CRITICAL PATH INTERMS OF INTUITIONISTIC TRIANGULAR FUZZY NUMBERS USING MAXIMUM EDGE DISTANCE METHOD

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Abstract

We live in a contemporary world where successful project management strategies are complex to manipulate the projects for project managers and decision-makers. It is essential to pinpoint strategies so that managers can accomplish projects and polish off them within a predetermined period of time and resource restrain. This research assists us to detect the critical path in an acyclic network in terms of intuitionistic triangular fuzzy numbers, we have proposed the "maximum edge distance" method. Forward and backward algorithms are designed to find the optimal path for the proposed method. Numerical examples are also illustrated for the same. Verification is done using the path length ranking technique. Simulation results are included by the use of the C program and MATLAB. Finally, the comparison is made with the traditional forward and backward pass (existing method) technique to point out the conclusion.

Keywords: Critical path problem, Triangular fuzzy number, Intuitionistic Triangular Fuzzy Number, Acyclic network.

I. Introduction

A project is understood as a set of interconnected operations that must be executed out in a particular manner to generate a significant profit. A complex project implicates many interlinked activities depend upon labor force, machines, and materials; it was unfeasible for organizers to assemble and achieve an optimum schedule. However, due to the complexity of few projects come across in the late 1950's it was essential to introduce a new technique that will be more adequate and efficacious strategies. Two techniques were adopted by Operations Research namely, Project Evaluation and Review Technique and Critical Path Method. The former was developed by the US Naval Forces in 1957 while the latter was developed by James E. Kelley and Morgan R. walker [9]. CPM was first applied in 1966 for the construction of a major Skyscraper that is the former World Trade Center Twin Towers in New York City. CPM has more boons which were implemented by Mauchly Associates. CPM and PERT are predominantly time-oriented methods. The most noteworthy dissimilarity between CPM and PERT was in the utilization of the time estimates. The value of time assigns to be probabilistic in PERT although they were deterministic in CPM. It was widely known as a valuable tool for the look and programming for huge come. The concept of the critical path allows the decision-maker to control the project's cost and schedule, and it can improve the quality of the work. This method is commonly utilized in various industries to analyze and improve the efficiency of a project.

Adequate project management strategies are censorious to organizers and decision-makers to approach projects in the conflicting domain. Project Managers are required to observe which techniques can accomplish projects and execute them within a particular period [7]. Actually, owing to the uncertainty of data in addition to the discrepancy of quantities framework, it was often difficult to secure the designated activity time. Hence, Lofti Asker Zadeh proposed the theory called fuzzy set theory which plays a significant aspect in this type of decision-making world [14]. There were several methods reported to solve the fuzzy critical path (FCP) problem in the open literature. The first method to find an optimal path called Fuzzy Program Evaluation and Review Technique (FPERT) was proposed by Chanas and Kamburowski [5]. FPERT assumes the time to find the critical path, whereas when the project managers have deterministic data to find a critical path they can use the Fuzzy Critical Path Method (FCPM). Stephen Dinakar and Rameshan [13] presented an approach to analyzing the critical path in a project network with octagonal fuzzy numbers. Balaganesan and Ganesan [3] proposed a new methodology to find the critical path where the imprecise parameters in the network diagram take the intuitionistic triangular fuzzy numbers instead of crisp numbers. N. Jose Parvin Praveena et.al proposed a new method called the new JOSE Algorithm to find FCP. This method was designed according to find the fuzzy critical path using 13 Parameters with a ranking method, namely the Euclidean ranking method. The dynamic encoding recursion of the critical path in terms of triskaidecagonal and Triskaidecagonal fuzzy numbers fuzzy critical Path was found [8]. Ravi Shankar Nowpada et.al presented a new analytical method for finding critical paths using a fuzzy project network. They have proposed a new defuzzification formula for trapezoidal fuzzy numbers and applied it to the float time for each activity in that project network and tabulated the values. With the use of table values, they found the critical path [11]. Thus, numerous papers are published on fuzzy critical path problems. Few among them are [1, 6 and 12].

The paper is organized as follows: In section 2, we review the basic definitions of fuzzy set theory. Section 3; focus on two different algorithms which are utilized to identify the intuitionistic fuzzy critical path. Numerical examples are illustrated to perform the proposed approach. The simulation result is included for one of the developed algorithms. Under results and discussions in section 4, the comparison is made with the existing method (Forward and backward pass computations). Section 5 concludes the paper.

II. Preliminaries

Definition 2.1 Fuzzy set [14]

Fuzzy sets are sets that are characterized by imprecise data with boundaries to express a degree of membership function in the closed unit interval [0, 1].

Let P be a non-empty set. Then a fuzzy set X is a set having the form of ordered pairs X = {(p, α_A (p)): p ∈ P} where the function $\alpha_x : P \to [0, 1]$ is called the membership function and α_x (p) is called the degree of membership of each element p ∈ P.

Definition 2.2 Intuitionistic Fuzzy set [2]

Let a set P be fixed. An intuitionistic fuzzy set X in P is an object having the form X = {(p, α_x (p), γ_x (p)) : p ∈ P} where the function $\alpha_x : P \to [0, 1]$ and $\beta_x : P \to [0, 1]$ defined the degree of membership and degree of non-membership respectively of the element p ∈ P to the set X, which is a subset of P, and for every p ∈ P, $0 \leq \alpha_x (p) + \beta_x (p) \leq 1$.

The amount $\alpha_x (p) = 1 - \alpha_x (p) - \beta_x (p)$ is called the hesitation part, which may be either membership value or non-membership value or both.

Definition 2.3 Fuzzy Number [14]

Let Ṕ is said to be a fuzzy number if it satisfies the following condition,

- **(i)** α_x (p) is piecewise continuous
- **(ii)** α_x (p) is convex,
- **(iii)** α_x (p) is normal.

Definition 2.4 Triangular fuzzy number [14]

A triangular fuzzy number X can be defined by a triplet (m, n, o; 1), where m < n < o; m, n, o ϵ R. The membership function $\alpha_x(p)$ is given as follows:

$$\alpha_x(\mathrm{p}) = \begin{cases} \frac{x-m}{n-m}, & m \le x < n \\ 1, & x = n \\ \frac{o-x}{o-n}, & n < x \le o \\ 0, & otherwise \end{cases}$$

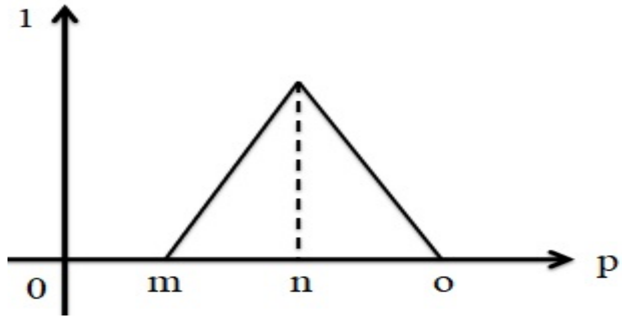


Fig.1. *Triangular Fuzzy Number*

Definition 2.5 Intuitionistic Triangular Fuzzy Number [2]

A intuitionistic triangular fuzzy number Ẋ can be defined by a triplet (ṁ, n, ȯ; 1), where ṁ<m < n < o < ȯ; ṁ, n, ȯ ϵ R. The membership function is alike given in Definition 2.4. The non-membership function $\beta_x(p)$ is given as follows:

$$\beta_x(\mathrm{p}) = \begin{cases} \frac{x-\dot{m}}{\mathrm{n}-\dot{m}}, & \dot{m} \le x < \mathrm{n} \\ 1, & x = \mathrm{n} \\ \frac{\dot{o}-x}{\dot{o}-\mathrm{n}}, & \mathrm{n} < x \le \dot{o} \\ 0, & otherwise \end{cases}$$

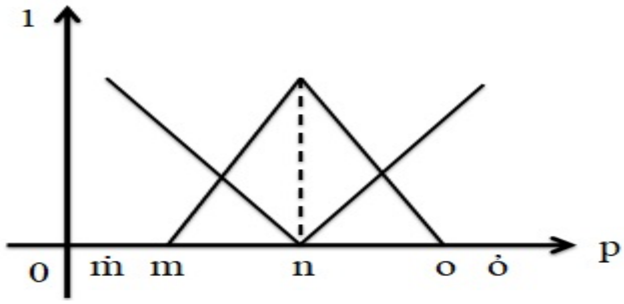


Fig.2. Intuitionistic Triangular Fuzzy Number

Definition 2.6 Addition operation on triangular fuzzy numbers [14]

Let A = (m_1, n_1, o_1) and B = (m_2, n_2, o_2) be two triangular fuzzy numbers, then A $\oplus$ B = ($m_1 + m_2, n_1 + n_2, o_1 + o_2$).

Definition 2.7 Subtraction operation on triangular fuzzy numbers [14]

Let A = (m_1, n_1, o_1) and B = (m_2, n_2, o_2) be two triangular fuzzy numbers, then A− B = ($m_1 - o_2, n_1 - n_2, o_1 - m_2$).

Definition 2.8 Maximum operation for triangular fuzzy numbers [14]

Let A = (m_1, n_1, o_1) and B = (m_2, n_2, o_2) be two triangular fuzzy numbers then L_{max} = max (A, B) = (max (m_1, m_2), max (n_1, n_2), max (o_1, o_2)).

Definition 2.9 Minimum operation for triangular fuzzy numbers [14]

Let A = (m_1, n_1, o_1) and B = (m_2, n_2, o_2) be two triangular fuzzy numbers then L_{min} = min (A, B) = (min (m_1, m_2), min (n_1, n_2), min (o_1, o_2)).

Definition 2.10 Acyclic network [4]

A digraph is a graph each of whose edges are directed. Hence an acyclic digraph is a directed graph without cycle.

III. Methodology

I. General Algorithm for intuitionistic fuzzy critical path problem using intuitionistic triangular fuzzy numbers

Step 1:
Construct an acyclic network G(V, E), where V is the set of vertices and E is the set of edges. Each arc lengths or edge weights corresponds to the cost, time etc., in practical problems.

Step 2:
Calculate all possible paths P_i, i = 1 to n from the source vertex 's' to the destination vertex 'd'.

Step 3:
The corresponding path lengths L_i, i = 1 to n using definition 2.6.
To calculate path length $P_i = \sum_{i=1}^{n} L_i$

Step 4:
After Calculating the path length for each possible path L_i, i = 1 to n, then find the path having the maximum value and rank it as first rank. The path which is ranked first is identified as the intuitionistic fuzzy critical path.

Numerical Example:

Step 1:
Construct an acyclic network G (V, E) of Type V graph fuzziness using definition 2.12, where the edge weights are taken as an intuitionistic triangular fuzzy number. [3] [6]

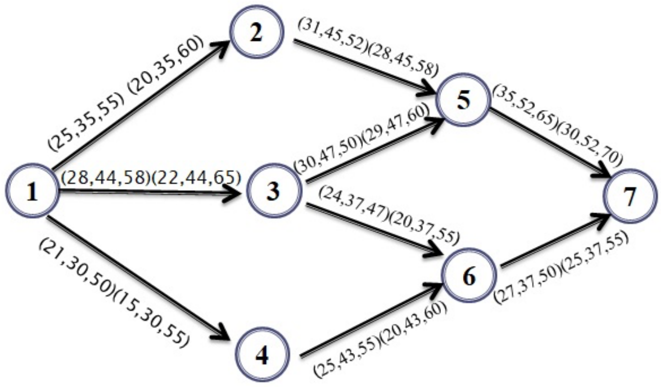


Fig. 3 *Intuitionistic triangular fuzzy network*

Step 2:
The possible paths are P_1 is 1-2-5-7, P_2 is 1-3-5-7, P_3 is 1-3-5-7 and P_4 is 1-4-6-7.

Step 3:
Consider Fig.3, calculate the path length, $P(L_i) = \sum_{i=1}^{n} L_i$. Calculated Values are tabulated below.

Table. 1 *Results of the Network*

Path (P_i)	Path length (L_i)	Ranking
P_1: 1-2-5-7	(91, 132, 172)(78, 132, 188)	2
P_2: 1-3-5-7	(93, 143, 173)(81, 143, 195)	1
P_3: 1-3-6-7	(79, 118, 155)(67, 118, 175)	3
P_4: 1-4-6-7	(73, 110, 155)(60, 110, 170)	4

From the table, the path P_1: 1-3-5-7, is identified as the intuitionistic fuzzy critical path because it has the highest value while calculating path length.

II Proposed Algorithm

Maximum Edge Distance Algorithm for intuitionistic fuzzy critical path (IFCP) problem using intuitionistic triangular fuzzy numbers

Notations used:
EL – Edge length
d[u, v] – Duration of the activity (u, v)
Adj [u] – Adjacent to node u
s – Source node
t – Destination node

(i) Forward procedure to calculate the IFCP

Step 1: Place all the vertices in Q = priority queue (1, 2, …. . , n-1, n).

Step 2: Choose s = u = 1, choose the source node as permanent node. Set EL[u] = (0,0,0)(0,0,0).

Step 3: Extract the maximum edge distance.
For all vϵAdj[u] that is for all edges emerging from u, calculate the following:

(i) If u is incident to only one node v then, EL[v] = EL[u] $\oplus$ d[u, v] using definition 2.6
(ii) If u is incident to more than one node v then, EL [v] = $Max_{v\in s}$ [(EL[u]$\oplus$d[u,v])] using definition 2.8

The new permanent node = v. Now, form the new priority queue by removing the source node s = u = 1 and the other nodes adjacent to u which are different from v.
Repeat step 3, until the permanent node = t. If so, terminate the execution of the algorithm.

Step 4: The intuitionistic fuzzy distance along the intuitionistic fuzzy critical path P namely intuitionistic fuzzy critical path length is denoted by D(P) and is defined as D(P) = $\sum_{(u,v)\epsilon P} l_{uv}$, where l_{uv} is the path length. It is calculated using definition 2.6 and the corresponding path is the IFCP.

(ii) Backward procedure to calculate the IFCP

Step 1: Place all the vertices in Q = priority queue (n, n-1,…..2, 1).

Step 2: Choose t = u = n, that is choose the destination node as permanent node. Set EL[u] = (0,0,0)

Step 3: Extract the maximum edge distance.

For all vϵAdj[u] that is for all edges incident on u, calculate the following:

(i) If u is incident to only one node v then, EL[u] = EL[v] $\oplus$ d[u, v] using definition 2.6
(ii) If u is incident to more than one node v then, EL [u] = $Max_{v\in s}$ [(EL[v]$\oplus$d[u,v])] using definition 2.8

The new permanent node = v. Now remove the destination node u = t from the priority queue and the other nodes incident to u other than v.
Repeat step 3, until the permanent node = s. If so, terminate the execution of the algorithm.

Step 4: Calculate the edge distance by using step 4 as given in forward procedure to calculate the IFCP.

Numerical Example:

Consider fig. 3 to find the IFCP, Backward procedure of an algorithm 3.2 to calculate the IFCP will not work here, since the edges incident to the destination node as the same path length, because the network is constructed using directed graph. Hence, we apply forward procedure of an algorithm 3.2.

Step 1: Q = priority queue (1, 2, 3, 4, 5, 6, 7)

Step 2: Let S = u = 1 (source node). EL[1] = (0,0,0) (0, 0, 0).

Step 3: 2ϵAdj[1] 3ϵAdj[1] and 4ϵAdj[1]
EL[2] = EL[1] $\oplus$ d[1,2] = (0, 0, 0) (0, 0, 0) + (25, 35, 55)(20,35,60) = (25, 35, 55)(20,35,60),
EL[3] = EL[1] $\oplus$ d[1,3] = (0, 0, 0) (0, 0, 0) + (28,44,58)(22, 44, 65) = (28,44,58)(22, 44, 65),
EL[4] = EL[1] $\oplus$ d[1,4] = (0, 0, 0) (0, 0, 0) + (21, 30, 50)(15, 30, 55) = (21, 30, 50)(15, 30, 55)
EL[v] = Max { (EL[2] , EL[3] , EL[4]) }
= Max {(25, 35, 55)(20,35,60), (28,44,58)(22, 44, 65), (21, 30, 50)(15, 30, 55)}
EL[v] = (28,44,58)(22, 44, 65) = EL [3]
The new permanent node = 3

Remove source node 1, node 2 and node 4 from the priority queue.
New priority queue is Q = Priority queue (5, 6, 7, 8)

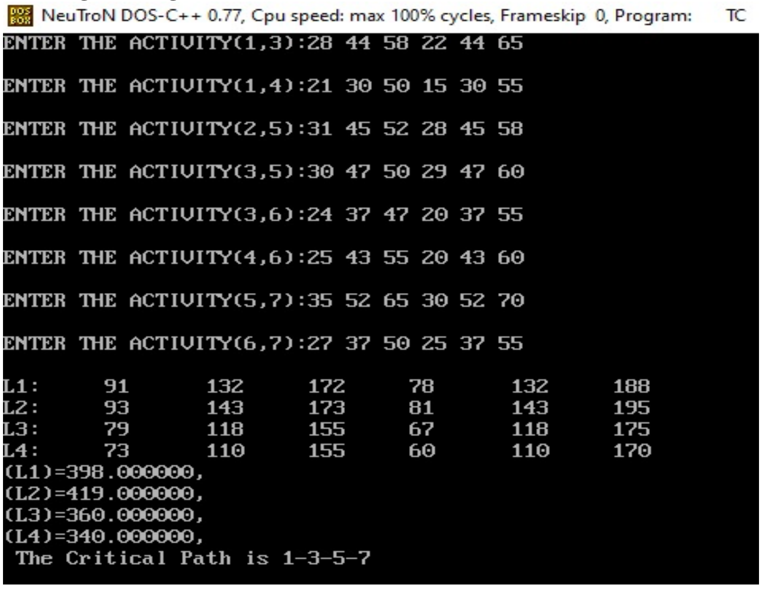
5 ϵ Adj [3], 6ϵAdj [3]
EL [5] = EL [3] $\oplus$ d [3, 5] = (28,44,58)(22,44,65) + (30,47,50)(29,47,60) = (58, 91, 108) (51, 91, 125)
EL [6] = EL [3]$\oplus$ d [3, 6] = (28,44,58) (22,44,65) + (24,37,47)(20,37,55) = (52, 81, 105) (42, 81, 120)
EL[v] = Max { EL [5] , EL[6] } = (58, 91, 108) (51, 91, 125) = EL [5]
The new permanent node = 5

Remove source node 6 from the priority queue.
New priority queue is Q = Priority queue (7)

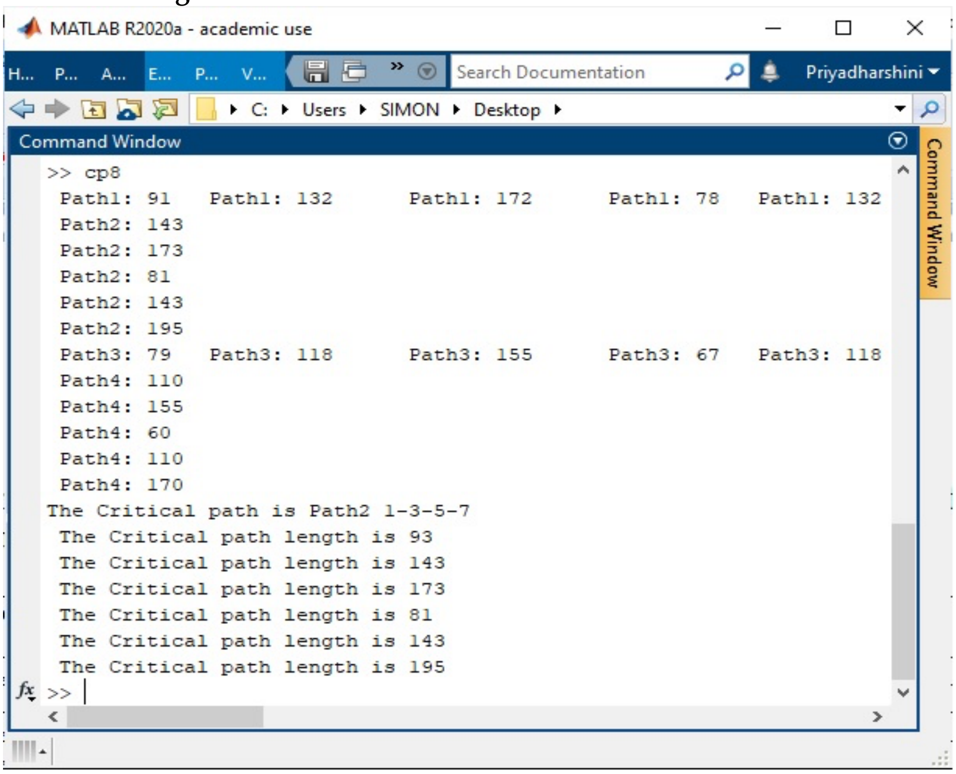
7ϵ Adj [5]
EL[7] = EL[5] $\oplus$ d[5,7] = (58,91,108)(51,91,125) + (35,52,65)(30,52,70) = (93,143,173) (81,143, 195)
The new permanent node = 7 = t = destination node.
Since, we reach the destination node we can stop the process.

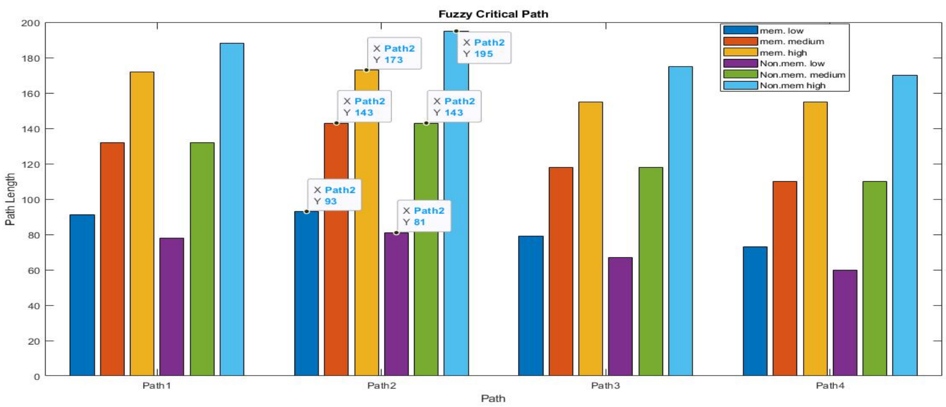
Step 4:
By using the formula stated in algorithm 3.2, D(P) = $\sum_{(u,v)\epsilon P} l_{uv}$. The intuitionistic fuzzy critical path length is calculated that is (93, 143, 173) (81, 143, 195) and the corresponding intuitionistic fuzzy critical path is 1-3-5-7.

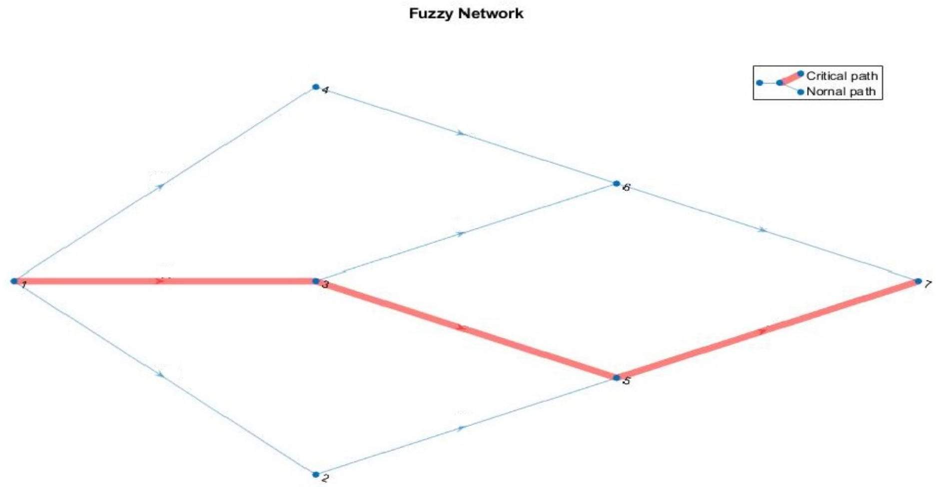
Simulation Result using C Program



Simulation Result using MATLAB







IV. Results and Discussions

Verification. For the sake of comparison, here verification is done using traditional forward and backward pass calculation.

Activity (i-j)	Duration	Earliest start Time(EST)	Earliest finish Time(EFT)	Latest finish Time(LFT)	Total Float
1-2	(25, 35, 55)(20, 35, 60)	(0,0,0) (0,0,0)	(25, 35, 55)(20, 35, 60)	(27,46,56)(23,46,67)	(2,11,1)(3,11,7)
1-3	(28,44,58)(22, 44, 65)	(0,0,0) (0,0,0)	(28,44,58)(22, 44, 65)	(28,44,58)(22,44,65)	**(0,0,0) (0,0,0)**
1-4	(21, 30, 50)(15, 30, 55)	(0,0,0) (0,0,0)	(21, 30, 50)(15, 30, 55)	(41,63,58)(36,63,80)	(20,33,8)(21,33,25)
2-5	(31, 45, 52)(28, 45, 58)	(25, 35, 55)(20, 35, 60)	(56,80,107)(48,80,118)	(58,91,108)(51,91,125)	(2,11,1)(3,11,7)
3-5	(30,47,50)(29,47,60)	(28,44,58)(22, 44, 65)	(58,91,108)(51,91,125)	(58,91,108)(51,91,125)	**(0,0,0) (0,0,0)**
3-6	(24,37,47)(20,37,55)	(28,44,58)(22, 44, 65)	(52,81,105)(42,81,120)	(66,106,123)(56,106,140)	(14,25,18)(14,25,20)
4-6	(25,43,55)(20,43,60)	(21, 30, 50)(15, 30, 55)	(46,73,105)(35,73,110)	(66,106,123)(56,106,140)	(20,33,18)(21,33,30)
5-7	(35,52,65)(30,52,70)	(58,91,108)(51,91,125)	(93,143,163)(81,143,195)	(93,143,163)(81,143,195)	**(0,0,0) (0,0,0)**
6-7	(27,37,50)(25,37,55)	(46,73,105)(35,73,115)	(73,110,155)(60,110,170)	(93,143,163)(81,143,195)	(20,30,58)(20,30,25)

Here path P_1:1-3-5-7 is identified as the intuitionistic fuzzy critical path**.**
The Comparison was done for the solution yield using the proposed method. Verification is done using the traditional forward and backward pass calculations. It is found that the result obtained in this paper, coincides with the result obtained through the existing methods. The iterations and time consumption used to find the critical path using maximum edge distance method was better that the existing method.

V. Conclusion

In this paper, we have developed a different algorithm namely the maximum edge distance method to find the optimal path in an intuitionistic fuzzy weighted directed graph with its edge weights as an intuitionistic triangular fuzzy number. The method proposed in this paper is an alternative way to identify the critical path in the fuzzy environment. This method has turned down the recurrence. The approximation of the project can be done effortlessly through this "Maximum edge distance" method. The reason to mention the word "effortless" is because that the completion time of the project given by this method will be optimized at its best as shown in the solution illustrated in the numerical example. Obviously and finally this method reduces the time consumption when compared to the regular methods used already (Forward and backward pass computations).

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