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A TWO NON-IDENTICAL UNIT PARALLEL SYSTEM WITH PRIORITY IN REPAIR AND CORRELATED LIFE TIMES

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Abstract

The paper analyses a two non-identical unit parallel system in respect of various measures of system effectiveness by using regenerative point techniques. It has been considered that the life times of both the units are correlated random variables and a single repairman is always available with the system to repair a failed unit.

Keywords: Transition probabilities, mean sojourn time, bi-variate exponential distribution, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

I. Introduction

Various authors including Sridharan & Kalyani (2002), Mokaddis & Sherbeny (2008) in the field of reliability theory have been analyzed two unit parallel system models under different sets of assumptions using regenerative point technique. Some of the authors using the concept of giving the priority to one of the unit in repair and compare to other, Malik et al. (2010), Kumar et al. (2018, 2021) developed a reliability model for a system of non-identical units parallel system with priority to repair . In all these systems models it is assumed that the lifetimes are uncorrelated random variables, but in practical situations this seems to be unrealistic because in many cases there may be some sort of correlation between the lifetimes of operating units. Singh & Poonia (2019) introduced the concept of correlation between failure and their times in the analysis of a single server two unit cold standby system. Later various papers including those by Gupta et al. (2010) have been analyzed the correlated failure and repair time distribution of a unit.

Gupta and co-workers [2008,2018] analyzed two unit parallel and standby system models under different sets of assumptions by taking the failure and repair times as correlated random variables having their joint distribution as bivariate exponential. They have considered only single type of failure in an operating unit. Some authors including [1999, 2013] analyzed two-unit parallel system models by taking the joint distribution of life times of the units working in parallel as bivariate exponential. They have also considered the single type of failure in an operating unit.

In the present paper we analyze a two non-identical unit parallel system model with priority in repair and correlated life times of the units working in parallel having their joint distribution as bivariate exponential distribution with different parameters as the form of the joint p.d.f. given below.

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}); \qquad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

Where, $I_0 = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$

is the modified Bessel function of type-I and order zero.

By using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and mean time to system failure (MTSF).
iii. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
iv. Expected busy period of repairman in the repair of unit-1 and unit-2 during time interval (0, t).
v. Net expected profit earned by the system in time interval (0, t).

II. System Description and Assumptions

1. The system consists of two non-identical units (unit-1 and unit-2). Initially, both the units work in parallel configuration.
2. Each unit of the system has two possible modes-Normal (N) and total failure (F).
3. The first unit gets priority in repair.
4. System failure occurs when both the units stop functioning.
5. A single repairman is always available with the system to repair a totally failed unit and repair discipline is first come, first served (FCFS).
6. If during the repair of a failed unit the other unit also fails, then the later failed unit waits for repair until the repair of the earlier failed unit is completed.
7. The repair times of both the units are uncorrelated random variables, each having a general distribution with different parameters.
8. Each repaired unit works as good as new.
9. The joint distribution of lifetimes (failure times) of both the units is taken to be bivariate exponential having a joint density function of the form ,

   $$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}); \quad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

   Where, $I_0 = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$
10. The arrival time distribution of repairman is general.

III. Notations and States of the System

We define the following symbols for generating the various states of the system-

N_{1O}, N_{2O} : Unit-1and unit-2 is in N-mode and operative in parallel.

F_{1r}, F_{2r} : Unit-1 and unit-2 is in F-mode and under repair.

F_{2w} : Unit-2 is in F-mode and under waiting for repair.

F^1_{1r} : First unit is in failure mode and its repair is continued from state S_1.

Considering the above symbols in view of assumptions stated in section-2, the possible states of the system are shown in the transition diagram represented by **Figure. 1**. It is to be noted that the epochs of transitions into the state S_1 from S_2 are non-regenerative, whereas all the other entrance epochs into the states of the systems are regenerative.

The other notations used are defined as follows:

E : Set of regenerative states.

$X_i\,(i=1,2)$: Random variables denoting the failure time of unit-1 N-mode and unit-2 respectively for (i=1, 2)

$f(x_1,x_2)$: Joint probability density function of $x_i; i=1,2$

$$=\alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1-\alpha_2 x_2} I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x_2})$$

$$;\ x_1,x_2,\alpha_1,\alpha_2>0;\quad 0\le r<1$$

Where, $I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x_2})=\sum_{k=0}^{\infty}\frac{(\alpha_1\alpha_2 r x_1 x_2)^k}{(k!)^2}$

$g_i(x)$: Marginal p.d.f. of $X_i = x$

$$=\alpha_i(1-r)e^{-\alpha_i(1-r)x}; x>0, \alpha>0$$

$K_i(\bullet|X)$: Conditional p.d.f. of $X_i \mid X_j = x; i\ne j, j=1,2$

$k_1(x_1 \mid X_2 = x_2)$: Conditional p.d.f. of $X_1 \mid X_2 = x$

$$=\alpha_1 e^{-(\alpha_1 x_1+\alpha_2 r x)} I_0(2\sqrt{\alpha_1\alpha_2 r x_1 x})$$

$k_2(x_2 \mid X_1 = x_1)$: Conditional p.d.f. of $X_2 \mid X_1 = x$

$$=\alpha_2 e^{-(\alpha_2 x_2+\alpha_1 r x)} I_0(2\sqrt{\alpha_1\alpha_2 r x x_2})$$

$g_i(\bullet), G_i(\bullet); i=1,2$: The repair time probability distribution function and cumulative distribution function of x_i

$g_{ij}(\bullet), g_{ij}^{(k)}(\bullet)$: P.d.f. of transition time from state S_i to S_j and S_i to S_j via S_k .

$p_{ij}(\bullet), p_{ij}^{(k)}(\bullet)$: Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k .

$p_{ij|x}(\bullet), p_{ij|x}^{(k)}(\bullet)$: Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k when it is known that the unit has worked for time x before its failure.

β_1 : Repair time of failed unit-2

* : Symbol for Laplace Transform i.e. $g_{ij}^*(s)=\int e^{-st}q_{ij}(t)dt$

~ : Symbol for Laplace Stieltjes Transform i.e. $\tilde{Q}_{ij}(s)=\int e^{-st}dQ_{ij}(t)t$

© : Symbol for ordinary convolution i.e.

$$A(t)©B(t)=\int_0^t A(u)B(t\text{-}u)du$$

†The limits of integration are 0 to ∞ whenever they are not mentioned.

TRANSITION DIAGRAM

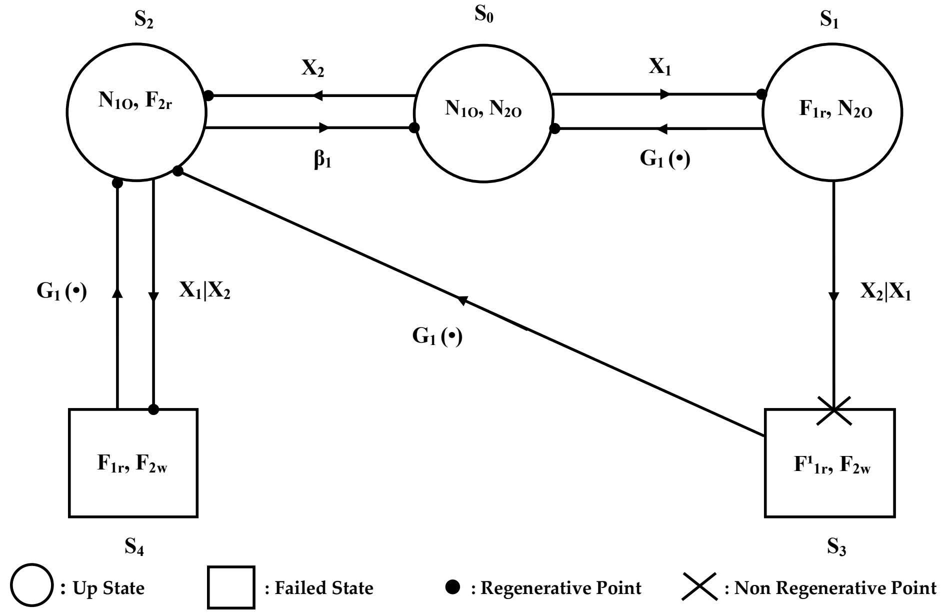

Figure.1

IV. Transition Probabilities and Sojourn Times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space $E = \{S_0 \text{to} S_4\}$.The various measures of system effectiveness are obtained in terms of steady-state transition probabilities and mean sojourn times in various states. First we obtain the direct conditional and unconditional transition probabilities in terms of

$$\alpha_1^{'} = \frac{\alpha_1}{\alpha_1 + \beta_1}, \qquad \alpha_2^{'} = \frac{\alpha_2}{\alpha_2 + \theta_1}$$

as follows-

$$p_{01} = \int \alpha_1 (1-r) e^{-\alpha_1 (1-r)t} e^{-\alpha_2 (1-r)t} dt = \frac{\alpha_1}{\alpha_1 + \alpha_2}$$

$$p_{02} = \int \alpha_2 (1-r) e^{-\alpha_2 (1-r)t} e^{-\alpha_1 (1-r)t} dt = \frac{\alpha_2}{\alpha_1 + \alpha_2}$$

$$p_{42} = \int dG_1(t) = 1$$

$$p_{10\,|\,x} = \int dG_1(t) \overline{K}_2(t \mid x) = \int_0^\infty dG_1(t) \int_u^\infty \alpha_2 e^{-(\alpha_2 y + \alpha_1 rx)} I_0 \left(2\sqrt{\alpha_1 \alpha_2 rxy}\right) dy$$

$$= \int_0^\infty dG_1(t) \left(\int_u^\infty \alpha_2 e^{-(\alpha_2 y + \alpha_1 rx)} \sum_{j=0}^\infty \frac{(\alpha_1 \alpha_2 rxy)^j}{(j!)^2} \right) du$$

$$= \alpha_2 e^{-\alpha_1 rx} \sum_{j=0}^\infty \frac{(\alpha_1 \alpha_2 rx)^j}{(j!)^2} \left(\int_0^\infty e^{-\alpha_2 y} y^j \left[\overline{G}_1(y) \right] dy \right)$$

Similarly,

$$p_{20|x} = 1-\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}, \qquad p_{24|x} = \alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}$$

$$p_{12|x}^{(3)} = 1-\left[\alpha_2 e^{-\alpha_1 rx}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\left(\int_0^{\infty} e^{-\alpha_2 y} y^j\left[\overline{G}_1(y)\right]dy\right)\right]$$

The unconditional transition probabilities with correlation coefficient from some of the above conditional transition probabilities can be obtained as follows:

$$p_{10} = \int p_{10|x}\ g_1(x)\ dx$$

$$= \alpha_1(1-r)\int e^{-\alpha_1(1-r)x}\ \alpha_2 e^{-\alpha_1 rx}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\left(\int_0^{\infty} e^{-\alpha_2 y} y^j\left[\overline{G}_1(y)\right]dy\right)\ dx$$

Similarly,

$$p_{20} = 1-\frac{\alpha_1'(1-r)}{(1-\alpha_1' r)}, \qquad p_{24} = \frac{\alpha_1'(1-r)}{(1-\alpha_1' r)}$$

$$p_{12}^{(3)} = \alpha_1(1-r)\int e^{-\alpha_1(1-r)x}\left(1-\left[\alpha_2 e^{-\alpha_1 rx}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\left(\int_0^{\infty} e^{-\alpha_2 y} y^j\left[\overline{G}_1(y)\right]dy\right)\right]\right)dx$$

It can be easily verified that,

$$p_{01}+p_{02}=1, \qquad p_{10}+p_{12}^{(3)}=1, \qquad p_{42}=1, \qquad p_{20}+p_{24}=1 \tag{1-4}$$

V. Mean Sojourn Time

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P\left[U_i > t\right]dt$$

Therefore, its values for various regenerative states are as follows-

$$\psi_0 = \int e^{-\alpha_1(1-r)t} e^{-\alpha_2(1-r)t} dt = \frac{1}{\left(\alpha_1+\alpha_2\right)(1-r)} \tag{5}$$

$$\psi_{1|x} = \int \overline{G}_1(t)\overline{K}_2\left(t\mid x\right)\ dt = \alpha_2 e^{-\alpha_1 rx}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rx)^j}{(j!)^2}\left(\int_0^{\infty} e^{-\alpha_2 y} y^j\left(\int_0^{y}\overline{G}_1(t)dt\right)dy\right)$$

So that,

$$\psi_1 = \int \psi_{1|x}\ g_1(x)\ dx$$

$$= \int \psi_{1|x}\ \alpha_1(1-r)e^{-\alpha_1(1-r)x}\ \ dx$$

$$= \alpha_1\alpha_2(1-r)\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 r)^j}{(j!)^2}\left[\left(\int e^{-\alpha_2 y} y^j\left(\int_0^{y}\overline{G}_1(t)dt\right)dy\right)\left(\int e^{-\alpha_1 rx} e^{-\alpha_1(1-r)x} x^j dx\right)\right] \tag{6}$$

$$= \int e^{-\alpha_2(1-r)t}\,\overline{G}_1(t)\,dt$$

$$\psi_{2|x} = \frac{1}{\beta_1}\left[1-\alpha_1' e^{-\alpha_2 rx(1-\alpha_1')}\right]$$

So that,

$$\psi_2 = \frac{1}{\beta_1}\left[1-\frac{\alpha_1'(1-r)}{(1-\alpha_1' r)}\right] \tag{7}$$

$$\psi_4 = \int \overline{G}_1\left(t\right)\ dt \tag{8}$$

VI. Analysis of Characteristics

I. Reliability and MTSF

Let $R_i(t)$ be the probability that the system operates during (0, t) given that at t=0 system starts from $S_i \in E$. To obtain it we assume the failed states S_2 and S_4 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + q_{02}^* Z_2^*}{1 - q_{01}^* q_{10}^* - q_{02}^* q_{20}^*} \tag{9}$$

We have omitted the argument's from $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. $Z_i^*(s)$; i = 0, 1, 2 are the L. T. of

$$Z_0(t) = e^{-(\alpha_1+\alpha_2)(1-r)t}, \qquad Z_1(t) = \overline{G}_1(t), \qquad Z_2(t) = e^{-\beta_1 t}$$

Taking the Inverse Laplace Transform of (9), one can get the reliability of the system when system initially starts from state S_0 .

The MTSF is given by,

$$E(T_0) = \int R_0(t) = \lim_{s\to 0} R_0^*(s) = \frac{\psi_0 + p_{01}\psi_1 + p_{02}\psi_2}{1 - p_{01}p_{10} - p_{02}p_{20}} \tag{10}$$

II. Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s) = \frac{N_1(s)}{D_1(s)} \tag{11}$$

Where,

$$N_1(s) = Z_0^*\left[1 - q_{24}^* q_{42}^*\right] + Z_1^* q_{01}^*\left[1 - q_{24}^* q_{42}^*\right] + Z_2^*\left[q_{01}^* q_{12}^{(3)*} + q_{02}^*\right]$$

and

$$D_1(s) = 1 - q_{24}^* q_{42}^* - q_{10}^* q_{01}^*\left(1 - q_{24}^* q_{42}^*\right) - q_{20}^*\left(q_{01}^* q_{12}^{(3)*} + q_{02}^*\right) \tag{12}$$

Where, $Z_i(t)$, i=0,1,2 are same as given in section VI(I).

The steady-state availability of the system is given by

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} s\, A_0^*(s) \tag{13}$$

We observe that

$$D_1(0) = 0$$

Therefore, by using L. Hospital's rule the steady state availability is given by

$$A_0 = \lim_{s\to 0}\frac{N_1(s)}{D_1'(s)} = \frac{N_1}{D_1'} \tag{14}$$

Where,

$$N_1 = \psi_0\left[1 - p_{24}p_{42}\right] + \psi_1 p_{01}\left[1 - p_{24}p_{42}\right] + \psi_2\left[p_{01}p_{12}^{(3)} + p_{02}\right]$$

and

$$D_1' = \psi_0 p_{20} + \psi_1 p_{01} p_{20} + \psi_2\left(1 - p_{10}p_{01}\right) + \psi_4 p_{24}\left(1 - p_{10}p_{01}\right) \tag{15}$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $\mu_{up}^*(s) = \frac{A_0^*(s)}{s}$ (16)

III. Busy Period Analysis

Let $B_i^1(t)$ and $B_i^2(t)$ be the respective probabilities that the repairman is busy in the repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair at epoch t, when initially the system starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L. T., one can obtain the values of above two probabilities in terms of their L. T. i.e. $B_i^{1*}(s)$ and $B_i^{2*}(s)$ as follows-

$$B_i^{1*}(s) = \frac{N_2(s)}{D_1(s)}, \qquad B_i^{2*}(s) = \frac{N_3(s)}{D_1(s)} \tag{17-18}$$

Where,

$$N_2(s) = \left(Z_1^* + q_{13}^* Z_3^*\right) q_{01}^* \left(1 - q_{42}^* q_{24}^*\right) - Z_4^* \left(q_{01}^* q_{12}^{(3)*} q_{24}^* + q_{02}^* q_{24}^*\right)$$

and

$$N_3(s) = Z_2^* \left(q_{01}^* q_{12}^{(3)*} + q_{02}^*\right)$$

and $D_1(s)$ is same as defined by the expression (12) of section VI(II).

The steady state results for the above two probabilities are given by-

$$B_0^1 = \lim_{s\to 0} s\, B_0^{1*}(s) = N_2 \setminus D_1' \quad \text{and} \quad B_0^2 = \lim_{s\to 0} s\, B_0^{2*}(s) = N_3 \setminus D_1' \tag{19-20}$$

Where,

$$N_2(0) = \left(\psi_1 + p_{13}\psi_3\right) p_{01} \left(1 - p_{42} p_{24}\right) - \psi_4 \left(p_{01} p_{12}^{(3)} p_{24} + p_{02} p_{24}\right) \tag{21}$$

$$N_3(0) = \psi_2 \left(p_{01} p_{12}^{(3)} + p_{02}\right) \tag{22}$$

and D_1' is same as given in the expression (15) of section VI(II).

The expected busy period in repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair during time interval (0, t) are respectively given by-

$$\mu_b^1(t) = \int_0^t B_0^1(u)\,du, \quad \text{and} \quad \mu_b^2(t) = \int_0^t B_0^2(u)\,du$$

So that,

$$\mu_b^{1*}(s) = \frac{B_0^{1*}(s)}{s} \quad \text{and} \quad \mu_b^{2*}(s) = \frac{B_0^{2*}(s)}{s} \tag{23-24}$$

IV. Profit Function Analysis

The net expected total cost incurred in time interval (0, t) is given by

P (t) = Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$= K_0 \mu_{up}(t) - K_1 \mu_b^1(t) - K_2 \mu_b^2(t) \tag{25}$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1 and K_2 are the amounts paid to the repairman per-unit of time when the system is busy in repair of unit-1 failed due to first repair with priority of unit-1 and unit-2 failed due to second repair respectively.

The expected total profit incurred in unit interval of time is $P = K_0 A_0 - K_1 B_0^1 - K_2 B_0^2$

VII. Particular Case

Let, $\overline{G}_1(y) = e^{-\theta_1 y}$

In view of above, the changed values of transition probabilities and mean sojourn times.

$$p_{10|x} = \alpha_2 e^{-\alpha_1 rx} \sum_{j=0}^{\infty} \frac{(\alpha_1 \alpha_2 rx)^j}{(j!)^2} \left(\int_0^{\infty} e^{-\alpha_2 y} y^j \left[\overline{G}_1(y) \right] dy \right) = \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}$$

$$p_{12|x}^{(3)} = 1 - \alpha_2' e^{-\alpha_1 rx(1-\alpha_2')}, \qquad p_{10} = \frac{\alpha_2'(1-r)}{(1-\alpha_2' r)}, \qquad p_{12}^{(3)} = 1 - \frac{\alpha_2'(1-r)}{(1-\alpha_2' r)}$$

$$\psi_1 = \frac{1}{\theta_1 + \alpha_2(1-r)}, \qquad \psi_4 = \frac{1}{\theta_1}$$

VIII. Graphical Study of Behaviour and Conclusions

For a more clear view of the behaviour of system characteristics with respect to the various parameters involved, we plot curves for **MTSF** and **profit function** in **Fig. 2** and **Fig. 3** w.r.t. $\boldsymbol{\alpha_1}$ for three different values of correlation coefficient $\boldsymbol{\alpha_2}$ **=0.1, 0.5, 0.9** and two different values of repair parameter **r =0.25, 0.6** while the other parameters are $\boldsymbol{\beta_1}$ **=0.085,** $\boldsymbol{\theta_1}$ **= 0.6**. It is clearly observed from **Fig. 2** that **MTSF** increases uniformly as the value of $\boldsymbol{\alpha_2}$ and **r** increase and it decrease with the increase in $\boldsymbol{\alpha_1}$. Further, to achieve **MTSF** at least 17 units we conclude for smooth curves that the values of $\boldsymbol{\alpha_1}$ must be less than **0.13, 0.23** and **0.45** respectively for $\boldsymbol{\alpha_2}$ **=0.1, 0.5, 0.9** when **r =0.6**. Whereas from dotted curves we conclude that the values of $\boldsymbol{\alpha_1}$ must be less than **0.12, 0.14** and **0.31** for $\boldsymbol{\alpha_2}$ **=0.1, 0.5, 0.9** when **r =0.25**.

Similarly, **Fig.3** reveals the variations in **profit (P)** with respect to $\boldsymbol{\alpha_1}$ for three different values of $\boldsymbol{\alpha_2}$ **= 0.3, 0.6, 0.9** and two different values of **r =0.3, 0.6,** when the values of other parameters $\boldsymbol{\beta_1}$ **=0.95,** $\boldsymbol{\theta_1}$ **= 0.09, K$_0$=200, K$_1$=95 and K$_2$=175**. Here also the same trends in respect of $\boldsymbol{\alpha_1}$ **,** $\boldsymbol{\alpha_2}$ an**d r** are observed in case of **MTSF**. Moreover, we conclude from the smooth curves that the system is profitable only if $\boldsymbol{\alpha_1}$ is less than **0.22, 0.42** and **0.66** respectively for $\boldsymbol{\alpha_2}$ **= 0.3, 0.6, 0.9** when **r =0.6.** From dotted curves, we conclude that the system is profitable only if $\boldsymbol{\alpha_1}$ is less than **0.19, 0.3** and **0.5** respectively for $\boldsymbol{\alpha_2}$ **= 0.3, 0.6, 0.9** when **r =0.3.**

Behaviour of **MTSF** w.r.t. $\boldsymbol{\alpha_1}$ for different values of $\boldsymbol{\alpha_2}$ and **r**

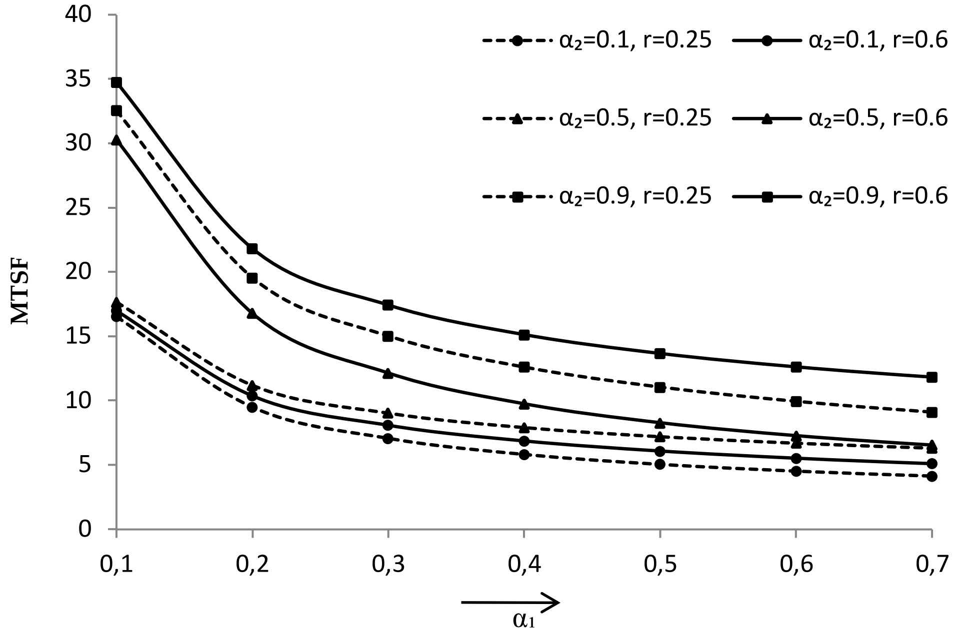

Figure.2

Behaviour of **PROFIT (P)** w.r.t. $\boldsymbol{\alpha_1}$ for different values of $\boldsymbol{\alpha_2}$ and **r**

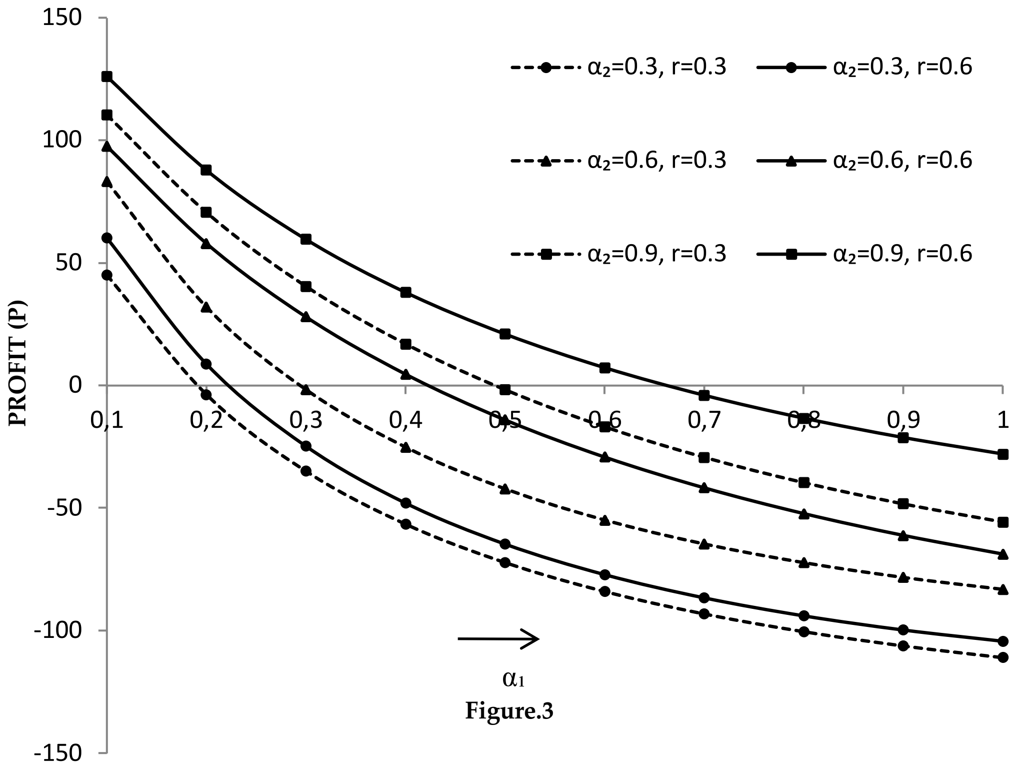

Figure.3

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