Analysis of Some Proposed Replacement Policies

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Abstract

This paper is coming up with an age replacement cost model under the standard age replacement policy (SARP) for some multi-unit systems. Furthermore, some two other age replacement cost models will be constructed for the multi-unit systems under some proposed policies (policy A and policy B). For simple illustration of the proposed age replacement cost models under SARP, policy A and policy B, numerical example was provided, and the result obtained will be beneficial to engineers, maintenance managers and plant management, in selecting and applying the optimal preventive maintenance policies.

Keywords: failure rate, proposed policies, multi-unit systems

I. Introduction

Multi-components systems deteriorate and subsequently fail due to age and usage. To reduce the occurrence of system failures, management of organizations are always interested in selecting and implementing the optimal preventive replacement policy for normal system operation. Furthermore, in describing the reliability of a multi-unit system, it is necessary to specify how the units of the system are connected and provide the rule of the operation. The simplest form of the system configuration is the series configuration. Designing systems in parallel configuration is done with the intention to improve systems reliability. In most practical situations, a combination of both series and parallel configurations is inevitable.

Enogwe *et al.* [1] used the distribution of the probability of failure times and come up with a replacement model for items that fails un-notice. Fallahnezhad and Najafian [2] investigated the number of spare parts and installations for a unit and parallel systems, so as cut down the average cost per unit time. Gertsbakh [3] described and presented some vital preventive maintenance models for some multi-component systems. Huang and Wang [4] constructed a time-replacement model for multistate systems, which can be used to determine the optimal time to replace the entire system, and this proposed approach provides further insight into the relationship between preventive maintenance policy setting and long-term system benefits. Jain *et al.* [5] developed Markov model for a multi-component system which is subjected to two types of failures, which are hardware failure and human error. Jain and Gupta [6] presented a preventive replacement model for a repairable system with multiple vacation and imperfect coverage. Lim *et al.* [7] presented some characteristics of some age substitution policies. Liu *et al.* [8] come up with mathematical models of uncertain reliability of some multi-component systems. Malki *et al.* [9] studied some age replacement policies of a parallel system with stochastic dependency. Murthy and Hwang [10] discussed that, the failures can be reduced through effective maintenance actions, and such maintenance actions can occur either at discrete time instants or continuously over time. Nakagawa [11] presented age replacement model for series and parallel system based on standard age replacement policy. Nakagawa *et al*. [12] presented the advantages of some proposed replacement policies. In an approach for analyzing the behavior of an industrial system under the cost free warranty policy, Niwas and Garg [13] developed a mathematical model of a system based on the Markov process, they also derived various parameters such as reliability, mean time to system failure, availability and expected profit for the system. Safaei *et al.* [14] studied the optimal preventive maintenance action for a system based on some conditions. Sudheesh *et al.* [15] studied age replacement model in discrete approach. Tsoukalas and Agrafiotis [16] presented a new replacement policy warrant for a system with correlated failure and usage time. Waziri and Yusuf [17] constructed an age replacement cost model for a parallel-series system based on some proposed policies, where they investigated the characteristics of the proposed policies. In trying to extending the optimal replacement time of multi-unit systems, Waziri et al. [18] come up with some proposed age replacement cost models involving discounting rate and minimal repair for a series system. Waziri [19] presented a discounted age replacement model for a unit based on discrete time. Wu *et al.* [20] proposed a new replacement policy and established corresponding replacement models for a deteriorating repairable system with multiple vacations of one repairman. Xie *et al*. [21] assessed the effects of safety barriers on the prevention of cascading failures. Zhao *et al.* [22] collected some recent results on age replacement policies and proposed some modified age replacement policies, such as optimal age replacement policy for a parallel system with a random number of units.

The literature review presented in this paper did not captured a way or strategy of extending the optimal replacement time of a multi-component system. This paper will proposed some proposed replacement cost models under some policies, so as to see the possibility of extending the optimal replacement time of some four multi-component systems, and this will be achieved through the following objectives:

1. By constructing age replacement cost model for series and parallel systems under the standard age replacement policy (SARP).
2. By constructing age replacement cost models for series and parallel systems under two proposed policies (policy A and policy B).
3. By providing a numerical example for simple illustration of the constructed replacement cost models.

II. Methods

Reliability measures namely reliability function and failure rates are used to obtain the expressions of replacement cost models for four systems under the standard age replacement policy (SARP) and under proposed two policies (policy A and policy B). A numerical example was given so to assess the three replacement policies.

III. Notations

- $r_i(t)$: Level I failure rate of unit D_i, for $i = 1, 2, 3, 4, 5, 6$.
- $r_i^*(t)$: Level II failure rate of unit D_i, for $i = 1, 2, 3, 4, 5, 6$.
- $R_i^*(t)$: Reliability function of Level II failure of unit B_i, for $i = 1, 2, 3, 4, 5, 6$.
- $SARP$: Standard age replacement policy.
- $R_{Si}^*(t)$: Reliability function of system S_i due to Level II failure, for $i = 1, 2, 3, 4$.
- $CS_i(T)$: Cost rate of system S_i under SARP, for $i = 1, 2, 3, 4$.
- $CYS_i(T)$: Cost rate of system S_i under policy A, for $i = 1, 2, 3, 4$.
- $CZ(T)$: Cost rate of system S_i under policy B, for $i = 1, 2, 3, 4$.
- X_{Si}^*: Optimal replacement time of system S_i under SARP, for $i = 1, 2, 3, 4$.
- Y_{Si}^*: Optimal replacement time of system S_i under policy A, for $i = 1, 2, 3, 4$.
- Z_{Si}^*: Optimal replacement time of system S_i under policy B, for $i = 1, 2, 3, 4$.
- C_{ir}: Cost of unplanned replacement of failed D_i due to Level II failure, for $i = 1, 2, 3, 4, 5, 6$.
- C_{im}: Cost of minimal repair of failed D_i due to Level II failure, for $i = 1, 2, 3, 4, 5, 6$.
- C_{sp}: Cost of planned replacement of system S_i at planned replacement time T, for $i = 1, 2, 3, 4$.
- C_{sr}: Cost of un-planned replacement of system S_i due to Level II failure, for $i = 1, 2, 3, 4$.

IV. Description of the Systems

Consider six units D_1, D_2, D_3, D_4, D_5 and D_6, arranged in four different configurations, so as to formed four different systems, which are series-parallel system (S_1), series-parallel system (S_2), parallel-series system (S_3) and parallel-series system (S_4). All the six units are subjected to Level I and Level II failures, such that, Level I failure is repairable one, while the Level II failure is non-repairable failure. Since all the six units are subjected to Level I and Level II failures, then, this implies that, all the four systems are also subjected to Level I and Level II failures. See the Figure 1, Figure 2 , Figure 3 and Figure 4 as the diagram of the four systems (S_1, S_2, S_3 and S_4).

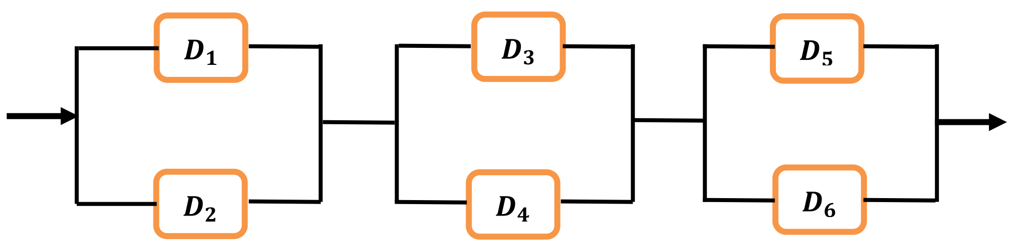

Figure 1: *Reliability block diagram of system* S_1

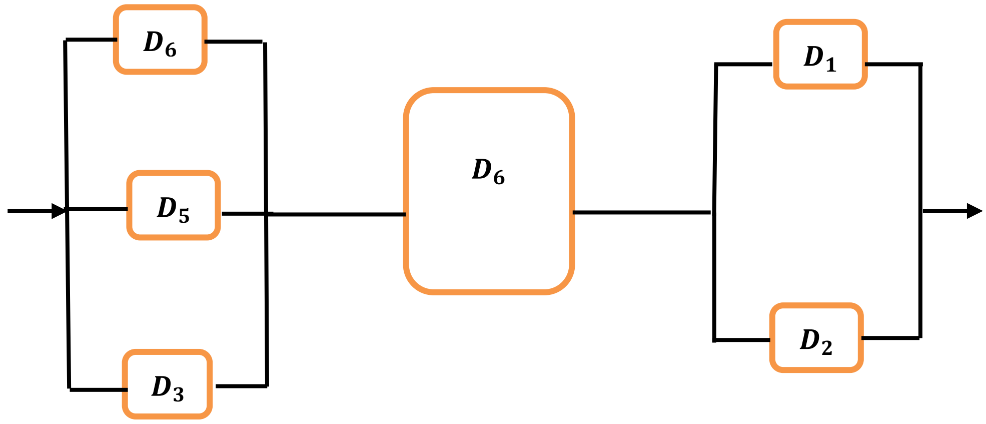

Figure 2: *Reliability block diagram of system S_2*

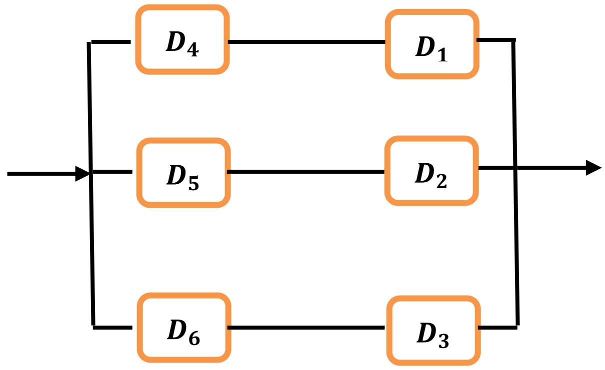

Figure 3: *Reliability block diagram of system S_3*

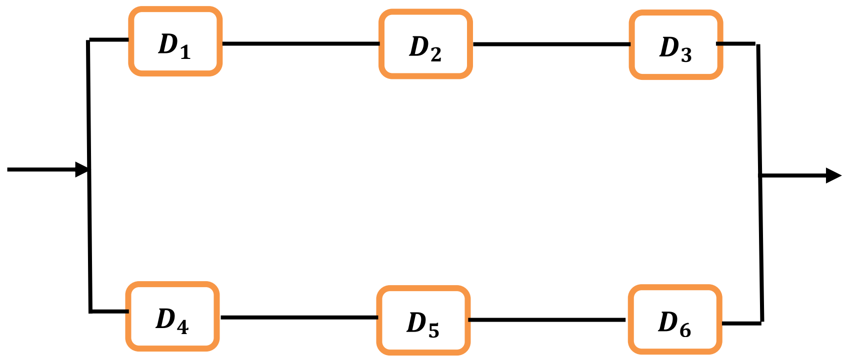

Figure 4: *Reliability block diagram of system S_4*

V. Replacement Cost Models Under SARP

Some Assumptions for SARP:

1. If a system fails due to Level I failure, then the system is minimally repaired.
2. If a system fails due to Level II failure, then the whole system replaced completely with new one.
3. Both the two levels of failures (Level I and Level II) of the six units arrives according to non-homogeneous Poisson process.
4. Rate of Level I failure of the six units follows the order: $r_1(t) \geq r_3(t) \geq r_5(t) \geq r_2(t) \geq r_4(t) \geq r_6(t)$.
5. Rate of Level II failure of the six units follows the order: $r_1^*(t) \geq r_3^*(t) \geq r_5^*(t) \geq r_2^*(t) \geq r_4^*(t) \geq r_6^*(t)$.
6. The cost of repair and replacement follows the order: $C_{im} < C_{Sp} < C_{Sr}$, for $i = 1, 2, 3, 4, 5, 6$.
7. A system is replaced at a planned time $T(T > 0)$ or at Level II failure, whichever occurs first.
8. The cost of planned replacement of a system is less than the cost of un-planned replacement.
9. The cost of repair of a failed unit is less than the cost of replacement of a unit.
10. All costs are positive numbers.

From the assumptions above, the probability that system S_1 will be replaced at planned replacement time T, before Level II failure occurs, is

$$R_{S1}^*(T) = (1 - \prod_{i=1}^{2}(1 - R_i^*(T))) \times (1 - \prod_{i=1}^{2}(1 - R_i^*(T))) \times (1 - \prod_{i=1}^{2}(1 - R_i^*(T))). \quad (1)$$

From the assumptions above, the probability that system S_2 will be replaced at planned replacement time T, before Level II failure occurs, is

$$R_{S2}^*(T) = (1 - \prod_{i=1}^{2}(1 - R_i^*(T))) \times (1 - \prod_{i=1}^{3}(1 - R_i^*(T))) \times R_6^*(T) . \quad (2)$$

From the assumptions above, the probability that system S_3 will be replaced at planned replacement time T, before Level II failure occurs, is

$$R_{S3}^*(T) = 1 - \left(1 - R_3^*(T)R_6^*(T)\right) \times \left(1 - R_2^*(T)R_5^*(T)\right) \times \left(1 - R_1^*(T)R_4^*(T)\right) . \quad (3)$$

From the assumptions above, the probability that system S_4 will be replaced at planned replacement time T, before Level II failure occurs, is

$$R_{S4}^*(T) = 1 - (1 - R_1^*(T)R_2^*(T)R_3^*(T)) \times (1 - R_4^*(T)R_5^*(T)R_6^*(T)) . \quad (4)$$

The mean time of systems S_1, S_2 , S_3 and S_4 under SARS, is

$$\int_0^T R_{Si}^*(t)dt \text{ , for } i = 1, 2, 3,4. \quad (5)$$

The cost of un-planned replacement (failure due to Level II failure) of S_1 and S_2 in one replacement cycle under SARP, is

$$C_{sr}\left(1 - R_{Si}^*(T)\right), \text{ for } i = 1, 2, 3, 4. \quad (6)$$

The cost of planned replacement at time T of S_1, S_2 , S_3 and S_4 in one replacement cycle under

SARS, is

$$C_{sp}R_{Si}^*(T), \quad \text{for} \quad i = 1,2,3,4\,. \tag{7}$$

The cost of minimal repair of components D_1, D_2, D_3, D_4, D_5 and D_6 due to Level I failure in one replacement cycle under SARP, is

$$J(T) = \int_0^T C_{1m}r_1(t)R_{Si}^*(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^*(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^*(t)dt \\ + \int_0^T C_{4m}r_4(t)R_{Si}^*(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^*(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^*(t)dt\,. \tag{8}$$

Using equations (5), (6), (7) and (8), the replacement cost rate of systems S_1, S_2 , S_3 and S_4 under SARP is

$$CS_i(T) = \frac{C_{sr}\left(1 - R_{Si}^*(T)\right) + C_{sp}\, R_{Si}^*(T) + \int_0^T J(t)\, R_{Si}^*(t)dt}{\int_0^T R_{Si}^*(t)dt}, \quad i = 1,2,3,4, \tag{9}$$

where

$$J(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t). \tag{10}$$

Noting that, $CS_i(T)$ for $i = 1,2,3,4$, is adopted as an objective function of an optimization problem, and the main goal is to obtain an optimal replacement time T_{Si}^* that minimizes $CS_i(T)$, for $i = 1,2,3,4$.

VI. Replacement Cost Models Under Policy A

From assumption 4, observe that, Level II failure of units D_1, D_3 and D_5 is higher than that of units D_2, D_4 or D_6. Policy A is a preventive maintenance policy, in which the un-planned replacement of a system, which depends on the failure of units D_1, D_3 and D_5 due to Level II. Noting that, the reliability function of a system due to policy A, depends on the location of units D_1, D_3 and D_5 in a system. But when any of the units D_2, D_4 or D_6 fails due to Level II failure, the failed unit is replace completely with new one and allow the system to continue operating from where it stopped.

Under policy A, we have the following descriptions:

1. System S_1 : the system is replace completely with new one when at least one of the components D_1, D_3 or D_5 fails due to Level II failure. Now, the probability that system S_1 will be replaced at planned replacement time T, before Level II failure occurs due to policy A, is

$$R_{S1}^{a*}(T) = R_1^*(T)R_3^*(T)R_5^*(T). \tag{11}$$

2. System S_2 : the system is replace completely with new one when all the three units D_1, D_3 and D_5 fails due to Level II failure. Now, the probability that system S_2 will be replaced at planned replacement time T, before Level II failure occurs due to policy A, is

$$R_{S2}^{a*}(T) = R_1^*(T)\left(1 - (1 - R_3^*(T))(1 - R_5^*(T))\right)\,. \tag{12}$$

3. System S_3 : the system is replace completely with new one when at least one of the units D_1, D_3 or D_5 fails due to Level II failure. Now, the probability that system S_3 will be replaced at planned replacement time T, before Level II failure occurs due to policy A, is

$$R_{S3}^{a*}(T) = \left(1 - (1 - R_1^*(T))(1 - R_3^*(T))(1 - R_5^*(T))\right). \tag{13}$$

4. System S_4 : the system is replace completely with new one when any of the combination fails: D_1 and D_5, or D_3 and D_5 fails. Now, the probability that system S_3 will be replaced at planned replacement time T, before Level II failure occurs due to policy A, is

$$R_{S4}^{a*}(T) = 1 - (1 - R_1^*(T)R_3^*(T))(1 - R_5^*(T)). \tag{14}$$

The mean time of systems of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy A, is

$$\int_0^T R_{Si}^{a*}(t)dt \text{ , for } \; i = 1, 2, 3,4.. \tag{15}$$

The cost of un-planned replacement (failure due to Level II failure) of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy A, is

$$C_{sr}\left(1 - R_{Si}^{a*}(T)\right), \text{ for } \; i = 1, 2, 3, 4. \tag{16}$$

The cost of planned replacement at time T of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy A, is

$$C_{sp}R_{Si}^{a*}(T), \quad \text{for } \; i = 1, 2, 3, 4. \tag{17}$$

The cost of minimal repair of components D_1, D_2, D_3, D_4, D_5 and D_6 due to Level I failure in one replacement cycle under policy A, is

$$\int_0^T C_{1m}r_1(t)R_{Si}^{a*}(t)dt + \int_0^T C_{2m}r_2(t)R_{Si}^{a*}(t)dt + \int_0^T C_{3m}r_3(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4m}r_4(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4m}r_5(t)R_{Si}^{a*}(t)dt + \int_0^T C_{5m}r_6(t)R_{Si}^{a*}(t)dt. \tag{18}$$

The cost of replacement of components D_2, D_4 and D_6 due to Level II failure in one replacement cycle under policy A, is

$$\int_0^T C_{2r}r_2^*(t)R_{Si}^{a*}(t)dt + \int_0^T C_{4r}r_4^*(t)R_{Si}^{a*}(t)dt \int_0^T C_{6r}r_6^*(t)R_{Si}^{a*}(t)dt. \tag{19}$$

Using equations (15), (16), (17), (18) and (19), the replacement cost rate of S_1, S_2 , S_3 and S_4 under policy A, is

$$CYS_i(T) = \frac{C_{sr}\left(1 - R_{Si}^{a*}(T)\right) + C_{sp}R_{Si}^{a*}(T) + \int_0^T K(t)R_{Si}^{a*}(t)dt + \int_0^T L(t)R_{Si}^{a*}(t)dt}{\int_0^T R_{Si}^{a*}(t)dt}, \text{ for } \; i = 1, 2, 3, 4 \tag{20}$$

where

$$K(t) = C_{1m}r_1(t) + C_{2m}r_2(t) + C_{3m}r_3(t) + C_{4m}r_4(t) + C_{5m}r_5(t) + C_{6m}r_6(t), \tag{21}$$

and

$$L(t) = C_{2r}r_2^*(t) + C_{4r}r_4^*(t) + C_{6r}r_6^*(t). \tag{22}$$

Noting that, $CYS_i(T)$ for $i = 1, 2, 3, 4,$ is adopted as an objective function of an optimization problem, and the main goal is to obtain an optimal replacement time Y_{Si}^* that minimizes $CYS_i(T)$, for $i = 1, 2, 3, 4$.

VII. Replacement Cost Models Under Policy B

Observe from assumption 4, that Level II failure of units $D_2,\ D_4$ and D_6 is lower than that of units D_1, D_3 or D_5. Policy B is a preventive maintenance policy, in which the un-planned replacement of a whole system depends on the failure of units $D_2,\ D_4$ and D_6 due to Level II. Noting that, the reliability function of a system due to policy B, depends on the location of units $D_2,\ D_4$ and D_6 in a system. But when any of the units $D_1,\ D_3$ or D_5 fails due to Level II failure, the failed unit is replace completely with new one and allow the system to continue operating from where it stopped.

Under policy B, we have the following descriptions:

1. System S_1 : the system is replace completely with new one when at least one of the units D_2, D_4 or D_6 fails due to Level II failure. Now, the probability that system S_1 will be replaced at planned replacement time T, before Level II failure occurs due to policy B, is

$$R_{S1}^{b*}(T) = R_2^*(T)R_4^*(T)R_6^*(T). \tag{23}$$

2. System S_2 : the system is replace completely with new one when all the three units D_2, D_4 or D_6 fails due to Level II failure. Now, the probability that system S_2 will be replaced at planned replacement time T, before Level II failure occurs due to policy B, is

$$R_{S2}^{b*}(T) = R_2^*(T)R_4^*(T)R_6^*(T). \tag{24}$$

3. System S_3 : the system is replace completely with new one when at least one of the components D_2, D_4 or D_6 fails due to Level II failure. Now, the probability that system S_3 will be replaced at planned replacement time T, before Level II failure occurs due to policy B, is

$$R_{S3}^{b*}(T) = 1 - (1 - R_2^*(T))(1 - R_4^*(T))(1 - R_6^*(T))\ . \tag{25}$$

4. System S_4 : the system is replace completely with new one when any of the combination fails: D_4 and D_2, or D_6 and D_2 fails. Now, the probability that system S_3 will be replaced at planned replacement time T, before Level II failure occurs due to policy B, is

$$R_{S4}^{b*}(T) = 1 - (1 - R_4^*(T)R_6^*(T))(1 - R_2^*(T))\ . \tag{26}$$

The mean time of systems of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy B, is

$$\int_0^T R_{Si}^{b*}(t)dt\ , \text{for} \quad i = 1, 2, 3, 4\ . \tag{27}$$

The cost of un-planned replacement (failure due to Level II failure) of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy B, is

$$C_{sr}\left(1 - R_{Si}^{b*}(T)\right), \text{for} \quad i = 1, 2, 3, 4\ . \tag{28}$$

The cost of planned replacement at time T of S_1, S_2 , S_3 and S_4 in one replacement cycle under policy B, is

$$C_{sp}R_{Si}^{b*}(T), \quad \text{for} \quad i = 1, 2, 3, 4\ . \tag{29}$$

The cost of minimal repair of components D_1, D_2, D_3, D_4, D_5 and D_6 due to Level I failure in one replacement cycle under policy B, is

$$\int_0^T C_{1m} r_1(t) R_{Si}^{b*}(t)dt + \int_0^T C_{2m} r_2(t) R_{Si}^{b*}(t)dt + \int_0^T C_{3m} r_3(t) R_{Si}^{b*}(t)dt + \int_0^T C_{4m} r_4(t) R_{Si}^{b*}(t)dt + \int_0^T C_{4m} r_5(t) R_{Si}^{b*}(t)dt + \int_0^T C_{5m} r_6(t) R_{Si}^{b*}(t)dt\ . \quad (30)$$

The cost of replacement of components D_1, D_3 and D_5 due to Level II failure in one replacement cycle under policy B, is

$$\int_0^T C_{1r} r_1^*(t) R_{Si}^{b*}(t)dt + \int_0^T C_{3r} r_3^*(t) R_{Si}^{b*}(t)dt \int_0^T C_{5r} r_5^*(t) R_{Si}^{b*}(t)dt. \quad (31)$$

Using equations (27), (28), (29), (30) and (31), the replacement cost rate of systems S_1, S_2 , S_3 and S_4 under policy B, is

$$CZS_i(T) = \frac{C_{Sr}\left(1 - R_{Si}^{b*}(T)\right) + C_{Sp} R_{Si}^{b*}(T) + \int_0^T M(t) R_{Si}^{b*}(t)dt + \int_0^T N(t) R_{Si}^{b*}(t)dt}{\int_0^T R_{Si}^{b*}(t)dt},\ \text{for}\ \ i = 1, 2, 3, 4, \quad (32)$$

where

$$M(t) = C_{1m} r_1(t) + C_{2m} r_2(t) + C_{3m} r_3(t) + C_{4m} r_4(t) + C_{5m} r_5(t) + C_{6m} r_6(t), \quad (33)$$

and

$$N(t) = C_{1r} r_1^*(t) + C_{3r} r_3^*(t) + C_{5r} r_5^*(t)\ . \quad (34)$$

Noting that, $CZS_i(T)$ for $i = 1, 2, 3, 4$, is adopted as an objective function of an optimization problem, and the main goal is to obtain an optimal replacement time Z_{Si}^* that minimizes $CZS_i(T)$, for $i = 1, 2, 3, 4$.

VIII. Numerical Example

To illustrate the characteristics of the constructed replacement cost models under SARP, policies A and B. Let the time of Level I failure for the six units follows Weibull distribution:

$$r_i(t) = \lambda_i \propto_i t^{\propto_i - 1},\ \text{for}\ i = 1, 2, 3, 4, 5, 6, \quad (35)$$

where $\propto_i > 1$ and $t \geq 0$.

Also, let the time of Level II failure for the six units follows Weibull distribution:

$$r_i^*(t) = \lambda_i^* \propto_i^* t^{\propto_i^* - 1},\ \text{for}\ i = 1, 2, 3, 4, 5, 6, \quad (36)$$

where $\propto_i > 1$and $t \geq 0$.

Let the set of parameters and cost of repair and replacement be used throughout this particular example:

1. $\propto_1 = 4$, $\propto_2 = 3$, $\propto_3 = 3$, $\propto_4 = 3$, $\propto_5 = 4$ and $\propto_6 = 2$.
2. $\lambda_1 = 0.03$, $\lambda_2 = 0.002$, $\lambda_3 = 0.03$, $\lambda_4 = 0.001$, $\lambda_5 = 0.001$ and $\lambda_6 = 0.001$.

3. $\propto_1^* = 4, \propto_2^* = 3.5, \propto_3^* = 4, \propto_4^* = 3.5, \propto_5^* = 4$, and $\propto_6^* = 3.5$.
4. $\lambda_1^* = 0.00033, \lambda_2^* = 0.00025, \lambda_3^* = 0.00030, \lambda_4^* = 0.00023, \lambda_5^* = 0.00025$ and $\lambda_6^* = 0.0002$.
5. $C_{sr} = 70$, $C_{sp} = 50$ and $C_{im} = 0.5$, for $i = 1, 2, 3, 4, 5, 6$.

By substituting the parameters in equations (35) and (36), the following equations (Level I and Level II failures) below are obtained as follows:

$$r_1(t) = 0.12t^3. \tag{37}$$

$$r_2(t) = 0.06t. \tag{38}$$

$$r_3(t) = 0.09t^2. \tag{39}$$

$$r_4(t) = 0.003t^2. \tag{40}$$

$$r_5(t) = 0.004t^3. \tag{41}$$

$$r_6(t) = 0.002t. \tag{42}$$

$$r_1^*(t) = 0.00132t^3. \tag{43}$$

$$r_2^*(t) = 0.000875t^{2.5}. \tag{44}$$

$$r_3^*(t) = 0.00012t^3. \tag{45}$$

$$r_4^*(t) = 0.000805t^{2.5}. \tag{46}$$

$$r_5^*(t) = 0.001t^3. \tag{47}$$

$$r_6^*(t) = 0.0007t^{2.5}. \tag{48}$$

Tables 1, 2 and 3 below are obtained, by substituting the assumed cost of replacement/repair and rates of Level I and Level II failures obtained above (equations (37) to (48)) in the replacement cost models constructed above (equations (9), (20) and (32)), so as to determine the optimal replacement times of the four systems.

Table 1. *Results obtained from evaluating the replacement cost rates of systems S_1, S_2, S_3 and S_4 under SARP.*

T	$CS_1(T)$	$CS_2(T)$	$CS_3(T)$	$CS_4(T)$
1	240.04	240.09	240.04	240.03
2	120.16	120.43	120.16	120.11
3	80.42	81.14	80.37	80.41
4	61.06	62.38	60.75	61.61
5	50.68	52.48	49.65	53.17
6	46.28	48.04	44.23	52.86
7	46.64	48.13	44.70	59.29
8	53.74	52.16	51.31	67.98
9	61.01	58.01	60.74	72.17
10	63.97	58.91	64.89	74.11
11	70.03	61.84	65.31	76.16
12	73.92	68.98	67.87	78.97

Table 2. *Results obtained from evaluating the replacement cost rates of systems S_1, S_2, S_3 and S_4 under policy A.*

T	$CYS_1(T)$	$CYS_2(T)$	$CYS_3(T)$	$CYS_4(T)$
1	240.78	240.65	240.57	240.57
2	122.87	121.82	121.18	121.19
3	87.58	84.10	81.96	82.03
4	76.01	68.08	62.89	63.41
5	76.62	62.52	52.13	54.13
6	83.71	64.18	46.27	51.36
7	89.44	70.76	44.88	53.95
8	90.24	77.06	48.34	59.77
9	92.73	77.01	55.49	64.92
10	94.99	73.65	61.79	66.28
11	95.45	76.44	62.77	67.90
12	98.00	79.00	65.57	69.72

Table 3. *Results obtained from evaluating the replacement cost rates of systems S_1, S_2, S_3 and S_4 under policy B.*

T	$CZS_1(T)$	$CZS_2(T)$	$CZS_3(T)$	$CZS_4(T)$
1	240.80	243.3	240.64	240.64
2	122.54	125.04	121.62	121.63
3	85.61	88.11	83.11	83.13
4	70.10	72.6	65.11	65.20
5	63.97	66.47	55.61	55.94
6	62.95	65.45	50.64	51.48
7	64.67	67.17	49.55	50.22
8	67.21	69.71	48.55	51.27
9	68.67	71.17	50.20	53.79
10	70.74	73.24	53.05	56.80
11	74.46	76.96	56.43	59.10
12	76.97	79.47	59.34	59.61

Table 4. *The optimal replacement times of systems S_1, S_2, S_3 and S_4 under SARP, policy A and policy B from tables 1, 2 and 3.*

System	Under SARP	Under policy A	Under policy B
S_1	$X_{S1}^* = 6.00$	$Y_{S1}^* = 4.00$	$Z_{S1}^* = 6.00$
S_2	$X_{S2}^* = 6.00$	$Y_{S2}^* = 5.00$	$Z_{S2}^* = 6.00$
S_3	$X_{S3}^* = 6.00$	$Y_{S3}^* = 7.00$	$Z_{S3}^* = 8.00$
S_4	$X_{S4}^* = 6.00$	$Y_{S4}^* = 6.00$	$Z_{S4}^* = 7.00$

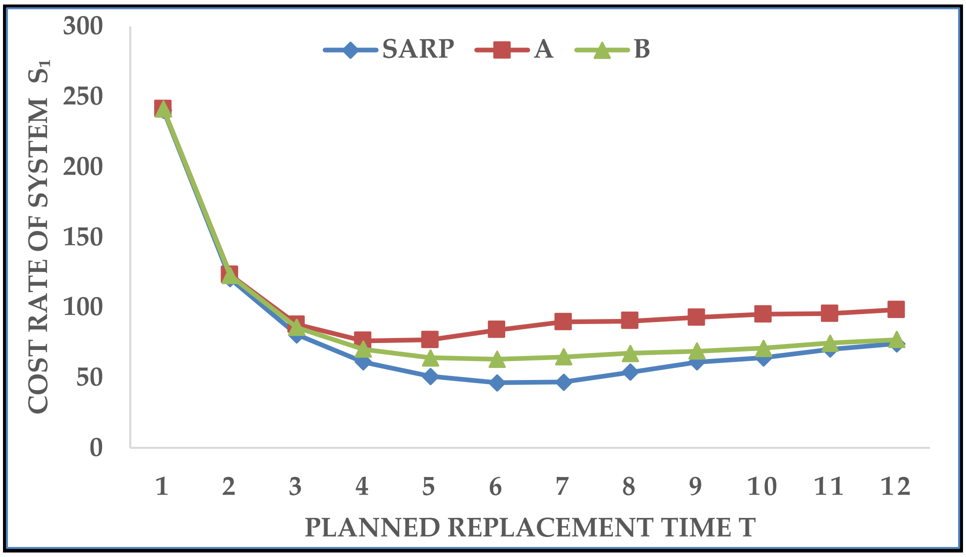

Figure 5: *The plot of cost rates of system S_1 under SARP, policy A and policy B against planned replacement time T.*

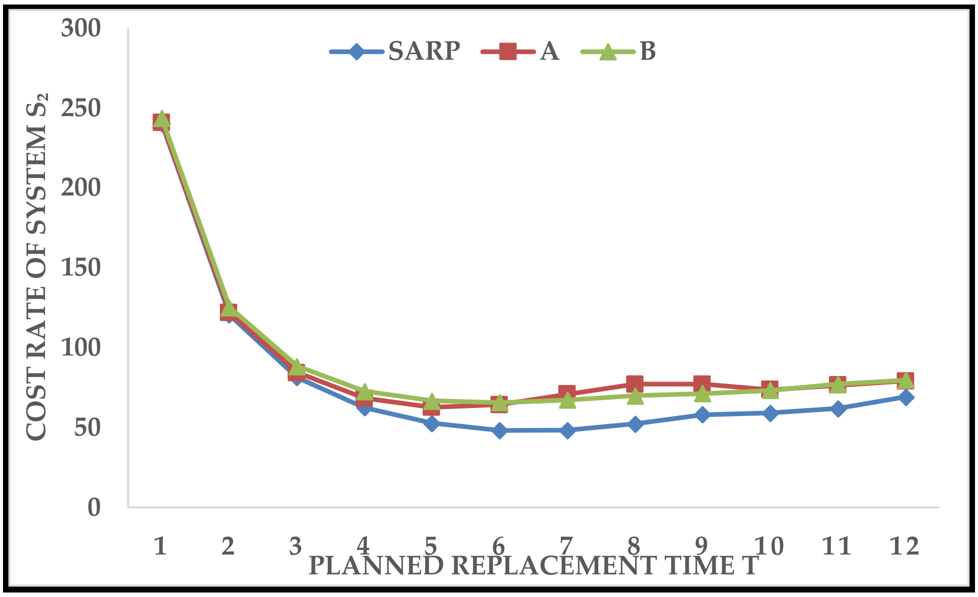

Figure 6: *The plot of cost rates of system S_2 under SARP, policy A and policy B against planned replacement time T.*

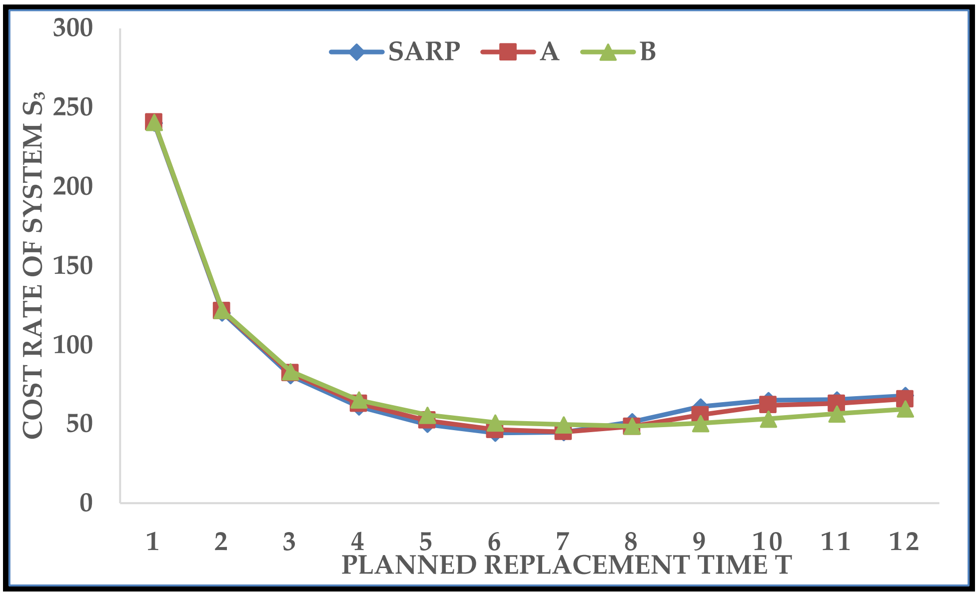

Figure 7: *The plot of cost rates of system S_3 under SARP, policy A and policy B against planned replacement time T.*

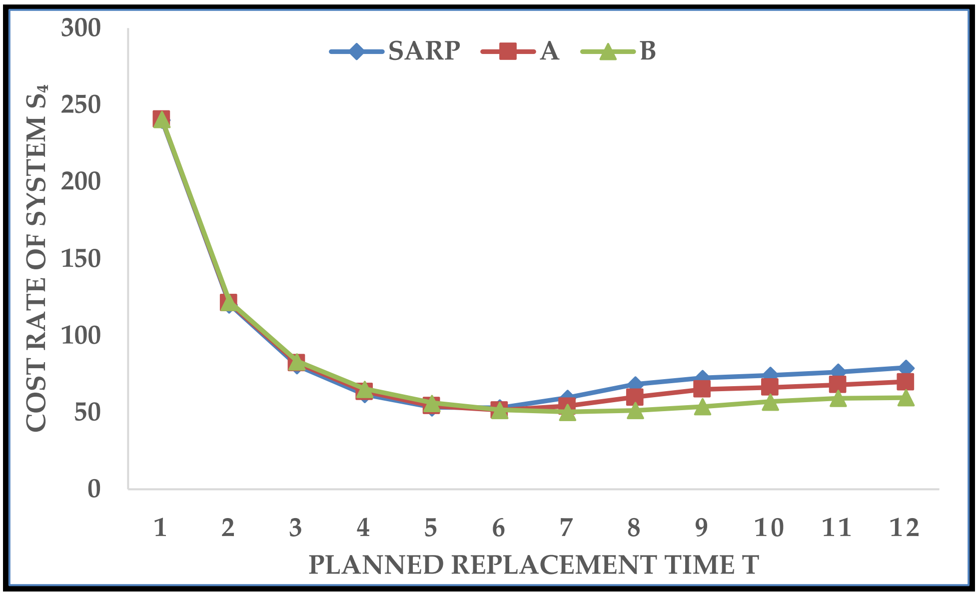

Figure 8: *The plot of cost rates of system S_4 under SARP, policy A and policy B against planned replacement time T.*

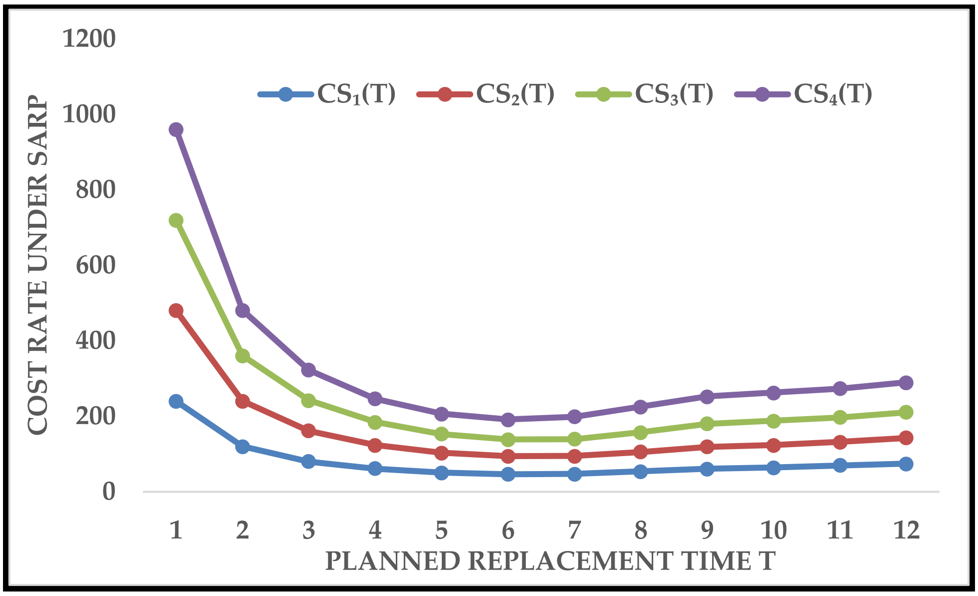

Figure 9 : *The plot of cost rates of the four systems under SARP against planned replacement time T.*

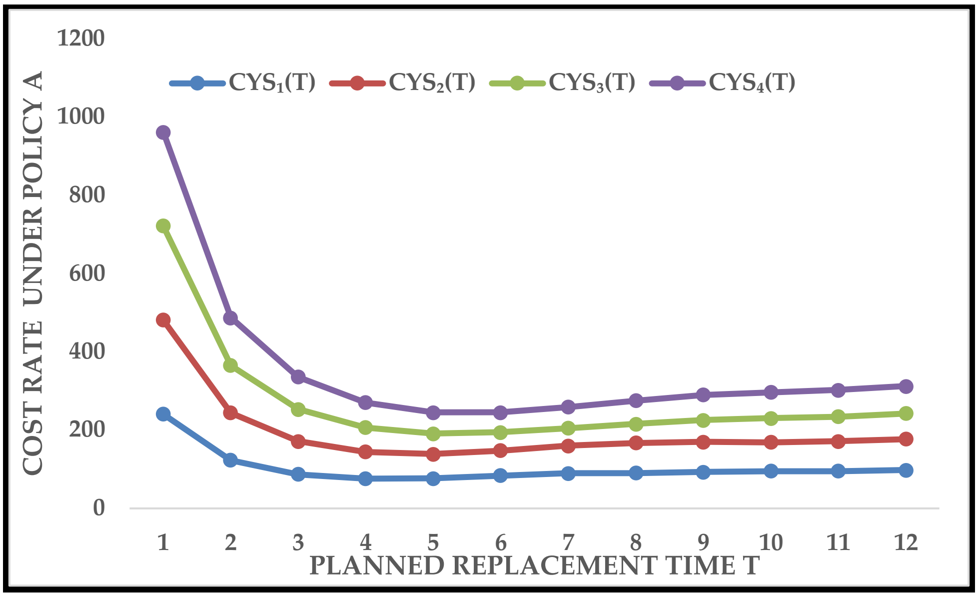

Figure 10 : *The plot of cost rates of the four systems under SARP against planned replacement time T.*

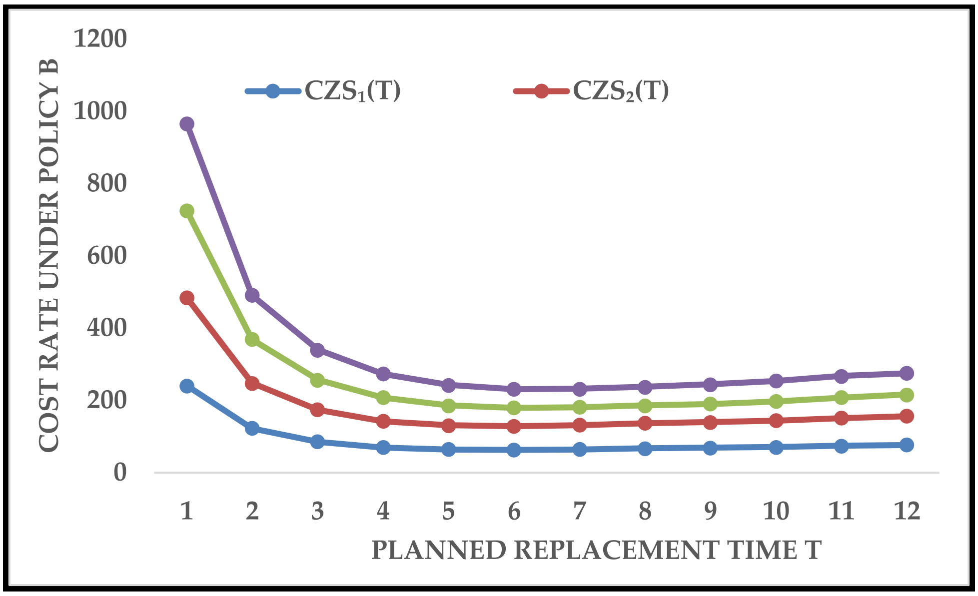

Figure 11 : *The plot of cost rates of the four systems under SARP against planned replacement time T.*

Some observations from the results obtained are as follows

1. From Table 4, observe that, optimal replacement time of systems S_3 and S_4 under policy B is higher than that of SARP and policy A.
2. From Table 4, observe that, optimal replacement time of systems S_1 and S_2 under SARP and policy B are the same.
3. From Table 4, observe that, optimal replacement time of all the four systems under SARP are the same.
4. From Figure 5, observe that:

$$CS_1(T) \leq CZS_1(T) \leq CYS_1(T). \tag{49}$$

5. From Figure 6, observe that:

$$CS_2(T) \leq CZS_2(T) \leq CYS_2(T). \tag{50}$$

6. From Figure 7, observe that:

$$CZS_3(T) \leq CYS_3(T) \leq CS_3(T). \tag{51}$$

7. From Figure 8, observe that:

$$CZS_4(T) \leq CYS_4(T) \leq CS_4(T). \tag{52}$$

8. From Figure 9, observe that:

$$CS_1(T) \leq CS_2(T) \leq CS_3(T) \leq CS_4(T). \tag{53}$$

9. From Figure 10, observe that:

$$CYS_1(T) \leq CYS_2(T) \leq CYS_3(T) \leq CYS_4(T). \tag{54}$$

10. From Figure 11, observe that:

$$CZS_1(T) \leq CZS_2(T) \leq CZS_3(T) \leq CZS_4(T). \tag{55}$$

IX. Discussion of Results Obtained

From the results obtained, we have the following observations:

1. It can be seen that, the optimal replacement time of the parallel - series systems (S_3 and S_4) under policy B, is higher than that of SARP and policy A. Furthermore, the results also showed that, the cost rates of the parallel - series systems (S_3 and S_4) under policy B is lower than that of SARP and policy A. With these reasons, preventive maintenance of the parallel - series systems under policy B is optimal when compared to preventive maintenance of parallel - series systems under SARP and policy A.
2. It can be seen that, the optimal replacement time of series - parallel systems (S_1 and S_2) under SARP and policy B are the same or very closed. While, the cost rates of the series - parallel systems (S_1 and S_2) under SARP is lower than that of under policies A and B. With these reasons, preventive maintenance of the series-parallel systems under SARP is optimal when compared to preventive maintenance of the series-parallel systems under policies A and B.

Hence, from the observations above, we suggest maintenance managers and plant management to adopt policy B as an optimal preventive policy of maintaining multi-unit systems which are in parallel-series configuration. While for systems with series-parallel configuration, SARP should be adopted as an optimal preventive replacement policy.

X. Conclusion

In trying to come up with some modifications and extension of the age replacement policy, this paper presented some proposed age replacement cost models for multi-unit systems under standard age replacement policy (SARP), policy A and policy B. The results obtained, showed that, preventive replacement of parallel-series systems under policy B is optimal when compared to SARP and policy A. While, preventive replacement of series-parallel systems under SARP is optimal when compared to policies A and B. Thus, the results is beneficial to maintenance managers, in selecting the optimal preventive replacement policy. All the replacement cost models and the results presented in this paper are vital to engineers, maintenance managers and plant management for proper maintenance analysis, decision and safety for multi-unit systems.

References

[1] Enogwe, S. U., Oruh, B. I. and Ekpenyong E. J. (2018). A modified replacement model for items that fail suddenly with variable replacement costs. *American Journal of Operations Research*, 8:457–473.

[2] Fallahnezhad, M. S. and Najafian, E. (2017). A model of preventive maintenance for parallel, series and single item replacement systems based on statistical analysis. *Communications in Statistics-Simulation and Computation*, 46:5846–5859.

[3] Gertsbakh, I. Reliability Theory with Applications to Preventive Maintenance. *Springer-Verlag*, London Limited, 2002.

[4] Huang, C. H. and Wang, C. H. (2019). A time-replacement policy for multistate systems with aging components under maintenance, from a Component Perspective. *Mathematical Problems in*

Engineering, https://doi.org/10.1155/2019/9651489.

[5] Jain, M., Agrawal, S. C. and Rani, S. (2010). Availability analysis of embedded computer System with two types of failure and common cause failure. *Journal of International Academy of Physical Sciences*, 14(4): 439–448.

[6] Jain, M. and Gupta, R. (2013). Optimal replacement policy for a repairable system with multiple vacations and imperfect fault coverage. *Computers and Industrial Engineering,* 66: 710–719.

[7] Lim, J. H., Qu, J. and Zuo, J. M. (2016). Age replacement policy based on imperfect repair with random probability. *Reliability Engineering and System Safety,* 149: 24–33.

[8] Liu, Y., Ma, Y., Qu, Z. and Li, X. (2018). Reliability mathematical models of repairable Systems with uncertain lifetimes and repair time. *IEEE,* 6: 71285–71295.

[9] Malki, Z., Ait, D. A. and Ouali, M. S. (2015). Age replacement policies for two-component systems with stochastic dependence. *Journal of Quality in Maintenance Engineering,* 20(3):346–357.

[10] Murthy, D. N. P. and Hwang, M. C. (2007). Optimal discrete and continuous maintenance policy for a complex unreliable machine. *International Journal of Systems Science*, 6(1):35–52.

[11] Nakagawa, T. Maintenance Theory of Reliability: *Springer-Verlag*, London Limited, 2005.

[12] Nakagawa, T., Chen, M. and Zhao, X. (2018). Note on history of age replacement policies. *International Journal of Mathematical, Engineering and Management Sciences,* 3(2): 151–161.

[13] Niwas, R. and Garg, H. (2018). An approach for analyzing the reliability and profit of an industrial system based on the cost free warranty policy. J Braz Soc Mech Sci Eng, 40(5): 5. 10.1007/s40430- 018-1167-8.

[14] Safaei, F., Ahmadi, J., and Balakrishnan, N. (2018). A repair and replacement policy for systems based on probability and mean of profits. *Reliability Engineering and System Safety,* 183: 143–152.

[15] Sudheesh, K. K., Asha, G. and Krishna, K. M. J. (2019). On the mean time to failure of an age replacement model in discrete time. *Communications in Statistics - Theory and Methods,* 50 (11): 2569–2585.

[16] Tsoukalas, M. Z. and Agrafiotis, G. K. (2013). A new replacement warranty policy indexed by the product's correlated failure and usage time. *Computers and Industrial Engineering,* 66:203–211.

[17] Waziri, T. A. and Yusuf, I. (2020). On age replacement policy of system involving minimal repair. *Reliability Theory and Application,* 4(59): 54–62.

[18] Waziri, T. A., Yakasai, B. M. and Yusuf, I. (2019). On Some Discounted Replacement Models of a Series System. *Life Cycle Reliability and Safety Engineering, doi*.org/10.1007/s41872-019-00101-3.

[19] Waziri, T. A. (2021). On discounted discrete scheduled replacement model**.** Annals of Optimization Theory and Practice, 4(2): 69–82.

[20] Wu, W., Song, J., Jiang, K. and Li, H. (2021). Optimal replacement policy based on the effective age of the system for a deteriorating repairable system with multiple vacations. *Journal of Quality in Maintenance Engineering,* https://doi.org/10.1108/JQME-06-2014-0036.

[21] Xie, L., Lundteigen, M. A. and Liu, Y. (2020). Reliability and barrier assessment of series-parallel systems subject to cascading failures. *Journal of Risk and Reliability,* 00(0): 1–15.

[22] Zhao, X., Al-Khalifa, N. K., Hamouda, A. M. and Nakagawa, T. (2017). Age replacement models: a summary of new perspectives and methods. *Reliability Engineering and System Safety*, 161: 95–105.