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A Discrete Parametric Markov-Chain Model of a Two Non-Identical Units Warm Standby Repairable System with Two Types of Failure

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Abstract

The paper deals with the stochastic analysis of two non-identical units (unit-1 and unit-2). Initially, one unit is operative and other is kept into warm standby. Each unit of the system has two possible modes-Normal (N) and Total Failure (F). A single repairman is always available with the system to repair a failed unit. The operative unit is non-repairable, hence upon failure it goes for replacement. The system failure occurs when both the units are in total failure mode. Failure and repair times of a unit are taken as independent random variables of discrete nature having geometric distributions with different parameters.

Keywords: Transition probabilities, mean sojourn time, geometric distribution, regenerative point technique, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

1. Introduction

The Two non-identical units warm standby system have been widely studied in the literature of reliability as they are frequently used in modern business and industries. It is obvious that the standby unit is switched to operate when the operating unit fails and the switching device which is used to put the standby unit into operation may be perfect at the time of need. Some authors including [5, 9, 10 and 13] analyzed two unit warm standby and two non-identical units warm standby redundant system models using different concepts. All the above system models have been analyzed by considering continuous distributions of all the random variables involved.

In many realistic situations, some writers [4 and 7] analyzed a two identical unit and two non-identical units cold standby system with two types of failure and later on [3] analyzed two non-identical units parallel system subject to two types of failure and correlated life times. Some authors [11, 12] analyzed the deferent concepts of assumptions. So in case of discrete random variable, discrete distribution is considered to be appropriate for obtaining the effectiveness of different reliability measures.

In the area of reliability using discrete distribution had given their ideas by analyzing two non-identical unit parallel system with geometric failure and repair time distribution. Since there is always a possibility for failure of any system during in its operative conditions in different measures. So to detect the type of failure inspection is very much required which had been always ignored by the researchers, whether using continuous or discrete distributions.

This system model is based on discrete parametric Markov-chain. Moreover, [1, 2, 6 and 8] introduced the concept of discrete parametric Markov-chain in analyzing the system models in the field of reliability modeling. The following economic related measures of system effectiveness are obtained by using regenerative point technique-

i. Transition probabilities and mean sojourn times in various states.
ii. Reliability and mean time to system failure.
iii. Point-wise and steady-state availability of the system during time (0, t-1).
iv. Expected busy period of repairman during time (0, t-1).
v. Net expected profit incurred by the system during a finite and steady-state are obtained.

2. System Description and Assumptions

1. The system comprises of two non-identical units. Initially, one unit is operative and other is kept into warm standby.
2. Each unit of the system has two modes- Normal (N) and total failure (F).
3. A single repairman is always available with the system to repair a failed unit.
4. The operative unit is non-repairable, hence upon failure it goes for replacement.
5. The system failure occurs when both the units are in total failure mode.
6. The repaired unit works as good as new.
7. Failure and repair times of the units follow independent geometric distributions with different parameters.

3. Notations and States of the System

3.1 Notations :

$p_i q_i^x$: p.m.f. of failure time of type-1 and type-2 respectively for i=1,2,3 and $p_i + q_i = 1$.

$r_i s_i^x$: p.m.f. of repair time by repairman of type-1 and type-2 respectively for i=1, 2 and $r_i + s_i = 1$.

$p' q'^x$: p.m.f. of repair time of first unit; $p' + q' = 1$.

$\theta, \bar{\theta}$: probability that the replacement of a second unit respectively; $\theta + \bar{\theta} = 1$

$q_{ij}(\cdot), Q_{ij}(\cdot)$: p.m.f. and C.d.f. of one step or direct transition time from state S_i to S_j.

p_{ij} : steady state transition probability from state S_i to S_j.

$$p_{ij} = Q_{ij}(\infty)$$

$Z_i(t)$: probability that the system sojourn in state S_i up to epoch (t-1).

ψ_i : Mean sojourn time in state S_i.

$*, h$: symbol and dummy variable used in geometric transform e. g.

$$GT\left[q_{ij}(t)\right] = q_{ij}^*(h) = \sum_{t=0}^{\infty} h^t q_{ij}(t)$$

3.2 Symbols for the states of the system

N_o^i : unit-i is in normal mode(N) and operative; i=1,2

N_{ws}^i : unit-i is in normal mode(N) and warm standby.; i=1,2

F_R^2 / F_{wR}^2 : unit-2 is in total failure (F) mode and under replacement/waits for replacement.

F_r^1 / F_{2r}^1 : unit-1 is in total failure (F) mode and under repair.

The transition diagram of the system model is shown in Figure. 1.

TRANSITION DIAGRAM

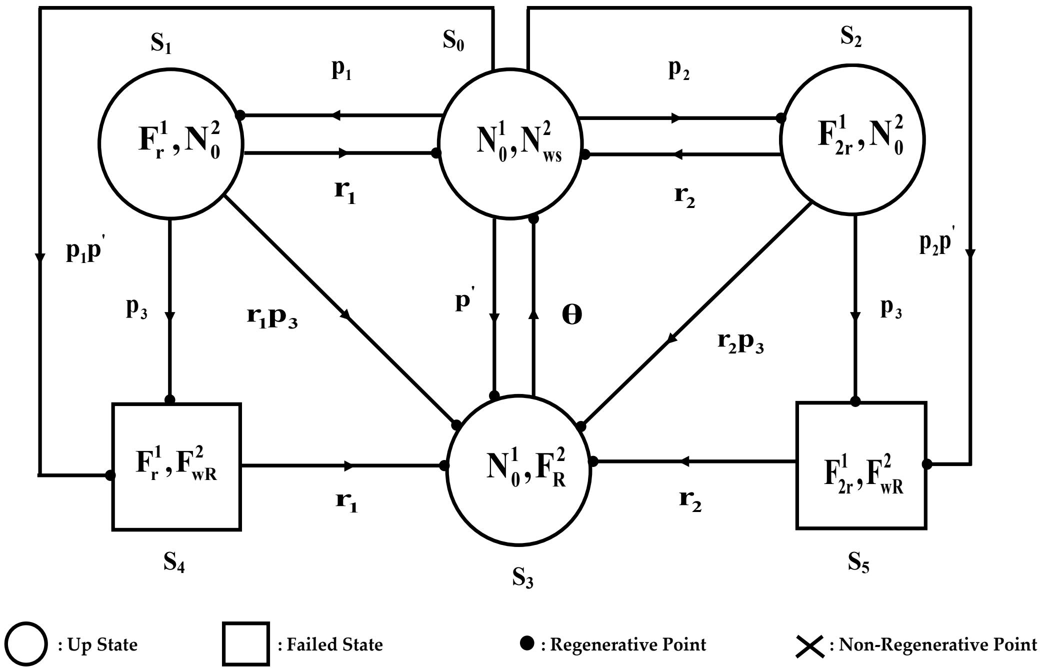

Figure.1

With the help of above symbols the possible states of the system are:

$S_0 \equiv \left(N_o^1, N_{ws}^2\right)$, $S_1 \equiv \left(F_r^1, N_o^2\right)$, $S_2 \equiv \left(F_{2r}^1, N_o^2\right)$

$S_3 \equiv \left(N_o^1, F_R^2\right)$, $S_4 \equiv \left(F_r^1, F_{wR}^2\right)$, $S_5 \equiv \left(F_{2r}^1, F_{wR}^2\right)$

The states S_0, S_1, S_2, S_3 are up states; S_4, S_5 are failed states.

4. Transition Probabilities and Sojourn Times

Let $Q_{ij}(t)$ be the probability that the system transits from state S_i to S_j during time interval (0, t) i.e., if T_{ij} is the transition time from state S_i to S_j then

$$Q_{ij}(t) = P\lfloor T_{ij} \le t \rfloor$$

By using simple probabilistic arguments we have,

$$Q_{01}(t) = \frac{p_1 q' q_2}{1 - q' q_1 q_2}\left[1 - \left(q' q_1 q_2\right)^{t+1}\right], \qquad Q_{02}(t) = \frac{p_2 q' q_1}{1 - q' q_1 q_2}\left[1 - \left(q' q_1 q_2\right)^{t+1}\right]$$

$$Q_{03}(t) = \frac{p' q_1 q_2}{1 - q' q_1 q_2}\left[1 - \left(q' q_1 q_2\right)^{t+1}\right], \qquad Q_{04}(t) = \frac{p' p_1 q_2}{1 - q' q_1 q_2}\left[1 - \left(q' q_1 q_2\right)^{t+1}\right]$$

$$Q_{05}(t) = \frac{p' p_2 q_1}{1 - q' q_1 q_2}\left[1 - \left(q' q_1 q_2\right)^{t+1}\right], \qquad Q_{10}(t) = \frac{r_1 q_3}{1 - s_1 q_3}\left[1 - \left(s_1 q_3\right)^{t+1}\right]$$

$$Q_{13}(t) = \frac{r_1 p_3}{1 - s_1 q_3}\left[1 - \left(s_1 q_3\right)^{t+1}\right], \qquad Q_{14}(t) = \frac{s_1 p_3}{1 - s_1 q_3}\left[1 - \left(s_1 q_3\right)^{t+1}\right]$$

$$Q_{20}(t)=\frac{r_2q_3}{1-s_2q_3}\left[1-(s_2q_3)^{t+1}\right], \qquad Q_{23}(t)=\frac{r_2p_3}{1-s_2q_3}\left[1-(s_2q_3)^{t+1}\right]$$

$$Q_{25}(t)=\frac{s_2p_3}{1-s_2q_3}\left[1-(s_2q_3)^{t+1}\right], \qquad Q_{30}(t)=\left\lfloor 1-(\bar{\theta})^{t+1}\right\rfloor$$

$$Q_{43}(t)=\left\lfloor 1-(s_1)^{t+1}\right\rfloor, \qquad Q_{53}(t)=\left\lfloor 1-(s_2)^{t+1}\right\rfloor \tag{1-14}$$

The steady state transition probabilities from state S_i to S_j can be obtained from (1-14) by taking t $\to\infty$, as follows:

$$p_{01}=\frac{p_1q^{'}q_2}{1-q^{'}q_1q_2}, \quad p_{02}=\frac{p_2q^{'}q_1}{1-q^{'}q_1q_2}, \quad p_{03}=\frac{p^{'}q_1q_2}{1-q^{'}q_1q_2}, \quad p_{04}=\frac{p^{'}p_1q_2}{1-q^{'}q_1q_2}$$

$$p_{05}=\frac{p'p_2q_1}{1-q^{'}q_1q_2}, \quad p_{10}=\frac{r_1q_3}{1-s_1q_3}, \quad p_{13}=\frac{r_1p_3}{1-s_1q_3}, \quad p_{14}=\frac{s_1p_3}{1-s_1q_3}$$

$$p_{20}=\frac{r_2q_3}{1-s_2q_3}, \quad p_{23}=\frac{r_2p_3}{1-s_2q_3}, \quad p_{25}=\frac{s_2p_3}{1-s_2q_3}$$

We observe that the following relations hold-

$$p_{30}=p_{43}=p_{53}=1, \qquad p_{01}+p_{02}+p_{03}+p_{04}+p_{05}=1$$

$$p_{10}+p_{13}+p_{14}=1, \qquad p_{20}+p_{23}+p_{25}=1, \tag{15-18}$$

5. Mean Sojourn Time

Let T_i be the sojourn time in state S_i (i=0-5) then ψ_i mean sojourn time in state S_i is given by

$$\psi_i=E(T_i)=\sum_{t=1}^{\infty}P\left[T_i\ge t-1\right]$$

In particular,

$$\psi_0=\frac{q^{'}q_1q_2}{1-q^{'}q_1q_2}, \quad \psi_1=\frac{s_1q_3}{1-s_1q_3}, \quad \psi_2=\frac{s_2q_3}{1-s_2q_3}$$

$$\psi_3=\frac{\bar{\theta}}{\theta}, \quad \psi_4=\frac{s_1}{r_1}, \quad \psi_5=\frac{s_2}{r_2} \tag{19-24}$$

6. Methodology for Developing Equations

In order to obtain various interesting measures of system effectiveness we developed the recurrence relations for reliability, availability and busy period of repairman as follows-

6.1 Reliability of the system

Here we define $R_i(t)$ as the probability that the system does not fail up to epochs 0, 1, 2,.., (t-1) when it is initially started from up state S_i . To determine it, we regard the failed states S_4, S_5 as absorbing state. Now, the expression for $R_i(t)$; i=0, 1, 2, 3; we have the following set of convolution equations.

$$R_0(t)=q^{'t}q_1^tq_2^t+\sum_{u=0}^{t-1}q_{01}(u)R_1(t-1-u)+\sum_{u=0}^{t-1}q_{02}(u)R_2(t-1-u)$$
$$=Z_0(t)+q_{01}(t-1)©R_1(t-1)+q_{02}(t-1)©R_2(t-1)+q_{03}(t-1)©R_3(t-1)$$

Similarly,

$$R_1(t) = Z_1(t) + q_{10}(t-1)©R_0(t-1) + q_{13}(t-1)©R_3(t-1)$$
$$R_2(t) = Z_2(t) + q_{20}(t-1)©R_0(t-1) + q_{23}(t-1)©R_3(t-1)$$
$$R_3(t) = Z_3(t) + q_{30}(t-1)©R_0(t-1) \tag{25-28}$$

Where,

$$Z_1(t) = s_1^t q_3^t, \qquad Z_2(t) = s_2^t q_3^t, \qquad Z_3(t) = q'^t$$

6.2 Availability of the System

Let $A_i(t)$ be the probability that the system is up at epoch (t-1), when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $A_i(t)$; i=0 to 5.

$$A_0(t) = q'^t q_1^t q_2^t + \sum_{u=0}^{t-1} q_{01}(u)©A_1(t-1-u) + \sum_{u=0}^{t-1} q_{02}(t-1)©A_2(t-1) + \sum_{u=0}^{t-1} q_{03}(t-1)©A_3(t-1)$$
$$+ \sum_{u=0}^{t-1} q_{04}(t-1)©A_4(t-1) + \sum_{u=0}^{t-1} q_{05}(t-1)©A_5(t-1)$$
$$= Z_0(t) + q_{01}(t-1)©A_1(t-1) + q_{02}(t-1)©A_2(t-1) + q_{03}(t-1)©A_3(t-1)$$
$$+ q_{04}(t-1)©A_4(t-1) + q_{05}(t-1)©A_5(t-1)$$

Similarly,

$$A_1(t) = Z_1(t) + q_{10}(t-1)©A_0(t-1) + q_{13}(t-1)©A_3(t-1) + q_{14}(t-1)©A_4(t-1)$$
$$A_2(t) = Z_2(t) + q_{20}(t-1)©A_0(t-1) + q_{23}(t-1)©A_3(t-1) + q_{25}(t-1)©A_5(t-1)$$
$$A_3(t) = Z_3(t) + q_{30}(t-1)©A_0(t-1)$$
$$A_4(t) = q_{43}(t-1)©A_3(t-1)$$
$$A_5(t) = q_{53}(t-1)©A_3(t-1) \tag{29-34}$$

Where the values of $Z_i(t)$; i=0 to 3 are same as given in section 6.1.

6.3 Busy Period of Repairman

Let $B_i^r(t)$ and $B_i^R(t)$ be the probability that the repairman is busy in the repair and replacement of a failed unit at epoch t-1, when it initially started from state S_i. Then, by using simple probabilistic arguments, as in case of reliability the following recurrence relations can be easily developed for $B_i^r(t)$ and $B_i^R(t)$; i=0 to 5.

$$B_0^r(t) = q_{01}(t-1)©B_1^r(t-1) + q_{02}(t-1)©B_2^r(t-1) + q_{03}(t-1)©B_3^r(t-1) + q_{04}(t-1)©B_4^r(t-1)$$
$$+ q_{05}(t-1)©B_5^r(t-1)$$
$$B_1^r(t) = Z_1(t) + q_{10}(t-1)©B_0^r(t-1) + q_{13}(t-1)©B_3^r(t-1) + q_{14}(t-1)©B_4^r(t-1)$$
$$B_2^r(t) = Z_2(t) + q_{20}(t-1)©B_0^r(t-1) + q_{23}(t-1)©B_3^r(t-1) + q_{25}(t-1)©B_5^r(t-1)$$
$$B_3^r(t) = q_{30}(t-1)©B_0^r(t-1)$$
$$B_4^r(t) = Z_4(t) + q_{43}(t-1)©B_3^r(t-1)$$
$$B_5^r(t) = Z_5(t) + q_{53}(t-1)©B_3^r(t-1) \tag{35-40}$$

Where,

The values of $Z_1(t)$ and $Z_2(t)$ are same as given in section 6.1, $Z_4(t)=s_1^t$ and $Z_5(t)=s_2^t$.

$$B_0^R(t)=q_{01}(t-1)©B_1^R(t-1)+q_{02}(t-1)©B_2^R(t-1)+q_{03}(t-1)©B_3^R(t-1)+q_{04}(t-1)©B_4^R(t-1)+q_{05}(t-1)©B_5^R(t-1)$$
$$B_1^R(t)=q_{10}(t-1)©B_0^R(t-1)+q_{13}(t-1)©B_3^R(t-1)+q_{14}(t-1)©B_4^R(t-1)$$
$$B_2^R(t)=q_{20}(t-1)©B_0^R(t-1)+q_{23}(t-1)©B_3^R(t-1)+q_{25}(t-1)©B_5^R(t-1)$$
$$B_3^R(t)=Z_3(t)+q_{30}(t-1)©B_0^R(t-1)$$
$$B_4^R(t)=q_{43}(t-1)©B_3^R(t-1)$$
$$B_5^R(t)=q_{53}(t-1)©B_3^R(t-1) \tag{41-46}$$

Where,

The value of $Z_3(t)$ is same as given in section 6.1.

7. Analysis of Reliability and MTSF

Taking geometric transform of (25-28) and simplifying the resulting set of algebraic equations for $R_0^*(h)$ we get

$$R_0^*(h)=\frac{N_1(h)}{D_1(h)} \tag{47}$$

Where,

$$N_1(h)=Z_0^*+hq_{01}^*Z_1^*+hq_{02}^*Z_2^*+\left\lfloor h^2q_{01}^*q_{13}^*+h^2q_{02}^*q_{23}^*+hq_{03}^*\right\rfloor Z_3^*$$
$$D_1(h)=1-h^2q_{01}^*q_{10}^*-h^2q_{02}^*q_{20}^*-h^2q_{03}^*q_{30}^*-h^3q_{01}^*q_{13}^*q_{30}^*-h^3q_{02}^*q_{23}^*q_{30}^*$$

Collecting the coefficient of h^t from expression (47), we can get the reliability of the system $R_0(t)$. The MTSF is given by-

$$E(T)=\lim_{h\to1}\sum_{t=1}^{\infty}h^tR(t)=\frac{N_1(1)}{D_1(1)}-1 \tag{48}$$

Where,

$$N_1(1)=\psi_0+p_{01}\psi_1+p_{02}\psi_2+\left[p_{01}p_{13}+p_{02}p_{23}+p_{03}\right]\psi_3$$
$$D_1(1)=1-p_{01}p_{10}-p_{02}p_{20}-p_{03}-p_{01}p_{13}-p_{02}p_{23}$$

8. Availability Analysis

On taking geometric transform of (29-34) and simplifying the resulting equations for we get,

$$A_0^*(h)=\frac{N_2(h)}{D_2(h)} \tag{49}$$

Where,

$$N_2(h)=\begin{vmatrix} Z_0^* & -hq_{01}^* & -hq_{02}^* & -hq_{03}^* & -hq_{04}^* & -hq_{05}^* \\ Z_1^* & 1 & 0 & -hq_{13}^* & -hq_{14}^* & 0 \\ Z_2^* & 0 & 1 & -hq_{23}^* & 0 & -hq_{25}^* \\ Z_3^* & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -hq_{43}^* & 1 & 0 \\ 0 & 0 & 0 & -hq_{53}^* & 0 & 1 \end{vmatrix}$$

and

$$D_2(h) = \begin{vmatrix} 1 & -hq_{01}^* & -hq_{02}^* & -hq_{03}^* & -hq_{04}^* & -hq_{05}^* \\ -hq_{10}^* & 1 & 0 & -hq_{13}^* & -hq_{14}^* & 0 \\ -hq_{20}^* & 0 & 1 & -hq_{23}^* & 0 & -hq_{25}^* \\ -hq_{30}^* & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -hq_{43}^* & 1 & 0 \\ 0 & 0 & 0 & -hq_{53}^* & 0 & 1 \end{vmatrix}$$

The steady state availabilities of the system due to operation of unit –

$$A_0 = \lim_{t\to\infty} A_0(t) = \lim_{h\to 1}(1-h)\frac{N_2(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$A_0 = -\frac{N_2(1)}{D_2'(1)} \tag{50}$$

Where,

$$N_2(1) = \psi_0 + p_{01}\psi_1 + p_{02}\psi_2 + [p_{01}p_{13} + p_{01}p_{14} + p_{02}p_{23} + p_{02}p_{25} + p_{03} + p_{04} + p_{05}]\psi_3$$

and

$$D_2'(1) = \psi_0 + p_{01}\psi_1 + p_{02}\psi_2$$

Now the expected uptime of the system due to operative unit upto epoch (t-1) are given by

$$\mu_{up}(t) = \sum_{x=0}^{t-1} A_0(x)$$

So that

$$\mu_{up}^*(h) = \frac{A_0^*(h)}{(1-h)} \tag{51}$$

9. Busy Period Analysis

On taking geometric transforms of (35-40) and (41-46), simplifying the resulting equations, we get

$$B_0^{r*}(h) = \frac{N_3(h)}{D_2(h)} \quad \text{and} \quad B_0^{R*}(h) = \frac{N_4(h)}{D_2(h)} \tag{52-53}$$

Where,

$$N_3(h) = Z_1^* hq_{01}^* + Z_2^* hq_{02}^* + Z_4^* h^2 q_{01}^* q_{14}^* + Z_4^* hq_{04}^* + Z_5^* h^2 q_{02}^* q_{25}^* + Z_5^* hq_{05}^*$$

and

$$N_4(h) = Z_3^* \left\lfloor h^2 q_{01}^* q_{13}^* + h^3 q_{01}^* q_{14}^* q_{43}^* + h^2 q_{02}^* q_{23}^* + h^3 q_{02}^* q_{25}^* q_{53}^* + hq_{03}^* + h^2 q_{04}^* q_{43}^* + h^2 q_{05}^* q_{53}^* \right\rfloor$$

and $D_2(h)$ is same as in availability analysis.

In the long run the respective probabilities that the repairman is busy in the repair and replacement of a failed unit are given by-

$$B_0^r = \lim_{t\to\infty} B_o^r(t) = \lim_{h\to 1}(1-h)\frac{N_3(h)}{D_2(h)} \quad \text{and} \quad B_0^R = \lim_{t\to\infty} B_o^R(t) = \lim_{h\to 1}(1-h)\frac{N_4(h)}{D_2(h)}$$

But $D_2(h)$ at h=1 is zero, therefore by applying L. Hospital rule, we get

$$B_0^r = -\frac{N_3(1)}{D_2'(1)} \quad \text{and} \quad B_0^R = -\frac{N_4(1)}{D_2'(1)} \tag{54-55}$$

Where,

$$N_3(1) = p_{01}\psi_1 + p_{02}\psi_2 + (p_{01}p_{14} + p_{04})\psi_4 + (p_{02}p_{25} + p_{05})\psi_5$$

and

$$N_4(1) = \psi_3\left[p_{01}p_{13} + p_{01}p_{14} + p_{02}p_{23} + p_{02}p_{25} + p_{03} + p_{04} + p_{05}\right]$$

and $D_2'(1)$ is same as in availability analysis.

Now the expected busy period of the repairman in repair of a failed unit up to epoch (t-1) are respectively given by-

$$\mu_b^r(t) = \sum_{x=0}^{t-1} B_0^r(x) \qquad \text{and} \qquad \mu_b^R(t) = \sum_{x=0}^{t-1} B_0^R(x) \tag{56-57}$$

10. Profit Function Analysis

We are now in the position to obtain the net expected profit incurred up to epoch (t-1) by considering the characteristics obtained in earlier section. Let us consider,

$K_0 =$ revenue per-unit time by the system due to operative unit.

$K_1 =$ cost per-unit time when repairman is busy in the repair of failed unit.

$K_2 =$ cost per-unit time when repairman is busy in the replacement of a failed unit.

Then, the net expected profit incurred up to epoch (t-1) is given by,

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^r(t) - K_2\mu_b^R(t) \tag{58}$$

The expected profit per unit time in steady state is given by-

$$\begin{aligned} P &= \lim_{t\to\infty}\frac{P(t)}{t} = \lim_{h\to 1}(1-h)^2 P^*(h) \\ &= K_0 \lim_{h\to 1}(1-h)^2\frac{A_0^*(h)}{(1-h)} - K_1 \lim_{h\to 1}(1-h)^2\frac{B_0^{r*}(h)}{(1-h)} - K_2 \lim_{h\to 1}(1-h)^2\frac{B_0^{R*}(h)}{(1-h)} \\ &= K_0A_0 - K_1B_0^r - K_2B_0^R \end{aligned} \tag{59}$$

11. Graphical Representation

The curves for MTSF and profit function have been drown for different values of failure parameters. Fig.2 depicts the variation in MTSF with respect to failure rate (p_1) for different values of repair rate (p_2) of a unit and constant repair rate $(p^{'})$ when values of other parameters are kept fixed as $p_3 = 0.001$, $r_1 = 0.5$, $r_2 = 0.7$ and $\theta = 0.01$. From the curves we conclude that expected life of the system decrease with increase in p_1. Further, increases as the values of p_2 and $p^{'}$ increases.

Similarly, Fig.3 reveals the variations in profit (P) with respect to p_1 for varying values of p_2 and $p^{'}$, when other parameters are kept fixed as $p_3 = 0.01$, $r_1 = 0.92$, $r_2 = 0.99$ and $\theta = 0.01$, $K_0 = 100$, $K_1 = 100$, $K_2 = 400$ and $K_3 = 300$. From the figure it is clearly observed from the smooth curves, that the system is profitable if the value of parameter p_1 is greater than 0.2, 0.33 and 0.5 respectively for $p_2 = 0.4$, 0.6 and 0.8 for fixed value of $p^{'} = 0.15$. From dotted curves, we conclude that system is profitable only if value of parameter p_1 is greater than 0.27, 0.39 and 0.6 respectively for $p_2 = 0.4$, 0.6 and 0.8 for fixed value of $p^{'} = 0.3$.

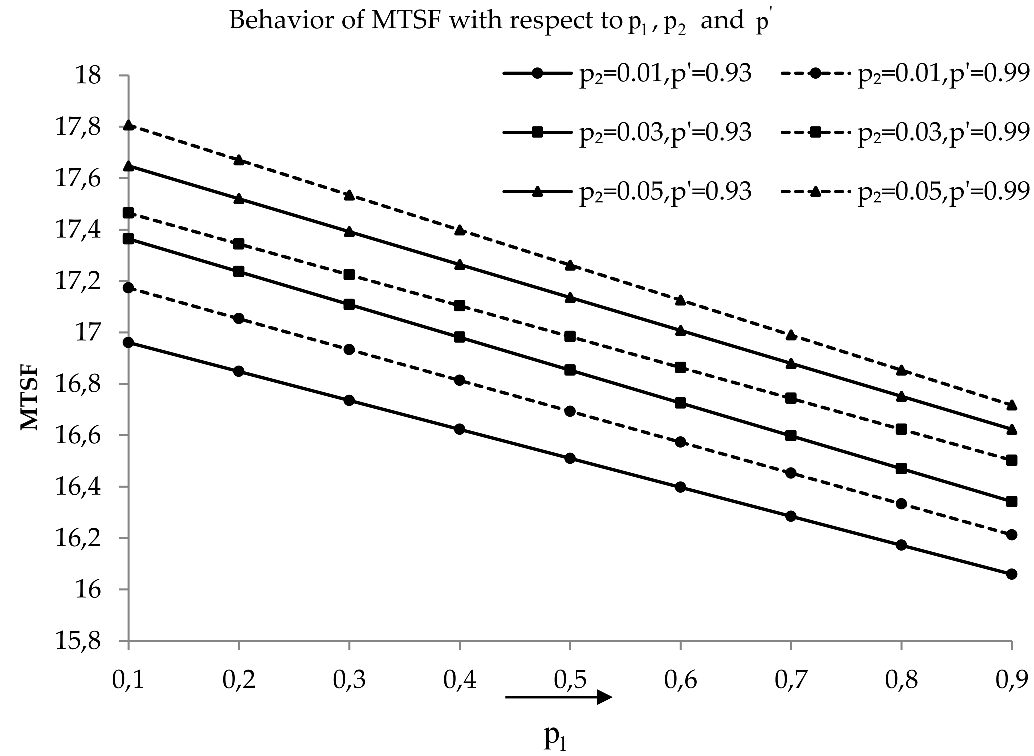

Figure.2

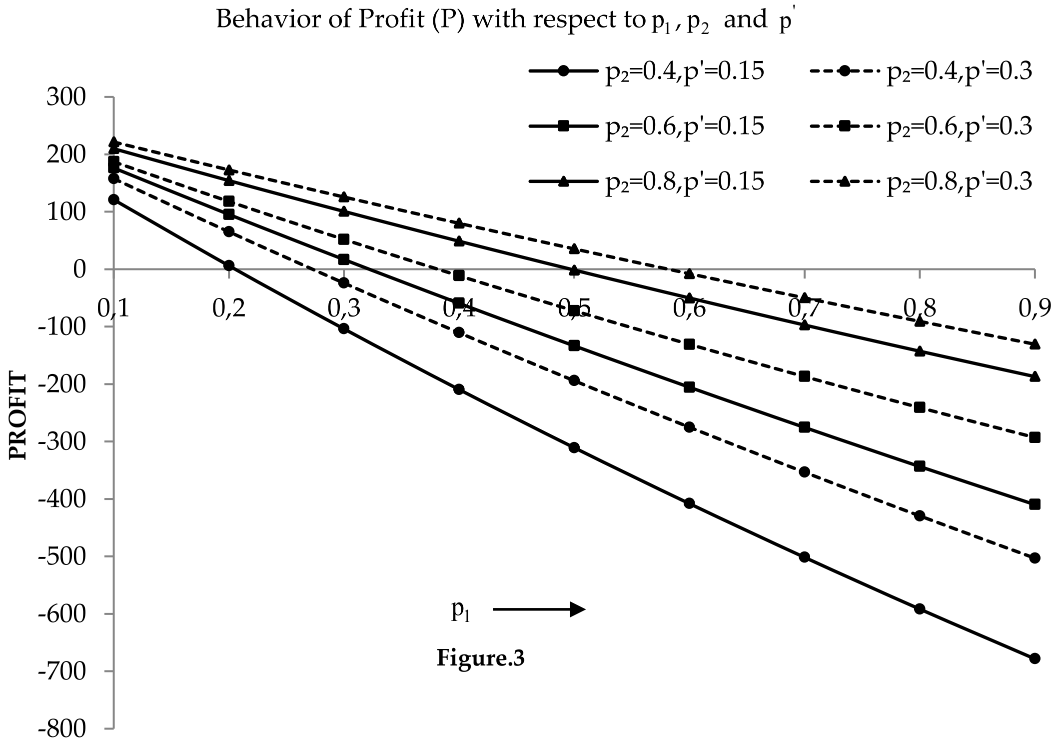

Figure.3

12. Conclusions

1. It is indicated in fig.2 that we can easily obtain the upper limit of "p_1" to achieve at least a particular value of MTSF. As an illustration to get at least MTSF 16.8 unit, the failure rate "p_1" must be less than 0.24, 0.56 and 0.79 respectively for repair rate $p_2 = 0.01$, 0.03 and 0.05 when

activation rate is kept fixed as $p' = 0.93$. Similarly, when $p' = 0.99$ is kept fixed as "p_1" must be less than 0.43, 0.68 and 0.87 corresponding to $p_2 = 0.01$, 0.03 and 0.05.

2. In fig. 3 it is reveled from the smooth curves, that the system is profitable if the value of parameter p_1 is greater than 0.2, 0.33 and 0.5 respectively for $p_2 = 0.4$, 0.6 and 0.8 for fixed value of $p' = 0.15$. From dotted curves, we conclude that system is profitable only if value of parameter p_1 is greater than 0.27, 0.39 and 0.6 respectively for $p_2 = 0.4$, 0.6 and 0.8 for fixed value of $p' = 0.3$.

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