65	24434	450.33	0.07%
I _e	-50%	0.065	70	28257	460.7	2%
	-25%	0.0975	67	20659	438.96	-2%
	0%	0.13	69	24434	450	0%
	25%	0.1625	72	32103	469.9	4.4%
	50%	0.195	67	20658	439.01	-2.44%

We applied our program to a major cosmetics retailer store in a mega-city to explain the proposed model. In advertising products, including TV/Internet, sunscreens, powders, lipsticks, and baby products, these products were initially promoted, but the products' sales declined slightly. For the validation of the model numerically, we consider the values of the parameters are as follows:

$A = 250, \alpha = 100, \beta = 0.85, M = 0.065, C_2 = 20, H = 0.5, C_1 = 1.3, a = 0.2, \theta = 0.02, I_p = 0.16, I_e = 0.13$.
Using Mathematica software, we get the optimal values are as follows:
(Z^*) = 450, $p^* = 24434$, and $T^* = 69$.

IV. Discussion

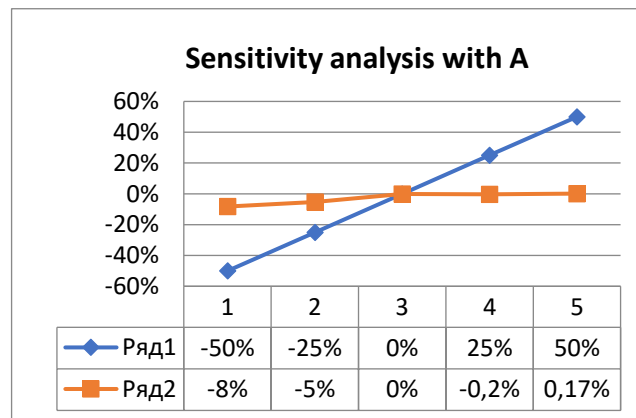
For the justification of the proposed model numerically, data for the numerical section, and using Mathematica software, we obtained the optimal values are as follows:

Profit (Z^*) = 450, Price (p^*) = 24434, and Time (T^*) = 69.

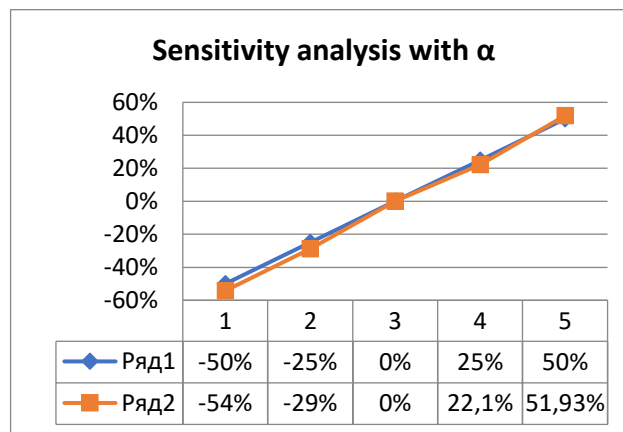
An affectability investigation is performed to contemplate the impacts of parameter values on the optimal solution. The Sensitivity analysis table shows the consequences of the model. The following decisions are as follows:

- ❖ If the parameters β, C_2 , and I_e are changed at the rate of 50%, 25%, -25%, and -50%, then the profit function Z decreases.
- ❖ If the parameters $A, \alpha, M, H, C_1, a, \theta$, and I_p , are changes at the rate of 50%, 25%, -25%, and -50%, then profit function Z increases.

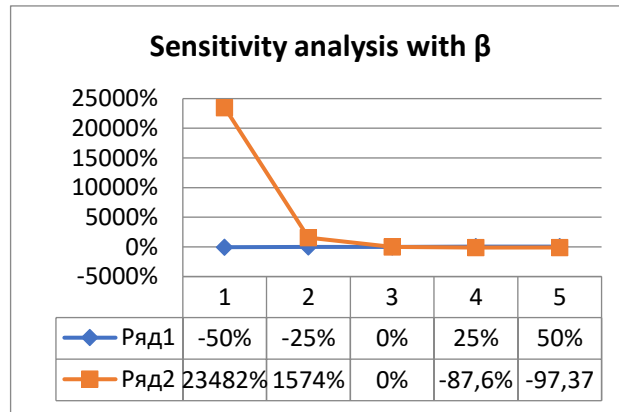
The following figures shows the affectability investigation as for parameters: $A, \alpha, \beta, M, H, C_2, C_1, a, \theta, I_p, I_e$:



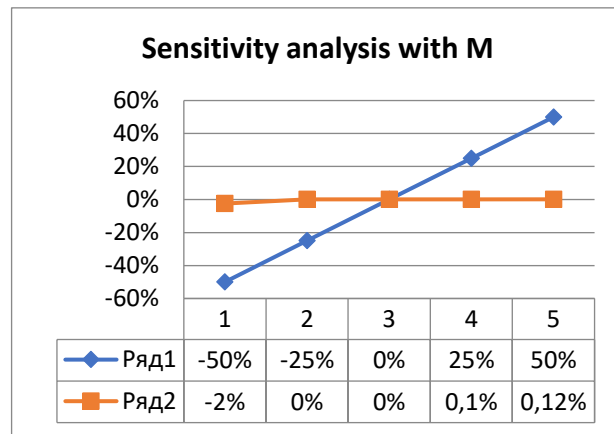
(Figure 1: *w. r. to parameter A*)



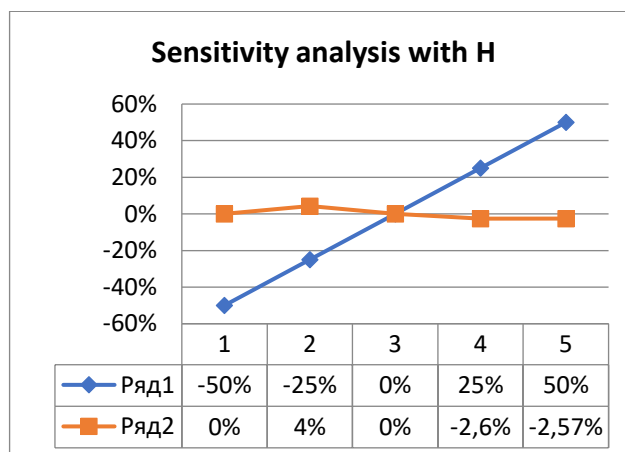
(Figure 2: *w. r. to parameter alpha*)



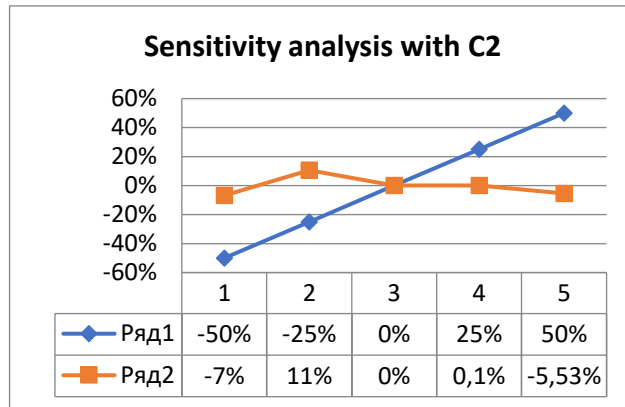
(Figure 3: *w. r. to parameter β*)



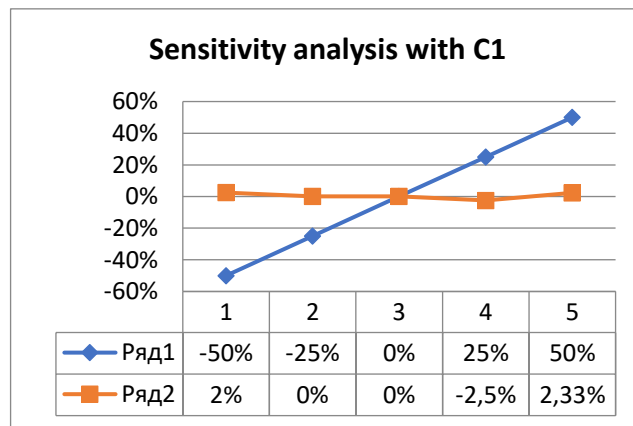
(Figure 4: *w. r. to parameter M*)



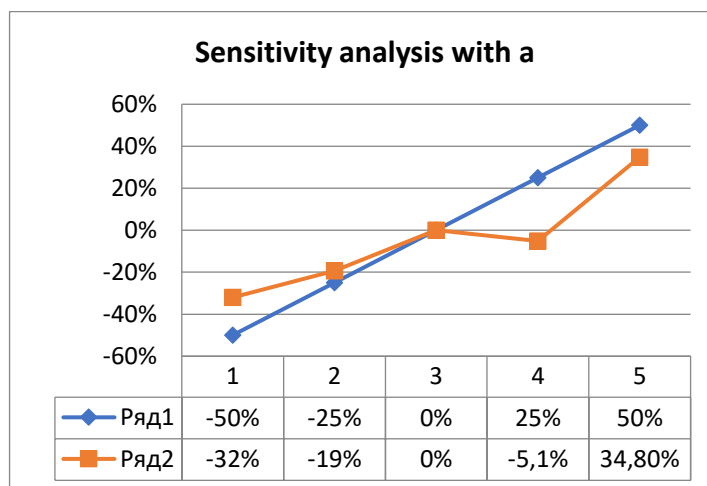
(Figure 5: *w. r. to parameter H*)



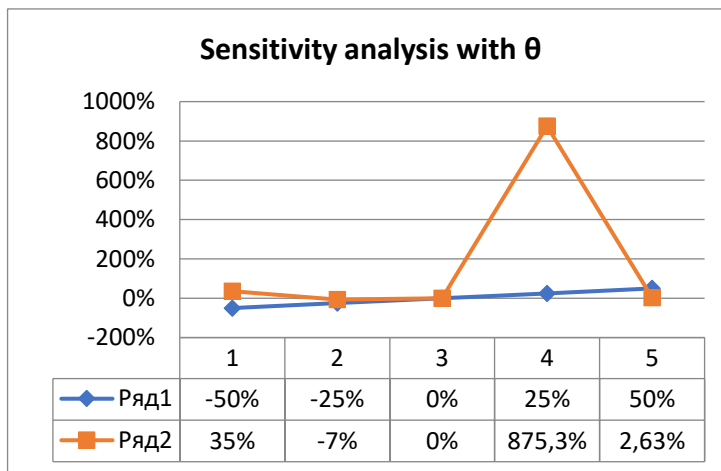
(Figure 6: w. r. to parameter C₂)



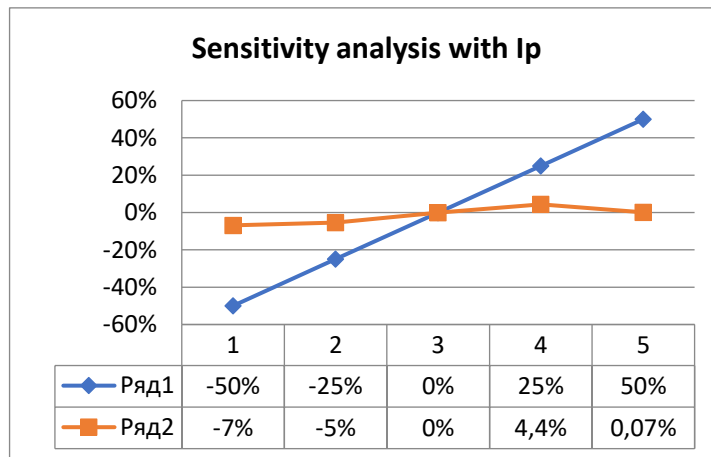
(Figure 7: w. r. to parameter C₁)



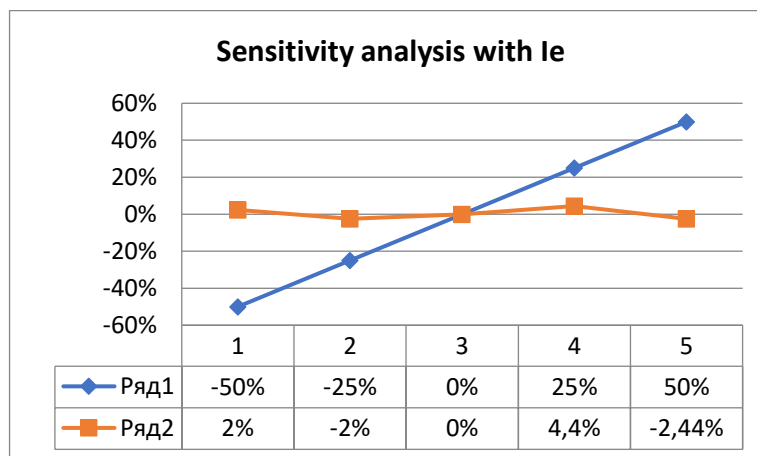
(Figure 8: w. r. to parameter a)



(Figure 9: w. r. to parameter θ)



(Figure 10: w. r. to parameter I_p)



(Figure 11: w. r. to parameter I_e)

V. Conclusions

This study established an inventory model for the linear decline rate and the price-related demand rate under the holding cost. Shortages are permitted and are entirely reproduced and allowed a delayed payment period. Supplier offers a credit limit to the customer during which there is no interest charged, but the supplier will charge some interest upon the prescribed time limit expiry. However, the retailer has stored to make the installment, choosing to profit by as far as possible. Also, approve the model with the assistance of a mathematical model and study the sensitivity analysis. This model further developed with the inflation rate.

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