Random processes imitation in fatigue studies

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Abstract

Modelling random processes traditionally supposes working with the spectral density. Although many engineering problems require the knowledge of spectral density, the specific character of fatigue damage accumulation dictates the different approach – namely, the consideration of the distribution of random values of the local extremes, which is responsible for fatigue damage accumulation. There is a need in developing the methods of random loading imitation in the experimental and numerical study of fatigue. According to the up-to-date situation in science in fatigue, both opposing approaches should be considered - the time domain and the frequency domain. The proposed method, which consists of two stages, meets that requirement. The performed case study based on laboratory fatigue testing confirms its applicability.

Keywords: metal fatigue, random loading, imitation,

1. Introduction

The quality of machines and equipment depends on the advanced quality management. The repair plan is also important [1]. The reliability of the industrial production should be guaranteed by reliable testing and design methods for estimation durability. Objective hazards that threaten the performance and durability of machine parts are the processes of degradation of their elements. It is necessary to consider the fatigue process caused by natural factors, namely, alternating loading. Therefore, the engineers need a tool for loads assessing, taking into account their random nature.

In fatigue studies under random loading, two main approaches are widely used [2]. They correspond to the time domain and the frequency domain. Both have their own areas of applications. Investigations in the frequency domain [3] are important while considering problems with studying the impacts of the particular frequencies (resonant effect), modal analysis. They are also important while treating the enormous data storage while using the method of Critical Plane Approach [4]. On the other hand, the well-proved fatigue accumulation problems mostly based on the information about cycles, their extremum values, their order of appearance. It is worth mentioning that the problem of the fatigue crack propagation almost ultimately based on information about extremums (not the frequencies) [5]. Applicable to automobile parts, the study [6belec] consider the random loading strictly in a time domain. A discussion goes on [7,8], which of the two methods is correct: the time-domain methods (mostly, Rainflow [9 endo]) or the frequency domain [3]. It goes out that choosing the particular domain for investigation dictates some additional requirements [10]. Registration of the random loading process is different [11]. It follows from the fact that treating in the time domain supposes the higher precision of peaks registration.

It is important to have a tool to estimate the spectral density of the processes, which are given only by a sequence of extrema. The example of such task is in paper [10]. The aim of this paper to provide such opportunity for the researchers.

2. Methods

This problem partly was first studied in [12]. During testing and numerical modelling in fatigue studies, the investigators should introduce the randomness in one or another way because it exists in service. The conditions of exploitation vary. Some factors are hard to control, etc. The proposed method consists of two steps, namely:

2.1. Random sequence of extremums generation

As mentioned in the Introduction, the most direct path to fatigue estimation is the Rainflow [2, 9], which operates with the extremums' sequence, namely local maximums and minimums of the random process. It would not be justified to repeat this sequence without change in fatigue studies because it would not have reflected the randomness in service and takes a lot of time. The good decision was proposed earlier in [13]. It introduced the so-called Markov's matrixes to create the variability.

Later, the method of target Markov's matrixes was developed, intended to reflect the service conditions of a particular object of investigation. The main idea of filling up the matrix is schematically presented in Fig.1. One by one, the half-cycles (ranges) are entered into the Markov's matrix (Fig.1, b). All important information is being presented in this way, namely, the maximum amplitude in realization, distributions of the values of the half-cycles, the number of their repetition during the period of investigation. The information concerning the sequence of the events is being lost during this procedure. The investigation of the impact of the sequence effect on the crack-propagation stage was reported in [14].

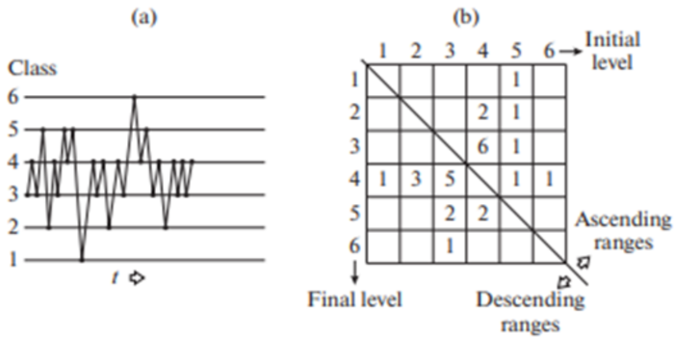

Figure 1: *Filling up the target matrix*

Next, the numerical modelling basing on the filled up earlier matrix is performed. The random number generator is used here. In this way, the engineers get the tool for the experimental or numerical study of fatigue by taking into account the random character of the loading, which is intrinsic to loading in service. As the result of this modelling, the investigators get the sequence of extremums. This sequence is sufficient for estimating the longevity of the objects using the time domain loading approach like Rainflow, [2,9].

2.2. Construction of the continuous function

Unfortunately, it is impossible to estimate the spectral density only the sequence of extremums generated by described above method. The function does not possess the property of smoothness; its firsts derivate is not continuous. It is worth mentioning that there is a need to estimate spectral density [10] of the modelled processes, particularly for the application of spectral methods (frequency domain) for longevity estimation. Although those methods are at some extend doubtful, they are still widespread [3]. As we mentioned above, they also have their own field for application: like des [4].

To overcome this problem, a method for introducing smoothness into the process was developed [12]. The adjacent peaks of the sequence are proposed to connect by half-cosines at the period (0,π). At the next step these half-cosines parts are concatenated.

The equations of half-cosines:

$$x(t) = A \cos (wt+\varphi) \quad (1)$$

where x(t)– is the part of the continuous extrapolating function, which is defined on the domain t=0… π/w, because the period of the cosine function is T=2π/w, s.

For each half-wave (1) starting from the successive extremum *MAX* or *MIN*, the parameters *A, w*, and φ are unique. The stress amplitude *A* [MPa] is defined as half of the range (modulus) of successive extremes:

$$A = mod\,(MAX_i - MIN_{i+1})/2 \ \vee\ A = mod\,(MAX_i - MIN_i)/2 \quad (2)$$

The obtained in this way sequence of the random reading forms the continuous random process with continuous first derivate. The concordances and peculiarities of the modelled processes will be analyzed later in the Case study. The main idea of this modelling – that is the values of the turning points, and their sequence remains unchanged. This point is paramount for fatigue estimation not only on the stage of crack initiation in fatigue but also during the crack propagation stage [5].

3. Case study

Following an engineering problem, the task was initially formulated to investigate the fatigue resistance of the metal specimens under the impact of the random process with the particular spectral density, shown in Fig. 4 [15]:

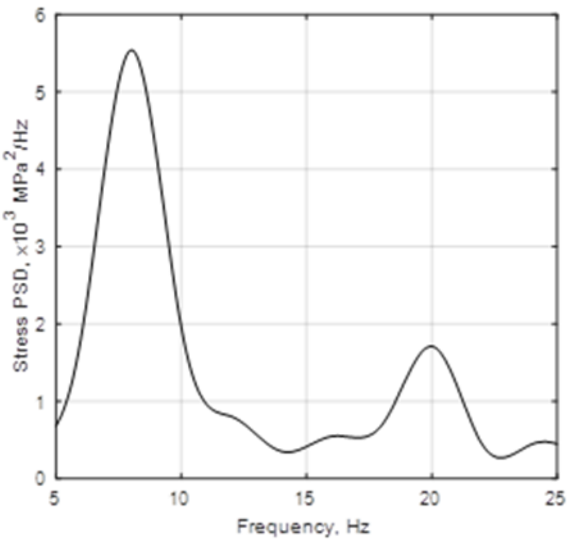

Figure 2: *Target spectral density for testing*

In Fig.3 the testing equipment is shown. Six Al specimens were tested simultaneously.

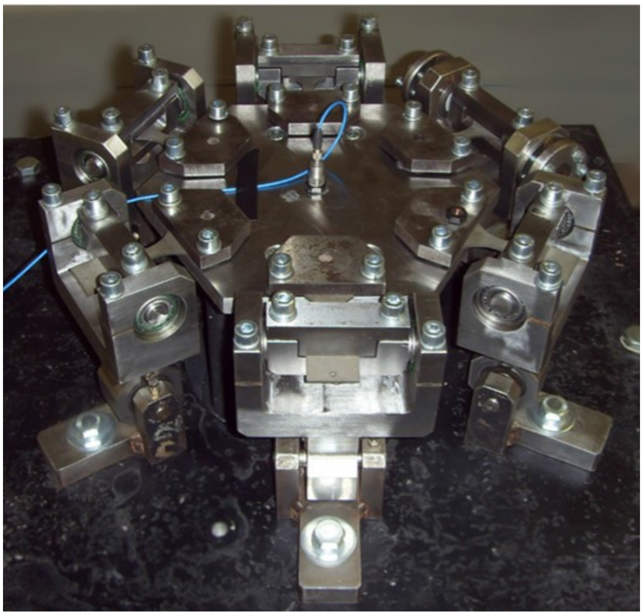

Figure 3. Testing equipment for regular and random cantilever loading

Fatigue experiment [15] was performed on 4 levels of loading to build the so-called Gassner curve (see also [7]). The example of the loading history is shown in Fig.5.

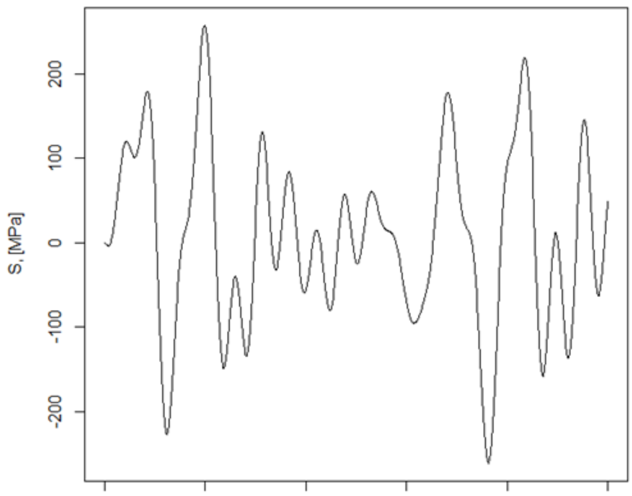

Figure 4. *1-second recording of the stress at the level of root mean square RMS=108 MPa.*

The main characteristics of the imitated random processes were obtained numerically and are shown in Table 1. With the aim of the study, the modelling was based on the laboratory records. For the sake of representativity several modelled trials on the base of laboratory realization were executed. In Table 1 the mean stress of the block is shown, RMS (the root mean square of realization), *I* – is irregularity factor [2] (*I*=*No*/*Ne*, *I*<1; where *No* is the number of crossings of the middle level line and *Ne* is the number of extremums. It is worth mentioning, that value *I* depends on the level number, in other words, on the registration precision. Spectra fullness *V* is defined as follows:

$$V = \sqrt[m]{\frac{1}{n}\sum h_i (\frac{\sigma_{ai}}{\sigma_{amax}})^{\wedge m}} \tag{3}$$

The value of V < 1 and is dimensionless. In formula (3) *m* is the slope coefficient of the fatigue curve; *n* is the total number of cycles in the block; h_i is the number of cycles at the *i*-th step; σ_{ai} is the current value of the stress amplitude; σ_{amax} is the maximum amplitude in the block. As can be seen from the formula, the fullness ratio of the spectrum *V* depends not only on the spectra form, but also on fatigue exponent *m*.

Table 1: *The main characteristics of random realizations: initial and modelled ones*

	Mean value [MPa]	RMS, [MPa]	*I*	*V*
Experimental realization	0.0244	108	0.67	0.56
1-st modelled realization	-0.0123	107	0.66	0.53
2-nd	-0.0056	99	0.68	0.55
…				
10-th	0.0342	104	0.66	0.57

Unlike the widespread practice of ignoring the time factor during the cycle counting, in this study, following the aim of the investigation, not only the peaks were selected, but also the half-periods of quasi-cycles.

According to the proposed method, each stress range: *r* [MPa] is associated with the following half period: *hT* [s]. The scatterplot of two random variables is shown in Fig. 6. The estimated correlation was cor=0.76. Also, the regression equation by the least square method was estimated:

$$r = -161+10.77\, hT \tag{4}$$

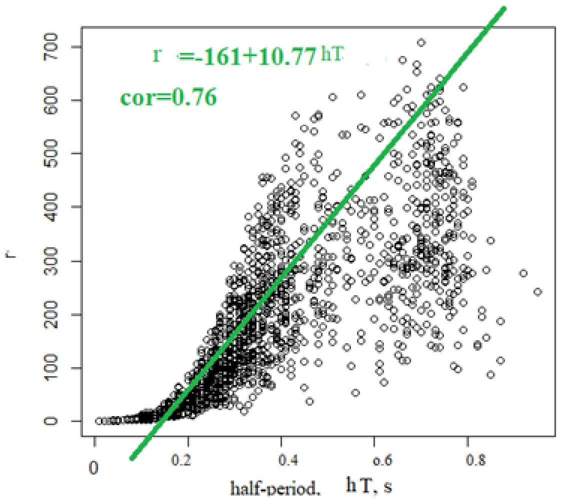

Figure 5. *The Scatterplot of half-periods hT and ranges r (with a regression line).*

The regression equation (3) was used for modelling the cosine curves in extrapolation. Unlike equation (1), in which the frequency is assumed to be the same for all half-cycles, a variable frequency value is used to form a more realistic process. To determine the frequency, the regression equation (3) was used. Equation (4) was used during modelling.

4. Results and discussion

After performing the tests with sufficient repetition of specimens at each loading level, the Gassner curve has been built [7,15]. This curve is analogous to the fatigue curve, but instead of the constant amplitudes, a maximum value in the block is shown. The realization of the random process, modelled on the base of spectral density (Fig. 4), was recorded, and the Rainflow cycle counting procedure [9] was performed.

The main statistical characteristics of a few modelled realizations were shown in Table 1. The spectral density of the modelled realization #1 is slightly differ from target spectral density. Anyway, the first and second frequencies coincide.

A much better coincidence is shown for Rainflow distributions. Several replicas were compared among each other as well with the distribution of initial realization. The good coincidence of parameters is also evident from Table 1.

5. CONCLUSIONS

Machine parts would be deteriorated due to fatigue produced by repeated loading, so the engineers should consider the fatigue impact.

Those impacts are of random nature, so the special imitation method was developed. This method works in time, as well in the frequency domain.

The method was approbated in the Case study based during the laboratory fatigue experiment under random loading.

The better coincidence takes place in the time domain. To improve modelling in a way it works equally well in both domain, addition experiment and researches are needed.

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