SELECTION OF LIFE TEST SAMPLING INSPECTION PLANS FOR CONTINUOUS PRODUCTION

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Abstract

Reliability sampling is the methodology often used in manufacturing industries for making decision about the disposition of lots of finished products based on the information generated from a life test. Such a methodology can be applied effectively for isolated lots as well as for a continuous stream of lots through the life tests to ensure control over the quality characteristics that are mainly related to the functioning of the manufacturing items in time. Sampling inspection plans for isolated lots are classified under lot by lot inspection procedures. Cumulative results plans are classified under the sampling inspection for continuous production, which results in continuous stream of lots. This paper presents the notion of life tests for cumulative results plans with a particular reference to chain sampling inspection plans when the lots are formulated from a continuous stream of production. The operating characteristic (OC) function of chain sampling plans for life tests is presented as a measure of performance when the lifetime random variable follows an exponential distribution. A procedure for designing the proposed plans indexed by two points on the characteristic curve for providing protection to the producer and consumer is discussed with illustrations. Tables yielding the parameters of the optimum plans are also provided.

Keywords: Acceptable mean life, Chain sampling plan, Consumer's risk, Cumulative results plan, Exponential distribution, Producer's risk.

1. Introduction

Reliability sampling, one of the decision-making procedures in statistical product control, is effectively implemented in the production and engineering processes to make an assessment about the finished products and to decide on the disposition of lot(s) of items. It involves a life test, which is an experiment performed on each of the items selected randomly from the lot(s) to observe lifetimes of the items as the values of the quality characteristic. It consists in a sampling procedure, called life test sampling plan, which is employed by drawing a random sample of test units from the lot and inspecting the units for deciding whether the lot is accepted or rejected based on the information provided by the test results. The focal point of any specific life test sampling plan is to determine whether the lifetimes of items attain the required standard or not based on the observations made from the sampled lifetime data. Such sampling plans can be developed considering the lifetime of the products as the quality characteristic as well as the random variable, which is hypothesized to follow a suitable probability distribution, like exponential, Weibull, lognormal, or gamma distribution rather than the normal distribution.

Analogous to the general classification of sampling inspection given in [1], life test sampling plans can be categorized primarily into two types, namely, lot-by-lot sampling plans for life tests and cumulative results sampling plans for life tests when production is continuous. Cumulative results plans are generally classified under the sampling inspection for continuous production, which results in continuous stream of lots. While the literature in product control cites voluminous references on the applications of many continuous-type probability distributions in the studies concerned with the development of various lot by lot sampling inspection plans for life tests, only a very few works on life test sampling plans for continuous production are noticeable. The earlier works, which laid the foundation for the expansion of various types of sampling plans, would include the theory of reliability sampling proposed and developed in [2] to [9]. Significant contributions in the development of life test sampling plans employing exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data have also been made in the past four decades. A detailed account of such plans was provided in [10].

The recent advances in the theory and applications of life test sampling plans are provided in [11] to [26]. The exponential distribution, which is a special case of gamma family of distributions as demonstrated in [27], has a wider application in the fields of queueing theory, reliability theory and engineering, and hydrology. It is used to model the performance of components that have a constant failure rate and is applied to the cases involving items that do not degrade with time or do not result in wear out failures. Examples include components of high-quality integrated circuits, such as diodes, transistors, resistors, and capacitors. The exponential distribution is considered as a perfect model for the long and constant period of low failure risk that characterizes the useful life of the product and represents the intrinsic failure phase in the field of reliability.

Earlier literature outlines the application of exponential distribution in the fields of actuarial, biological and engineering sciences. One may refer to [28] to [32] for more details. The designing of life test sampling plans under the conditions of Marshall – Olkin extended exponential distribution has been discussed in [33]. While the exponential distribution is appropriate for modeling the lifetime of an item, it is commonly applied for the inferential aspect of utilizing life information. Hence, as a member of the lifetime continuous probability distributions, the exponential distribution can be considered as an apt probability model to adopt in real life situations.

This paper presents the concept of life tests for cumulative results plans with a particular reference to chain sampling inspection plans when the lots are formed from the items resulted from a continuous stream of production. The operating characteristic (OC) function of chain sampling plans for life tests is presented as a measure of performance when the lifetime random variable follows an exponential distribution A procedure for designing the proposed plans indexed by two points on the characteristic curve, namely acceptable mean (or median) life and unacceptable mean (or median) life associated with producer's risk and consumer's risk, respectively, is discussed with illustrations. Tables yielding the parameters of the optimum chain sampling plans are also provided.

2. Cumulative Results Sampling Plans for Life Tests

Cumulative results sampling inspection plans generally use the current as well as past sample information from product entities in making a decision about the current product entities (see [34]). A class of cumulative results plans is developed based on the procedure and the concepts introduced in [35]. The basic procedure is labeled as chain sampling plan and is designated as ChSP-1. The cumulative results plans, including ChSP-1, are applied under the following conditions:

(a) The production is reasonably steady so that results on current and preceding lots are broadly indicative of a continuing process;

(b) Samples from lots are obtained essentially in the order of production;

(c) Inspection is by attributes with quality defined in terms of a fraction nonconforming; and

(d) lots are expected to be essentially of same quality.

ChSP-1, a special purpose attributes sampling plan, was devised in [35] for continuous production. A detailed discussion on the significance and designing of ChSP-1 has been made in [36] – [38] and in [10]. Under this plan, there is a provision to utilize only small acceptance numbers such as 0 or 1, and to make use of the information provided by a fixed number of preceding lots for deciding about the disposition of the present lot. A salient feature of this plan is that it provides greater protection to the producer and consumer against rejection of satisfactory lot quality and acceptance of unsatisfactory lot quality, respectively, when compared to lot-by-lot single sampling plans by attributes with small acceptance numbers, say 0 and 1.

In the context of life tests, one may define the acceptance number as the allowable number of failures whereas in the traditional acceptance sampling it is the allowable number of nonconforming items. Similarly, the lot quality under life testing is defined by mean or median lifetime of the product whereas in the customary sampling inspection it is defined by fraction nonconforming. Consider the following conditions:

(a) Sampling plans for life tests are required to be set up for product characteristics that involve costly or destructive testing.

(b) Situations warrant small samples to be drawn from the lot.

Under these conditions, i.e., when lots or batches are produced continuously by a production process and very small sample sizes are required to be selected from each lot or batch due to destructive or costly nature of inspections, sampling plans with small acceptance numbers, say only a fewer number of failures are desirable. More importantly, for small sample sizes such as n = 4, 5, 6 or even n = 10, only zero failure is practicable. It has been demonstrated in [39] and [40] that, under sampling inspection by attributes, single sampling plans for life tests with zero failures or zero acceptance number, designated by $SSP-(n, 0)$, are unattractive as they fail to provide protection to the producer against the acceptable mean or median life of the product. The operating characteristic curves of such sampling plans having zero failures are quite often in undesirable shapes and hence, they seldom ensures protection to producers, but ensures protection to consumers against unacceptable mean or median life of the product. It can also be demonstrated that single sampling plans admitting one or more failures in a sample of items improve upon the undesirable characteristics of $SSP-(n, 0)$, but may require larger sample sizes. In order to overcome this shortcoming, the chain sampling plans of type ChSP-1 for life tests allowing not more than one failure in the random samples drawn from the submitted lot can be adopted.

Thus, ChSP-1 for life tests can be employed in situations that warrant small samples when costly or destructive testing is involved. Specified by two parameters, viz., the sample size n, and the clearance number i, ChSP-1 can be implemented using the following operating procedure:

Step 1: For each lot, take a random sample of *n* items and observe the number, *d*, of failures.

Step 2: Accept the lot, if *d* = 0 and reject the lot, if *d* > 1. If *d* = 1, accept the lot, provided there are no failures in the immediately preceding *i* random samples of size *n*.

2.1 Exponential Distribution

Let *T* be a random variable representing the lifetime of the components. Assume that *T* follows an exponential distribution with scale parameter θ. The probability density function and the cumulative distribution function of *T* are, respectively, defined as follows:

$$f(t;\theta) = \frac{1}{\theta}\exp\left(-\frac{t}{\theta}\right), 0 \le t < \infty; \theta > 0 \quad (1)$$

$$F(t;\theta) = 1 - \exp\left(-\frac{t}{\theta}\right). \quad (2)$$

The mean life time, the median life time, the reliability function and hazard function for specified time *t* under the exponential distribution are, respectively, given below:

$$\mu = E(t) = \theta \quad (3)$$

$$\mu_d = \theta \ln(2) \quad (4)$$

$$R(t;\theta) = \exp\left(-\frac{t}{\theta}\right) \quad (5)$$

$$Z(t;\theta) = \frac{1}{\theta},\ 0 \le t < \infty \quad (6)$$

It is known that the reliable life is the life beyond which some specified proportion of items in the lot will survive. Associated with the exponential distribution, it is defined by

$$\rho = -\theta \ln(R), \quad (7)$$

where R is the proportion of items surviving to time ρ.

The proportion, p, of product failing before time t, is defined by the cumulative probability distribution of T and is expressed by

$$p = F(t;\theta) = 1 - \exp\left(-\frac{t}{\theta}\right). \quad (8)$$

2.2 Operating Characteristic Function of ChSP-1 for Life Tests

One of the measures for assessing the performance of any sampling inspection plan is its operating characteristic (OC) function. It is defined as the probability of acceptance of the lot under the sampling plan and is a function of the lot quality or the proportion, p, of product failing before time t or the failure probability. According to [35], the OC function associated with ChSP-1 plans for life tests would represent the proportion of lots that will be accepted under the plan and is expressed as a function of p by

$$P_a(p) = P_{0,n} + P_{1,n}\left(P_{0,n}\right)^i, \quad (9)$$

where $P_{0,n}$ is the probability of having zero failures in a sample of size n and $P_{1,n}$ is the probability of having one failure in sample of size n. It may be noted that $P_{0,n}$, $P_{1,n}$ and $P_a(p)$ are defined under the conditions of binomial distribution as given below:

$$P_{0,n} = (1-p)^n,$$

$$P_{1,n} = np(1-p)^{n-1}$$

$$P_a(p) = (1-p)^n + np(1-p)^{n(i+1)-1}. \quad (10)$$

Under the conditions of Poisson distribution, the expressions for $P_{0,n}$, $P_{1,n}$ and $P_a(p)$ are respectively, are as follows:

$$P_{0,n} = e^{-np},$$

$$P_{1,n} = npe^{-np}$$

$$P_a(p) = e^{-np} + npe^{-np(i+1)}. \quad (11)$$

In the context of sampling plans for life tests, it is to be observed that the failure probability, p, is defined by the proportion of product failing before time t, and hence, the expression for p is defined by the cumulative probability distribution of T given as (8). Associated with a specific value of p, there exists a unique value of t / θ. Since the mean life is $\mu = \theta$, p is related to t / μ. In a similar way, for a specified value of t / μ, the value of p could be obtained. As the value of p is associated with t / μ, the operating characteristic function of a life test sampling plan can be considered as a function of t / μ, rather than p, and hence, the OC curve of the plan could be obtained by plotting the acceptance probabilities against the values of t/μ. If the median life is to be considered for the operating characteristics of the desired plan, p can be associated with $t/\mu_d = (t/\theta)(\ln(2))^{-1}$.

3. Procedure for the Selection of ChSP–1 for Life Tests with Desired Discrimination

It can be observed that when the lifetime random variable follows an exponential distribution, ChSP-1 for life tests would be designated by the parameters, *viz*., the sample size, n, and the clearance number, i., which depend on the desired mean or median life. Hence, under an exponential distribution, when the mean life criterion is to be involved, a specific ChSP-1 can be determined by specifying the requirements that the OC curve should pass through two prescribed points, namely, $(\mu_0, 1-\alpha)$ and (μ_1, β), where μ_0 and μ_1 are the acceptable and unacceptable mean life, respectively, which are associated with the producer's risk, α, and the consumer's risk, β.

Corresponding to μ_0 and μ_1, one may define p_0 and p_1 as the acceptable and unacceptable proportions of the lot failing before time, t, respectively. Here, p_0 and p_1 may be considered as the producer's quality level and consumer's quality level with α and β as the associated producer's and consumer's risks, respectively.

Further, associated with p_0 and p_1 are the dimensionless ratios t/μ_0 and t/μ_1, respectively. The specification of these quality levels would ensure protection to the producer against rejection of satisfactory lots as well as the consumer against acceptance of unsatisfactory lots, and would be considered to fix the *OC* curve in accordance with a desired degree of discrimination. The operating ratio, defined by $OR = \mu_0/\mu_1$, is used as a measure of discrimination. An optimum life test sampling plan for specified points $(\mu_0, 1-\alpha)$ and (μ_1, β) can be determined by satisfying the following two conditions so that the maximum producer's and consumer's risks will be fixed at α and β, respectively:

$$P_a(\mu_0) \geq 1-\alpha \tag{12}$$

and

$$P_a(\mu_1) \leq \beta. \tag{13}$$

It may be noted that the specification of $(p_0, 1-\alpha)$ and (p_1, β) is equivalent to the specification of the points $(\mu_0, 1-\alpha)$ and (μ_1, β) or $(t/\mu_0, 1-\alpha)$ and $(t/\mu_1, \beta)$. When median life criterion is desired, $(\mu_{d_0}, 1-\alpha)$ and (μ_{d_1}, β) are specified as the requirements for ensuring protection to the producer and the consumer, and the operation ratio, $OR = \mu_{d_0}/\mu_{d_1}$, is used as the measure of discrimination, where μ_{d_0} is the acceptable median life and μ_{d_1} is the unacceptable median life. An integrated approach to determine ChSP-1 for life tests satisfying the prescribed requirements under mean life criterion when the underlying distribution of the life time random variable follows an exponential distribution and its implementation is described below:

Step 1: Specify the values of t/μ_0 and t/μ_1 with $\alpha = 0.05$ and $\beta = 0.10$, respectively.

Step 2: Find p_0 and p_1 corresponding to t/μ_0 and t/μ_1 using the relationship existing between p and t/μ.

Step 3: Obtain the optimum values of n and i for the specified strength $(\mu_0, 1-\alpha)$ and (μ_1, β) satisfying the conditions (12) and (13) either through (10) or through (11) with the values of p_0 and p_1.

Step 4: Perform the life test considering t as the test termination time and μ as the expected mean life. Observe the number, d, of failures.

Step 5: Terminate the life test if either the termination time t is reached or the event of more than one failure occurs before time t.

Step 6: Accept the lot if there are no failures; reject the lot if more than one failure occurs before reaching time t.

Step 7: If one failure occurs, accept the present lot, provided no failures have occurred in the immediately preceding i samples of size n.

Following the first three steps of the above procedure, Tables 1 and 2 which yield the optimum ChSP-1 for life tests under mean and median life criteria are constructed. The plan parameters given in the tables are determined for a wide range of values of the dimensionless ratios corresponding to mean and median life criteria and satisfy the prescribed conditions with the maximum producer's risk of 5 percent and the maximum consumer's risk of 10 percent.

3.1 Numerical Illustration

Assume that a ChSP-1 for life tests is to be instituted. It is assumed that the lifetime of the mobile phone battery is a random variable which is distributed according to an exponential distribution. It is expected that the plan shall provide the desired degree of discrimination measured in terms of the operating ratio OR = 16, ensuring protection to the producer and the consumer in terms of the acceptable mean life and unacceptable mean life of the battery given respectively $\mu_0 = 75000$ minutes and $\mu_1 = 4680$ minutes with the associated producer's risk of 5 percent and the consumer's risk of 10 percent. Suppose that the experimenter wishes to terminate the life test at t = 300 minutes. As $OR = 16.025 \approx 16$ and $t/\mu_0 = 0.004$, entering Table 1 with these values, one finds n = 38 and i = 2, as the sample size and clearance number of the desired ChSP-1. Thus, the plan for the given conditions is implemented as given below:

1. Select a random sample of 38 items from the present lot.
2. Conduct the life test on each of the sampled items and observe the number of failures before reaching the termination time fixed as t = 300 minutes.
3. Terminate the life test once the termination time, t = 300 minutes, is reached or when one or more failures occur before reaching the termination time.
4. Accept the present lot, if no failure is observed; reject the lot, if more than one failure occurs; accept the lot, if one failure is observed and no failures were found in the preceding two samples.
5. Treat the items which survive beyond the specified time t = 300 minutes as passed.

3.2 Numerical Illustration

The time to failure (in hours) of monolithic integrated circuits can be modeled by an exponential distribution with failure rate fixed at 0.0003. Assume that the components are resulted from a continuous stream of production. The producer's and consumer's requirements are defined in terms of acceptable mean life, μ_0, of 9000 hours and unacceptable mean life, μ_1, of 500 hours. The producer's risk of rejecting the lot having $\mu_0 = 9000$ hours and the consumer's risk of accepting the lot having $\mu_1 = 500$ hours are fixed at 5 percent and 10 percent, respectively. The total time duration of life test is fixed at t = 13.5 hours. For the given requirements, the measure of discrimination is found to be OR = 18 and $t/\mu_0 = 0.0015$. Entering Table 1 with the values of R and t/μ_0, the optimum ChSP-1 is determined as n = 87 and i = 3.

Table 1: Optimum ChSP-1 for Life Tests Based on Exponential Distribution Indexed by Acceptable Mean Life and Unacceptable Mean Life with a Maximum Producer's Risk of 5 Percent and Consumer's Risk of 10 Percent [Key: *n*, *i*, where *n* is the sample size and *i* is the clearance number]

OR	t/μ_0											
	0.001	0.00125	0.0015	0.00175	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
15.5	152, 2	122, 2	102, 2	87, 2	77, 2	52, 2	39, 2	32, 2	27, 2	23, 2	20, 2	18, 2
16.0	147, 2	118, 2	99, 2	85, 2	74, 2	50, 2	38, 2	31, 2	26, 2	22, 2	20, 2	18, 2
16.5	143, 2	114, 2	96, 2	82, 2	72, 2	49, 2	37, 2	30, 2	25, 2	22, 2	19, 2	17, 2
17.0	137, 3	110, 3	92, 3	79, 3	69, 3	47, 2	36, 2	29, 2	24, 2	21, 2	19, 2	17, 2
17.5	133, 3	107, 3	89, 3	77, 3	68, 2	46, 2	35, 2	28, 2	24, 2	21, 2	18, 2	16, 2
18.0	130, 3	104, 3	87, 3	75, 2	66, 2	44, 3	34, 2	27, 3	23, 2	20, 2	18, 2	16, 2
18.5	126, 3	101, 3	85, 2	73, 2	64, 2	43, 3	33, 2	27, 2	22, 3	19, 3	17, 2	16, 2
19.0	123, 3	99, 3	82, 4	71, 3	62, 3	42, 2	32, 2	26, 2	22, 2	19, 2	17, 2	15, 2
19.5	120, 3	96, 3	80, 3	69, 3	61, 2	41, 2	31, 2	25, 3	21, 3	19, 2	16, 3	15, 2
20.0	117, 3	94, 3	78, 3	67, 4	59, 3	40, 2	30, 3	25, 2	21, 2	18, 2	16, 2	14, 3
20.5	114, 3	92, 2	77, 2	66, 2	58, 2	39, 2	30, 2	24, 2	20, 3	18, 2	16, 2	14, 2
21.0	111, 3	89, 3	75, 2	64, 3	56, 4	38, 3	29, 2	24, 2	20, 2	17, 3	15, 3	14, 2
21.5	109, 3	87, 3	73, 3	63, 2	55, 3	37, 3	28, 3	23, 2	20, 2	17, 2	15, 2	14, 2
22.0	106, 3	85, 3	71, 4	61, 4	54, 3	37, 2	28, 2	23, 2	19, 2	17, 2	15, 2	13, 2
22.5	104, 3	84, 2	70, 3	60, 3	53, 2	36, 2	27, 3	22, 2	19, 2	16, 2	14, 3	13, 2
23.0	102, 3	82, 3	68, 3	59, 2	52, 2	35, 2	27, 2	22, 2	18, 3	16, 2	14, 2	13, 2
23.5	100, 3	80, 3	67, 3	58, 2	51, 2	34, 3	26, 2	21, 2	18, 2	16, 2	14, 2	13, 2
24.0	98, 3	78, 3	66, 2	56, 4	50, 2	34, 2	26, 2	21, 2	18, 2	15, 3	14, 2	12, 2
24.5	96, 3	77, 3	64, 3	55, 3	49, 2	33, 2	25, 2	20, 3	17, 3	15, 2	13, 3	12, 2
25.0	94, 3	75, 3	63, 3	54, 3	48, 2	32, 3	25, 2	20, 2	17, 2	15, 2	13, 2	12, 2
25.5	92, 3	74, 3	62, 2	53, 3	47, 2	32, 2	24, 2	20, 2	17, 2	15, 2	13, 2	12, 2
26.0	90, 3	73, 2	61, 2	52, 3	46, 2	31, 2	24, 2	19, 3	16, 3	14, 2	13, 2	12, 1
26.5	89, 2	71, 3	60, 2	51, 3	45, 3	31, 2	23, 3	19, 2	16, 2	14, 2	13, 2	11, 2
27.0	87, 3	70, 3	59, 2	50, 3	44, 3	30, 2	23, 2	19, 2	16, 2	14, 2	12, 2	11, 2
28.0	84, 3	67, 4	56, 4	49, 2	43, 2	29, 2	22, 2	18, 2	15, 3	13, 3	12, 2	11, 2
29.0	81, 3	65, 3	55, 2	47, 2	41, 3	28, 2	22, 2	18, 2	15, 2	13, 2	12, 2	11, 1
30.0	78, 3	63, 3	53, 2	46, 2	40, 2	27, 3	21, 2	17, 2	14, 3	13, 2	11, 2	10, 2
31.0	76, 3	61, 3	51, 3	44, 3	39, 2	26, 3	20, 2	17, 2	14, 2	12, 2	11, 2	10, 2
32.0	74, 2	59, 3	50, 2	43, 2	38, 2	26, 2	20, 2	16, 2	14, 2	12, 2	11, 2	10, 1
33.0	71, 4	57, 4	48, 3	42, 2	37, 2	25, 2	19, 2	16, 2	13, 2	12, 2	10, 3	9, 3
34.0	69, 3	56, 2	47, 2	40, 3	36, 2	24, 2	19, 2	15, 2	13, 2	11, 2	10, 2	9, 2
35.0	67, 4	54, 3	46, 2	39, 3	35, 2	24, 2	18, 2	15, 2	13, 2	11, 2	10, 2	9, 2
36.0	66, 2	53, 2	44, 3	38, 3	34, 2	23, 2	18, 2	14, 3	12, 2	11, 2	10, 1	9, 1
37.0	64, 2	51, 3	43, 3	37, 3	33, 2	22, 3	17, 2	14, 2	12, 2	11, 1	9, 3	9, 1
38.0	62, 3	50, 3	42, 2	36, 3	32, 2	22, 2	17, 2	14, 2	12, 2	10, 2	9, 2	8, 3

The plan obtained in the present numerical illustration is implemented as given below:

1. A random sample of 87 integrated circuits is drawn from the current lot.
2. All 87 circuits are placed for life test simultaneously for the time duration of 13.5 hours.

3. When no failures are observed until 13.5 hours, the current lot is accepted; when more than one failure is observed before the termination time, the lot is rejected; when exactly one failure occurs in the total duration of 13.5 hours, the current lot is accepted, only if no failure was observed in the immediately preceding 3 samples of size n = 87.

Table 2: Optimum ChSP-1 for Life Tests Based on Exponential Distribution Indexed by Acceptable Median Life and Unacceptable Median Life with a Maximum Producer's Risk of 5 Percent and Consumer's Risk of 10 Percent [Key: *n*, *i*, where *n* is the sample size and *i* is the clearance number]

OR	t / μ_{d_0}										
	0.0015	0.00175	0.002	0.0025	0.003	0.004	0.005	0.006	0.007	0.008	0.009	0.01
15.0	146, 2	125, 2	110,2	88, 2	74, 2	56, 2	45, 2	38, 2	33, 2	29, 2	26, 2	23, 2
15.5	141, 2	121, 2	106,2	86, 2	72, 2	54, 2	44, 2	37, 2	32, 2	28, 2	25, 2	23, 2
16.0	137, 2	118, 2	103,2	83, 2	69, 2	52, 2	42, 2	36, 2	31, 2	27, 2	24, 2	22, 2
16.5	132, 3	113, 3	99, 3	80, 3	67, 2	51, 2	41, 2	35, 2	30, 2	26, 2	24, 2	21, 2
17.0	128, 3	110, 3	97, 2	78, 2	65, 3	49, 3	40, 2	33, 3	29, 2	25, 3	23, 2	21, 2
17.5	125, 3	107, 3	94, 3	76, 2	63, 3	48, 2	39, 2	32, 3	28, 2	25, 2	22, 2	20, 2
18.0	121, 3	104, 3	92, 2	74, 2	62, 2	47, 2	38, 2	32, 2	27, 3	24, 2	22, 2	20, 2
18.5	118, 3	102, 3	89, 3	72, 2	60, 3	45, 3	37, 2	31, 2	27, 2	24, 2	21, 2	19, 2
19.0	115, 3	99, 3	87, 3	70, 2	58, 4	44, 3	36, 2	30, 2	26, 2	23, 2	21, 2	19, 2
19.5	112, 3	97, 2	85, 3	68, 3	57, 3	43, 3	35, 2	29, 3	25, 3	22, 3	20, 2	18, 2
20.0	110, 3	94, 3	83, 2	66, 4	56, 2	42, 3	34, 2	29, 2	25, 2	22, 2	20, 2	18, 2
20.5	107, 3	92, 3	81, 3	65, 3	54, 3	41, 3	33, 3	28, 2	24, 2	21, 3	19, 2	17, 4
21.0	105, 3	90, 3	79, 3	63, 4	53, 3	40, 3	33, 2	27, 3	24, 2	21, 2	19, 2	17, 2
21.5	102, 3	88, 3	77, 3	62, 3	52, 2	39, 3	32, 2	27, 2	23, 2	21, 2	18, 3	17, 2
22.0	100, 3	86, 3	75, 4	61, 2	51, 2	39, 2	31, 2	26, 3	23, 2	20, 2	18, 2	16, 3
22.5	98, 3	84, 3	74, 3	59, 3	50, 2	38, 2	31, 2	26, 2	22, 3	20, 2	18, 2	16, 2
23.0	96, 3	82, 4	72, 3	58, 3	49, 2	37, 2	30, 2	25, 2	22, 2	19, 3	17 3	16, 2
23.5	94, 3	81, 3	71, 3	57, 3	48, 2	36, 3	29, 3	25, 2	21, 3	19, 2	17, 2	16, 2
24.0	92, 3	79, 3	69, 4	56, 2	47, 2	36, 2	29, 2	24, 2	21, 2	19, 2	17, 2	15, 2
24.5	90, 3	78, 2	68, 3	55, 2	46, 2	35, 2	28, 3	24, 2	21, 2	18, 2	16, 3	15, 2
25.0	89, 2	76, 3	67, 2	54, 2	45, 3	34, 3	28, 2	23, 3	20, 2	18, 2	16, 2	15, 2
25.5	87, 3	75, 2	66, 2	53, 2	44, 3	34, 2	27, 2	23, 2	20, 2	18, 2	16, 2	14, 3
26.0	85, 3	73, 3	64, 3	52, 2	43, 3	33, 2	27, 2	23, 2	20, 2	17, 3	16, 2	14, 2
27.0	84, 2	72, 3	63, 3	51, 2	43, 2	32, 3	26, 3	22, 2	19, 2	17, 2	15, 2	14, 2
28.0	81, 3	69, 4	61, 3	49, 3	41, 3	31, 3	25, 3	21, 3	19, 2	17, 2	15, 2	14, 2
29.0	78, 3	67, 3	59, 2	47, 4	40, 2	30, 3	25, 2	21, 2	18, 2	16, 2	14, 3	13, 2
30.0	75, 4	65, 3	57, 3	46, 2	39, 2	29, 3	24, 2	20, 2	17, 4	16, 2	14, 2	13, 2
31.0	73, 3	63, 2	55, 3	45, 2	37, 3	28, 3	23, 2	20, 2	17, 2	15, 2	14, 2	12, 3
32.0	71, 3	61, 3	54, 2	43, 3	36, 3	28, 2	22, 3	19, 2	17, 2	15, 2	13, 2	12, 2
33.0	69, 2	59, 3	52, 2	42, 2	35, 3	27, 2	22, 2	18, 3	16, 2	14, 2	13, 2	12, 2
34.0	67, 2	57, 4	51, 2	41, 2	34, 3	26, 2	21, 2	18, 2	16, 2	14, 2	13, 2	11, 3
35.0	65, 3	56, 2	49, 3	40, 2	33, 3	25, 3	21, 2	17, 4	15, 2	14, 2	12, 2	11, 2
36.0	63, 3	54, 3	48, 2	39, 2	32, 3	25, 2	20, 2	17, 2	15, 2	13, 2	12, 2	11, 2
37.0	62, 2	53, 2	47, 2	38, 2	32, 2	24, 2	20, 2	17, 2	15, 2	13, 2	12, 2	11, 2
38.0	60, 3	52, 2	45, 3	37, 2	31, 2	24, 2	19, 2	16, 2	14, 2	13, 2	11, 3	10, 3

3.3 Numerical Illustration

Suppose that an experimenter is interested to implement ChSP-1 for life tests to make a decision about the disposition of a current lot of manufactured products whose life time follows an exponential distribution. It is assumed that the life test will be terminated at $t = 200$ hours. It is expected that the plan shall yield the desired degree of discrimination when the median life criterion is used providing protection to the producer and the consumer in terms of the acceptable median life and unacceptable median life fixed as $\mu_{d_0} = 80000$ hours and $\mu_{d_1} = 4100$ hours, associated with the producer's risk of 5 percent and consumer's risk of 10 percent. For the specified requirements, one obtains $t / \mu_{d_0} = 0.0025$, $t / \mu_{d_1} = 0.0488$ and R = 19.51. Hence, from Table 2, corresponding to OR = 19.5 and $t / \mu_{d_0} = 0.0025$, the optimum ChSP-1 is determined with its parameters specified by n = 68 and i = 3.

4. Conclusion

The concept of cumulative results plans for life tests is introduced with reference to chain sampling inspection plans involving the formation of lots from the items resulted from a continuous stream of production. It consists in the methodical procedure for making a decision about the present (current) lot based on the inspection of a random sample drawn from the lot and make use of the information provided by previous samples when exactly one failure occurs in the sample drawn from the current lot. The operating characteristic function of chain sampling plans for life tests is defined as a measure of performance under the condition that lifetime random variable follows an exponential distribution. A procedure for designing the optimum plans for life tests under mean and median life criteria is described. The optimum plans indexed by two points on the characteristic curve, namely acceptable mean (or median) life and unacceptable mean (or median) life associated with producer's risk and consumer's risk, respectively, are provided along with suitable illustrations.

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