# Inferences on Stress Strength Reliability in Multicomponent System for Type I Generalized Half-Logistic Distribution

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**Abstract**

*This article deals with inferences on stress strength reliability in a multicomponent system for Type I generalized half-logistic distribution. It is assumed that the strength and stress components are independently distributed. In this work, we develop some statistical properties of the type I generalized half-logistic distribution. Furthermore, the expression for stress strength reliability for a multicomponent setup was obtained and studied. Two methods to estimate the multicomponent stress-strength reliability - maximum likelihood and Bayesian estimation were employed. The Bayes estimates of the multicomponent stress strength reliability are obtained under squared error loss function and using gamma priors for the parameters. Simulation studies were conducted to assess the efficiency of the methods. The importance of this model was studied by applying it to a real life data set.*

**Keywords:** Type I generalized half-logistic distribution; multicomponent system; stress strength reliability; beta function.

## 1. Introduction

Researchers and statisticians have paid a lot of attention to stress-strength reliability. Their vast range of applications includes industries ranging from transportation and communications to medicine and healthcare. If the system's strength is higher than the stress it is subjected to, it is called trustworthy. Random stress is given to an appliance, $Y$, and the strength is $X$ then a measure of the reliability of a system is given by $R = Pr\{X > Y\}$. There has been a great deal of effort done on estimating $R$ using various $X$ and $Y$ distributions and estimate methodologies. Kotz *et al*. (2003) provide an overview of the applications and theories in this field . Raqab *et al*. (2008) and Kundu and Raqab (2009) found $R$ where $X$ and $Y$ are independent three-parameter generalized exponential and three-parameter Weibull random variables, respectively. Kundu and Raqab (2013) have calculated the stress-strength reliability for a three-parameter generalized Rayleigh distribution. Using the phase-type distribution and a discrete distribution, Jose *et al*. (2020) calculated stress-strength reliability and Jose and Drisya (2020) evaluated time-dependent reliability using the phase-type distribution, respectively.

The development of multicomponent stress-strength reliability has also received considerable attention. For example, consider a system with $k$ statistically independent and identically distributed strength components subjected to a shared load. When $s(1 \leq s \leq k)$ or more components concurrently survive, this multicomponent stress-strength system is activated. This was initially explored by Bhattacharya and Johnson (1974). A wide range of industrial and military applications can benefit from such systems. When $s = k$ and $s = 1$ respectively, the following system corresponds to series and parallel. Using a panel of $k$ identical solar cells, Johnson (1988) showed that this set-up may be used in practice to ensure that the mission's power requirements are met even if only $s$ of the cells are in use at any one time. Some cells may be unable to function properly due to severe temperatures, and this extreme temperature may be a factor in a cell's strength. Dey *et al*. (2016) used this model to estimate the multicomponent stress-strength reliability for the Kumaraswamy distribution, among many other practical uses. An example of a log-logistic distribution of strength and stress was studied by Rao and Kantam (2010). The dependability of multicomponent stress-strength models was calculated by assuming generalized exponential and Burr XII distributions for the components in Kizilaslan and Nadar (2015), Rao (2012), and Rao *et al*. (2014).

Olapade developed the type I generalized half-logistic model, which is shown below (2014). A generalized version of the half-logistic distribution suggested by Balakrishnan, the distribution is used in this case (Balakrishnan, 1985). If a random variable $X$ has the density function $f(x)$ of the type I generalized half-logistic (TIGHL) distribution, it is said to have the type I generalized half-logistic (TIGHL) distribution if

$$f(x) = \frac{b2^b}{\sigma} \frac{e^{\frac{x}{\sigma}}}{\left(1 + e^{\frac{x}{\sigma}}\right)^{b+1}}; 0 \leq x < \infty, b > 0, \sigma > 0 \tag{1}$$

and $f(x) = 0$ elsewhere with cumulative distribution function as

$$F(x) = 1 - \left(\frac{2}{1 + e^{\frac{x}{\sigma}}}\right)^b \tag{2}$$

where $\sigma$ and $b$ are the scale and shape parameters, respectively. Jose and Manoharan's approach is a particular case of the model in (1) (Jose and Manoharan, 2016). This model's dependability qualities haven't been well studied in the literature, which prompted us to investigate them and come up with clearer formulations. In addition to Bello *et al*. (2017), Awodutire and Awodutire *et al*. (2020a) and others, the type I generalized half-logistic model has been further generalized. According to Jose *et al*. (2019) the stress-strength reliability of Kumaraswamy half-logistic distribution was analyzed. Furthermore, a power-transformed half-logistic distribution was used to estimate stress-strength reliability in single and multicomponent system (Xavier and Jose, 2020a, 2020b).

The following is the article's flow: The type-I generalized half-logistic model's dependability features are discussed in Section 2. Under a multicomponent arrangement, the calculation of the distribution's strength stress reliability is discussed in Section 3. Maximum likelihood estimates and Bayesian estimates are developed. Gamma priors are used for Bayesian estimation under the squared error loss function. In Section 4, characteristics were tested in a series of computer simulations. In the same section, a real-world dataset is used to demonstrate the model's capabilities. Section 5 is the final conclusions of the paper.

## 2. Reliability Properties

Olapade(2014) had studied some properties of the Type I generalized half logistic distribution. In this section, more research is carried out in order to derive precise formulations for a number of dependability characteristics. Moment generating function, mean failure time, mean residual life function, and Renyi and Shannon entropies are among the properties that are further studied.

### 2.1. Moment generating function

The moment generating function can be obtained as

$$M_x(t) = E(\mathrm{e}^{tx}) = \frac{b2^b}{\sigma}\int_0^\infty \mathrm{e}^{tx}\frac{\mathrm{e}^{\frac{x}{\sigma}}}{\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^{b+1}}\mathrm{d}x$$

Now consider the transformation $\frac{1}{\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)} = u$, then

$$\begin{aligned} M_x(t) = E(\mathrm{e}^{tx}) &= b2^b\int_0^1 u^{b-t\sigma-1}(1-u)^{t\sigma}\mathrm{d}u \\ &= 2^b\frac{\Gamma(b-t\sigma)\Gamma(t\sigma+1)}{\Gamma(b)};\ \ \Re(b-t\sigma)>0 \end{aligned} \tag{3}$$

where $\Gamma(.)$ is called the gamma function defined as $\Gamma(a) = \int_0^\infty x^{a-1}\mathrm{e}^{-x}dx; \Re(a) > 0$.

### 2.2. Mean Time to Failure Function

Then we can have mean time to failure (MTTF) or $E(X)$ as $E(X) = \frac{\mathrm{d}}{\mathrm{d}t}M_x(t)|_{t=0}$

$$\begin{aligned} E(X) &= 2^b\frac{\Gamma(b-t\sigma)\Gamma(t\sigma+1)}{\Gamma(b)}\left[\sigma\psi(b-t\sigma)+\sigma\psi(t\sigma+1)\right]|_{t=o} \\ &= 2^b\sigma\left[\psi(b)+\psi(1)\right] \end{aligned} \tag{4}$$

Here $\psi(.)$ called the digamma function is the logarithmic derivative of the gamma function, that is $\psi(.) = \frac{\Gamma'(.)}{\Gamma(.)}$.

### 2.3. Mean residual life function

The mean residual life function for a non-negative continuous random variable $X$ is defined as $\eta(x) = E(X-x|X>x)$ and can be obtained by

$$\begin{aligned} \eta(x) &= \frac{1}{S(x)}\int_x^\infty S(y)\mathrm{d}y \\ &= \frac{\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^b}{2^b}2^b\int_x^\infty \frac{1}{\left(1+\mathrm{e}^{\frac{y}{\sigma}}\right)^b}\mathrm{d}y \end{aligned}$$

Now consider the transformation $\frac{1}{\left(1+\mathrm{e}^{\frac{y}{\sigma}}\right)} = u$, then

$$\begin{aligned} \eta(x) &= \sigma\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^b\int_0^{\frac{1}{1+\mathrm{e}^{\frac{x}{\sigma}}}}\frac{u^{b-1}}{1-u}\mathrm{d}u \\ &= \sigma\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^b\sum_{k=0}^\infty\frac{(1)_k}{k!}\int_0^{\frac{1}{1+\mathrm{e}^{\frac{x}{\sigma}}}}u^{b+k-1}\mathrm{d}u \\ &= \sigma\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^b\sum_{k=0}^\infty\frac{(1)_k}{(b+k)k!}\left(\frac{1}{1+\mathrm{e}^{\frac{x}{\sigma}}}\right)^{b+k} \\ &= \frac{\sigma}{b}\sum_{k=0}^\infty\frac{(1)_k(b)_k}{(b+1)_k k!}\left(\frac{1}{1+\mathrm{e}^{\frac{x}{\sigma}}}\right)^k \\ &= \frac{\sigma}{b}{}_2F_1\left(1,b;b+1;\frac{1}{1+\mathrm{e}^{\frac{x}{\sigma}}}\right) \end{aligned} \tag{5}$$

where $(t)_m = t(t+1)...(t+m-1)$ and ${}_pF_q(z)$ is the generalized hypergeometric function. The generalized hypergeometric function ${}_pF_q(z)$ is defined as

$$ {}_pF_q(a_1,...,a_p;b_1,...,b_q;z) = \sum_{k=0}^{\infty} \frac{(a_1)_k...(a_p)_k}{(b_1)_k...(b_q)_k} \frac{z^k}{k!} $$

where $b_j \neq 0,-1,-2,...;\ i=1,2,...,p;\ j=1,2,...,q,$. The convergence conditions and other details are available from books on special functions, see for example Mathai and Haubold (2008).

## 2.4. Renyi and Shannon entropies

The entropy of a random variable $X$ is a measure of variation of the uncertainty. Renyi entropy is defined as $I_r(\gamma) = \frac{1}{1-\gamma}\ln\left\{\int_{\Re} f^{\gamma}(x)\mathrm{d}x\right\}$, where $\gamma > 0$ and $\gamma \neq 1$.

$$
\begin{aligned}
I_r(\gamma) &= \frac{1}{1-\gamma}\ln\left[\frac{b^{\gamma}2^{\gamma b}}{\sigma^{\gamma}}\int_0^{\infty}\frac{\mathrm{e}^{\frac{\gamma(x)}{\sigma}}}{\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^{\gamma(b+1)}}\mathrm{d}x\right] \\
&= \frac{1}{1-\gamma}\ln\left[\frac{b^{\gamma}2^{\gamma b}}{\sigma^{\gamma}}\int_0^{1}u^{\gamma b-1}(1-u)^{\gamma-1}\mathrm{d}u\right] \\
&= \frac{1}{1-\gamma}\ln\left[\left(\frac{b2^{b}}{\sigma}\right)^{\gamma}B(\gamma b,\gamma)\right];\gamma>0,\gamma\neq 1 \qquad (6)
\end{aligned}
$$

The Shannon entropy is defined as $E[-\ln f(x)]$ and can be obtained as

$$ E[-\ln f(x)] = -E[\ln b + b\ln 2 - \ln\sigma] - E\left[\frac{x}{\sigma}\right] + (b+1)E\left[\ln(1+\mathrm{e}^{\frac{x}{\sigma}})\right] $$

Now

$$
\begin{aligned}
E\left[\frac{x}{\sigma}\right] &= 2^{b}(\psi(b)-\psi(1)) \\
E\left[\ln(1+\mathrm{e}^{\frac{x}{\sigma}})\right] &= \frac{b(b+1)2^{b}}{\sigma}\int_0^{\infty}\frac{\mathrm{e}^{\frac{x}{\sigma}}\ln\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)}{\left(1+\mathrm{e}^{\frac{x}{\sigma}}\right)^{b+1}}\mathrm{d}x \\
&= b(b+1)2^{b}\int_0^{1}u^{b-1}\ln\left(\frac{1}{u}\right)\mathrm{d}u \\
&= \frac{(b+1)2^{b}}{b}
\end{aligned}
$$

Hence, the Shannon entropy reduces to

$$ E[-\ln f(x)] = -[\ln b + b\ln 2 - \ln\sigma] + 2^{b}(\psi(b)-\psi(1)) + \frac{(b+1)2^{b}}{b} \qquad (7) $$

# 3. Multicomponent Strength Stress Reliability

The stress-strength reliability of a system is defined as the chance that the system will continue to work effectively until the strength surpasses the stress. When the system is placed to use and subjected to a random stress, the system's strength changes as a result of the manufacturing variability and unpredictable circumstances. Material, production technique, humidity, temperature, and other variables may all be exploited to create manufacturing variations and unpredictability in products. There are several studies in the literature that have attempted to estimate the multicomponent stress-strength reliability for different statistical distributions.

Reliability of multicomponent stress strength is established by Bhattacharyya and Johnson (1974) as

$$\begin{aligned} R_{s,k} &= Pr\left\{\text{at least } s \text{ of } (X_1, X_2, ..., X_k) \text{ exceed } Y\right\} \\ &= \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty [1-F(y)]^i [F(y)]^{k-i} dG(y) \end{aligned} \tag{8}$$

where $X_1, X_2, X_3, ..., X_k$ are independently distributed with the cumulative distribution function $F(x)$ and are subjected to common random stress $Y$ with cumulative distribution function $G(y)$. Let $X_1, X_2, ..., X_k$ be iid with $TIGHL(q, \sigma)$ and $Y \sim TIGHL(b, \sigma)$ be independently distributed. Thus, by putting equations (1) and (2) into (8), we can derive the multicomponent system's stress strength reliability using the type I generalized half logistic distribution as

$$R_{s,k} = \frac{b2^b}{\sigma} \sum_{i=s}^{k} \binom{k}{i} \int_0^\infty \left(\frac{2}{1+e^{\frac{x}{\sigma}}}\right)^{qi} \left(1 - \left(\frac{2}{1+e^{\frac{x}{\sigma}}}\right)^q\right)^{k-i} \frac{e^{\frac{x}{\sigma}}}{(1+e^{\frac{x}{\sigma}})^{b+1}} dx$$

Let $t = \left(\frac{2}{1+e^{\frac{x}{\sigma}}}\right)^q$ then $dt = -\frac{q2^q e^{\frac{x}{\sigma}}}{\sigma(1+e^{\frac{x}{\sigma}})^{q+1}} dx$ and when $x \to 0, t \to 1$ and when $x \to \infty, t \to 0$.

$$\begin{aligned} R_{s,k} &= \frac{b}{q} \sum_{i=s}^{k} \binom{k}{i} \int_0^1 t^i [1-t]^{k-i} t^{\frac{b}{q}} dt \\ &= \frac{b}{q} \sum_{i=s}^{k} \binom{k}{i} \int_0^1 t^{\frac{b}{q}+i} [1-t]^{k-i} dt \\ &= \delta \sum_{i=s}^{k} \binom{k}{i} B(\delta + i + 1, k - i + 1) \end{aligned} \tag{9}$$

where $\delta = \frac{b}{q}$ and $B(.)$ is the beta function defined as $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1} dt$; $\Re(x) > 0, \Re(y) > 0$.

As special case, consider $X_1, X_2, ..., X_k$ are connected in parallel, then $s = 1$ and $R_{s,k}$ will be

$$R_{1,k} = \delta \sum_{i=1}^{k} \binom{k}{i} B(\delta + i + 1, k - i + 1) \tag{10}$$

Consider $X_1, X_2, ..., X_k$ are connected in series, then $s = k$ and $R_{k,k}$ will be

$$R_{k,k} = \delta \sum_{i=k}^{k} \binom{k}{i} B(\delta + i + 1, k - i + 1) = \frac{\delta}{\delta + k + 1} \tag{11}$$

### 3.1. Maximum Likelihood Estimation of $R_{s,k}$

Consider two random samples of size m and n, respectively, drawn from the variables strength, X, and stress, Y, each of which follows a type I generalized half-logistic distribution with shape parameters q and b, respectively, and a common scale parameter $\sigma$, then the log-likelihood function of the observed data is as follows:

$$\begin{aligned} l &= m(\ln q + q \ln 2 - \ln \sigma) + n(\ln b + b \ln 2 - \ln \sigma) + \sum_{i=1}^{m} \frac{x_i}{\sigma} + \sum_{j=1}^{n} \frac{y_j}{\sigma} \\ &- (q+1) \sum_{i=1}^{m} \ln(1 + e^{\frac{x_i}{\sigma}}) - (b+1) \sum_{j=1}^{n} \ln(1 + e^{\frac{y_j}{\sigma}}) \end{aligned}$$

$$\frac{\partial l}{\partial q} = \frac{m}{q} + m\ln 2 - \sum_{i=1}^{m} \ln(1+\mathrm{e}^{\frac{x_i}{\sigma}}) \Rightarrow \hat{q} = \frac{m}{\sum_{i=1}^{m}\ln(1+\mathrm{e}^{\frac{x_i}{\sigma}}) - m\ln 2} \tag{12}$$

$$\frac{\partial l}{\partial b} = \frac{n}{q} + n\ln 2 - \sum_{j=1}^{n} \ln(1+\mathrm{e}^{\frac{y_j}{\sigma}}) \Rightarrow \hat{b} = \frac{n}{\sum_{j=1}^{n}\ln(1+\mathrm{e}^{\frac{y_j}{\sigma}}) - n\ln 2} \tag{13}$$

$$\begin{aligned}\frac{\partial l}{\partial \sigma} &= (q+1)\sum_{i=1}^{m} \frac{x_i \mathrm{e}^{\frac{x_i}{\sigma}}}{\sigma^2(1+\mathrm{e}^{\frac{x_i}{\sigma}})} + (b+1)\sum_{j=i}^{n} \frac{y_j \mathrm{e}^{\frac{y_j}{\sigma}}}{\sigma^2(1+\mathrm{e}^{\frac{y_j}{\sigma}})} \\ &- \frac{1}{\sigma^2}\left(\sum_{i=1}^{m} x_i + \sum_{j=1}^{n} y_j\right) - \frac{(m+n)}{\sigma} \end{aligned} \tag{14}$$

$\widehat{\sigma}$ can be obtained by iteratively solving the equation $\frac{\partial l}{\partial \sigma} = 0$. Given the estimates, the MLE of $R_{s,k}$ becomes

$$\hat{R}_{s,k} = \hat{\delta}\sum_{i=s}^{k}\binom{k}{i} B(\hat{\delta}+i+1, k-i+1) \tag{15}$$

The asymptotic variance of the estimate of $R_{s,k}$ as defined by Rao (1973) is

$$AVar(\hat{R}_{s,k}) = Var(\hat{q})\left(\frac{\partial R_{s,k}}{\partial q}\right)^2 + Var(\hat{b})\left(\frac{\partial R_{s,k}}{\partial b}\right)^2 \tag{16}$$

where

$$Var(\hat{q}) = E\left[-\frac{\partial^2 l}{\partial q^2}\right]^{-1} = \frac{q^2}{m}$$

$$Var(\hat{b}) = E\left[-\frac{\partial^2 l}{\partial b^2}\right]^{-1} = \frac{b^2}{n}$$

$$\frac{\partial R_{s,k}}{\partial q} = -\frac{R_{s,k}}{q} + \frac{\delta^2}{q}\sum_{i=s}^{k}\binom{k}{i} B(\delta+i+1, k-i+1)\{\psi(\delta+k+2) - \psi(\delta+i+1)\}$$

$$\frac{\partial R_{s,k}}{\partial b} = \frac{R_{s,k}}{b} + \frac{\delta}{b}\sum_{i=s}^{k}\binom{k}{i} B(\delta+i+1, k-i+1)\{\psi(\delta+i+1) - \psi(\delta+k+2)\}$$

Then the asymptotic 95% confidence interval for the system reliability, $R_{s,k}$ can be obtained as $R_{s,k} \mp 1.96\sqrt{AVar(\hat{R_{s,k}})}$.

## 3.2. Bayes estimation of $R_{s,k}$

Parameters are assumed to be constants in the conventional estimate technique. For example, the parameters in the model may not be constant over the entire testing time, therefore they must be handled as random variables. The prior distribution of the parameters may be utilized as information on the uncertainty associated with them in Bayesian estimation, which is a method for overcoming this. This section is devoted to estimating $R_{s,k}$. by use of a Bayesian approach. Here, we assume that the parameters $q, b$ and $\sigma$ have gamma prior distributions with $(c_i, d_i)$,$i$= 1, 2, 3 correspondingly. Assume random variable Z has parameters $(c_i, d_i)$ with the following gamma density is

$$h(z) = \frac{d_i^{c_i}}{\Gamma(c_i)} z^{c_i-1}\mathrm{e}^{-zd_i};\ 0 \le z < \infty, c_i > 0, d_i > 0, i = 1,2,3$$

and $h(z) = 0$ elsewhere. The joint prior of $b, q$ and $\sigma$ can be written as

$$\pi(b,q,\sigma) \propto q^{c_1-1}b^{c_2-1}\sigma^{c_3-1}\mathrm{e}^{-d_1q-d_2b-d_3\sigma} \tag{17}$$

On the basis of the squared error loss function, it was possible to generate Bayes estimates of $R_{s,k}$ when the likelihood function and the prior posterior distribution of the parameters $q, b$ and $\sigma$ are combined and the following result is obtained:

$$\begin{aligned}
\pi_0^*(\sigma|b,q,\text{data}) &\propto \sigma^{c_3-m-n-1}e^{\frac{1}{\sigma}\left(\sum_{r=1}^m x_r+\sum_{s=1}^n y_s\right)-d_3\sigma} \\
&\times \prod_{r=1}^{m}\left(1+e^{\frac{x_r}{\sigma}}\right)^{-(q+1)}\prod_{s=1}^{n}\left(1+e^{\frac{y_s}{\sigma}}\right)^{-(b+1)} \\
\pi_1^*(q|\sigma,\text{data}) &\propto Gamma\left(m+c_1, d_1-m\ \ln 2+\sum_{r=1}^{m}\ln\left(1+e^{\frac{x_r}{\sigma}}\right)\right) \\
\pi_2^*(b|\sigma,\text{data}) &\propto Gamma\left(n+c_2, d_2-n\ \ln 2+\sum_{s=1}^{n}\ln\left(1+e^{\frac{y_s}{\sigma}}\right)\right)
\end{aligned}$$

Any well-known distribution cannot be reduced to the posterior distribution of $\sigma$. Random samples are generated using the Markov chain Monte Carlo (MCMC) method because posterior distributions cannot be reduced into closed forms. It is possible to estimate the posterior density functions if they are unimodal and generally symmetric; for details, see Gelman *et al* (2003). When a previous is log-concave, then a posterior is similarly log-concave, according to Kundu (2008) Metropolis-Hasting and the normal proposal distribution will be utilized to generate random samples from posterior distributions of $\sigma$. Bayesian $R_{s,k}$ estimation is given in the following manner:

**Step 1:** Set the initial values $\sigma^0$ and $i = 1$. Let Let $\gamma = \sigma^{i-1}$.

**Step 2:** Generate $q$ from $Gamma\left(m+c_1, d_1-m\ \ln 2+\sum_{r=1}^{m}\ln\left(1+e^{\frac{x_r}{\sigma}}\right)\right)$.

**Step 3:** Generate $b$ from $Gamma\left(n+c_2, d_2-n\ \ln 2+\sum_{s=1}^{n}\ln\left(1+e^{\frac{y_s}{\sigma}}\right)\right)$.

**Step 4:** Using the proposal density $h(\sigma) \equiv N(\sigma^{i-1}, 1), \sigma > 0$, generate $\sigma^i$ from $\pi_0^*(\sigma|b^{i-1}, q^{i-1}, \text{data})$ using step 5.

**Step 5:** From the proposal density, generate a sample, $\tau$. Generate $U$ from Uniform $(0,1)$ and if $U \le min\left\{1, \frac{\pi_0^*(\tau)h(\gamma)}{\pi_0^*(\gamma)h(\tau)}\right\}$, accept $\tau$ and set $\sigma^i = \tau$.

**Step 6:** Compute $R_{s,k}^i$ and set $i$ to $i+1$.

**Step 7:** Repeat steps 2 to 6, $K$ times and obtain the Bayesian estimates of $q, b, \sigma$ and $R_{s,k}$ as $\sum_{i=1}^{K}\frac{q^i}{K}, \sum_{i=1}^{K}\frac{b^i}{K}, \sum_{i=1}^{K}\frac{\sigma^i}{K}$ and $\sum_{i=1}^{K}\frac{R_{s,k}^i}{K}$ respectively.

The method of Chen and Shao (1999) can be used to construct the $100(1-\alpha)\%$ high posterior density (HPD) credible interval of $R_{s,k}$.

## 4. Simulation study and data analysis

Here, we compare the performances of $R_{s,k}$ for different sample sizes. Random samples of sizes 15, 20, 30, 40, and 50 with 1000 replications each from the strength and stress populations were generated for $(q, b) = \{(3.5, 1.0), (2.5, 1.0), (1.5, 1.0), (1.0, 1.5), (1.0, 2.5), (1.0, 3.5)\}$ respectively. The value of $\sigma$ was fixed at 2 for all simulation results. The ML estimators of $\hat{q}$ and $\hat{b}$ were then substituted to obtain the estimate for $R_{s,k}$ with $(s, k) = \{(1, 3), (2, 4)\}$. The bias, MSE, and asymptotic confidence intervals of the MLE of $R_{s,k}$ are presented in Table 1. The MSE values decrease as the sample size increases for both $(s, k)$ which verifies the consistency property of the MLE of $R_{s,k}$.

**Table 1:** *Bias and MSE for MLE of $R_{s,k}$*

| $(q,b)$ | n | $R_{1,3}$ | $\hat{R}_{1,3}$ | Bias(e-5) | MSE | ACI | $R_{2,4}$ | $\hat{R}_{2,4}$ | Bias(e-5) | MSE | ACI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (3.5,1) | 10 | 0.180797101 | 0.180796973 | -0.01280 | 0.00200 | (0.09312,0.26847) | 0.150567959 | 0.150567848 | -0.01110 | 0.00115 | (0.08385,0.21728) |
| | 20 | | 0.180797069 | -0.00320 | 0.00149 | (0.10496,0.25663) | | 0.150567931 | -0.00280 | 0.00086 | (0.09286,0.20826) |
| | 30 | | 0.180797039 | -0.00620 | 0.00099 | (0.11898,0.24260) | | 0.150567905 | -0.00540 | 0.00057 | (0.10354,0.19758) |
| | 40 | | 0.180797055 | -0.00460 | 0.00074 | (0.12731,0.23427) | | 0.1505679195 | 0.0.0039 | 0.00043 | (0.10988,0.19124) |
| | 50 | | 0.180797055 | -0.00460 | 0.00059 | (0.13299,0.22860) | | 0.1505679190 | -0.00400 | 0.00034 | (0.11420,0.18693) |
| (2.5,1) | 10 | 0.237967914 | 0.237967743 | -0.01710 | 0.00324 | (0.12631,0.34962) | 0.200831848 | 0.200831695 | -0.01530 | 0.0019 | (0.11488,0.28677) |
| | 20 | | 0.2379678833 | -0.00307 | 0.00241 | (0.14139,0.33453) | | 0.2008318202 | -0.00278 | 0.00143 | (0.12651,0.27514) |
| | 30 | | 0.2379679029 | -0.00111 | 0.00161 | (0.15927,0.31666) | | 0.2008318376 | -0.00104 | 0.00095 | (0.14028,0.26137) |
| | 40 | | 0.2379678962 | -0.00074 | 0.00158 | (0.16988,0.30604) | | 0.2008318316 | -0.00084 | 0.00071 | (0.14845,0.25320) |
| | 50 | | 0.2379679 | -0.00014 | 0.00096 | (0.17711,0.29882) | | 0.2008318 | -0.00480 | 0.00057 | (0.15402,0.24764) |
| (1.5,1) | 10 | 0.347402597 | 0.347402677 | 0.00800 | 0.00615 | (0.19358,0.50122) | 0.300993125 | 0.300993199 | 0.00740 | 0.00397 | (0.17744,0.42453) |
| | 20 | | 0.3474025377 | -0.00593 | 0.00460 | (0.21439,0.48040) | | 0.3009930684 | -0.00566 | 0.00296 | (0.19419,0.40779) |
| | 30 | | 0.347402578 | -0.00190 | 0.00305 | (0.23905,0.45575) | | 0.300993106 | -0.00190 | 0.00196 | (0.21402,0.38796) |
| | 40 | | 0.347402670 | -0.00108 | 0.00228 | (0.25367,0.44112) | | 0.300993192 | -0.00108 | 0.00147 | (0.22577,0.37620) |
| | 50 | | 0.347402511 | 0.00129 | 0.00182 | (0.26363,0.43116) | | 0.300993044 | 0.00129 | 0.00117 | (0.23377,0.36820) |
| (1,1.5) | 10 | 0.558441558 | 0.558441470 | -0.00088 | 0.01379 | (0.32821,0.78866) | 0.510489510 | 0.510489417 | 0.00133 | 0.0120 | (0.29563,0.72534) |
| | 20 | | 0.5584416232 | 0.00065 | 0.01031 | (0.35941,0.75746) | | 0.5104895784 | 0.00068 | 0.00898 | (0.32471,0.69626) |
| | 30 | | 0.5584415802 | 0.00022 | 0.00683 | (0.39635,0.72052) | | 0.5104895333 | 0.00023 | 0.00596 | (0.35916,0.66181) |
| | 40 | | 0.558441478 | -0.00084 | 0.00511 | (0.41826,0.69862) | | 0.510489426 | 0.00068 | 0.00446 | (0.37959,0.64138) |
| | 50 | | 0.558441653 | 0.00095 | 0.00408 | (0.43316,0.68371) | | 0.510489610 | 0.00100 | 0.00356 | (0.39350,0.62747) |
| (1,2.5) | 10 | 0.687645688 | 0.687645901 | 0.00213 | 0.02103 | (0.40335,0.97193) | 0.650349650 | 0.650349888 | 0.00238 | 0.02266 | (0.35524,0.94545) |
| | 20 | | 0.687645726 | 0.00038 | 0.01573 | (0.44182,0.93347) | | 0.650349694 | 0.00044 | 0.01696 | (0.39505,0.90564) |
| | 30 | | 0.6876457020 | 0.00014 | 0.01043 | (0.48738,0.88790) | | 0.6503496664 | 0.00016 | 0.01127 | (0.44222,0.85847) |
| | 40 | | 0.6876457103 | 0.00022 | 0.00781 | (0.51441,0.86087) | | 0.6503496757 | 0.00025 | 0.00844 | (0.47025,0.83044) |
| | 50 | | 0.687645700 | 0.00012 | 0.00623 | (0.53282,0.84246) | | 0.6503497 | 0.00050 | 0.00674 | (0.48934,0.81134) |
| (1,3.5) | 10 | 0.76037296 | 0.760373130 | 0.01700 | 0.02690 | (0.43886,1.08188) | 0.73273002 | 0.73273021 | 0.01900 | 0.03229 | (0.38051,1.08494) |
| | 20 | | 0.7603730025 | 0.00425 | 0.02013 | (0.48228,1.03846) | | 0.7327300634 | 0.00434 | 0.02418 | (0.42791,1.03754) |
| | 30 | | 0.760373041 | 0.00441 | 0.01337 | (0.53373,0.98701) | | 0.732730108 | 0.00880 | 0.01609 | (0.48411,0.98134) |
| | 40 | | 0.7603730198 | 0.00598 | 0.01000 | (0.56428,0.95646) | | 0.7327300833 | 0.00633 | 0.01205 | (0.51753,0.94792) |
| | 50 | | 0.7603730206 | 0.00606 | 0.00799 | (0.58509,0.93565) | | 0.7327300842 | 0.00642 | 0.00963 | (0.54031,0.92514) |

**Table 2:** *Bias, Risk and HPDCI for Bayes estimate of $R_{s,k}$ under prior 1*

| $(q, b, )$ | $(m, n)$ | $R_{1,3}$ | $\hat{R}_{1,3}$ | Bias | MSE | ACI | $R_{2,4}$ | $\hat{R}_{2,4}$ | Bias | MSE | ACI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (3.5, 1) | (15, 15) | 0.1808 | 0.2063 | 0.0255 | 0.0040 | (0.1122, 0.3301) | 0.1506 | 0.1749 | 0.0243 | 0.0034 | (0.0897, 0.2960) |
| | (20, 20) | | 0.2052 | 0.0244 | 0.0033 | (0.1156, 0.3190) | | 0.1693 | 0.0187 | 0.0025 | (0.0967, 0.2766) |
| | (30, 30) | | 0.1908 | 0.0100 | 0.0017 | (0.1225, 0.2740) | | 0.1618 | 0.0112 | 0.0012 | (0.1046, 0.2390) |
| | (40, 40) | | 0.1869 | 0.0061 | 0.0012 | (0.1289, 0.2625) | | 0.1595 | 0.0089 | 0.0010 | (0.1067, 0.2267) |
| | (50, 50) | | 0.1886 | 0.0078 | 0.0010 | (0.1328, 0.2552) | | 0.1575 | 0.0069 | 0.0007 | (0.1096, 0.2153) |
| (2.5, 1) | (15, 15) | 0.2380 | 0.2690 | 0.0311 | 0.0057 | (0.1549, 0.4209) | 0.2008 | 0.2259 | 0.0251 | 0.0044 | (0.1224, 0.3634) |
| | (20, 20) | | 0.2578 | 0.0198 | 0.0039 | (0.1582, 0.3791) | | 0.2195 | 0.0187 | 0.0035 | (0.1318, 0.3480) |
| | (30, 30) | | 0.2520 | 0.0141 | 0.0026 | (0.1671, 0.3542) | | 0.2108 | 0.0100 | 0.0019 | (0.1401, 0.3016) |
| | (40, 40) | | 0.2479 | 0.0100 | 0.0020 | (0.1669, 0.3366) | | 0.2108 | 0.0100 | 0.0016 | (0.1439, 0.29954) |
| | (50, 50) | | 0.2464 | 0.0084 | 0.0014 | (0.1811, 0.3232) | | 0.2106 | 0.0098 | 0.0012 | (0.1534, 0.2774) |
| (1.5, 1) | (15, 15) | 0.3474 | 0.3704 | 0.0230 | 0.0076 | (0.2203, 0.5473) | 0.3010 | 0.3229 | 0.0219 | 0.0074 | (0.1846, 0.5010) |
| | (20,20) | | 0.3656 | 0.0182 | 0.0057 | (0.2314, 0.5150) | | 0.3217 | 0.0207 | 0.0056 | (0.1975, 0.4827) |
| | (30, 30) | | 0.3593 | 0.0119 | 0.0038 | (0.2460, 0.4843) | | 0.3114 | 0.0104 | 0.0037 | (0.2077, 0.4390) |
| | (40, 40) | | 0.3553 | 0.0079 | 0.0028 | (0.2584, 0.4639) | | 0.3107 | 0.0097 | 0.0027 | (0.2155, 0.4171) |
| | (50, 50) | | 0.3582 | 0.0108 | 0.0023 | (0.2675, 0.4565) | | 0.3090 | 0.0080 | 0.0020 | (0.2305, 0.4017) |
| (1, 1.5) | (15, 15) | 0.5584 | 0.5653 | 0.0069 | 0.0085 | (0.3754, 0.7398) | 0.5105 | 0.5254 | 0.0149 | 0.0093 | (0.3387, 0.7152) |
| | (20, 20) | | 0.5667 | 0.0083 | 0.0066 | (0.4072, 0.7224) | | 0.5171 | 0.0067 | 0.0071 | (0.3499, 0.6851) |
| | (30, 30) | | 0.5653 | 0.0068 | 0.0043 | (0.4384, 0.6904) | | 0.5164 | 0.0060 | 0.0049 | (0.3810, 0.6521) |
| | (40, 40) | | 0.5608 | 0.0024 | 0.0033 | (0.4429, 0.6680) | | 0.5142 | 0.0037 | 0.0039 | (0.3935, 0.6389) |
| | (50, 50) | | 0.5620 | 0.0036 | 0.0025 | (0.4631, 0.6562) | | 0.5144 | 0.0039 | 0.0031 | (0.4063, 0.6213) |
| (1, 2.5) | (15, 15) | 0.6876 | 0.6867 | -0.0009 | 0.0064 | (0.5090, 0.8277) | 0.6503 | 0.6475 | -0.0028 | 0.0078 | (0.4519, 0.8012) |
| | (20, 20) | | 0.6889 | 0.0012 | 0.0048 | (0.5392, 0.8047) | | 0.6461 | -0.0042 | 0.0061 | (0.4865, 0.7896) |
| | (30, 30) | | 0.6860 | -0.0017 | 0.0034 | (0.5634, 0.7917) | | 0.6532 | 0.0029 | 0.0040 | (0.5240, 0.7653) |
| | (40, 40) | | 0.6845 | -0.0031 | 0.0026 | (0.5813, 0.7786) | | 0.6506 | 0.0003 | 0.0033 | (0.5352, 0.7556) |
| | (50, 50) | | 0.6882 | 0.0005 | 0.0021 | (0.5929, 0.7748) | | 0.6495 | -0.0008 | 0.0027 | (0.5469, 0.7446) |
| (1, 3.5) | (15, 15) | 0.7604 | 0.7578 | -0.0025 | 0.0046 | (0.6086, 0.8808) | 0.7327 | 0.7242 | -0.0085 | 0.0062 | (0.5492, 0.8564) |
| | (20, 20) | | 0.7532 | -0.0071 | 0.0038 | (0.6198, 0.8610) | | 0.7250 | -0.0078 | 0.0046 | (0.5709, 0.8369) |
| | (30, 30) | | 0.7579 | -0.0024 | 0.0026 | (0.6410, 0.8441) | | 0.7365 | 0.0038 | 0.0033 | (0.6137, 0.8373) |
| | (40, 40) | | 0.7612 | 0.0008 | 0.0018 | (0.6722, 0.8354) | | 0.7323 | -0.0004 | 0.0023 | (0.6298, 0.8216) |
| | (50, 50) | | 0.7546 | -0.0058 | 0.0017 | (0.6672, 0.8267) | | 0.7300 | -0.0027 | 0.0020 | (0.6388, 0.8117) |

**Table 3:** *Bias, Risk and HPDCI for Bayes estimate of $R_{s,k}$ under prior 2*

| $(q, b,)$ | $(m, n)$ | $R_{1,3}$ | $\hat{R}_{1,3}$ | Bias | MSE | ACI | $R_{2,4}$ | $\hat{R}_{2,4}$ | Bias | MSE | ACI |
|---|---|---|---|---|---|---|---|---|---|---|---|
| (3.5, 1) | (15, 15) | 0.1808 | 0.2329 | 0.0521 | 0.0069 | (0.1259, 0.3698) | 0.1506 | 0.2029 | 0.0523 | 0.0064 | (0.1025, 0.3363) |
| | (20, 20) | | 0.2218 | 0.0410 | 0.0047 | (0.1252, 0.3406) | | 0.1842 | 0.0337 | 0.0034 | (0.1008, 0.2876) |
| | (30, 30) | | 0.2072 | 0.0264 | 0.0025 | (0.1328, 0.2998) | | 0.1785 | 0.0283 | 0.0023 | (0.1142, 0.2586) |
| | (40, 40) | | 0.2044 | 0.0237 | 0.0020 | (0.1370, 0.2845) | | 0.1695 | 0.0189 | 0.0014 | (0.1100, 0.2375) |
| | (50, 50) | | 0.2006 | 0.0198 | 0.0014 | (0.1441, 0.2715) | | 0.1627 | 0.0121 | 0.0008 | (0.1163, 0.2230) |
| (2.5, 1) | (15, 15) | 0.2380 | 0.2995 | 0.0615 | 0.0096 | (0.1637, 0.4635) | 0.2009 | 0.2632 | 0.0624 | 0.0096 | (0.1397, 0.4275) |
| | (20, 20) | | 0.2777 | 0.0397 | 0.0054 | (0.1660, 0.4109) | | 0.2321 | 0.0312 | 0.0043 | (0.1347, 0.3694) |
| | (30, 30) | | 0.2746 | 0.0366 | 0.0042 | (0.1843, 0.3973) | | 0.2309 | 0.0300 | 0.0031 | (0.1494, 0.3300) |
| | (40, 40) | | 0.2599 | 0.0219 | 0.0024 | (0.1843, 0.3550) | | 0.2199 | 0.0191 | 0.0018 | (0.1537, 0.3018) |
| | (50, 50) | | 0.2556 | 0.0177 | 0.0018 | (0.1863, 0.3335) | | 0.2191 | 0.0183 | 0.0016 | (0.1554, 0.2953) |
| (1.5, 1) | (15, 15) | 0.3474 | 0.4127 | 0.0653 | 0.0120 | (0.2599, 0.5983) | 0.3010 | 0.3657 | 0.0647 | 0.0113 | (0.2146, 0.5522) |
| | (20,20) | | 0.3846 | 0.0371 | 0.0073 | (0.2498, 0.5392) | | 0.3422 | 0.0412 | 0.0076 | (0.2017, 0.5048) |
| | (30, 30) | | 0.3771 | 0.0297 | 0.0049 | (0.2622, 0.5094) | | 0.3281 | 0.0271 | 0.0044 | (0.2238, 0.4586) |
| | (40, 40) | | 0.3652 | 0.0178 | 0.0031 | (0.2627, 0.4722) | | 0.3209 | 0.0199 | 0.0029 | (0.2312, 0.4255) |
| | (50, 50) | | 0.3614 | 0.0140 | 0.0025 | (0.2705, 0.4636) | | 0.3155 | 0.0145 | 0.0022 | (0.2321, 0.4091) |
| (1, 1.5) | (15, 15) | 0.5584 | 0.5897 | 0.0312 | 0.0094 | (0.3988, 0.7579) | 0.5105 | 0.5459 | 0.0354 | 0.0103 | (0.3564, 0.7243) |
| | (20, 20) | | 0.5845 | 0.0260 | 0.0066 | (0.4366, 0.7249) | | 0.5393 | 0.0288 | 0.0075 | (0.3816, 0.6390) |
| | (30, 30) | | 0.5736 | 0.0152 | 0.0040 | (0.4472, 0.6888) | | 0.5314 | 0.0209 | 0.0051 | (0.3987, 0.6637) |
| | (40, 40) | | 0.5726 | 0.0142 | 0.0035 | (0.4570, 0.6815) | | 0.5258 | 0.0154 | 0.0037 | (0.4068, 0.6412) |
| | (50, 50) | | 0.5690 | 0.0106 | 0.0025 | (0.4675, 0.6601) | | 0.5243 | 0.0138 | 0.0033 | (0.4137, 0.6393) |
| (1, 2.5) | (15, 15) | 0.6876 | 0.7008 | 0.0131 | 0.0061 | (0.5438, 0.8389) | 0.6503 | 0.6675 | 0.0172 | 0.0076 | (0.4918, 0.8174) |
| | (20, 20) | | 0.7012 | 0.0136 | 0.0048 | (0.5517, 0.8227) | | 0.6718 | 0.0214 | 0.0061 | (0.5107, 0.8027) |
| | (30, 30) | | 0.6966 | 0.0090 | 0.0036 | (0.5774, 0.8078) | | 0.6634 | 0.0130 | 0.0045 | (0.5211, 0.7788) |
| | (40, 40) | | 0.6965 | 0.0088 | 0.0026 | (0.5899, 0.7912) | | 0.6578 | 0.0075 | 0.0030 | (0.5463, 0.7563) |
| | (50, 50) | | 0.6929 | 0.0053 | 0.0021 | (0.5990, 0.7755) | | 0.6576 | 0.0072 | 0.0026 | (0.5558, 0.7556) |
| (1, 3.5) | (15, 15) | 0.7604 | 0.7662 | 0.0059 | 0.0043 | (0.6170, 0.8781) | 0.7327 | 0.7471 | 0.0143 | 0.0058 | (0.5757, 0.8656) |
| | (20, 20) | | 0.7666 | 0.0062 | 0.0038 | (0.6263, 0.8746) | | 0.7380 | 0.0053 | 0.0045 | (0.6015, 0.8555) |
| | (30, 30) | | 0.7686 | 0.0082 | 0.0024 | (0.6662, 0.8529) | | 0.7389 | 0.0062 | 0.0031 | (0.6158, 0.8368) |
| | (40, 40) | | 0.7659 | 0.0055 | 0.0018 | (0.6674, 0.8383) | | 0.7351 | 0.0024 | 0.0026 | (0.6248, 0.8269) |
| | (50, 50) | | 0.7589 | -0.0014 | 0.0015 | (0.6723, 0.8258) | | 0.7368 | 0.0041 | 0.0020 | (0.6384, 0.8180) |

The Bayesian estimates were derived using the MCMC technique with two priors. The Bayesian estimates were derived using the MCMC technique with two priors. Prior 1: (c 1,d 1)=(1,0.5), (c 2,d 2)=(2,0.5), (c 3,d 3)=(1,1) and Prior 2: (c 1,d 1)=(1,1.5), (c 2,d 2)=(2.5,0.5), (c 3,d 3)=(1,1) (2,1). We ran the MCMC chains with a variety of beginning values and generated a total of 10000 iterations. The first 9000 iterations were deleted to reduce the distribution's initial influence. This is referred to as burn-in. Tables 2 and 3 show the bias, Bayes risk, and HPD confidence ranges for $R_{s,k}$ estimations. With increasing sample size, the risk and interval lengths are seen to decrease. We ran the MCMC chains with a variety of beginning values and generated a total of 10000 iterations. The first 9000 iterations were deleted to reduce the distribution's initial influence. This is referred to as burn-in. Tables 2 and 3 show the bias, Bayes risk, and HPD confidence ranges for $R_{s,k}$estimations. With increasing sample size, the risk and interval lengths are seen to decrease.

## 4.1. Data analysis

In this part, a real-world dataset is examined to demonstrate how the produced conclusions may be used. Al-Mutairi et al. (2013) and Rao (2014) considered the dataset, the amount of time (in minutes) that clients had to wait before being served. As an example, suppose bank A has five service points, say $X_1, X_2, ..., X_5$, while bank B has one service point, say Y with $m$= 100 and $n$= 60 as the sample sizes, respectively. For your convenience, the dataset is displayed here.

**D**ata X: 0.8,0.8,1.3,1.5,1.8,1.9,1.9,2.1,2.6,2.7,2.9,3.1,3.2,3.3,3.5,3.6,4.0,4.1,4.2, 4.2,4.3,4.3,4.4,4.4, 4.6,4.7,4.7,4.8,4.9,4.9,5.0,5.3,5.5,5.7,5.7,6.1,6.2,6.2,6.2,6.3, 6.7,6.9,7.1,7.1,7.1,7.1,7.4,7.6,7.7,8.0, 8.2,8.6,8.6,8.6,8.8,8.8,8.9,8.9,9.5,9.6,9.7,9.8,10.7,10.9,11.0,11.0,11.1,11.2,11.2,11.5,11.9,12.4,12.5, 12.9,13.0,13.1,13.3,13.6,13.7,13.9,14.1,15.4,15.4,17.3,17.3,18.1,18.2,18.4,18.9,19.0,19.9,20.6,21.3, 21.4,21.9,23.0,27.0,31.6,33.1,38.5

**D**ata Y: 0.1,0.2,0.3,0.7,0.9,1.1,1.2,1.8,1.9,2.0,2.2,2.3,2.3,2.3,2.5,2.6,2.7,2.7,2.9,3.1,3.1,3.2,3.4,3.4, 3.5,3.9,4.0,4.2,4.5,4.7,5.3,5.6,5.6,6.2,6.3,6.6,6.8,7.3,7.5,7.7,7.7,8.0,8.0,8.5,8.5,8.7,9.5,10.7,10.9,11.0, 12.1,12.3,12.8,12.9,13.2,13.7,14.5,16.0,16.5,28.0

In order to match the datasets, we used the Type I generalized half-logistic distribution, and it can be shown that the model fits the data quite well. $q$ and $\sigma$ have MLE values of 0.41 and 3.33 for Data $X$. The MLEs of $b$ and $\sigma$ for Data $Y$ are 0.69 and 3.33. Table 4 contains the results of the KS-test as well as the relevant $p$-values. Figure 1 shows a histogram of the fit, which shows how well the model fits. The values $s$= 5 and $k$= 5 are used for example reasons only, which means that the service points in Bank A are connected in a series fashion. A series connection of the service points might be read as consumers offering services for all five of the service points that are now accessible. The estimate of $R_{5,5}$ is obtained as 0.2160 with a 95 percent asymptotic confidence range of (0.1580,0.2740).

**Table 4:** *Goodness for fit for data set*

| | Shape parameter | Scale parameter | K-S Statistic | $p$-value |
|---|---|---|---|---|
| Data $X$ | 0.41 | 3.33 | 0.1136 | 0.1513 |
| Data $Y$ | 0.69 | 3.33 | 0.0728 | 0.9083 |

The MCMC technique under two priors was used to produce the Bayesian estimates in this case. Preliminary estimates for the following priors are used:

Prior 1: $(c_1, d_1) = (2, 1), (c_2, d_2) = (2, 1), (c_3, d_3) = (0.5, 0.5)$, and

Prior 2: $(c_1, d_1) = (1, 0.5), (c_2, d_2) = (2, 0.5), (c_3, d_3) = (1, 1)$.

We ran the MCMC chains and generated 20000 iterations, the first 10000 iterations of the distribution were removed in order to reduce the initial influence of the distribution. In order to break the

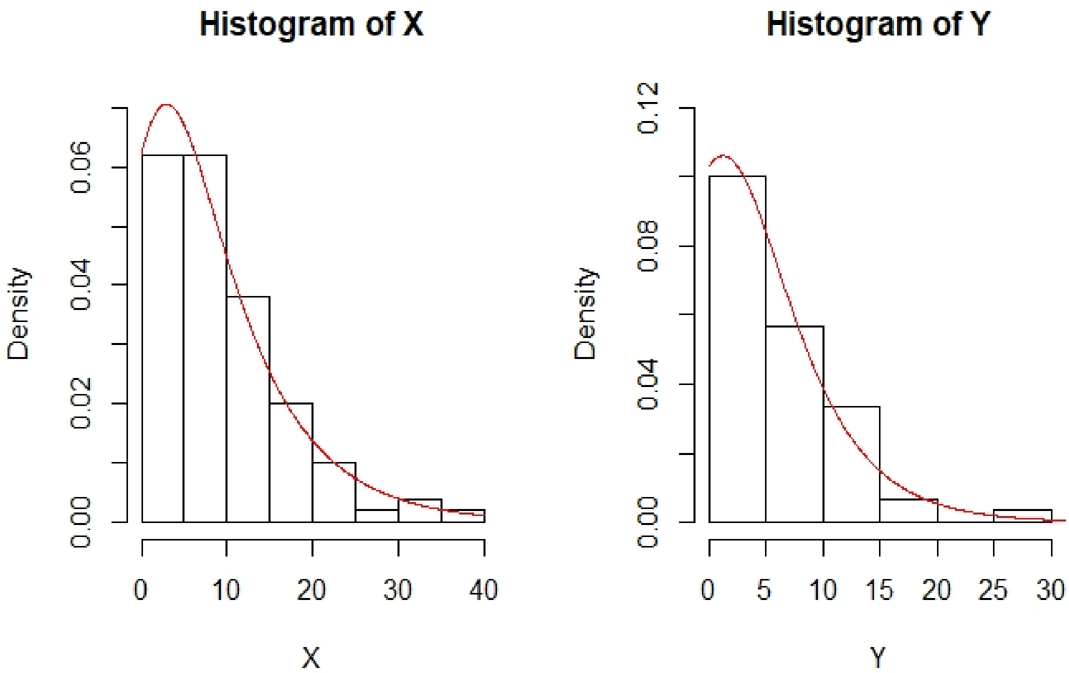


**Figure 1:** *The fitted density for X and Y*

reliance among the produced samples, we select a sample every tenth one. This results in a final chain of 1000 samples. According to the preceding condition, the multicomponent stress-strength reliability is derived as $\hat{R}_{5,5}$= 0.2189 with 95 percent credible interval as (0.1660,0.2766). Figure 2 shows the trace plot of and histogram of the $R_{s,k}$ values. Prior 2 yields the multicomponent stress-strength reliability as $\hat{R}_{5,5}$= 0.2192 with 95 percent credible interval as (0.1701,0.2760). Figure 3 depicts a trace plot and histogram of the $R_{s,k}$ values.

## 5. Conclusions

Using the Type I generalized half-logistic model, we may derive explicit formulas for several of the model's dependability features. Additional point and interval estimates of the multicomponent stress strength reliability, $R_{s,k}$ where the strength of its constituents and the stress applied to it are statistically independent and follow a Type I generalized half-logistic distribution are presented. The maximum likelihood estimates and Bayesian estimates under the squared error loss function are generated. The results of the simulations indicate that the estimations were compatible with one another. Furthermore, as the sample size was increased, the length of the confidence interval shrank as a result of this. As an example of how the proposed conclusions can be put into practice, a real-life scenario is explored.

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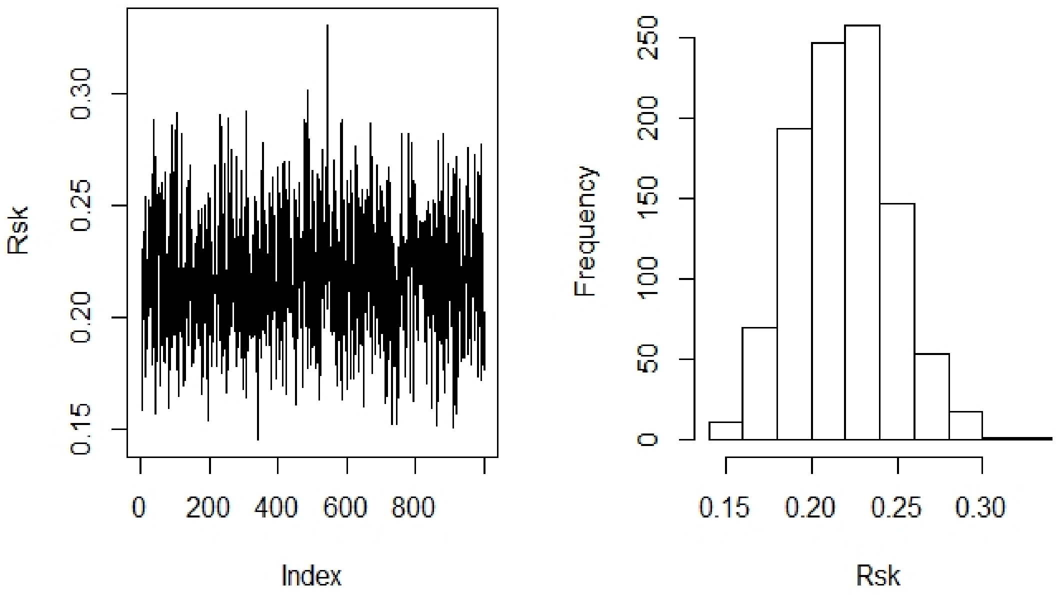


**Figure 2:** *Trace plot and histogram of $R_{s,k}$ values under prior 1*

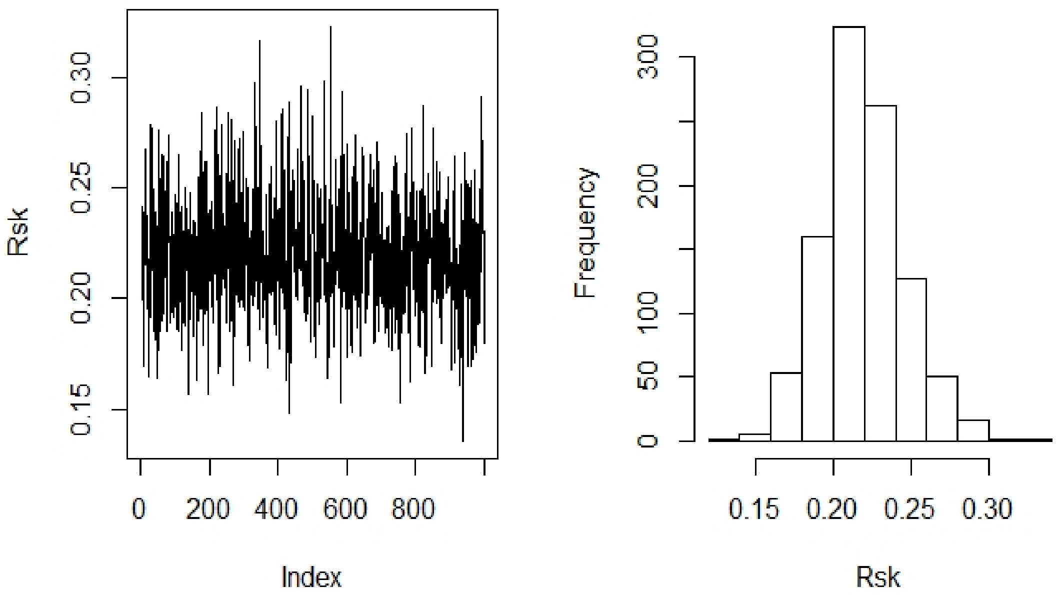


**Figure 3:** *Trace plot and histogram of $R_{s,k}$ values under prior 2*

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