An Inventory Model with Quantity Dependent Trade Credit for Stock and Price Dependent Demand, Variable Holding Cost and Partial Backlogging

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Abstract

Here, the modelling of an inventory system for price and stock reliant demand with the combination of quantity discount and credit limit policy has been presented. Price and stock level are the key sources that always affect the demand of any product. In present study the cost of holding is considered as a time varying function. Vendors usually offer different policies or discounts to attract more customers. Different possible cases for offered trade credit period are discussed in the model. Shortages with partial backlogging are considered here in the development of the model. The different possible cases in this model is exemplified numerically with the help of software Mathematica 11.3 and a sensitivity analysis with respect to distinct system parameters is also presented.

Keywords: Inventory Model, Quantity Discount, Deterioration, Stock and Price Dependent, Demand, Trade Credit, Partial Backordering, Variable Holding Cost, Shortages.

I. Introduction

The aim of the present paper is to develop an inventory model for deteriorating items using price and stock dependent demand. Most of the previous inventory models were developed by assuming constant, time dependent, stock dependent and many different demand patterns. In general, there are more marketing schemes and strategies that influence the market demand. Available stock and selling price are the main factors that affect the demand of customers.

Deterioration is another important factor whose role in the construction of an inventory model is very useful. Deterioration can be defined as the reduction in the original quality and value of the product in any term. Mostly, all available products deteriorate, the only difference is that in some products the rate of deterioration is large and in some it is very low. So, for more accuracy in result, the deterioration should be considered in the modeling of inventory models.

Some researchers worked on inventory models without considering shortages. It cannot be predicted that the stocked units will always be enough to satisfy the demand of all the customers. So, shortage is also an important concept in inventory model that should be taken into account. In the construction of an inventory model the assumption that during stock out the occurring shortages are either completely lost or completely backlogged, is not realistic. So there will be a partial backlogging of the demand during stock out.

Further trade credit period is the useful incentive policy to attract more customers. In this time period vendor allows a certain time limit to retailer to pay all his dues. If the retailer pays all his dues before the credit limit then there will be no interest otherwise interest will be charged on unpaid amount. Retailer can increase his profit by earning interest on sales revenue.

In today's high competitive market, vendors usually offer new schemes or policies to promote their business. In the present paper quantity discount policy is considered with trade credit period and both of these works as a promotional tool for the business. To make the study more realistic and to improve the efficiency of the model, holding cost is also taken as a linear function of time.

II. Literature Review

Skouri and Papakristos [1] proposed an inventory model with quantity discount policy using price dependent demand. Chang [2] introduced an inventory model based on price-dependent demand under quantity and freight discounts. Alfares [3] developed an inventory policy under stock level dependent demand, time varying holding cost and quantity discount. Tripathi et al [4] investigated a partial backordering inventory model for deteriorating items under quantity discount scheme.

Geetha and Udayakumar [5] proposed a non-instantaneous deteriorating model for price and advertisement dependent demand with partial backorder. Sanni and Chigbu [6] developed a three-parameter Weibull distribution deteriorating inventory model under stock level dependent demand with shortage backordering. Li and Teng [7] introduced pricing and lot-sizing strategies for perishable products when demand depends on stock level, selling price, product freshness and reference price. Rastogi et al. [8] developed an inventory model for non-instantaneous deteriorating items with price sensitive demand and partial backlogging. Shaikh et al. [9] studied an EOQ model for decaying products using time dependent demand under shortage backordering and trade credit. Tayal et al. [10] presented deteriorating inventory model for two level of shortage using stock dependent demand and fractional backlogging. Rani et al. [11] studied a green supply chain inventory model for decaying items with credit period dependent demand. Handa et al. [12] worked on an inventory model under trade credit policy and shortages in which stock level plays a major role for demand.

In present paper holding cost is taken as a linear function of time. Jaggi [13] proposed a non-instantaneous deteriorating inventory model with variable holding cost in which demand depends upon price, and holding cost is taken as a variable. Tayal et al. [14] introduced an EPQ model with exponential demand rate and time dependent holding cost. Rastogi et al. [15] developed a deteriorating inventory model for price sensitive demand, linear holding cost and trade credit period. Aggarwal et al. [16] proposed an inventory model for price dependent demand, linear holding cost and partial backlogging under inflation.

Skouri et al. [17] formulated an inventory model with Weibull distribution deterioration and ramp type demand rate. Dutta and Kumar [18] studied a deteriorating inventory model in which demand and holding cost is considered as a function of time and permitted shortages are partially backlogged. Mahapatra et al. [19] introduced a model for deteriorating items based on reliability dependent demand under partial backlogging. Singh et al. [20] worked on replenishment policy for decaying items with partial backordering under credit financing and inflation.

Patra [21] investigated effect of inflation and time value of money for two warehouse inventory model under shortages. Bhojak and Gothi [22] introduced Weibull distributed deteriorating inventory model for time reliant demand with deficiency and backordering. Singh and Sharma [23] proposed a reverse logistic supply chain inventory model for imperfect production/remanufacturing with partial backordering and inflation. Kumar et al. [24] studied the effect of preservation and learning on partial backordering inventory model for deteriorating items with the effect of Covid-19 pandemic.

Kumar et al. [25] worked on an inventory model for two-level storage under the effect of learning and inflation. Wang et al. [26] studied a supply chain inventory model for decaying products under seller's optimal credit financing. Singh et al. [27] formulated an inventory model under preservation technology using stock dependent demand with credit financing. Shaikh [28] introduced a deteriorating inventory model based on price and advertisement dependent demand under shortage backordering and mixed type of trade credit. Sundararajan and Uthayakumar [29] formulated an optimal inventory policy with promotional efforts and backordering of shortages under trade credit period. Mishra and Talati [30] studied quantity discount inventory policy in which demand depends upon the frequency of advertisement and stock with preservation and backordering of shortages.

This study represents an inventory model considering variable demand, quantity discount and partial backlogging. To make the study more realistic, holding cost is taken as the function of time. Different cases for allowed trade credit period are also elaborated in the model. To improve the efficiency of the model numerical example for different cases and sensitivity analysis for distinct value of parameters have been discussed.

III. Assumptions and notations

$I(t)$ level of inventory at any time t

d, β, γ coefficients of demand

Q_1 initial stock level

Q_2 back order quantity during stock out

$\phi(\eta)$ rate of backlogging

k rate of deterioration

η waiting time up to next arrival

T cycle time

u_1 time at which level of inventory becomes zero

h_a, h_b parameters of holding cost

s_r per unit shortage cost

λ per unit deterioration cost

l_r lost sale cost per unit

c purchasing cost per unit

A per order ordering cost

p selling price per unit

$U.T.P_x.$ unit time profit

M allowed trade credit period

I_c rate of interest charged

I_e rate of interest earned

These following are the assumptions used here:

- Products considered in this model are of deteriorating nature.

- Demand rate is a function of price and stock and is given by $D=(d-\beta p+\gamma I_1(t))$
- No replacement policy is allowed for deteriorating products in whole cycle.
- The system allows shortages and partial backlogging.
- Deterioration rate is constant.
- In the model all-units quantity discounts and the length of credit periods are defined as follow:

$$M_i = \begin{cases} M_1 & 1 \le X_1 < Y_1 \\ M_2 & Y_1 \le X_2 < Y_2 \\ M_3 & Y_2 \ge X_3 \end{cases}$$

where X_1, X_2, X_3 denote the boundaries of quantity in units and $M_1 > M_2 > M_3$.

- Backlogging rate is assumed as a function of waiting time.
- Holding cost is considered as a linear function of time i.e., $(h_a + h_b(t))$.
- Trade credit is allowed for different time period.
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4. Mathematical Modelling

Figure 1. Represents the behaviour of inventory system with respect to time. Here Q_1 denotes the initial inventory level at $t=0$. The level of inventory depletes in the interval $[0,u_1]$ due to demand and deterioration. At $t=u_1$, inventory reaches to zero level and after that shortages occur. The depletion of the inventory is represented by the following Fig 1.

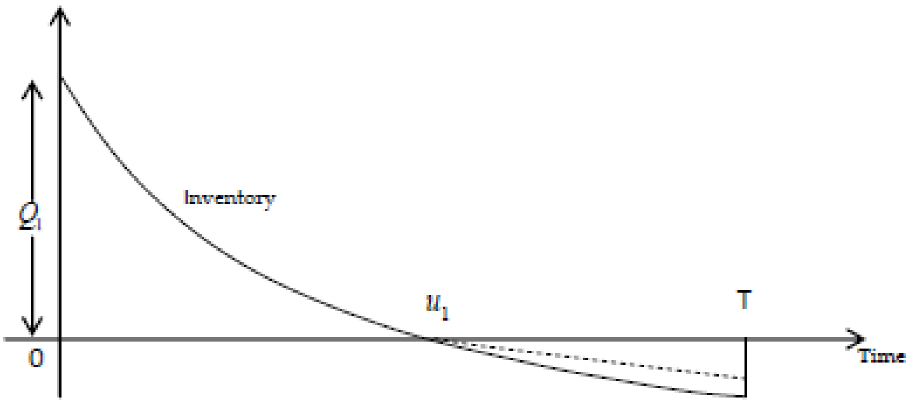

Figure 1 *Inventory time graph of system*

Inventory system can be represented by the following differential equations:

$$\frac{dI_1}{dt} + kI_1 = -(d - \beta p + \gamma I_1(t)) \quad 0 \le t \le u_1 \tag{1}$$

$$\frac{dI_2}{dt} = -(d - \beta p) \; \ldots\ldots\ldots\ldots \; u_1 \le t \le T \tag{2}$$

Boundary equations are given as follow:

$$I_1(u_1) = I_2(u_1) = 0 \tag{3}$$

Solution of the equations (1) and (2) are given by

$$I_1(t) = [(u_1 - t) + (k+\gamma)(u_1^2 - t^2)]e^{-(k+\gamma)t} \quad 0 \le t \le u_1 \tag{4}$$

$$I_2(t) = (d - \beta p)(u_1 - t) \quad u_1 \le t \le T \tag{5}$$

V Associated Costs

V.I. Ordering Cost

Ordering cost per order of the system is as follow:

$$O.C_x. = A \tag{6}$$

V.II. Purchasing Cost

If c is the purchasing cost per unit and Q_1, Q_2 are the ordering quantity and backordered quantity respectively then purchasing cost of the system will be:

$P.C_x. = \{Q_1 + Q_2\}c$

Where

$$Q = I(0) = (d - \beta p)(u_1 + (k+\gamma)\frac{u_1^2}{2}) \tag{7}$$

Here I(0) denotes the initial inventory level at the starting of the cycle and Q2 is the backordered quantity during stock out of the inventory.

$$Q_2 = \int_{u_1}^{T} (d - \beta\gamma)\phi(\eta)dt$$

$$Q_2 = \frac{(d - \beta p)(T^2 - u_1^2)}{2T} \tag{8}$$

Hence, the purchasing cost of the system will be

$$P.C_x = \{(u_1 + (k+\gamma)\frac{u_1^2}{2}) + \frac{(T^2 - u_1^2)}{2T}\}(d - \beta p)c \tag{9}$$

V.III. Holding Cost

Holding cost is considered in the duration of positive inventory. It is a linear function of time and is given by:

$$H.C_x = \int_0^{u_1} (h_a + h_b t) I_1(t)dt$$

$$H.C_x. = (d - \beta p)\{h_a(\frac{u_1^2}{2} + (k+\gamma)\frac{u_1^3}{6} - (k+\gamma)^2\frac{u_1^4}{8})) + h_b(\frac{u_1^3}{6} + (k+\gamma)\frac{u_1^4}{24} - (k+\gamma)^2\frac{u_1^5}{15})\} \tag{10}$$

V.IV. Shortage Cost

In the inventory system shortage occurs during the stock out of inventory. Shortage cost of the system will be as follow:

$$S.C_x. = s_r \int_{u_1}^{T} (d - \beta p)dt$$

$$S.C_x. = s_r (d - \beta p)(T - u_1) \tag{11}$$

V.V. Lost Sale Cost

In the inventory system lost sale cost occurs when some customers fulfil their demand from other places, during the stock out conditions.

$$L.S.C_x. = l_r \int_{u_1}^{T} (d - \beta p)(1 - \phi(\eta))dt$$

$$L.S.C_x. = l_r\{\frac{(d - \beta p)(T - v)^2}{2T}\} \tag{12}$$

V.VI Deterioration cost

Deterioration cost is considered for those products that are deteriorated or decayed in the system. The deterioration cost for the system is as follow:

$$D.C_x. = \lambda\{I_1(0) - \int_0^{u_1} (d - \beta p)dt\}$$

$$D.C_x. = \lambda(d - \beta p)\{(k + \gamma - 1)\frac{u_1^2}{2} - (k + \gamma)\frac{u_1^3}{6} + (k + \gamma)^2 \frac{u_1^4}{8}\} \tag{13}$$

V.VII Sales revenue

Sales revenue:

$S.R_x. = (Q_1 + Q_2)p$

Hence, the sale revenue of the system is given by

$$S.R_x = \{(u_1 + (k + \gamma)\frac{u_1^2}{2}) + \frac{(T^2 - u_1^2)}{2T}\}(d - \beta p)p \tag{14}$$

VVIII.Permissible delay

Two cases for allowed trade credit period are given as follow:

Case 1: When $M \geq u_1$

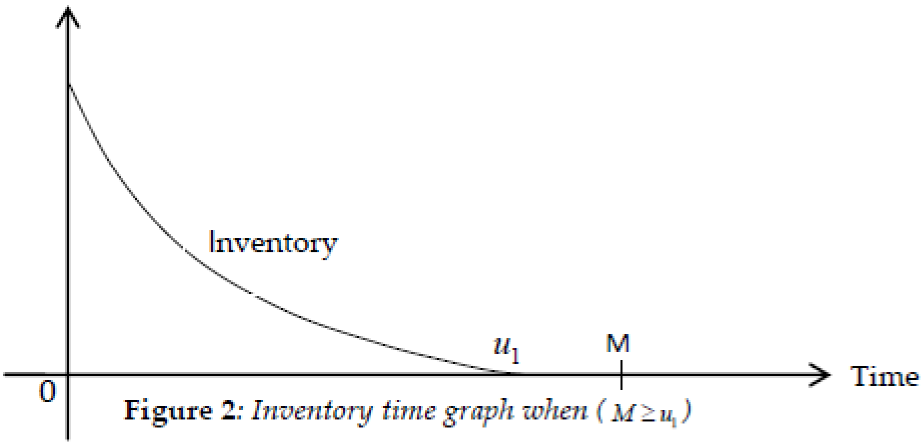

Figure 2: *Inventory time graph when* ($M \geq u_1$)

For this case retailer has an adequate amount of funds to clear up all his dues since the credit limit period is more than the time of positive inventory.

Interest charged in this case will be:

$I.C_1 = 0 = 0$

Interest earned in the duration of [0, M] is given by:

$$IE_1. = pI_e \int_0^{u_1} (d - \beta p + \gamma I(t))tdt + (M - u_1)\int_0^{u_1} (d - \beta p + \gamma I(t))dt$$

$$= pI_e(d - \beta p)\{(\frac{u_1^2}{2} + \gamma(\frac{u_1^3}{3} + (k + \gamma)\frac{u_1^4}{24} - \frac{(k + \gamma)^2}{15}u_1^5) + (M - u_1)(u_1 + \gamma(\frac{u_1^2}{2} + (k + \gamma)\frac{u_1^3}{6} - \frac{(k + \gamma)^2}{8}u_1^4)\} \tag{15}$$

Case 2: When $M < u_1$

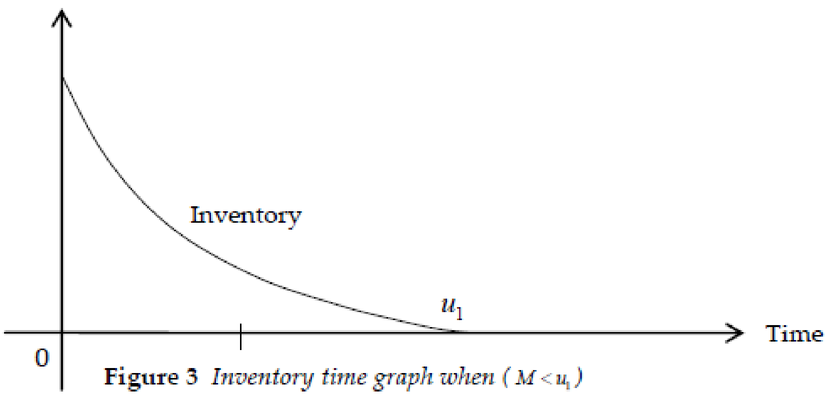

Figure 3 *Inventory time graph when* ($M < u_1$)

For this case the retailer has to settle all his payment before zero stock. On the basis of interest earned and interest charged, following two cases arise.

Case 2.1: When $M < u_1$ and

$$pD[0,M] + IE_{2.1}[0,M] \geq cI(0) \tag{16}$$

For this case retailer has enough money to settle all his payments.
Interest charged is given by

$$I.C_{2.1} = 0 \tag{17}$$

Interest earned in the duration [0, M] is given by

$$I.E_{2.1} = pI_e \int_0^M (d - \beta p + \gamma I(t))t dt$$

$$= pI_e(d - \beta p)\{\frac{M^2}{2} + \gamma(\frac{M^3}{6} + (k+\gamma)\frac{M^3}{24} - \frac{(k+\gamma)^2}{15}M^5\} \tag{18}$$

Case 2.2: When $M < u_1$ and

$$pD[0,M] + IE_{2.2}[0,M] < cI(0) \tag{19}$$

For this case retailer has not enough money to settle all his payments.
Interest earned in the duration [0, M] is given by

$$I.E_{2.2} = pI_e \int_0^M (d - \beta p + \gamma I(t))t dt$$

$$I.E_{2.2} = pI_e(d - \beta p)\{\frac{M^2}{2} + \gamma(\frac{M^3}{6} + (k+\gamma)\frac{M^3}{24} - \frac{(k+\gamma)^2}{15}M^5\} \tag{20}$$

Interest charged on unpaid amount is given by

$$I.C_{2.2} = B.I_c \tag{21}$$

where B is given by:

$$B = cE_1(0) - \{pD[0,M] + IE_{2.2}[0,M]\}$$

$$B = I_c(d - \beta p)\{c(u_1 + (k+\gamma)\frac{u_1^2}{2}) - (pI_e(\frac{M^2}{2} + \gamma(\frac{M^3}{6} + (k+\gamma)\frac{M^3}{24} - \frac{(k+\gamma)^2}{15}M^5))$$
$$+ p(u_1 + \gamma(\frac{u_1^2}{2} + (k+\gamma)\frac{u_1^3}{6} - \frac{(k+\gamma)^2}{8}u_1^4)\}$$

V.IX. Unit Time Profit

$$U.T.P_x = \frac{1}{T}\{S.R_x. - P.C_x. - H.C_x. - D.C_x. - L.S.C_x. - S.C_x. - O.C_x. - IC + IE\} \tag{22}$$

VI. Numerical Example

Case 1: When $M \geq u_1$

A=500 per/order, c=35 Rs./unit, d=200 units, k=0.001, T=30 days, M=23 days, β =2.2, γ =0.1 Rs./unit, l_r =18 Rs./unit, s_r =15 Rs./unit, h_a =0.8 Rs./unit, h_b =0.45 Rs./unit, λ =18 Rs./unit I_e =0.02,
After solving this model with the help of corresponding parameters optimal value of
p =46.7032 Rs, u_1 =21.7042 Rs. and $U.T.P_x$ =8146.47 Rs.
The behavior of the system for $U.T.P_x$ is given by the figure 4 with the help of Mathematica 11.3.

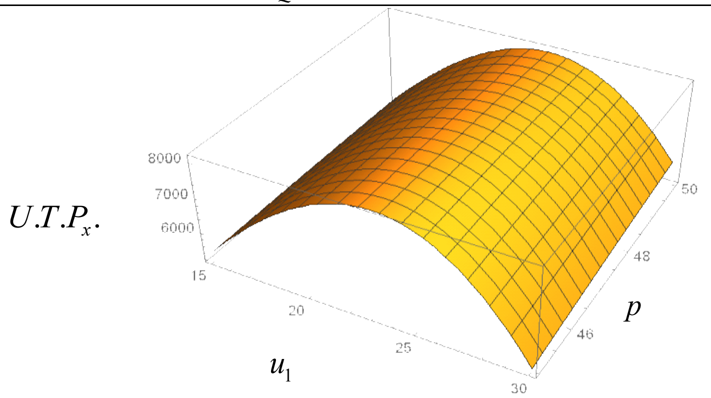

Fig. 4: *Optimality of the system for case 1*

Case 2.1: When $M < u_1$ and

$pD[0,M] + IE_{2.1}[0,M] \geq cI(0)$:

A=500 per/order, c=35 Rs./unit, d=200 units, k=0.001, T=30 days, M=20 days, β =2.2, γ =0.1 l_r =18 Rs./unit, s_r =15 Rs./unit, h_a =0.8 Rs./unit, h_b =0.45 Rs./unit, λ =18 Rs./unit, I_e =0.02,

After solving this model with the help of corresponding parameters optimal value of
p =47.8656 Rs., u_1 =23.2287 days and $U.T.P_x$ =8155.1 Rs.

The behavior of the system for $U.T.P_x$ is given by the figure 5 with the help of Mathematica 11.3.

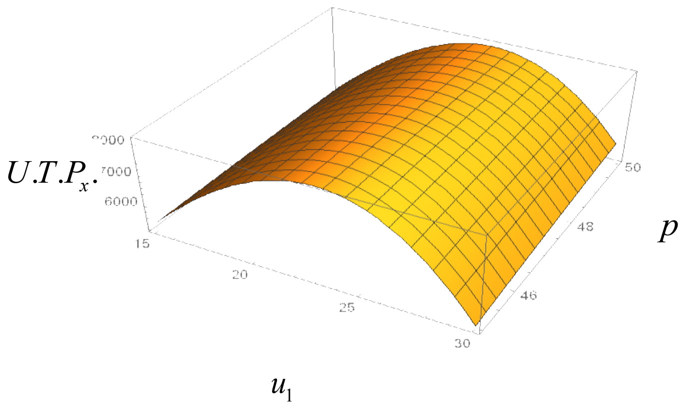

Fig. 5: *Optimality of the system for case 2.1*

Case 2.2: When $M < u_1$ and

$pD[0,M] + IE_{2.2}[0,M] < cI(0)$:

A=500 per/order, c=35 Rs./unit, d=200, k=0.001, T=30, M=17 days, β =2.2, γ =0.1, l_r =18 Rs./unit, s_r =15 Rs./unit, h_a =0.8 Rs./unit, h_b =0.45 Rs./unit, λ =18 Rs./unit I_e =0.02, I_c =0.03,

After solving this model with the help of corresponding parameters optimal value of p =48.2336, u_1 =23.1396 days and $U.T.P_x$ =8066.84 Rs

The behavior of the system for $U.T.P_x$ is given by the figure 6 with the help of Mathematica 11.3.

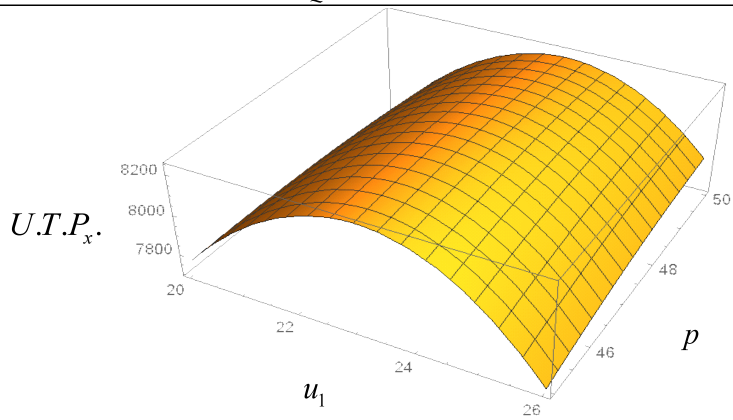

Fig. 6: *Optimality of the system for case 2.2*

The Algorithm

The solution procedure, to maximize the $U.T.P_x.$ for optimal ordering quantity is given as follows:
Here U.T.P. is the function of two variables 'u1' and 'p'. So, to maximize U.T.P. we put the partial derivatives

$\frac{\partial U.T.P_x.(u_1, p)}{\partial u_1} = 0$ and $\frac{\partial U.T.P_x.(u_1, p)}{\partial p} = 0$

After solving these equations, system gives the optimal value of v and p

$\frac{\partial^2 U.T.P_x.(u_1, p)}{\partial^2 u_1} < 0$, $\frac{\partial^2 U.T.P_x.(u_1, p)}{\partial^2 p} < 0$

provided

$$\left(\frac{\partial^2 U.T.P_x.(u_1, p)}{\partial^2 u_1}\right)\left(\frac{\partial^2 U.T.P_x.(u_1, p)}{\partial^2 p}\right) - \left(\frac{\partial^2 U.T.P_x.(u_1, p)}{\partial u_1 \partial p}\right)^2 > 0$$

Find the value of u_1 and p for every credit period length.
Calculate X_i i.e., ordering quantity for every value of u_1 and p.
Calculate a valid quantity $X_i = X^*$.
Find out the unit time profit for this X^* .
Evaluate $U.T.P_x.$ for all given credit period lengths and also for the value which is greater than X_i^* .

Quantity Discount Approach

Model is demonstrated numerically for the different trade credit period with the help of Mathematica 11.3.

$$M_i = \begin{cases} 23 \ days & 1 \le X_1 < 2700 \\ 20 \ days & 2700 \le X_2 < 5500 \\ 17 \ days & X_3 \ge 5500 \end{cases}$$

With respect to these credit periods and the values of parameters discussed above, optimal ordering quantity i.e., $X^* = 5348.81$ units, $p = 47.8659$ Rs. and u_1 =23.2287 days.

And U.T.Px = 8155.1 Rs. Also $(U.T.P_x.)_{Y_2} = 8099.45$ Rs., Clearly

$(U.T.P_x.)_{X^*} > (U.T.P_x.)_{Y_2}$

$X^* = 5348.81$ is the optimal value of ordering quantity

$(U.T.P_x.)_{X^*} = 8155.1$ Rs. is the optimal value of unit time profit for the system.

Also, optimal value of u_1 =23.2287 days and $p = 47.8659$ Rs.

VII. Sensitivity Analysis

Sensitivity analysis for distinct parameters is specified as follows. In this the effect of different system parameters on unit time profit is calculated to check the stability of the system.

Case 1: When $M \geq u_1$

Table 1: Variation in optimal solution for demand parameter (d):

% change in (d)	(d)	u_1	p	$U.T.P_x.$
-20%	160	21.3076	37.3959	5141.32
-15%	170	21.4102	39.7209	5826.7
-10%	180	21.5104	42.0471	6555.9
-5%	190	21.6084	44.3746	7329.19
0%	200	21.7042	46.7032	8146.47
5%	210	21.7979	49.0328	9007.97
10%	220	21.8896	51.3634	9913.83
15%	230	21.9793	53.6947	10864.2
20%	240	22.0672	56.0268	11859.1

Table 2: Variation in optimal solution for shortage parameter (β):

% change in (β)	(β)	u_1	p	$U.T.P_x.$
-20%	1.76	22.1533	58.3596	10315.7
-15%	1.87	22.0261	54.9293	9675.04
-10%	1.98	21.9097	51.8814	9107.39
-5%	2.09	21.8028	49.1555	8600.98
0%	2.2	21.7042	46.7032	8146.47
5%	2.31	21.6130	44.4855	7736.29
10%	2.42	21.5284	42.4702	7364.30
15%	2.53	21.4497	40.6310	7025.42
20%	2.64	21.3762	38.9457	6715.43

Table 3: Variation in optimal solution for lost sale cost parameter (γ):

% change in (γ)	(γ)	u_1	p	$U.T.P_x.$
-20%	0.08	28.3126	37.3189	13184.9
-15%	0.085	26.1631	40.2893	11471.9
-10%	0.09	24.4238	42.7617	10127.1
-5%	0.095	22.9620	44.8712	9041.34
0%	0.1	21.7042	46.7032	8146.47
5%	0.105	20.6039	48.3159	7396.91
10%	0.11	19.6293	49.7510	6760.70
15%	0.115	18.7576	51.0395	6214.61
20%	0.12	17.9717	52.2053	5741.37

Table 4: Variation in optimal solution for deterioration cost parameter (l_r):

% change in (l_r)	(l_r)	u_1	p	$U.T.P_x.$
-20%	14.4	21.6753	46.6497	8159.91
-15%	15.3	21.6827	46.6631	8156.54
-10%	16.2	21.6899	46.6765	8153.17
-5%	17.1	21.6970	46.6899	8149.82
0%	18	21.7042	46.7032	8146.47
5%	18.9	21.7113	46.7165	8143.12
10%	19.8	21.7184	46.7298	8139.79
15%	20.7	21.7255	46.7430	8136.43
20%	21.6	21.7326	46.7562	8133.14

Table 5: Variation in optimal solution for deterioration parameter (s_r):

% change in (s_r)	(s_r)	u_1	p	$U.T.P_x.$
-20%	12	21.6131	46.4286	8227.84
-15%	12.75	21.6359	46.4977	8207.37
-10%	13.5	21.6588	46.5665	8186.98
-5%	14.25	21.6815	46.6350	8166.68
0%	15	21.7042	46.7032	8146.47
5%	15.75	21.7268	46.7712	8126.34
10%	16.5	21.7494	46.8389	8106.30
15%	17.25	21.7719	46.9063	8086.34
20%	18	21.7943	46.9735	8066.47

Table 6: Variation in optimal solution for interest earned parameter (I_e):

% change in (I_e)	(I_e)	u_1	p	$U.T.P_x.$
-20%	0.016	21.9530	46.8795	8019.16
-15%	0.017	21.8887	46.8323	8050.30
-10%	0.018	21.8258	46.7873	8081.91
-5%	0.019	2.4643	46.7443	8113.97
0%	0.02	21.7042	46.7032	8146.47
5%	0.021	21.6453	46.6640	8179.39
10%	0.022	21.5877	46.6265	8212.72
15%	0.023	21.5306	46.5906	8246.49
20%	0.024	21.4760	46.5563	8280.55

Case 2: When M $M < u_1$

Table 7: Variation in optimal solution for demand parameter (d):

% change in (d)	(d)	u_1	p	$U.T.P_x.$
-20%	160	22.446	38.1208	5090.59
-15%	170	22.6416	40.5444	5784.96
-10%	180	22.8372	42.9764	6526.76
-5%	190	23.0329	45.4168	7316.64
0%	200	23.2287	47.8656	8155.1
5%	210	23.4247	50.3228	9042.74
10%	220	23.6209	52.7883	9980.14
15%	230	23.8175	55.2622	10967.9
20%	240	24.0146	57.7445	12006.8

Table 8: Variation in optimal solution for shortage parameter (β):

% change in (β)	(β)	u_1	p	$U.T.P_x.$
-20%	1.76	24.2121	60.2352	10474.1
-15%	1.87	23.9218	56.5753	9782.19
-10%	1.98	23.6646	53.3374	9174.07
-5%	2.09	23.435	50.4524	8635.46
0%	2.2	23.2287	47.8656	8155.1
5%	2.31	23.0422	45.5332	7724.07
10%	2.42	23.8728	43.4195	7335.16
15%	2.53	22.7182	41.495	6982.49
20%	2.64	22.5764	39.7356	6661.24

Table 9: Variation in optimal solution for lost sale cost parameter (γ):

% change in (γ)	(γ)	u_1	p	$U.T.P_x.$
-20%	0.08	31.417	40.0413	14157.7
-15%	0.085	28.5285	42.1984	12079.8
-10%	0.09	26.394	44.2577	10483.8
-5%	0.095	24.6733	46.1438	9206.48
0%	0.1	23.2287	47.8656	8155.1
5%	0.105	21.9865	49.4465	7270.26
10%	0.11	20.9008	50.9116	6511.38
15%	0.115	19.9409	52.2844	5849.5
20%	0.12	19.0845	53.5868	5263.38

Table 10: Variation in optimal solution for deterioration cost parameter (l_r):

% change in (l_r)	(l_r)	u_1	p	$U.T.P_x$.
-20%	14.4	23.1993	47.8156	8163.83
-15%	15.3	23.2067	47.8282	8161.64
-10%	16.2	23.214	47.8407	8159.45
-5%	17.1	23.2214	47.8532	8157.28
0%	18	23.2287	47.8656	8155.1
5%	18.9	23.236	47.8780	8152.93
10%	19.8	23.2432	47.8904	8150.77
15%	20.7	23.2505	47.9028	8148.61
20%	21.6	23.2577	47.9151	8146.46

Table 11: Variation in optimal solution for deterioration parameter (s_r):

% change in (s_r)	(s_r)	u_1	p	$U.T.P_x$.
-20%	12	23.1130	47.588	8219.98
-15%	12.75	23.1420	47.6579	8203.62
-10%	13.5	23.1710	47.7275	8187.68
-5%	14.25	23.1999	47.7968	8171.68
0%	15	23.2287	47.8656	8155.1
5%	15.75	23.2574	47.9341	8139.12
10%	16.5	23.2861	48.0023	8123.23
15%	17.25	23.3147	48.0701	8107.43
20%	18	23.3432	48.1376	8091.72

Table 12: Variation in optimal solution for interest earned parameter (I_e):

% change in (I_e)	(I_e)	u_1	p	$U.T.P_x$.
-20%	0.016	23.2312	47.8977	8044.48
-15%	0.017	23.2306	47.8896	8072.14
-10%	0.018	23.230	47.8816	8099.79
-5%	0.019	23.2293	47.8736	8127.45
0%	0.02	23.2287	47.8656	8155.1
5%	0.021	23.2281	47.8577	8182.76
10%	0.022	23.2274	47.8499	8210.42
15%	0.023	23.2268	47.8421	8238.07
20%	0.024	23.2262	47.8344	8265.73

VIII. Observations

- Table 1 and 7 represent the effect of demand d on critical time u_1, selling price p and on $U.T.P_x$. It is observed that with an increment in d, there is also an increment in u_1, p and in $U.T.P_x$.
- Table 2 and 8 show the effect of β on u_1, p and on $U.T.P_x$, it is observed that after an increment in β, a pattern of decrement is observed in u_1, p and in $U.T.P_x$ in both the tables.
- Table 3 and 9 list the variation in γ and its effect on u_1, p and $U.T.P_x$, it is observed that as the value of γ increases the values of u_1 and $U.T.P_x$ decreases, while the value of p in both the tables increases.
- Table 4 and 10 represent the effect of l_r on u_1, p and on $U.T.P_x$, it is observed that after an increment in l_r, some increment in u_1 and p is observed while some decrement in $U.T.P_x$ in both the tables is detected.
- Table 5 and 11 list the variation in parameter s_r on optimal value of u_1, p and on $U.T.P_x$. It is observed that after an increment in s_r, some increment in u_1 and p while some decrement in $U.T.P_x$ in both the tables are detected.
- Table 6 and 12 represent the effect of I_e on u_1, p and on $U.T.P_x$, it is observed that after an increment in I_e, some decrement in u_1 and p while some increment in $U.T.P_x$ in both the tables are detected.

IX. Conclusion

Present paper considers an inventory model for price and stock-dependent demand under some real-life situations like variable holding cost and credit financing policies. In today's period when there is the high competition in the market, vendors usually offer new schemes or policies to customers to promote their business. In present study quantity discount policy is also applied because it works as an incentive and a promotional tool for any business. Shortages are allowed with partial backordering. To improve the efficiency of the model numerical examples for different cases and sensitivity analysis for distinct parameters have been discussed with the help of Mathematica 11.3. This Model further can be extended for different demand patterns, deterioration, backlogging rate and also for different realistic approaches like preservation, inflationary environment and green supply chain.

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