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A Production Inventory Model for Deteriorating Items with Effect of Price Discount under the Stock Dependent Demand

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Abstract

In most of the production-based model, the effect of deterioration at initial stage of the production is not assumed even if assumed than considered to be constant, in few productions-based model deteriorations is assumed time-dependent but during initial level of production its effect was not considered. For a product whose life is high this phenomenon is justifying but for those products whose life is very less this is not faring to not assume the effect of deterioration. In the present study, author have considered the effect of deterioration at the initial level of the model and for the rest cycle of the model as well. After a fixed time, a price discount is also offered for partially deteriorated items where a discount rate is offer at the original one, Shortages are not allowed, production is assumed as demand dependent, and demand is assumed as stock dependent where holding cost of the inventory is assumed as a function of time. A numerical example is also discussed.

Keywords: Stock Dependent Demand, Economic Production Quantity, Time dependent Holding Cost.

I. Introduction

Present study contains an inventory control model that decides the amount of a single commodity that satisfies the market requirement over an infinite planning horizon.

One of the important aspects of the production inventory model is deterioration. In previous research, so many researchers have been taken into an account on deterioration. The four major concerns of any manufacturing firm are production, planning, quality, and maintenance. In the present time, each business has a competition day by day for a superior quality item and provides better customer services. Because of globalization and new specialized advancement, the manufacturing infrastructure likewise changes quickly.

In this paper, the non-instantaneous deterioration model under production policy has been considered. Here the deterioration starts with the beginning time (from one day) because the proper storage condition is not good, so it will start to decline. In this circumstance to encourage the sales with more attractive offer on the decayed unit is known as the rebate rate. In the current paper, author studied and derived a production-based inventory model under the effect of stock dependent demand rate and fixed deterioration rate. Also, author have taken a linear holding cost. The production inventory system has completed in four stages in each cycle.

Author assume that the production starts with zero inventory level in each cycle. After a certain time, demands will be fulfilled then break the production. The collected inventory is then quietly empty out and after some time it reached zero due to the effect of demand and deterioration. The decaying item which loss self-original value sells at a discounted rate. When the inventory falls down to zero then one cycle is complete, the same process will be repeated in the whole process.

Assuming constant rate of deterioration [1] analyzed the effect of Economic Production Quantity model of decaying items, [2] elaborate on the production model of deteriorating items that minimize the total cost. Also [3] focused on the inventory model for the exponential demand rate. An approach based on inventory model developed by [4] and explained a multi-lot size inventory-based system with constant demand and production rates. For different demand rates for different stage of inventory [5], [6], [7], [8] and [9] studied the Economic Production Quantity model under the consideration of partial backlogged.

[10] studied a deteriorating inventory model with exponentially decreasing demand, where author have assumed a finite planning horizon in the model. [11] developed an Economic Production Quantity model with time-varying demand and partially backlogged model. [12] explored the inventory production-based inventory model by considering shortages. [13], investigated and formulated an optimal returned policy-based model by considering reverse logistics with backorders where [14] extended these models with new criteria of limited storage facility under inflation. In all inventory models, author seen that the product almost all items either software and hardware have fixed shelf life. Because of the expiration, new technology, time-consuming, more efforts, non-auto start, etc. substantial decay of inventory system cannot be neglected, it was another major feature of the real world. In this field, [15] developed an Economic Production Quantity model with constant demand and exponentially decaying items.

The stock decreases due to demand which is a function of the on-hand inventory and deterioration which is constant. It is seen that items have a lifetime which cessation when advantages become zero of the on-hand inventories. It is noticed that products have a lifetime which cessation when the benefit becomes zero. Considering the concept of permissible delay in payment, [16] introduced a perishable inventory model with a parabolic rate of demand along with partial backlogging.

For the product of low cycle [17] presented a quality consideration deteriorating inventory model. [18] considered a multivariate demand model for decaying items having shortages. [19] presented a demand dependent production inventory model with price-sensitive demand and shortages. In the last few decades researcher pay attention to the deterioration-based models, they considered the different deterioration rates for different environment conditions. Now author discuss the non – instantaneous deteriorating items in our study so many researchers contribute to this field, but author have discussed some of them.

[20] developed an inventory control model under stock dependent consummation rate. [21] also did work on Gupta’s model. [22] and considered different inventory models based on the time value of money and inflation. After that [23] developed the deterministic model where non-linear holding cost was considered for decay items with stock dependent consummation rate. [24] explored an inventory model with a stock dependent rate of demand and holding cost was time-dependent. After that [25] investigated the model in which demand and holding cost both are stock dependent.

[26] built up the echelon inventory model for decay products and assumed variable holding cost with stock dependent demand. In view of non-instantaneous deterioration, under the impact of inflation [27] analyzed and developed an inventory model with completely backlogged shortages

where price and stock dependent demand was taken. By considering trade-credit strategy and two warehouse [28] presented an Economic Production Quantity model for non-instantaneous deteriorating items. [29] analyzed and developed a two-shop based stock model for non-instantaneous and assumed variable holding cost with stock dependent demand. Cardenas Barron et al. (2020) built up an Economic Production Quantity model and examined a uniform inventory model with non- non-linear stock dependent holding cost and demand. Based on Price discount facility [30] developed an EOQ model for deteriorating items with stock-dependent demand and partial backlogging. For multi-item deteriorating [31] presented a two-echelon inventory model with price-and stock-dependent demand.

For deteriorating items [32] described discount facility and discussed a study of inventory model for deteriorating items with price and stock dependent demand under-price discount facility. In some of the study a concept of overtime production was considered in the same way [33] presented an inventory model for deteriorating items under overtime production for deteriorating items with nonlinear price and stock dependent demand.

For the product with time varying holding cost by offering some quantity discounts [34] developed and discussed partial backlogging inventory model with price and stock level dependent demand.

In most of the inventory models, it is commonly seen that the ordering cost is assumed to be fixed but it is not certifiable in the present emulative market. Under this presumption, many attempts were made by various research scholars in the direction to developed inventory control model for decaying items under the effect of exponential demand where holding cost has been time-dependent. But in this study deterioration was considered at the initial level of the production. In the current paper, considering the more practical phenomena author have discussed the stock dependent rate of demand and time-varying holding cost with rebate rate in the production model and author have assumed the impact of decay at the initial level of production.

II. Assumptions and Notations

- The demand rate for the product is assumed to be stock-based.
- The produced unit of product is always available to face the demand of the market.
- Products start deteriorates at the initial level of the model but price discount on deteriorated items is offered only when production is stopped.
- Shortages are not permitted in the model.
- There is no replacement or repair of perishable items.
- The holding cost of the inventory units is a function of time.
- $p = k.D$, where P is the production rate and k ≥ 1, where D is the demand.

The notations are as follows used in the present model

A = Setup cost

C_h = Holding cost per unit per unit time

C_p = production cost per unit

C_d = Deterioration cost per unit

p = Production rate per unit time

r = Price discount per unit cost

t_1 = The time where production was stopped

T = Duration of complete production cycle

$I_1(t)$= Level of inventory at time t between the intervals $0 \le t \le t_1$

$I_2(t)$= Level of inventory at time t between the intervals $t_1 \le t \le T$

III. Mathematical Formulation of the Model

The stock level develops during the interval $[0, t_1]$. Thus, the stock level in a creation period is represented by the differential equation (1) where the inventory is increasing due to production and reached to the highest level at t_1. The stock level exhausts along the $[t_1, T]$. The stock level in a non-creation period is represented by the differential equation (2) also in the time slot due to combined effect of demand and deterioration the inventory level is decreasing and reached to the lowest point at T. This model is shown by the following Figure. 1

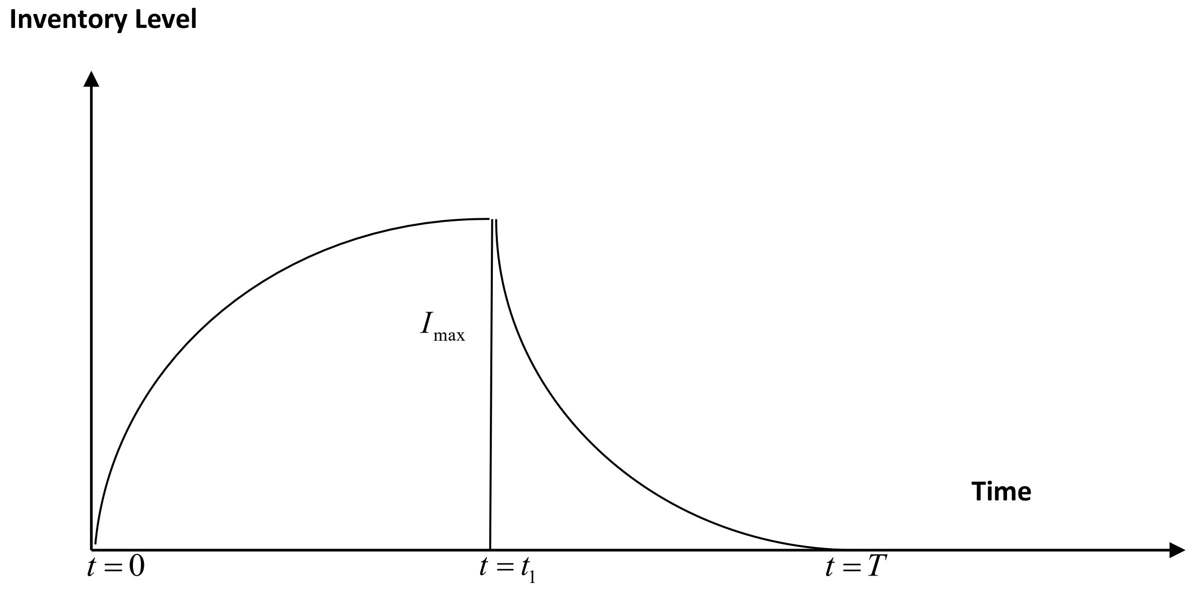

Figure 1 Graphical representation of the inventory with respect to time

$$\frac{dI_1(t)}{dt} + \theta t I_1(t) = p - \left(a + bI_1(t)\right) \qquad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_2(t)}{dt} + \theta t I_2(t) = -\left(a + bI_2(t)\right) \qquad t_1 \le t \le T \tag{2}$$

Author have the following boundary conditions

$I_1(0) = 0 = I_2(T)$

By using the boundary conditions,

$$I_1(t) = \frac{(a-p)}{b+\theta}\left(e^{-t(b+\theta)} - 1\right) \qquad 0 \le t \le t_1 \tag{3}$$

$$I_2(t) = \frac{a}{b+\theta} e^{-t(b+\theta)}\left(e^{T(b+\theta)} - e^{t(b+\theta)}\right) \qquad t_1 \le t \le T \tag{4}$$

Maximum inventory level

$$I_{Max} = I_1(t_1) = \frac{(a-p)}{b+\theta}\left(e^{-t_1(b+\theta)} - 1\right) \tag{5}$$

Using the condition of continuity

$I_1(t_1) = I_2(t_1)$

$$\frac{(a-p)}{b+\theta}\left(e^{t_1(-b-\theta)} - 1\right) = \frac{a}{b+\theta}\left(e^{T(b+\theta)}e^{-t_1(b+\theta)} - 1\right)$$

$$\left(e^{t_1(-b-\theta)} - 1\right)(a-p) = a\left(e^{(b+\theta)(T-t_1)} - 1\right)$$

$$t_1 = \frac{a}{p}T \tag{6}$$

Now various costs associated with the models are: -

Production cost

$$PC = C_p p t_1 \tag{7}$$

Setup cost

$$SC = A \tag{8}$$

Cost for Holding the Inventory

$$HC = C_h\left[\int_0^{t_1} I_1(t)dt + \int_{t_1}^{T} I_2(t)dt\right]$$

$$HC = \frac{a}{2(b+\theta)^3}\begin{pmatrix} -2h_2 - (b+\theta)\begin{pmatrix} 2\big(1+(T-t_1)(b+\theta)\big)h_1 \\ +\big(2T+T^2(b+\theta)-{t_1}^2(b+\theta)\big)h_2 \end{pmatrix} \\ +2e^{(T-t_1)(b+\theta)}\big(h_2+(b+\theta)(h_1+t_1h_2)\big) \end{pmatrix}$$
$$-\frac{(a-p)}{2(b+\theta)^3}\begin{pmatrix} -2\big((b+\theta)h_1+h_2\big)+t_1(b+\theta)^2(2h_1+t_1h_2) \\ +2e^{-t_1(b+\theta)}\big(h_2+(b+\theta)(h_1+t_1h_2)\big) \end{pmatrix} \tag{9}$$

Deterioration cost

$$DC = C_d\left[\int_0^{t_1} \theta I_1(t)dt + \int_{t_1}^{T} \theta I_2(t)dt\right]$$
$$= \frac{C_d\theta}{(\theta+b)}\left((a-p)\left(\frac{1-e^{-t_1(b+\theta)}}{\theta+b} - t_1\right) - \frac{a}{(\theta+b)}\left((T-t_1)(\theta+b)+1-e^{(T-t_1)(b+\theta)}\right)\right) \tag{10}$$

Price Discount

$$PD = C_p r\left[\int_0^{t_1}\big(a+bI_1(t)\big)dt + \int_{t_1}^{T}\big(a+bI_2(t)\big)dt\right]$$
$$= C_p r\left[at_1 + \frac{b(a-p)}{b+\theta}\left(\frac{1-e^{-t_1(b+\theta)}}{b+\theta} - t_1\right) + \frac{a}{(b+\theta)^2}\left((T-t_1)\theta^2 + b\left(e^{(T-t_1)(b+\theta)}+(T-t_1)\theta-1\right)\right)\right] \tag{11}$$

The average Total Cost per unit time

$$TC(t_1,T) = \frac{1}{T}[PC+SC+HC+DC+PD]$$

$$= \frac{1}{T}\left[\begin{matrix} C_p p t_1 + A + \frac{a}{2(b+\theta)^3}\begin{pmatrix} -2h_2-(b+\theta)\begin{pmatrix} 2\big(1+(T-t_1)(b+\theta)\big)h_1 \\ +\big(2T+T^2(b+\theta)-{t_1}^2(b+\theta)\big)h_2 \end{pmatrix} \\ +2e^{(T-t_1)(b+\theta)}\big(h_2+(b+\theta)(h_1+t_1h_2)\big) \end{pmatrix} \\ -\frac{(a-p)}{2(b+\theta)^3}\begin{pmatrix} -2\big((b+\theta)h_1+h_2\big)+t_1(b+\theta)^2(2h_1+t_1h_2) \\ +2e^{-t_1(b+\theta)}\big(h_2+(b+\theta)(h_1+t_1h_2)\big) \end{pmatrix} \\ +\frac{C_d\theta}{(\theta+b)}\left((a-p)\left(\frac{1-e^{-t_1(b+\theta)}}{\theta+b}-t_1\right)-\frac{a}{(\theta+b)}\left((T-t_1)(\theta+b)+1-e^{(T-t_1)(b+\theta)}\right)\right) \\ +C_p r\left(at_1+\frac{b(a-p)}{b+\theta}\left(\frac{1-e^{-t_1(b+\theta)}}{b+\theta}-t_1\right)+\frac{a}{(b+\theta)^2}\left((T-t_1)\theta^2+b\left(e^{(T-t_1)(b+\theta)}+T\theta-t_1\theta-1\right)\right)\right) \end{matrix}\right] \tag{12}$$

Substituting the value of $t_1 = \frac{a}{p}T$,by (6) in (12),

Total Cost per unit time $TC = \frac{A}{T} + aC_p$

$$+\frac{1}{2T(b+\theta)^3}\left\{\begin{matrix} (a-p)\begin{pmatrix} 2\big((b+\theta)h_1+h_2\big)-\frac{aT(b+\theta)^2}{p}\left(2h_1+\frac{aTh_2}{p}\right) \\ -2e^{-\frac{aT(b+\theta)}{p}}\left(h_2+(b+\theta)\left(h_1+\frac{aTh_2}{p}\right)\right) \end{pmatrix} \\ -a\begin{pmatrix} 2h_2+(b+\theta)\begin{pmatrix} 2\left(1+\left(T-\frac{aT}{p}\right)(b+\theta)\right)h_1 + \\ \left(2T+(b+\theta)\left(T^2-\frac{a^2T^2}{p^2}\right)\right)h_2 \end{pmatrix} \\ -2e^{\left(T-\frac{aT}{p}\right)(b+\theta)}\left(h_2+(b+\theta)\left(h_1+\frac{aTh_2}{p}\right)\right) \end{pmatrix} \end{matrix}\right\}$$

$$+\frac{C_d\theta}{T(b+\theta)}\left((a-p)\left(-\frac{aT}{p}+\frac{1-e^{-\frac{aT(b+\theta)}{p}}}{b+\theta}\right)-\frac{a}{(b+\theta)}\left(1-e^{\left(T-\frac{aT}{p}\right)(b+\theta)}+\left(T-\frac{aT}{p}\right)(b+\theta)\right)\right)$$
$$+\frac{C_p r}{T}\left(\frac{a^2T}{p}+\frac{b(a-p)\left(\frac{1-e^{-\frac{aT(b+\theta)}{p}}}{b+\theta}-\frac{aT}{p}\right)}{b+\theta}+\frac{a\left(\left(T-\frac{aT}{p}\right)\theta^2+b\left(e^{\left(T-\frac{aT}{p}\right)(b+\theta)}+T\theta-\frac{aT\theta}{p}-1\right)\right)}{(b+\theta)^2}\right) \tag{13}$$

IV. Solution Procedure

Now the optimum value of T which minimize the total cost.
The values of T for which
$\frac{\partial TC(T)}{\partial T}=0$, Satisfying the condition $\frac{\partial^2 TC(T)}{\partial T^2}>0$
The optimal solution of the equation (13) is obtained by using Mathematica software. Above said process can also be seen through the following example.

V. Numerical Example

Considering $A\to 50, a\to 20, b\to 1.2, C_p\to 15, h_1\to 2, h_2\to 0.15, p\to 40, \theta\to .006, C_d\to 3, C_p\to 2, r\to .02$ in appropriate units. The optimal value of $T^*=2.20006, t_1^*=1.10003, TC^*=349.78825$

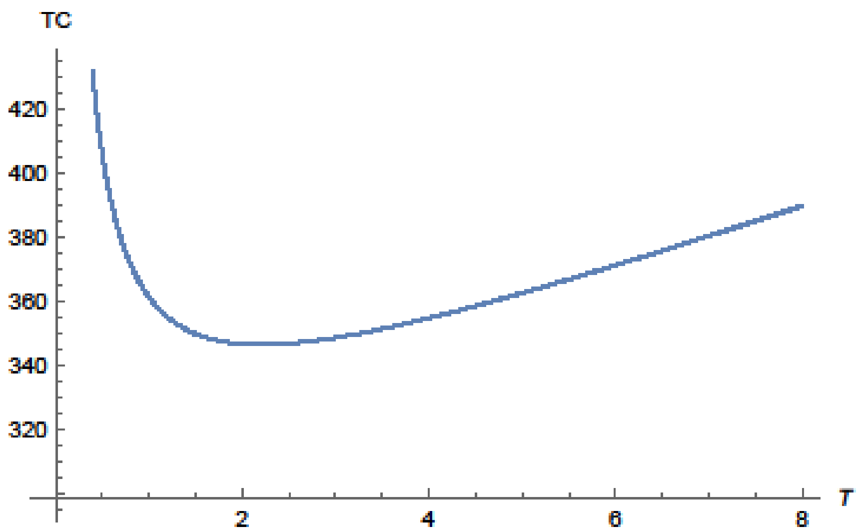

Figure 2. Concavity of the profit function.

VI. Sensitivity Analysis

Based on the values used in above example in the model author have examined the sensitivity analysis by changing some parameters one at a time and keeping the rest fixed.

Table 1 Sensitivity Analysis w.r.t. Various Parameters

Parameter	%	Changed value	t_1	T	$I_{\max}$	TC
A	+50%	75	1.34725	2.69451	169.0449	359.918
	+25%	62.5	1.22741	2.40402	130.22971	354.026
	0	50	1.10003	2.20006	114.77403	349.788
	-25%	37.5	0.97816	1.98821	71.21354	341.498
	-50%	25	0.87699	1.68724	32.21354	333.805
a	+50	30	1.43291	1.09105	19.33721	489.125
	+25	25	1.22886	1.10554	53.97692	431.226
	0	20	1.10003	2.20006	114.77403	349.788
	-25	15	0.92214	2.31269	148.29284	276.394
	-50	10	0.78764	2.39991	189.76542	200.947
b	+50	1.8	1.09371	2.18739	90.53786	354.985
	+25	1.5	1.09685	2.19369	102.73451	352.063
	0	1.2	1.10003	2.20006	114.77403	349.788
	-25	0.9	1.10324	2.20647	133.95031	348.072
	-50	0.6	1.11009	2.22018	149.24731	346.857
P	+50	60	0.53185	1.59475	107.98073	361.688
	+25	50	0.86818	1.82913	110.76742	357.296
	0	40	1.10003	2.20006	114.77403	349.788
	-25	30	1.33422	2.62627	143.91819	335.358
	-50	20	1.67151	2.91326	201.21387	317.963

Author have calculated the sensitivity analysis based on different parameters. The outcome of the result is compared.

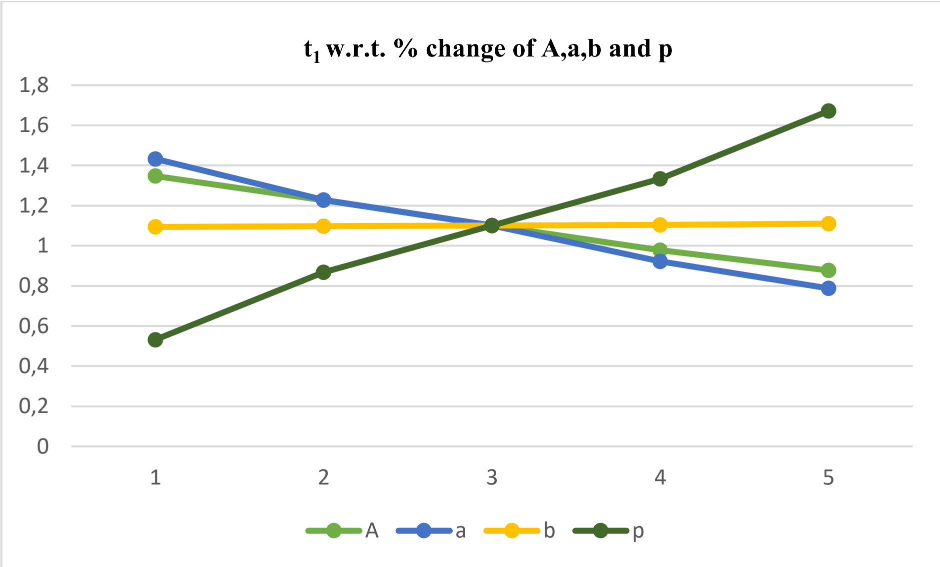

Figure 3. t_1 v/s change in parameters

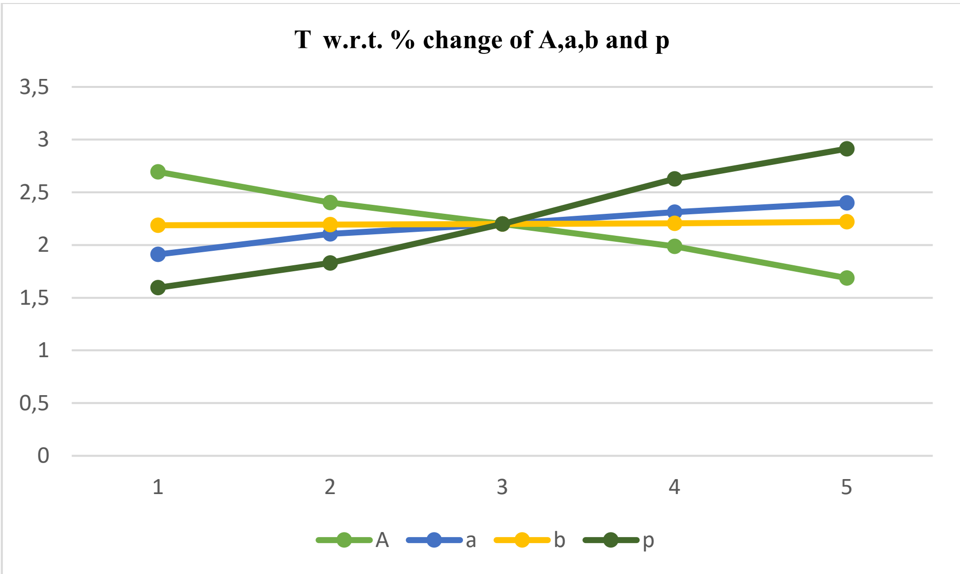

Figure 4. T v/s change in parameters

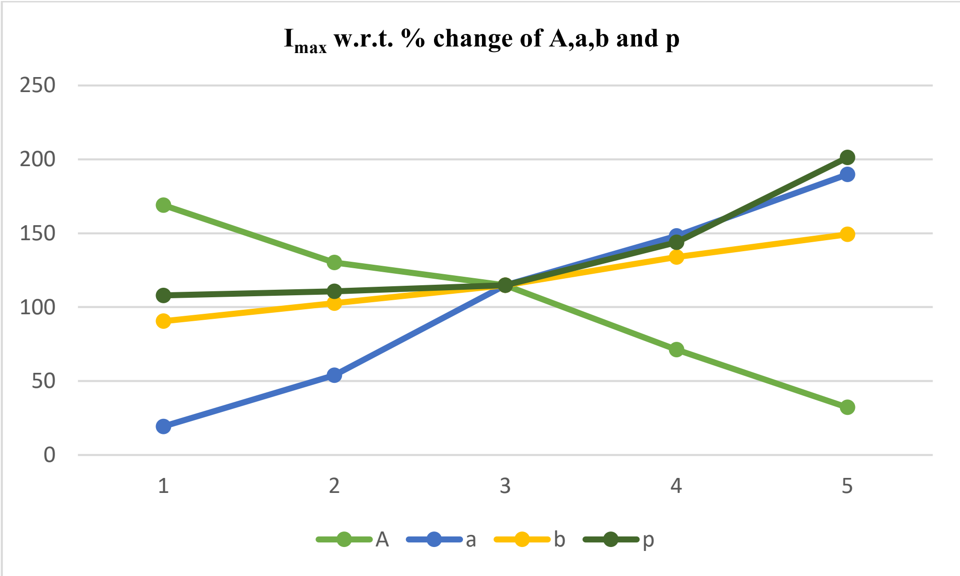

Figure 5. I_{max} v/s change in parameters

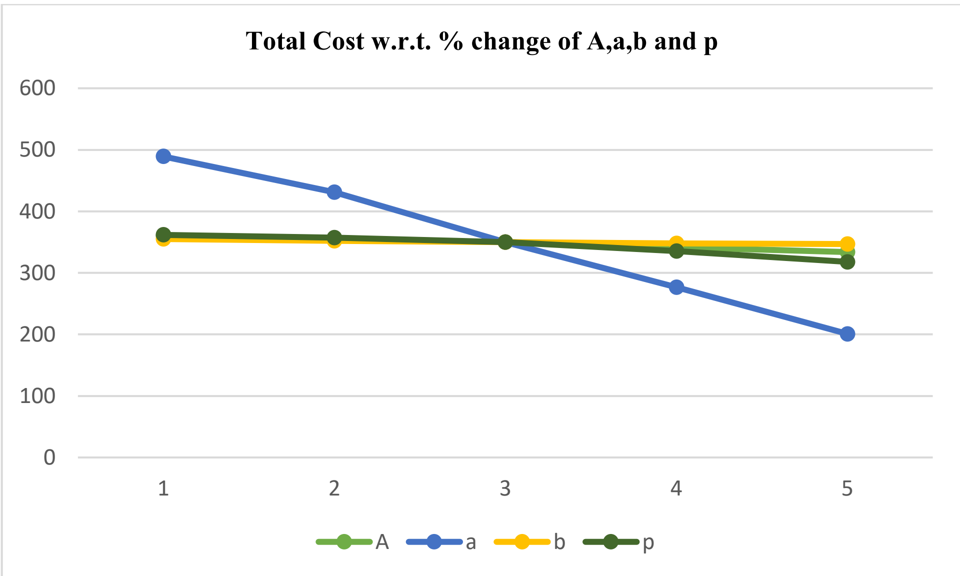

Figure 6. Total Cost v/s change in parameters

VII. Observations

In the present study, author have calculated the sensitivity analysis based on the parameters used in the study. Author have made changes in the parameters by (–50%, –25%, 0%, 25%, 50%). Some important inferences drawn from Table -1 and Figures 2 to 5 are as follows:

(i) Table No 1 shows that as the value of A goes from -50% to + 50%, the value of t_1,T, I_{max} , and total cost also increases.

(ii) Table No 1 shows that as the value of a goes from -50% to + 50%, the value of T and I_{max} decreases while the value of $t_1$and total cost increases rapidly.

(iii) Table No 1 shows that as the value of b goes from -50% to + 50%, the value of T total cost increases while the value of t_1, T and I_{max} decrease rapidly.

(iv) Table No 1 shows that as the value of p goes from -50% to + 50%, the value of T total cost increases while the value of t_1, T ,and I_{max} decreases.

As per above analysis it is obvious that it becomes a new tool to take initial deterioration and it is also observed that we have maximize the average total profit.

VIII. Conclusions

In the present study, author analyzed some important facts related to maximization of total cost related to inventory models and developed an inventory control production-based model for deteriorating items to reduce total cost. Also, the model has been developed by assuming the rate of demand depends on in-hand stock and holding cost is time-dependent.

In this model, author have assumed that produced items that have been partially degraded, customers have been discounted on their selling price, and products that were completely damaged or deteriorated have been discarded which is a more realistic assumption and helps to improve the profit. Also, in the present model, author have assumed that for the items having low life deterioration take place at the very initial stage of the cycle and developed this model under the effect of deterioration when production is going on. To validate the optimality, a numerical example has been taken and explained in the model. Sensitivity between the ranges -50% to 50% of different parameters have been carried out to check the deviation. The assumption of taking initial time-based deterioration makes this study more effective. Managerial aspect of the study is that this paper provide a good platform for the research scholars to use this study to investigate various changes in the deterioration perimeter and formulate new study.

IX. Future scope of the study

Present study may be further expanded by making some more changes in the main parameters of the study like this model can also be developed under a shortage, trade credits, and some price discount on the in-hand inventory may also be assumed.

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