CONSTRUCTION OF LIFE TEST SAMPLING INSPECTION PLANS BY ATTRIBUTES BASED ON MARSHALL - OLKIN EXTENDED EXPONENTIAL DISTRIBUTION

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Abstract

A life test is a random experiment conducted on the manufactured items such as electrical and electronic components for estimating their life time based on the inspection of randomly sampled items. Life time of the items is a random variable which follows a specific continuous-type distribution, called the lifetime distribution. Reliability sampling, which is one among the classifications of product control, deals with inspection procedures for sentencing one or more lots or batches of items submitted for inspection. In this paper, the concept of sampling plans for life tests involving two samples is introduced under the assumption that the life time random variable is modeled by Marshall - Olkin extended exponential distribution. A procedure is developed for designing the optimum plan with minimum sample sizes when two points on the desired operating characteristic curve are prescribed to ensure protection to the producer and the consumer.

Keywords: Life Test Sampling Plan, producer's risk, Marshall Olkin Extended Exponential Distribution, Consumers risk, OC function, Reliability sampling.

1. Introduction

Reliability sampling is the methodology that deals with sampling inspection procedures, called life test or reliability sampling plans essentially adopted in the industrial processes for taking decisions on the disposition of the lot(s) of items such as electric or electronic components based on the assessment of quality utilizing lifetimes of the items as the quality characteristics. A life test sampling plan is employed by drawing a random sample of test units, which are subjected to a set of test procedures, from the lot and inspecting the units for deciding whether the lot is accepted or rejected based on the information provided by the test results. A specific sampling plan focuses on the objective of determining whether the lifetimes of items reach the specified standard or not based on the observations made from the sampled lifetime data. Such sampling plans can be developed considering the lifetime of the products as the quality characteristic as well as the random variable, which is appropriately modeled by a probability distribution, like exponential, Weibull, lognormal, or gamma distribution rather than the normal distribution.

The literature in product control provides adequate references on the applications of many continuous-type probability distributions in the studies concerned with the development of sampling inspection plans for life tests. The earlier works, which laid the foundation for the expansion of various types of sampling plans, would include the theory of reliability sampling proposed and developed in [1], [2], [3], [4], [5], [6], [7] and [8]. Significant contributions in the development of life test sampling plans employing exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data have also been made in the past four decades. A detailed account of such plans was provided in [9]. Recent advances in the theory of life test sampling plans are provided in [10 – 25].

Marshall – Olkin extended exponential distribution (MOEED), introduced in [26] as a generalization of the exponential distribution, exhibits the property of monotone failure rate. In some applications of biological, agricultural and entomological studies, the failure rate function of the underlying distribution may be inverted bathtub – shaped hazard function or unimodal. When a probability distribution for life-time variable has a failure rate function that takes various shapes, it is the natural choice to adopt the distribution in practice. Further, MOEED has the failure rate that decreases with time, fairly constant failure rate and failure rate that increases with time, indicative of infantile or early-failures, useful life or random failures and wear-out failures, respectively.

Due to the possibility of various shapes of failure rate function, which is the case similar to gamma and Weibull distribution, MOEED is especially suitable for modeling life time of an item and is used commonly for the inferential aspect of utilizing life information. Hence, as a member of the lifetime continuous distributions, MOEED can be considered as an apt probability model to adopt in real life situations and may be used as an alternative to the gamma, Weibull and other exponentiated family of distributions. Considering its importance in reliability studies, Different criteria for designing life test sampling plans are discussed in [27] under the condition that the life test is evaluated in terms of mean life, hazard life and reliability life under the conditions for the application of MOEED.

In this paper, a special type of sampling plans which involves two random samples and allows a maximum of one failure in the combined samples for life tests is introduced under the assumption that the lifetime quality characteristic is modeled by MOEED. The method of designing optimum sampling plans indexed by two prescribed points on the operating characteristic curve, namely acceptable mean life and unacceptable mean life, associated with the producer's risk and the consumer's risk, respectively, is discussed with illustrations under the conditions for application of MOEED for desired degree of discrimination which would ensure protection to the producer and consumer.

2. Special Type of Sampling Plans for Life Tests

A special type of sampling plan, devised and discussed in [28 - 29], is a lot-by-lot sampling plan by attributes in which provisions are made to utilize only small acceptance numbers such as 0 or 1, and to inspect the submitted lot by drawing a second random sample even if the first random sample contains zero nonconforming (defective) items. A special feature of this plan is that the operating characteristics of the plan lies between those of $c = 0$ and $c = 1$ single sampling plans, and thus provides better discrimination over the single sampling plans with wide range of operating ratios. Consider the following conditions:

(a) Sampling plans for life tests are required to be set up for product characteristics that involve costly or destructive testing.

(b) Situations warrant small samples to be drawn from the lot.

In such conditions, a sampling plan with zero or fewer failures in the samples is quite reasonable to employ for the disposition of the lot. But, as demonstrated in [29] and [30], under sampling inspection by attributes, single sampling plans for life tests with zero failures or zero acceptance number, designated by $SSP-(n, 0)$, are unattractive as they fail to provide protection to the producer against the acceptable mean life of the product. The operating characteristic curves of such sampling plans having zero failures are quite often in undesirable shapes and hence, they seldom ensures protection to producers, but ensures protection to consumers against unacceptable mean life of the product.

It can be demonstrated that single sampling plans admitting one or more failures in a sample of items improve upon the undesirable characteristics of $SSP-(n, 0)$, but may require larger sample sizes. In order to overcome this shortcoming, the special type of sampling plans can be adopted for life tests allowing a maximum of one failure in the random samples drawn from the submitted lot.

Most often, situations involving small samples may warrant the use of single sampling plans with a fewer number of failures such as $c=0$ and $c=1$. But, the OC curves of $c=0$ and $c=1$ plans would indicate that there will be a conflicting interest between the producer and the consumer as $c=0$ plans always provide protection to the consumer with smaller risk of accepting the lot having unacceptable mean life of the product while $c=1$ plans favor the producer with smaller risk of rejecting the lot having acceptable mean life. This situation of conflict can be annulled if a suitable life test plan having its OC curve lying between the OC curves of $c=0$ and $c=1$ plans is designed. While introducing the special type of sampling plan, it was shown in [28] that there is a wide gap between the OC curves of $c=0$ and $c=1$ plans and established that the OC curves of the special plan lie between the OC curves of $c=0$ and $c=1$ plans. He also advocated that the sampling plans of similar kind could be used effectively in such situations. Hence, the special type of sampling plans could be the natural choice and could be considered as an alternative to single sampling plans having zero or fewer failures, such as c = 0 or c = 1. .

Before discussing the procedure for the selection of an optimum special type of sampling plan for life tests with the objective of providing protection to the producer and consumer against rejection of the lot for the specified acceptable mean life and against acceptance of the lot for the specified unacceptable mean life, the operating procedure of the plan is now described as given below:

A sample of n_1 items is taken from a given lot and inspected. If one or more failures are found, i.e., $m_1 \geq 1$, while inspecting n_1 items, then the lot is rejected; if no failure is found, i.e., $m_1 = 0$, a second sample of n_2 items is taken and the number of failures, m_2, is observed. If zero or one failure is found, i.e., $m_2 \leq 1$, while inspecting n_2 items, then the lot is accepted; if two or more failures are found, i.e., $m_2 > 1$, then the lot is rejected.

Thus, the special type of the sampling plans for life tests is specified by two parameters n_1 and n_2, which are the number of items in the first and second random samples, respectively.

3. Marshall - Olkin Extended Exponential Distribution

Let *T* be a random variable representing the lifetime of the components. Assume that *T* follows Marshall – Olkin extended exponential distribution (MOEED). The probability density function and the cumulative distribution function of *T* are, respectively, defined by

$$f(t;\gamma,\theta)=\frac{\gamma e^{-t/\theta}}{\theta(1-\bar{\gamma}e^{-t/\theta})^{2}},\ t,\gamma,\theta>0 \tag{1}$$

and

$$F(t;\gamma,\theta)=\frac{1-e^{-t/\theta}}{1-\bar{\gamma}e^{-t/\theta}},\ t,\gamma,\theta>0, \tag{2}$$

where $\bar{\gamma} = 1-\gamma$, γ is the shape parameter and θ is the scale parameter.

The mean life time, the reliability function and hazard function for specified time *t* under MOEED are, respectively, given by

$$\mu = E(T) = -\frac{\gamma\theta\log\gamma}{1-\gamma}, \tag{3}$$

$$R(t;\theta,\gamma) = \frac{\gamma}{e^{t/\theta}-\bar{\gamma}}, t \geq 0 \tag{4}$$

and $$Z(t;\theta,\gamma) = \frac{e^{t/\theta}}{\theta(e^{t/\theta}-\bar{\gamma})}, t \geq 0. \tag{5}$$

The reliable life is the life beyond which some specified proportion of items in the lot will survive. The reliable life associated with MOEED is defined and denoted by

$$\rho = \theta\log\left(\frac{\gamma+\bar{\gamma}R}{R}\right), \tag{6}$$

where *R* is the proportion of items surviving beyond life ρ.

The proportion, p, of product failing before time *t*, is defined by the cumulative probability distribution of *T* and is expressed by

$$p = P(T \leq t) = F(t;\gamma,\theta). \tag{7}$$

4. Operating Characteristic Function of Life Test Sampling Plans

Associated with the special of type of sampling plans are the performance measures, such as operating characteristic function and average sample number function, which are, respectively, expressed by

$$P_a(p) = f(0\,|\,n_1,p)[f(0\,|\,n_2,p)+f(1\,|\,n_2,p)] \tag{8}$$

and $$ASN(p) = n_1 + n_2 f(0\,|\,n_1,p), \tag{9}$$

where *p* is the proportion of product failing before time *t*, and $f(0\,|\,n_1,p)$, $f(0\,|\,n_2,p)$ and $f(1\,|\,n_1,p)$ are defined either from the binomial distribution or from the Poisson distribution. Under the conditions for application of Poisson model for the OC curve, the OC and ASN functions will have the following forms:

$$P_a(p) = e^{-n_1p}(e^{-n_2p}+n_2pe^{-n_2p}) \tag{10}$$

and $$ASN(p) = n_1 + n_2e^{-n_1p} \tag{11}$$

When the Binomial model is used, the OC and ASN functions are respectively given by

$$P_a(p) = (1-p)^{n_1}\left((1-p)^{n_2}+n_2p(1-p)^{n_2-1}\right) \tag{12}$$

and $$ASN(p) = n_1 + n_2(1-p)^{n_1} \tag{13}$$

In the context of sampling plans for life tests, it is to be observed that the failure probability, *p*, is defined by the proportion of product failing before time *t*, and hence, the expression for p is defined by the cumulative probability distribution of *T* given as (7). Associated with a specific value of *p*, there exists a unique value of t/μ, which is derived using (2), (3) and (7) as $t/\mu = -\left(\frac{t}{\theta}\right)\left(\frac{1-\gamma}{\gamma \log \gamma}\right)$.

In a similar way, for a specified value of t/μ, the value of *p* could be obtained. As the value of *p* is associated with t/μ, the operating characteristic function of a life test sampling plan can be considered as a function of t/μ, rather than *p*, and hence, the OC curve of the plan could be obtained by plotting the acceptance probabilities against the values of t/μ.

5. Procedure for the Selection of Life Test Sampling Plans

It can be observed that when the life time random variable follows a Marshall – Olkin extended exponential distribution, a life test sampling plan would be designated by the parameters, such as sample size(s) and acceptance number(s) of the sampling plan and the parameters of the distribution, like γ and θ (or μ). Hence, under MOEED, a specific life test sampling plan can be determined by specifying the requirements that the OC curve should pass through two prescribed points, namely, (μ_0, α) and (μ_1, β), where μ_0 and μ_1 are the acceptable and unacceptable mean life, respectively, which are associated with the risks α and β.

Corresponding to μ_0 and μ_1, one may define p_0 and p_1 as the acceptable and unacceptable proportions of the lot failing before time, *t*, respectively. Here, p_0 and p_1 may be considered as the producer's quality level and consumer's quality level with α and β as the associated producer's and consumer's risks, respectively.

Further, associated with p_0 and p_1 are the ratios t/μ_0 and t/μ_1, respectively. The specification of these quality levels would ensure protection to the producer against rejection of satisfactory lots as well as the consumer against acceptance of unsatisfactory lots, and would be considered to fix the *OC* curve in accordance with a desired degree of discrimination. An optimum life test sampling plan for specified points $(p_0, 1-\alpha)$ and (p_1, β) can be determined by satisfying the following two conditions so that the maximum producer's and consumer's risks will be fixed at α and β, respectively:

$$P_a(p_0) \geq 1-\alpha \tag{14}$$

and $$P_a(p_1) \leq \beta . \tag{15}$$

It may be noted that the specification of $(p_0, 1-\alpha)$ and (p_1, β) is equivalent to the specification of the points $(\mu_0, 1-\alpha)$ and (μ_1, β) or $(t/\mu_0, 1-\alpha)$ and $(t/\mu_1, \beta)$. The following integrated approach can be used to determine a life test sampling plan that meets the specified requirements under the conditions of MOEED and its implementation:

Step 1: Specify the value of the shape parameter γ or its estimate.

Step 2: Specify the values of t/μ_0 and t/μ_1 with $\alpha = 0.05$ and $\beta = 0.10$, respectively.

Step 3: Find p_0 and p_1 corresponding to t/μ_0 and t/μ_1 based on the procedure described in the previous section.

Step 4: Find the optimum values of n_1 and n_2 for the specified strength $(p_0, \alpha, p_1, \beta)$ satisfying the conditions (14) and (15), utilizing the expressions either (10) or (12).

Step 5: Draw randomly a set of n_1 items from the submitted lot.

Step 6: Conduct the life test considering t_0 as the test termination time and μ as the expected mean life. Observe the number, *m*, of failures.

Step 7: Terminate the life test if either time t_0 is reached or the condition $m \geq 1$ occurs before time t_0.

Step 8: Reject the lot if $m \geq 1$ at time t_0; If *m* = 0, draw a random sample of n_2 items from the remainder of the lot and observed the number of failures. If there is one or zero failure before time t_0, accept the lot; otherwise reject the lot..

5.1. Numerical Illustration

Assume that the lifetime random variable follows MOEED defined with the shape parameter γ and the shape parameter is estimated from the past history as γ = 1.5, it is desired to institute a life test sampling plan when the acceptable mean life and unacceptable mean life are prescribed as 75000 hours and 4285 hours, respectively. The producer's and consumer's risks are specified as $\alpha = 0.05$ and $\beta = 0.10$. The experimenter wishes to terminate the life test at *t* = 150 hours. For the given requirements, the values of t/μ_0 and t/μ_1 are obtained as 0.002 and 0.035, respectively. Based on the procedure described earlier, the parameters of the special type of sampling plan are determined as n_1 = 23 and n_2 = 109. Thus, the life test plan for the given conditions is implemented as given below:

1. Select a random sample of 23 items from a lot, conduct the life test on each of the sampled item and observe the number of failures while inspecting 23 items before reaching the termination time fixed as *t* = 150 hours.
2. Terminate the life test once the termination time, $t_0 = 150$ hours is reached or when the number of failures is 1 or more before reaching the termination time.
3. Reject the lot if the observed number of failures is one or more; if no failure is observed in 23 items before reaching the test termination time, select a random sample of n_2 = 109 items and conduct the life test on each of the sampled item. If the observed number of failures is one or less, accept the lot; otherwise reject the lot.
4. Treat the items which survive beyond time $t_0 = 150$ hours as passed.

5.2. Numerical Illustration

Suppose that an experimenter is interested to implement a life test sampling inspection plan for taking a decision about the disposition of a submitted lot of manufactured products whose life time follows MOEED. The value of γ, the shape parameter, is estimated as 2.5. It is assumed that the life test will be terminated at *t* = 18 hours. The acceptable and unacceptable proportions of the lot failing before time, *t*, are respectively prescribed as $p_0 = 0.001$ and $p_1 = 0.018$ with the associated risks fixed at the levels $\alpha = 0.05$ and $\beta = 0.10$. The values of t/μ corresponding to $p_0 = 0.001$ and $p_1 = 0.018$ are determined as $t/\mu_0 = 0.00175$ and $t/\mu_1 = 0.03$. Thus, optimum sample sizes for the special type of sampling plan for life tests satisfying the conditions (14) and (15), corresponding to $t/\mu_0 = 0.00175$ and $t/\mu_1 = 0.03$, are obtained as n_1 = 34 and n_2 = 168. The acceptable mean life and unacceptable mean life are also obtained as $\mu_0 = t/0.00175 = 10285$ hours and $\mu_1 = t/0.03 = 600$ hours, respectively.

6. Conclusion

A special type of sampling inspection plans for life-tests which involve two samples and allows a maximum of one failure is proposed when the lifetime quality characteristic is modeled by a Marshall – Olkin extended exponential distribution. A procedure for the selection of the proposed plan is discussed through numerical illustrations. The life test sampling plans which could be derived by the procedure discussed in this paper will ensure protection to the producer and consumer as the plans are indexed by acceptable and unacceptable proportion of product failing before the specified time, *t*. The practitioners can generate the required sampling plans for various choices of γ adopting the procedure.

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