STOCHASTIC ANALYSIS OF A REPAIRABLE SYSTEM OF NON-IDENTICAL UNITS WITH PRIORITY AND CONDITIONAL FAILURE OF REPAIRMAN

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Abstract

Here, we describe the stochastic analysis of a repairable system consisting two non-identical units called the main unit and the other is a duplicate unit. The units have direct complete failure from the operative state. A single repairman has been engaged to carry out the repair activities that can be failed while performing his jobs with the main unit. The repairman does repair activities of the duplicate unit without any problem. Priority for operation and repair to the duplicate unit is given over the main unit. The repairman performs with full efficiency after getting treatment. The distribution for failure rates of the units has been considered as negative exponential while arbitrary distributions have been taken for repair and treatment rates. The use of semi-Markov process and regenerative point technique has been made to study the probabilistic behavior of the system in different possible transition states. The reliability characteristics of the system model have been examined numerically and graphically for particular values of the parameters. The profit of the system has also been analyzed for some fixed values of the repair and other maintenance costs.

Keywords: System of Non-identical Units, Priority, Conditional Failure of Repairman and Stochastic Analysis

I. Introduction

Over the years the researchers in the field of the reliability have been struggling to identify the best possible structure of the components and the techniques which can be used to improve the performance of repairable systems. As a result of which some reliability improve techniques for the repairable systems have been emerged as the provision of redundancy, priority in repair discipline and configuration of the components such as series, parallel, series-parallel, parallel-series, k-out-of-n and other mixed mode structures. The technique of cold standby redundancy with different repair policies has been used most frequently during stochastic modeling of repairable systems. Subramanian and Natarajan [10] developed an N-Unit standby redundant system with R repair facilities. Cao and Wu [2] discussed a cold standby system of two unit with replaceable repair facility. Smith [9] highlighted the concept of regenerative stochastic processes. On the other hand, the objective of the manufacturers is not only to produce the systems with considerable reliability but also to launch the products in markets with optimal balance between reliability and the production costs. To cope with this situation it becomes necessary to use systems with non-identical units and appropriate repair facilities. The systems with non-identical units have also been studied in the past considering the ideas of priority in repair discipline. Kadyan et al. [4] discussed the stochastic modeling of a system of non-identical units with priority in different mode of failures. Salah and EL-Sherbeny [8] described a two unit non-identical parallel system subject to preventive maintenance and repairs. Kumar et al. [5] analyzed profit of a warm standby non-identical unit system with single server. Kadyan et al. [7] developed system models using the concept of priority. In the field of reliability research it is a common practice that the repair facility called server or repairman cannot fail while performing its assignments i.e. the jobs related to maintenance, repair of the faults and any other precautionary needs of the systems. This assumption on repair facility seems to be unrealistic when repair activities perturb due to the reasons which cause the failure of the service facility or any other catastrophic failure. Chen and Wang [3] analyzed a retrial machine repair problem with warm standbys and a single server with N-policy. Kumar and Nandal [6] developed a system of two non-identical units with conditional failure of repairman. Anuradha et al. [1] analyzed a 1-out-of-2: G System with Priority to Repair and Conditional Failure of Service Facility.

In view of the above observations and facts, the purpose of the present paper is to analyze stochastically a repairable system of non-identical units with the concept of priority and conditional failure of the repairman. The system has one main unit which is initially operative and the other unit is considered as duplicate in cold standby redundancy. The units have direct complete failure from the operative state. A single repairman is engaged to carry out the repair activities that can be failed while performing his jobs with the main unit. The repairman does repair activities of the duplicate unit without any problem. Priority for operation and repair to the duplicate unit is given over the main unit. The repairman performs with full efficiency after getting treatment. The distribution for failure rates of the units has been considered as negative exponential while arbitrary distributions have been taken for repair and treatment rates. The use of semi-Markov process and regenerative point technique has been made to study the probabilistic behavior of the system in different possible transition states. The reliability characteristics of the system model such as MTSF, availability, busy period of the server due to repair of the main and duplicate units, expected number of repairs of the units, expected number of the treatments given to the repairman and finally the profit function have been examined numerically and graphically for particular values of the parameters. The profit of the system has also been analyzed for some fixed values of the repair and other maintenance costs.

II. System Description

1. The system comprises of two non-identical units; one main unit which is initially operative and the other unit is considered as duplicate in cold standby redundancy.
2. The duplicate unit becomes operative after the failure of main unit.
3. A single repairman is engaged to carry out the repair activities that can be failed while performing his jobs with the main unit.
4. The repaired unit works as good as new.
5. Priority for operation and repair to the duplicate unit is given over the main unit.
6. The distribution for failure rates of the units has been considered as negative exponential while arbitrary distributions have been taken for repair and treatment rates

The state transition diagram shown in the figure 1 as:

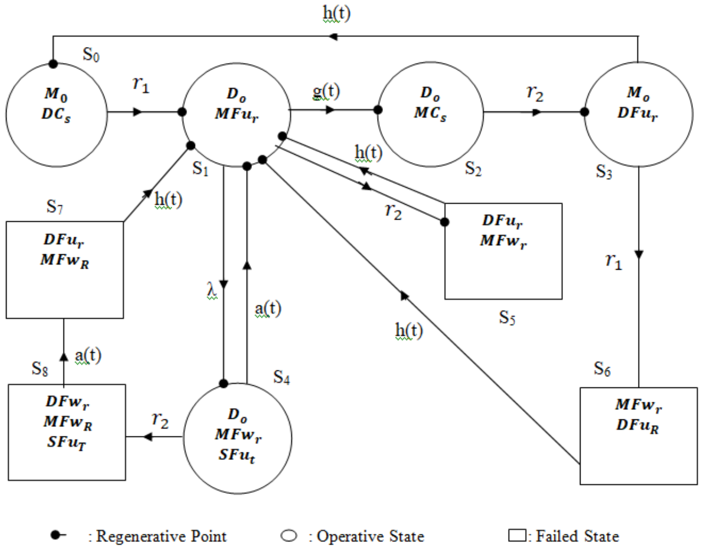

Figure 1:*State Transition Diagram*

a) Notations and Abbreviations

O	Operative state
▭	Failed State
●	Regenerative point
λ	Failure rate of the repairman
M_0	Main unit is Operative and in normal mode
D_{cs}	Duplicate unit is in cold standby
r_1	Failure rate of the main unit
r_2	Failure rate of the duplicate unit
$g(t)/G(t)$	pdf/cdf of the main unit repair time
$h(t)/H(t)$	pdf/cdf of the duplicate unit repair time
$a(t)/A(t)$	pdf/cdf of the treatment time of the repairman
MF_{ur}/MF_{UR}	Main unit failed under repair /continuously under repair from previous state
DF_{ur}/DF_{UR}	Duplicate unit failed under repair/continuously under repair from previous state
Mw_{ur}/Mw_{UR}	Main unit waiting for repair /continuously waiting for repair from previous state
Dw_{ur}/Dw_{UR}	Duplicate unit waiting for repair/continuously waiting for repair from previous state
SF_{ur}/SF_{UR}	Repairman failed under treatment/continuously under treatment from previous state
q_{ij}/Q_{ij}	pdf/cdf of transition from regenerative state (or non-regenerative state) S_i or to a failed state S_j without visiting any regenerative state in (0,t]
m_{ij}	Mean sojourn time (μ_i) in state S_i when system transits directly to state S_j so that $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*'}(0)$
μ_i	The mean sojourn time in state S_i
$M_i(t)$	Probability that the system up initially in state $S_i \in E$ is up at time t without visiting to any regenerative state
$W_i(t)$	Probability that the repairman is busy in the state S_i up to time t without making any transition to any other regenerative state or returning to the same state via one or more non regenerative states
$\phi_i(t)$	cdf of the first passage time from regenerative state S_i to a failed state
$A_i(t)$	Probability that the system is in upstate at instant t given that the system entered regenerative state S_i at t = 0
$B_i(t)$	Probability that the repairman is busy at instant t given that the system entered regenerative

	state S_i at t = 0
RM_i(t)	Expected number of repair of the main unit given to the repairman in (0, t] such that the system entered regenerative state $S_i$at t = 0
RD_i(t)	Expected number of repair of the duplicate unit given to the repairman in (0, t] such that the system entered regenerative state$S_i$at t = 0
TR_i(t)	Expected number of Treatment given to the repairman in (0, t] such that the system entered regenerative state S_i at t = 0
$\otimes$	Symbol for Stieltjes convolution
©	Symbol for Laplace convolution
*/**	Symbol for Laplace Transform/ Laplace Stieltjes Transform
P	Profit function of the system
K_0	Revenue per unit to the system
K_1	Cost per unit for which repairman is busy to repair the main unit.
K_2	Cost per unit for which repairman is busy to repair the duplicate unit.
K_3	Cost per unit for repair of main unit
K_4	Cost per unit for repair of duplicate unit
K_5	Cost per unit treatment given to the repairman
TP	Transition Probabilities
MSTs	Mean Sojourn Times
MTSF	Mean Time to System Failure
LT	Laplace Transform
LST	Laplace Stieltjes Transform
LIT	Laplace Inverse Transform
s-MP	semi-Markov Process
RPT	Regenerative Point Technique

III. Reliability Measures of the System

a) Transition Probabilities

Simple probabilistic considerations yield the following expression for the non-zero elements $p_{ij} = \lim_{t\to\infty} Q_{ij}(t) = \int_0^\infty q_{ij}(t)dt$ as:

$$dQ_{01}(t) = r_1 e^{-r_1 t}dt, \quad dQ_{12}(t) = g(t)e^{-(r_2+\lambda)t}dt, \quad dQ_{14}(t) = \lambda e^{-(r_2+\lambda)t}\overline{G(t)}dt,$$
$$dQ_{15}(t) = r_2 e^{-(r_2+\lambda)t}\overline{G(t)}dt, \quad dQ_{23}(t) = r_2 e^{-(r_2)t}dt, \quad dQ_{30}(t) = h(t)e^{-r_1 t}dt,$$
$$dQ_{36}(t) = r_1 e^{-r_1 t}\overline{H(t)}dt, dQ_{41}(t) = a(t)e^{-r_2 t}dt, dQ_{48}(t) = r_2 e^{-r_2 t}\overline{A(t)}dt$$
$$dQ_{51}(t) = h(t)dt, dQ_{61}(t) = h(t)dt, dQ_{71}(t) = h(t)dt, dQ_{87}(t) = a(t)dt$$

By taking t$\rightarrow\infty$ of the above expressions using $p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$, we get

$$p_{01} = 1, p_{12} = g^*(r_2+\lambda), p_{14} = \frac{\lambda}{(r_2+\lambda)}[1-g^*(r_2+\lambda)], p_{15} = \frac{r_2}{(r_2+\lambda)}[1-g^*(r_2+\lambda)],$$
$$p_{23} = 1, p_{30} = h^*(r_1), p_{36} = [1-h^*(r_1)], p_{41} = a^*(r_2), p_{48} = [1-a^*(r_2)],$$
$$p_{51} = h^*(0), p_{71} = h^*(0), p_{61} = h^*(0), p_{87} = a^*(0)$$

It is verified that:

$$p_{01} = p_{12} + p_{14} + p_{15} = p_{30} + p_{36} = p_{41} + p_{48} = p_{51} = p_{61} = p_{71} = p_{87} = 1$$

b) Mean Sojourn Times

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in the state. If T_i be the sojourn time in the state i, then the mean sojourn time in the state i is

$$\mu_i = \int_0^\infty Pr(T_i > t) or \mu_i = \sum_j m_{ij} \text{ But } m_{ij} = -\frac{d}{ds}\left[Q_{ij}^{**}(s)\right]_{s=0}$$

$$\mu_0 = m_0, \mu_1 = m_{12} + m_{14} + m_{15}, \mu_2 = m_{23}$$

$$\mu_3 = m_{30} + m_{36}, \mu_4 = m_{41} + m_{48}$$

$$\mu_3' = m_{30} + m_{31.6}, \mu_4' = m_{41} + m_{41.87}$$

c) Reliability and MTSF

Let $\emptyset_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have following recursive relations for $\emptyset_i(t)$:

$$\emptyset_i(t) = \sum_j Q_{ij}(t)®\emptyset_j(t) + \sum_k Q_{ik}(t) \quad (1)$$

where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Thus, the following equations are obtained by using (1) as:

$$\emptyset_o(t) = Q_{01}(t) \otimes \emptyset_1(t)$$

$$\emptyset_1(t) = Q_{12}(t) \otimes \emptyset_2(t) + Q_{14}(t) \otimes \emptyset_4(t) + Q_{15}(t)$$

$$\emptyset_2(t) = Q_{23}(t) \otimes \emptyset_3(t)$$

$$\emptyset_3(t) = Q_{30}(t) \otimes \emptyset_0(t) + Q_{36}(t)$$

$$\emptyset_4(t) = Q_{41}(t) \otimes \emptyset_1(t) + Q_{48}(t)$$

Taking LST of above relations to obtain $\emptyset_0^{**}(s)$ using this, we have

$$R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s}$$

Taking LIT of R*(s), we can obtain the reliability R(t) of the system model. The MTSF is given by

$$\text{MTSF} = \lim_{n\to 0} R^*(s) = \frac{N_1}{D_1}$$

where,

$$N_1 = \mu_0(p_{12} + p_{15} + p_{14}p_{48}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{12} + \mu_4 p_{14},$$

$$D_1 = (1 - p_{12}p_{30} - p_{14}p_{41})$$

d) Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at epoch 't' given that the computer system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{ij}^{(n)}(t)©A_j(t) \quad (2)$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Thus, the following equations are obtained by using (2) as:

$$A_o(t) = M_o(t) + q_{01}©A_1(t)$$

$$A_1(t) = M_1(t) + q_{12}(t)©A_2(t) + q_{14}(t)©A_4(t) + q_{15}(t)©A_5(t)$$

$$A_2(t) = M_2(t) + q_{23}©A_3(t)$$

$$A_3(t) = M_3(t) + q_{3o}(t)©A_o(t) + q_{31.6}(t)©A_1(t)$$

$$A_4(t) = M_4(t) + q_{41}(t)©A_1(t) + q_{41.87}(t)©A_1(t)$$

$$A_5(t) = q_{51}(t)©A_1(t)$$

where

$$M_0(t) = e^{-r_1 t}, M_1(t) = e^{-(r_2+\lambda)t}\overline{G(t)}, M_2(t) = e^{-r_2 t}, M_4(t) = e^{-r_2 t}\overline{A(t)}, M_3(t) = e^{-r_1 t}\overline{H(t)}$$

Taking L.T of above expressions and calculate the value of $A_0^*(s)$, we have

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2'}$$

where,

$$N_2 = \mu_0 - \mu_0 p_{12}p_{36} - \mu_0 p_{14} - \mu_0 p_{15} + \mu_1 + \mu_2 p_{12} + \mu_3 p_{12} - \mu_4 p_{14}$$

$$D_2' = \mu_0 p_{12}p_{30} + \mu_1 + \mu_2 p_{12} + \mu_3^1 p_{12} + \mu_4^1 p_{14} + \mu_5 p_{15}$$

e) Busy Period of the Repairman Due to Repairs

Let $B_i^R(t)$ be the probability that server is busy in repairing the unit at epoch 't' given that the system entered state S_i at $t = 0$. The recursive relations for $B_i^R(t)$ are given as:

$$B_i^R(t) = W_i^R(t) + \sum_j q_{ij}^{(n)}(t) © B_j^H(t) \quad (3)$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Thus, the following equations are obtained by using (3) as:

i) Repair of Main Unit

$$B_0^M(t) = q_{01} © B_1^M(t)$$
$$B_1^M(t) = W_1(t) + q_{12}(t) © B_2^M(t) + q_{11.5}(t) © B_1^M(t) + q_{14}(t) © B_4^M(t)$$
$$B_2^M(t) = q_{23} © B_3^M(t)$$
$$B_3^M(t) = q_{3o}(t) © B_0^M(t) + q_{31.6}(t) © B_1^M(t)$$
$$B_4^M(t) = q_{41}(t) © B_1^M(t) + q_{41.87}(t) © B_1^M(t)$$
$$B_5^M(t) = q_{51} © B_1^M(t)$$

where

$$W_1(t) = e^{-(r_2+\lambda)t}\overline{G(t)}$$

Taking L.T. of above expressions and calculate the value of $B_0^*(s)$, we have

$$B_0^M = \lim_{s\to 0} sB_0{}^*(s) = \frac{N_3}{D_2'}$$

Where, $N_3 = \mu_1$ and D_2' is already defined.

ii) Repair of Duplicate unit

$$B_0^D(t) = q_{01} © B_1^D(t)$$
$$B_1^D(t) = q_{10}(t) © B_0^D(t) + q_{14}(t) © B_4^D(t) + q_{11.5}(t) © B_1^D(t)$$
$$B_2^D(t) = q_{23} © B_3^D(t)$$
$$B_3^D(t) = W_3(t) + q_{3o}(t) © B_0^D(t) + q_{31.6}(t) © B_1^D(t)$$
$$B_4^D(t) = q_{41}(t) © B_1^D(t) + q_{41.87}(t) © B_1^D(t)$$
$$B_5^D(t) = W_5(t) + q_{51} © B_1^D(t)$$

where

$$W_3(t) = e^{-(r_1)t}\overline{H(t)} + [r_1 e^{r_1 t} © 1]\overline{H(t)}$$
$$W_5(t) = \overline{H(t)},$$

Taking L.T. of above expressions and calculate the value of B_0^*(s), we have

$$B_0^D = \lim_{s\to 0} sB_0{}^*(s) = \frac{N_{3D}}{D_2'}$$

where, $N_{3D} = W_3^*(0)p_{12}$ and D_2' is already defined.

f) Expected Number of Repairs of the Main Unit

Let $R_i^M(t)$ be the expected number of repairs of the unit by the repairman in (0, t] such that the system entered regenerative state i at t = 0. The recursive relation for $R_i^M(t)$ are given as:

$$R_i^M(t) = \sum_j Q_{i,j}^{(n)}(t) Ⓢ [\delta_j + R_i^M(t)] \quad (4)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$ if j is the regenerative state where the repairman does job afresh, otherwise, $\delta_j = 0$. Thus, the following equations are obtained by using (4) as:

$$R_0^M(t) = Q_{01} \otimes R_1^M(t)$$
$$R_1^M(t) = Q_{12}(t) \otimes [1 + R_2^M(t)] + Q_{14}(t) \otimes R_4^M(t) + Q_{11.5}(t) \otimes R_1^M(t)$$
$$R_2^M(t) = Q_{23} \otimes R_3^M(t)$$
$$R_3^M(t) = Q_{30}(t) \otimes R_0^M(t) + Q_{31.6}(t) \otimes R_1^M(t)$$
$$R_4^M(t) = Q_{41}(t) \otimes R_1^M(t) + Q_{41.87}(t) \otimes R_1^M(t)$$
$$R_5^M(t) = Q_{51} \otimes \underline{R_1^M}(t)$$

Taking L.S.T. of above expressions and calculating for $\overline{R_0^M}(s)$, we have

$$R_0^M = \lim_{s\to 0} s\overline{R_0^M}(s), = \frac{N_4}{D_2^{'}}$$

where, N_4 = p_{12} and $D_2^{'}$ is already defined.

g) Expected Number of Repairs of the Duplicate Unit

Let $R_i^D(t)$ be the expected number of repairs of the unit by the repairman in (0, t] such that the system entered regenerative state i at t = 0. The recursive relation for $R_i^D(t)$ are given as:

$$R_i^D(t) = \sum_j Q_{i,j}^{(n)}(t) \circledS [\delta_j + R_i^D(t)] \quad (5)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$ if j is the regenerative state where the repairman does job afresh, otherwise, $\delta_j = 0$.Thus, the following equations are obtained by using (5) as:

$$R_0^D(t) = Q_{01} \otimes R_1^D(t)$$
$$R_1^D(t) = Q_{12}(t) \otimes R_2^D(t) + Q_{14}(t) \otimes R_4^D(t) + Q_{11.5}(t) \otimes R_1^D(t)$$
$$R_2^D(t) = Q_{20} \otimes R_3^D(t)$$
$$R_3^D(t) = Q_{3o}(t) \otimes [1 + R_0^D(t)] + Q_{31.6}(t) \otimes [1 + R_1^D(t)]$$
$$R_4^D(t) = Q_{41}(t) \otimes R_1^D(t) + Q_{41.87}(t) \otimes [1 + R_1^D(t)]$$
$$R_5^D(t) = Q_{51} \otimes [1 + R_1^D(t)]$$

Taking L.S.T. of above expressions and calculating for $\overline{R_0^D}(s)$, we have

$$R_0^D = \lim_{s\to 0} s\overline{R_0^D}(s) = \frac{N_5}{D_2^{'}}$$

where, N_5 = p_{12} and $D_2^{'}$ is already defined.

h) Expected Number of Treatment Given to the Repairman

Let $T_i^R(t)$ be the expected number of repairs of the unit by the repairman in (0, t] such that the system entered regenerative state i at t = 0. The recursive relation for $T_i^R(t)$ are given as:

$$T_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) \circledS [\delta_j + T_i^R(t)] \quad (6)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$ if j is the regenerative state where the repairman does job afresh, otherwise, $\delta_j = 0$.Thus, the following equations are obtained by using (6) as:

$$T_0^R(t) = Q_{01} \otimes T_1^R(t)$$
$$T_1^R(t) = Q_{12}(t) \otimes T_2^R(t) + Q_{14}(t) \otimes T_4^R(t) + Q_{11.5}(t) \otimes T_1^R(t)$$
$$T_2^R(t) = Q_{23} \otimes T_3^R(t)$$
$$T_3^R(t) = Q_{30}(t) \otimes T_0^R(t) + Q_{31.6}(t) \otimes T_1^R(t)$$
$$T_4^R(t) = Q_{41}(t) \otimes [1 + T_1^R(t)] + Q_{41.87}(t) \otimes [1 + T_1^R(t)]$$
$$T_5^R(t) = Q_{51} \otimes T_1^R(t)$$

Taking L.S.T. of above expressions and calculating for $\overline{T_0^R}(s)$,we have

$$T_0^R = \lim_{s\to 0} s\overline{T_0^R}(s) = \frac{N_6}{D_2^{'}},$$

where, N_6=p_{14} and $D_2^{'}$ is already defined.

IV Profit Analysis

The following expression can be used to obtain Profit of the system model:

$$P_A = K_0A_0 - K_1B_0^M(t) - K_2B_0^D(t) - K_3R_0^M(t) - K_4R_0^D(t) - K_5T_0^R(t) \quad (7)$$

V. Particular Cases

Let us take

g(t) = $\alpha e^{-\alpha t}$, h(t) = $\beta e^{-\beta t}$, a(t) = $\Upsilon e^{-\Upsilon t}$,

$$p_{12} = \frac{\alpha}{(\alpha+\lambda+r_2)}, p_{15} = \frac{r_2}{(\alpha+\lambda+r_2)}, p_{14} = \frac{\lambda}{(\alpha+\lambda+r_2)}, p_{41} = \frac{\Upsilon}{(\Upsilon+r_2)}, p_{48} = \frac{r_2}{(\Upsilon+r_2)}, p_{30} = \frac{\beta}{(\beta+r_1)}, p_{36} = \frac{r_1}{(\beta+r_1)}$$

$$\mu_0 = \frac{1}{r_1}, \mu_1 = \frac{1}{(\alpha+\lambda+r_2)}, \mu_2 = \frac{1}{r_2}, \mu_3 = \frac{1}{(\beta+r_2)}, \mu_4 = \frac{1}{(\Upsilon+r_2)}, \mu_5 = \frac{1}{\beta}, \mu_4^{'} = \frac{r_2}{(\Upsilon+r_2)}\left[\frac{1}{\beta}+\frac{1}{\beta}\right], \mu_3^{'} = \frac{1}{\beta}\left(\frac{r_1}{(\beta+r_1)}\right),$$

MTSF = $\frac{N_1}{D_1}$, Availability = $\frac{N_2}{D_2^{'}}$,

Busy period of the repairman (B) = $\frac{N_3}{D_2^{'}}$,

Expected number of repairs of main unit (R_{M0}) = $\frac{N_5}{D_2^{'}}$,

Expected number of repairs of duplicate unit (R_{D0}) = $\frac{N_4}{D_2^{'}}$,

Expected number of Treatment given to the Repairman (T_{R0}) = $\frac{N_6}{D_2^{'}}$,

where,

$$N_1 = \frac{1}{r_1}\left[\frac{\alpha}{(\alpha+\lambda+r_2)} + \frac{r_2}{(\alpha+\lambda+r_2)} + \frac{\lambda}{(\alpha+\lambda+r_2)(\Upsilon+r_2)}\right] + \frac{1}{(\alpha+\lambda+r_2)} + \frac{1}{r_2}\frac{\alpha}{(\alpha+\lambda+r_2)} + \frac{1}{(\beta+r_2)}\frac{\alpha}{(\alpha+\lambda+r_2)} + \frac{\lambda}{(\alpha+\lambda+r_2)(\Upsilon+r_2)},$$

$$D_1 = 1 - \frac{\alpha}{(\alpha+\lambda+r_2)}\frac{\beta}{(\beta+r_1)} - \frac{\lambda\Upsilon}{(\alpha+\lambda+r_2)(\Upsilon+r_2)},$$

$$N_2 = \frac{1}{r_1} - \frac{r_1}{(\beta+r_1)}\frac{1}{r_1}\frac{\alpha}{(\alpha+\lambda+r_2)} - \frac{1}{r_1}\frac{\lambda}{(\alpha+\lambda+r_2)} - \frac{1}{r_1}\frac{r_2}{(\alpha+\lambda+r_2)} + \frac{1}{(\alpha+\lambda+r_2)} + \frac{1}{r_2}\frac{\alpha}{(\alpha+\lambda+r_2)} + \frac{\alpha}{(\alpha+\lambda+r_2)}\frac{1}{(\beta+r_1)} - \frac{\lambda}{(\alpha+\lambda+r_2)}\frac{1}{(\Upsilon+r_2)}$$

$$N_3 = \frac{1}{(\alpha+\lambda+r_2)}, N_{3D} = \frac{\alpha}{(\alpha+\lambda+r_2)}, N_4 = \frac{\alpha}{(\alpha+\lambda+r_2)}\, N_5 = \frac{\alpha}{(\alpha+\lambda+r_2)}\, N_6 = \frac{\lambda}{(\alpha+\lambda+r_2)}$$

$$D_2' = \frac{1}{(\alpha+\lambda+r_2)} + \frac{1}{r_1}\frac{\alpha}{(\alpha+\lambda+r_2)}\frac{\beta}{(\beta+r_1)} + \frac{1}{r_2}\frac{\alpha}{(\alpha+\lambda+r_2)} + \frac{1}{\beta}\frac{\alpha}{(\alpha+\lambda+r_2)}\frac{r_1}{(\beta+r_1)} + \frac{r_2}{(\Upsilon+r_2)}\frac{\lambda}{(\alpha+\lambda+r_2)}\left[\frac{1}{\Upsilon}+\frac{1}{\beta}\right] + \frac{1}{\beta}\frac{r_2}{(\alpha+\lambda+r_2)}$$

VI. Graphical Presentation

The graphical representation of MTSF, availability and profit function has been shown in figures 2,3 and 4 respectively to check their behavior with respect to the values of the parameters associated with failure and repair rates. From Figure 2, it is observed that the MTSF of the system decreases when failure rate of main unit is increased from 0.01 to 0.1. Also, MTSF increases with an increase inrepair rate of main unit, duplicate unit and treatment rate of the repairman.

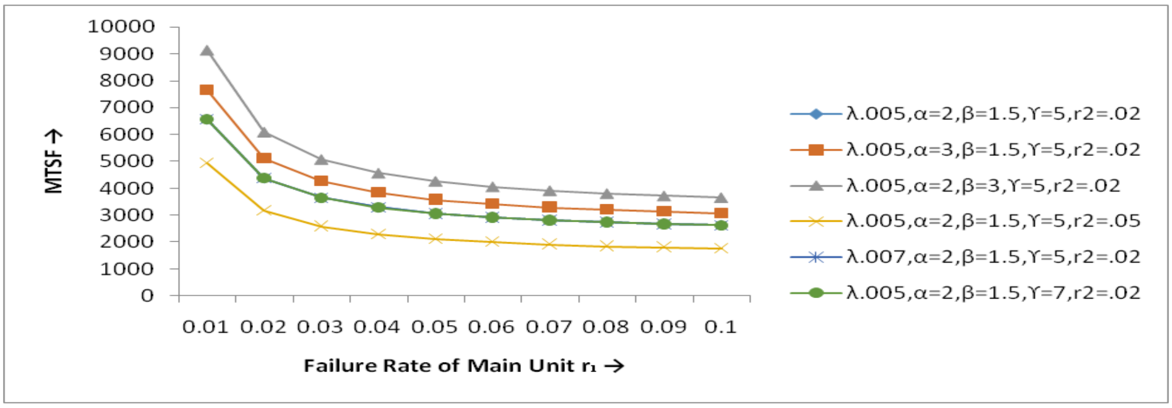

Figure 2: *MTSF Vs Failure Rate of Main Unit*

From Figure 3, it is clearly seen that the availability of the system decreases rapidly with increase of failure rate of main unit. Also, availability of the system increases with an increase inrepair rate of main unit, duplicate unit and treatment rate of the repairman.

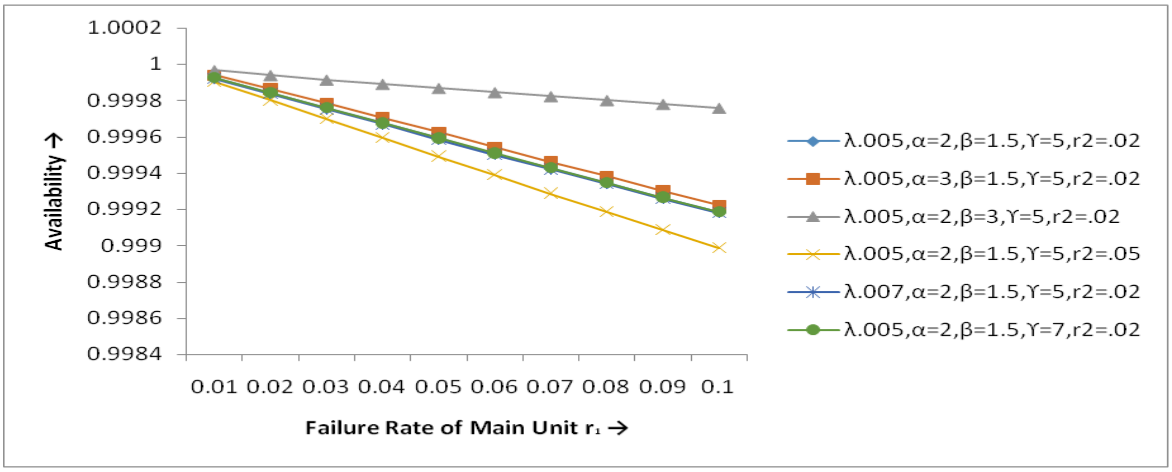

Figure 3: *Availability Vs Failure Rate of Main Unit*

From Figure 4, it is observed that the profit decreases when failure rate of the main unit increases. Also,the profit of the system is increases with an increase inrepair rate of main unit, duplicate unit and treatment rate of the repairman.

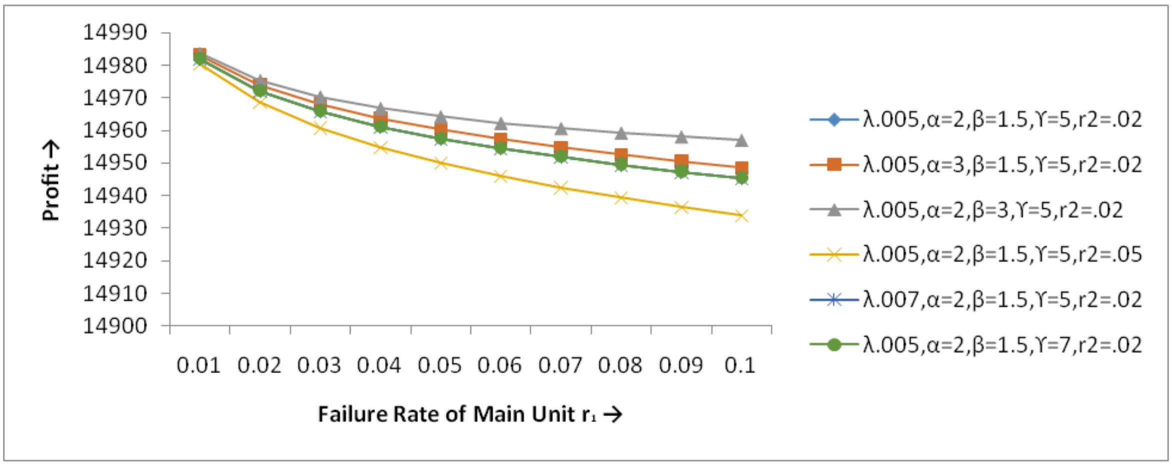

Figure 4: *Profit Vs Failure Rate of Main Unit*

VII. Conclusion

The idea of priority for repair and operation of the duplicate unit has been used to determine reliability characteristics of a stochastic model developed for a system of non-identical units with failure of repairman. The failure of repairman is called conditional failure as it fails only during the repair of the main unit. In this study reliability measures such as MTSF, availability and profit function are obtained and their behavior is shown respectively figures: Figure 2, Figure 3 and Figure 4. It is observed that MTSF, availability and profit function decline when failure rate increases. On the other hand, these measures increase with the increase of repair rate of main unit, duplicate unit and treatment rate of the repairman. Further, the study reveals that profit of the system model can be increased by increasing the repair rate of the duplicate unit.

VIII. Application

The oxygen supply system which is shown in figure 5 can be considered as a direct application of the present study. An acute shortage of medical oxygen and oxygen cylinders has been observed during COVID-19 pandemic situation everywhere throughout the World. The oxygen therapy was in dire need for the survival of patients during this pandemic. The scarcity of oxygen cylinders has also pushed up the demand for oxygen concentrators. Today, oxygen concentrators are in great demand after devices for oxygen therapy in home isolation. Therefore, the present study has been designed to analyze the oxygen supply system comprises oxygen concentrator as a main unit and the oxygen cylinder as its duplication. In case of electricity failure, it becomes necessary to give priority for operation and repair of the oxygen cylinder to cover the risk. Thus, it is a non-identical system of two units in which the concepts of priority and the failure of repairman have been considered to examine some important reliability characteristics so that the users of the oxygen supply system may take appropriate decision to minimize the risk.

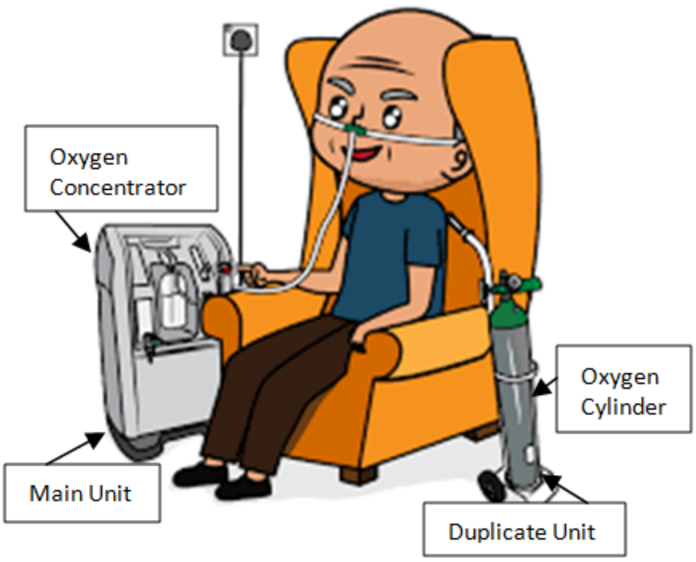

Figure 5: *Oxygen Supply System*

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