A NOVAL APPLICATION OF DUANE PROCESS FOR MODELING TWO GRADED MANPOWER SYSTEM WITH DIRECT RECRUITMENT IN BOTH THE GRADES

Ch. Ganapathi Swamy[1], K. Srinivasa Rao [2]

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[1]Department of Community Medicine, GSL Medical College, Rajahmundry, Andhra Pradesh, India. Email: ganesh051981@gmail.com
[2]Dept.of statistics, Andhra University, Visakhapatnam, Andhra Pradesh, India. Email: ksraoau@yahoo.co.in

Abstract

Human Resource Management and other companies rely heavily on manpower models. Manpower planning was a prerequisite for effective organization administration. The construction and analysis of two graded manpower models with direct Duane recruiting processes in both graduates is the subject of this paper. Duane's recruitment procedure was capable of identifying time-dependent recruitments. Poisson and non-homogeneous Poisson processes are used in the Duane recruitment process as precise instances for specified parameter values. It is assumed that the organization has two grades and that the recruitment procedure is based on the Duane Process. The processes of leaving and promotion are Poisson processes. The model's transient behavior was investigated by deriving unambiguous expressions for system characteristics such as the mean number of employees in each grade, the mean durational stay of an employee in each grade, and the variance of number of employees in each grade using differential equations. The model's sensitivity analysis of parameters shows that the Duane recruiting process has a substantial impact on system performance indicators. It was also noted that this model incorporates rates of recruitment that are increasing, decreasing, or stable. This model proved helpful in analyzing organizational manpower issues.

Keywords - Two graded Manpower model, Duane process, Time dependent recruitment rate, Sensitivity analysis.

I. Introduction

Planning the organization with the manpower structure in mind was a prerequisite for getting the most out of the resources. Due to their usefulness in creating strategies for Human Resource Development and resource allocation, much work has been reported on manpower models. Seal was a pioneer in the mathematical modelling of labour systems [1]. Silock looked at the observable fact of labour yield, which is related to the study of demography [2]. Bartholomew examined manpower models based on probability distributions of an employee's total service time in the organization [3] [4]. Ugwuowo and Mc Clean, as well as Wang, have examined manpower models and various approaches to their development [5] [6].

The parameters of the manpower model, such as the mean number of employees in each grade, the mean duration of stay an employee in each grade, and the variation of number of employees in each grade, were required for effective analysis and design of manpower systems Srinivasa Rao [7]. Kannan Nilakantan investigated the manpower models staffing rules and their extension to individual outsourcing [8]. Jeeva and Geetha looked at manpower models in a hazy environment [9]. Lalithadevi and Srinivasan used geometric process and shock models to investigate a single graded manpower system [10]. Osagiede and Ekhosuehi used continuous-time Markov chains and sparse stochastic measures to investigate Manpower models [11].

The graded manpower models with poisson processes have been examined by Srinivasa Rao, K. et al., Kondababu et al., and Govinda Rao et al. They implied that the hiring procedure was time-sensitive [12] [13] 14] [15] [16]. Parameswari, K., and Srinivasan [17] used a geometric technique to investigate the reduction in manpower for a two-graded system. Amudha.T and Srinivasan.A, discussed the problem of time to recruitment for a two-graded system, taking into account the loss of personnel in the form of an I.I.D Exponential random variable sequence [18]. Saral, L. et al. established a two-tiered personnel structure and a recruiting policy based on two thresholds [19]. Srividhya, K. et al. investigated manpower loss in a multi-graded organization [20]. Jayanthi et al. (2018) looked at a single graded manpower system and looked at the time to recruitment with a break-even point [21]. Tamas Banyai et al. used Markov chains to study a model for analyzing human resource use [22]. Arokkia Saibe,P et al. investigated two stochastic models based on the assumption that manpower shortages and inter-policy decision delays constitute two distinct sequences of independent and identically distributed random variables with two distinct breakdown thresholds [23]. They assumed that the hiring procedure was time-sensitive.

However, in many real-world circumstances in corporate offices and government agencies, the recruitment process was time-sensitive and did not necessarily follow the Poisson process. As a result, non stationary models must be considered for correct analysis. Srinivasa Rao et al. [24] recently developed two graded manpower models based on non-homogeneous Poisson recruitment. Srinivasa Rao,K et al. [25] have studied on two grade manpower model with Duane recruitment process. They realized that the recruitment rate was linearly proportional to time and that the duration between recruitment was distributed in an exponential manner. However, the recruiting rate in many organizations may increase/decrease/remain steady and time-dependent. The time-dependent non-stationary recruitment process can be fully characterized by the Duane process, which follows a Weibull distribution of inter-recruitment time. Little is reported in the literature on two hierarchical workforce models that use the Duane recruitment process directly at both grades. Therefore, this paper uses the Duane recruitment process of both graduates to develop and analyze a two-step manpower model. Poisson and non-homogeneous processes are two examples of the Duane process. The concept can be applied to a variety of organizations thanks to the recruitment in both grades. The remainder of the paper was laid out as follows:

II. Two graded manpower model with direct recruitment:

Consider a personnel system with two grades, each of which has its own recruitment process. The grade I recruiting process was supposed to be a Duane process, with the mean recruitment rate being a power function of time t and the form $\lambda_1(t) = a_1 b_1 t^{b_1 - 1}$. The grade II recruiting process was considered to be a Duane process with a mean recruitment rate of $\lambda_2(t) = a_2 b_2 t^{b_2 - 1}$. A Poisson process with parameter is used to promote students from grade I to grade II. Poisson processes with parameters and are used in the grade I and grade II leaving processes, respectively. "Figure 1" depicted a schematic diagram depicting the two-grade manpower concept.

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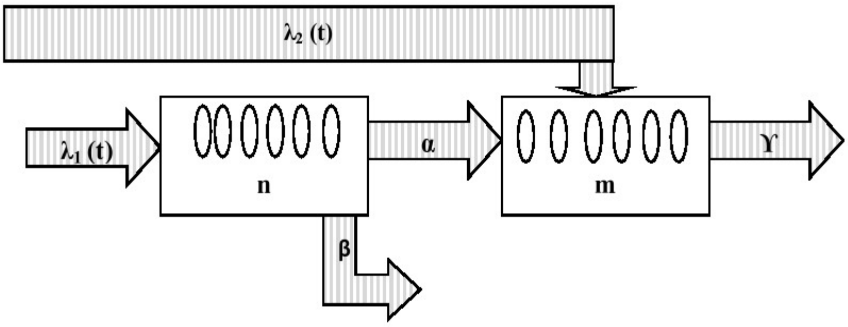

Figure 1: *Two grade manpower model with direct Duane recruitment process.*

Let $P_{n,m}(t)$ denote the probability that there are 'n' employees in grade-1 and 'm' employees in grade-2 at time 't' in the organization. Then the difference-differential equations of the model are

$$\frac{\partial Pn,m(t)}{\partial t} = -[\lambda_1(t) + \lambda_2(t) + n\alpha + n\beta + m\gamma]P_{n,m}(t) + [\lambda_1(t)]P_{n-1,m}(t)$$

$$+[\lambda_2(t)]P_{n,m-1}(t) + \alpha(n+1)P_{n+1,m-1}(t) \ +\beta(n+1)P_{n+1,m}(t) \ +(m+1)\gamma P_{n,m+1}(t) \quad (1)$$

$$\frac{\partial Pn,0(t)}{\partial t} = -[\lambda_1(t) + \lambda_2(t) + n\alpha + n\beta]P_{n,0}(t) + [\lambda_1(t)]P_{n-1,0}(t) + \beta(n+1)P_{n+1,0}(t) + \gamma P_{n,1}(t) \quad (2)$$

$$\frac{\partial P0,m(t)}{\partial t} = -[\lambda_1(t) + \lambda_2(t) + +m\gamma]P_{0,m}(t) + [\lambda_2(t)]P_{0,m-1}(t) + \alpha P_{1,m-1}(t) + \ P_{1,m}(t) + (m+1)\gamma P_{0,m+1}(t) \quad (3)$$

$$\frac{\partial P0,0(t)}{\partial t} = -[\lambda_1(t) + \lambda_2(t)]P_{0,0}(t) + \beta P_{1,0}(t) + (m+1)\gamma P_{0,1}(t) \quad (4)$$

Let $P(S_1,S_2; t)$ be the joint probability generating function then

$$P(S_1, S_2;\ t) = \sum_n \sum_m S_1^n S_2^m \, Pn, m\ (t) \quad (5)$$

Multiplying the equations (1) to (4) with corresponding $S_1^n S_2^m$ and summing over all n=0, 1, 2,…;

m= 0,1,2,…; we get

$$\frac{\partial P}{\partial t} = [\alpha(S_2 - S_1) + \beta(1 - S_1)]\frac{\partial P}{\partial S_1} + \gamma[1 - S_2]\frac{\partial P}{\partial S_2} - [\lambda_1(t)(1 - S_1) \ +\lambda_2(t)(1 - S_2)]P$$

$$\frac{\partial P}{\partial t} - [\alpha(S_2 - S_1) + \beta(1 - S_1)]\frac{\partial P}{\partial S_1} - \gamma[1 - S_2]\frac{\partial P}{\partial S_2} = [\lambda_1(t)(S_1 - 1) + \lambda_2(t)(S_2 - 1)]P \quad (6)$$

After simplification, we get

$$\frac{\partial P}{\partial t} + [\alpha(S_1 - S_2) + \beta(S_2 - 1)]\frac{\partial P}{\partial S_1} + \gamma[S_2 - 1]\frac{\partial P}{\partial S_2} = [\lambda_1(t)(S_1 - 1) \ + \lambda_2(t)(S_2 - 1)]P \quad (7)$$

Solving the equation (7) by Lagrangian's method, the auxiliary equations are

$$\frac{\partial t}{1} = \frac{\partial S_1}{\alpha(S_1 - S_2) + \beta(S_2 - 1)} = \frac{\partial S_2}{\gamma(S_2 - 1)} = \frac{\partial P}{[\lambda_1(t)(S_1 - 1) + \lambda_2(t)(S_2 - 1)]P} \quad (8)$$

With the initial conditions that there are N employees in grade-1 and M employees in grade-2 in the organization at time t=0. i.e., $P_{N,M}(0) = 1 \; and \; P_{0,0}(t) = 0$ for t > 0.

To solve the equation (8) the functional forms of $\lambda_1(t)$ and $\lambda_2(t)$ are required.

Since the recruitment processes follow Duane processes we have the mean recruitment rates of grade-1and grade-2 in the system are $\lambda_1(t) = a_1 b_1 t^{b1-1}$ and $\lambda_1(t) = a_2 b_2 t^{b2-1}$ respectively, where $\lambda>0$, a_1, b1, a_2 and b_2 are constants

Solve the equations in (8) we get

$$A = (S_2 - 1)e^{-\gamma t}$$

$$B = e^{-(\alpha+\beta)t}\left[(S_1 - 1) + \frac{\alpha(S_2 - 1)}{\gamma - (\alpha+\beta)}\right]$$

$$C = S_1^N S_2^M \left\{\frac{-a_1 b_1\, e^{-(\alpha+\beta)t}\,(s_1 - 1)}{[\alpha+\beta]} - \frac{a_1 b_1\ \alpha\, e^{-(\alpha+\beta)t}\,(s_2 - 1)}{[\gamma - (\alpha+\beta)][\alpha+\beta]} + \frac{a_1 b_1\ \alpha\, e^{-\gamma t}\,(s_2 - 1)}{[\gamma - (\alpha+\beta)][\gamma]} - \frac{a_2 b_2\, e^{-\gamma t}\,(s_2 - 1)}{\gamma}\right\}$$

where $S_1^N = \left[1 - (1 - S_1)e^{-(\alpha+\beta)t} - \frac{\alpha(1-S_2)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N$, $S_2^M = [1 - (1 - S_2)e^{-\gamma t}]^M$.

A,B and C are arbitrary constants. (9)

The joint probability generating function of the number of employee in grade-1 and in grade-2 is

$$P(S_1, S_2;\, t) = C.\text{Exp}\left\{a_1 b_1 (S_1 - 1)\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv\right] + \frac{a_1 b_1\,\alpha\,(S_2-1)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_1-1)}dv\right] + a_2 b_2 (S_2 - 1)\left[e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_2-1)}dv\right]\right\} \tag{10}$$

Substituting the value of 'C' from equation (9) in the equation (10), the joint probability generating function of the number of employees in the grade-1 and grade-2 are obtained as

$$P(S_1, S_2;\, t) = \left\{\exp\left\{a_1 b_1 (S_1 - 1)\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + \frac{a_1 b_1\,\alpha\,(S_2-1)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_1-1)}dv\right] + \frac{a_1 b_1\,\alpha\,(S_2-1)}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2 (S_2 - 1)\left[e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_2-1)}dv - \frac{e^{-\gamma t}}{\gamma}\right]\right\}[XY]\right\}$$

Where $X = S_1^N = \left[1 - (1 - S_1)e^{-(\alpha+\beta)t} - \frac{\alpha(1-S_2)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N$,

$$Y = S_2^M = [1 - (1 - S_2)e^{-\gamma t}]^M,\ |S_1| < 1, |S_2| < 1 \tag{11}$$

III. Characteristics of the model:

The characteristics of the model are obtained by using the equation (11).Expanding $P(S_1, S_2;\, t)$and collecting the constant terms, we get the probability that there is no employee in the organization as

$P_{0,0}(t) =$

$$\Big\{\exp\Big\{a_1 b_1(-1)\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big] + \frac{a_1 b_1\,\alpha(-1)}{\gamma-(\alpha+\beta)}\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_1-1)}dv\Big] + \Big[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big] + a_2 b_2(-1)\Big[e^{-\gamma t}\int_0^t e^{\gamma v}\, v^{(b_2-1)}dv - \frac{e^{-\gamma t}}{\gamma}\Big]\Big\}[X_1 Y_1]\Big\}$$

Where, $X_1 = \Big[1 - e^{-(\alpha+\beta)t} - \frac{\alpha}{\gamma-(\alpha+\beta)}\big[e^{-(\alpha+\beta)t} - e^{-\gamma t}\big]\Big]^N$ *and* $Y_1 = [1 - e^{-\gamma t}]^M$ (12)

Taking $S_2 = 1$ in equation (11), we get the probability generating function of the number of employees in grade-1 as

$$P(S_1;t) = \Big[\big[1-(1-S_1)e^{-(\alpha+\beta)t}\big]^N\Big]\Big[\exp\Big[a_1 b_1(S_1-1)\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big]\Big]\Big],$$

$|S_1| < 1$ (13)

Expanding $P(S_1;t)$ and collecting the constant terms, we get the probability that there is no employee in grade-1 as

$$P_0(t) = \Big[\big[1-e^{-(\alpha+\beta)t}\big]^N\Big]\Big[\exp\Big[a_1 b_1(-1)\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big]\Big]\Big] \quad (14)$$

The mean number of employees in grade-1 is

$$L_1(t) = \big[Ne^{-(\alpha+\beta)t}\big] + a_1 b_1\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big] \quad (15)$$

The probability that there is at least one employee in grade-1 is

$U_1(t) = 1 - P_0(t)$

$$U_1(t) = 1 - \Big[\big[1-e^{-(\alpha+\beta)t}\big]^N\Big]\Big[\exp\Big[a_1 b_1(-1)\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big]\Big]\Big] \quad (16)$$

The average duration of stay of an employee in grade-1 is

$W_1(t) = \frac{L_1(t)}{(\alpha+\beta)[1-P_0(t)]}$

$$W_1(t) = \frac{\big[Ne^{-(\alpha+\beta)t}\big] + a_1 b_1\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big]}{(\alpha+\beta)\Big[1 - \Big[\big[1-e^{-(\alpha+\beta)t}\big]^N\Big]\Big[\exp\Big[a_1 b_1(-1)\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big]\Big]\Big]\Big]} \quad (17)$$

The variance of the number of grade-1 is

$$V_1(t) = \big[Ne^{-(\alpha+\beta)t}\big]\big[1-e^{-(\alpha+\beta)t}\big] + a_1 b_1\Big[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v}\, v^{(b_1-1)}dv - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\Big] \quad (18)$$

Coefficient of variation of the number of employees in grade-1 is $cv_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)}$ where, $V_1(t)$ and $L_1(t)$ are given in equations (18) and (15) respectively (19)

Similarly, taking $S_1 = 1$ in equation (11), then we get the probability generating function of the number of employees in grade-2 as

$$P(S_2;t) = \left\{Exp\left[\frac{a_1 b_1 \alpha (S_2 - 1)}{\gamma - (\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right]\right.\right.$$

$$\left.+\frac{a_1 b_1 \alpha (S_2-1)}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + \frac{a_1 b_1 \alpha (S_2-1)}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right]\right\}\left[\left[1 - \frac{\alpha(1-S_2)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N\right]$$

$$[[1-(1-S_2)e^{-\gamma t}]^M] \quad \text{where, } |S_2| < 1 \tag{20}$$

Expanding $P(S_2;t)$ and collecting the constant terms, we get the probability that there is no employee in grade-2 as

$$P_0(t) = \left\{Exp\left[\frac{a_1 b_1 \alpha (-1)}{\gamma - (\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right]\right.\right.$$

$$\left.+\frac{a_1 b_1 \alpha (-1)}{\gamma - (\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2 (-1)\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right]\right\}$$

$$\left[1 - \frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N [[1-e^{-\gamma t}]^M] \tag{21}$$

The mean number of employees in grade-2 is

$$L_2(t) = Me^{-\gamma t} + N\left[\frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]$$

$$+\left[\frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right]\right.$$

$$\left.+\frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right]\right] \tag{22}$$

The probability that there is at least one employee in grade-2 is

$$U_2(t) = 1 - P_0(t)$$

$$= 1 - \left\{Exp\left[\frac{a_1 b_1 \alpha (-1)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right] + \frac{a_1 b_1 \alpha (-1)}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \right.\right.\right.$$

$$\left.\left.\left.\frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2 (-1)\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right]\right]\right\} \left[1 - \frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N [[1-e^{-\gamma t}]^M]$$

(23)

The average duration of stay of employees in grade-2 is

$$W_2(t) = \frac{Me^{-\gamma t} + N\left[\frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right] + \left[\frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right] + \frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right]\right]}{\gamma\left\{1 - \left[\text{Exp}\left[\frac{a_1 b_1 \alpha(-1)}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right] + \frac{a_1 b_1 \alpha(-1)}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2(-1)\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right]\right] \left[\left[1 - \frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]^N\right]\left[\left[1 - e^{-\gamma t}\right]^M\right]\right\}} \tag{24}$$

The variance of the number of employees in grade-2 is

$$V_2(t) = Me^{-\gamma t}(1 - e^{-\gamma t})$$

$$+N\left[\frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]\left[\left[1 - \left[\frac{\alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t} - e^{-\gamma t}\right]\right]\right]\right]$$

$$+\frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[e^{-(\alpha+\beta)t}\int_0^t e^{(\alpha+\beta)v} v^{(b_1-1)} dv - e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_1-1)} dv\right]$$

$$+\frac{a_1 b_1 \alpha}{\gamma-(\alpha+\beta)}\left[\frac{e^{-\gamma t}}{\gamma} - \frac{e^{-(\alpha+\beta)t}}{(\alpha+\beta)}\right] + a_2 b_2\left[e^{-\gamma t}\int_0^t e^{\gamma v} v^{(b_2-1)} dv - \frac{e^{-\gamma t}}{\gamma}\right] \tag{25}$$

Coefficient of variation of the number of employees in grade-2 is

$cv_2(t) = \frac{\sqrt{V_2(t)}}{L_2(t)}$ where, $V_2(t)$ and $L_2(t)$ are given in equations (25) and (22) respectively. (26)

The mean number of employees in the organization is $L = L_1 + L_2$ where, $L_1(t)$ and $L_2(t)$ are given in equations (15) and (22) respectively. (27)

IV. Numerical illustration and results

A numerical illustration was used to explain the model's behaviour in this subdivision. For the recruiting, advancement, and leaving rates of the organization, several values of the parameters were explored. Because the manpower model's performance characteristics were particularly time-sensitive, the transient behaviour of the model was investigated by computing performance measures with the following set of values for the model parameters: t= 1.5,2,2.5,3and 3.5; α=3,4,5,6 and 7; β=4,5,6,7 and 8; γ=5,6,7,8 and 9; a_1=5,10,15,20 and 25; b_1=3,4,5,6 and 7; a_2=5,10,15,20 and 25; b_2=3,4,5,6 and 7; N=1000,1100,1200,1300 and 1400 ; M=600,700,800,900 and 1000.

Performance measures such as the mean number of employees in grades I and II, the mean duration of stay of a grade I employee and in grade II, the variance of the number of employees in grades I and II, and the coefficient of variation of the number of employees in grades I and II were computed and presented in Tables 1 and 2. Figures 2a, 2b, 3a and 3b show the link between parameters and performance measures.

Table 1 demonstrated that the performance indicators in grades I and II were extremely time sensitive. The mean number of employees in grade I and grade 2 in the organization increased from 4.0180 to 24.1946 and 8.1050 to 45.6831, respectively, while time (t) varied from 1.5 to 3.5. When all other factors were held constant, the mean period of stay of an employee in grade I and grade II in the company increased from 0.5845 to 3.4564 and 1.7138 to 9.1367, respectively.

When all other parameters were held constant, the mean number of employees in grade I was not influenced and in grade II it increased from 8.4203 to 8.6416.When all other parameters were held constant, the mean duration of stay of an employee in grade I was not influenced and in grade II it increased from 1.7533 to 1.7848.

When all other parameters were held constant, the mean number of employees in grade I was not influenced and in grade II it increased from 8.4203 to 8.6416.When all others parameters were held constant, the mean duration of stay of an employee in grade I was not influenced and in grade II it increased from 1.7533 to 1.7848.

Table 1: *Values of L_1, L_2, W_1 and W_2 for different values of parameters*.

t	α	β	γ	a_1	b_1	a_2	b_2	N	M	L_1	L_2	W_1	W_2
1.5	3	4	5	5	3	5	3	1000	600	4.0180	8.1050	0.5845	1.7138
2	3	4	5	5	3	5	3	1000	600	7.4352	13.5421	1.0628	2.7769
2.5	3	4	5	5	3	5	3	1000	600	11.9497	22.0582	1.7071	4.4219
3	3	4	5	5	3	5	3	1000	600	17.5364	32.7960	2.5052	6.5599
3.5	3	4	5	5	3	5	3	1000	600	24.1946	45.6831	3.4564	9.1367
1.5	3	4	5	5	3	5	3	1000	600	4.0180	8.1050	0.5845	1.7138
1.5	4	4	5	5	3	5	3	1000	600	3.5804	8.4013	0.4604	1.7508
1.5	5	4	5	5	3	5	3	1000	600	3.2370	8.6505	0.3744	1.7862
1.5	6	4	5	5	3	5	3	1000	600	2.9553	8.8632	0.3118	1.8188
1.5	7	4	5	5	3	5	3	1000	600	2.7189	9.0462	0.2646	1.8483
1.5	7	5	5	5	3	5	3	1000	600	2.5174	8.7440	0.2282	1.8003
1.5	7	6	5	5	3	5	3	1000	600	2.3435	8.4931	0.1994	1.7634
1.5	7	7	5	5	3	5	3	1000	600	2.1921	8.2806	0.1763	1.7350
1.5	7	8	5	5	3	5	3	1000	600	2.0589	8.0977	0.1573	1.7130
1.5	7	8	6	5	3	5	3	1000	600	2.0589	6.5928	0.1573	1.2559
1.5	7	8	7	5	3	5	3	1000	600	2.0589	5.7212	0.1573	0.9933
1.5	7	8	8	5	3	5	3	1000	600	2.0589	5.0994	0.1573	0.8147
1.5	7	8	9	5	3	5	3	1000	600	2.0589	4.6108	0.1573	0.6856
1.5	7	8	9	10	3	5	3	1000	600	4.1178	5.9837	0.2791	0.7104
1.5	7	8	9	15	3	5	3	1000	600	6.1767	7.3565	0.4126	0.8309
1.5	7	8	9	20	3	5	3	1000	600	8.2356	8.7293	0.5492	0.9739
1.5	7	8	9	25	3	5	3	1000	600	10.2944	10.1021	0.6863	1.1236
1.5	7	8	9	25	4	5	3	1000	600	19.7548	15.5521	1.3170	1.7280
1.5	7	8	9	25	5	5	3	1000	600	35.6026	24.0877	2.3735	2.6764
1.5	7	8	9	25	6	5	3	1000	600	61.6965	37.3243	4.1131	4.1471
1.5	7	8	9	25	7	5	3	1000	600	104.0989	57.6922	6.9399	6.4102
1.5	7	8	9	25	7	10	3	1000	600	104.0989	60.9278	6.9399	6.7698
1.5	7	8	9	25	7	15	3	1000	600	104.0989	64.1634	6.9399	7.1293
1.5	7	8	9	25	7	20	3	1000	600	104.0989	67.3990	6.9399	7.4888
1.5	7	8	9	25	7	25	3	1000	600	104.0989	70.6346	6.9399	7.8483
1.5	7	8	9	25	7	25	4	1000	600	104.0989	84.7664	6.9399	9.4185
1.5	7	8	9	25	7	25	5	1000	600	104.0989	107.9303	6.9399	11.9923
1.5	7	8	9	25	7	25	6	1000	600	104.0989	145.3699	6.9399	16.1522
1.5	7	8	9	25	7	25	7	1000	600	104.0989	205.2306	6.9399	22.8034
1.5	3	4	5	5	3	10	3	1000	600	4.0180	8.1050	0.5845	1.7138
1.5	3	4	5	5	3	10	3	1100	600	4.0208	8.1838	0.5849	1.7230
1.5	3	4	5	5	3	10	3	1200	600	4.0235	8.2627	0.5853	1.7327
1.5	3	4	5	5	3	10	3	1300	600	4.0263	8.3415	0.5856	1.7428
1.5	3	4	5	5	3	10	3	1400	600	4.0290	8.4203	0.5860	1.7533
1.5	3	4	5	5	3	10	3	1400	700	4.0290	8.4756	0.5860	1.7610
1.5	3	4	5	5	3	10	3	1400	800	4.0290	8.5309	0.5860	1.7688
1.5	3	4	5	5	3	10	3	1400	900	4.0290	8.5863	0.5860	1.7767
1.5	3	4	5	5	3	10	3	1400	1000	4.0290	8.6416	0.5860	1.7848

The performance metrics in grade I and grade II employees in the organization were extremely sensitive to time, as shown in **Table 2**. When other parameters were held constant, it was discovered that time (t) varies from 1.5 to 3.5, the variance of the number of employees in grade I and grade II increased from 4.0180 to 24.1946 and 8.1042 to 45.6831, respectively, and the coefficient of variation of the number of employees in both grades decreased from 0.4989 to 0.2033 and 0.3512 to 0.1480.

When the promotion rate (α) from grade I to grade II increased from 3 to 7, the variance of the number of employees in grade I decreased from 4.0180 to 2.7189 and increased from 8.1042 to 9.0456, and the coefficient of variation of the number of employees in grade I increased from 0.4989 to 0.6065 and decreased from 0.3512 to 0.3325, when all other parameters remained constant.

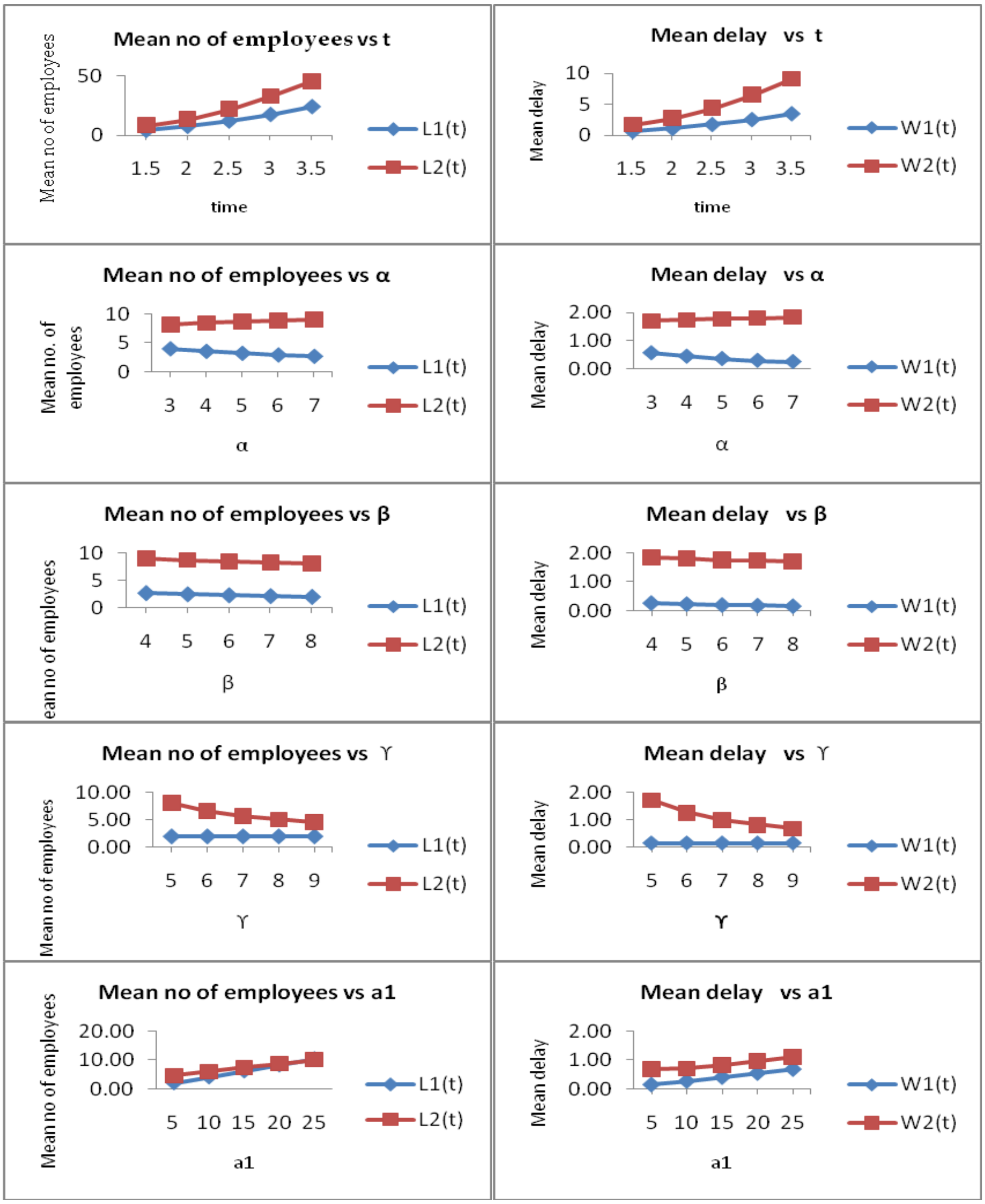

Figure 2a: *Relation between the parameters and performance measures*

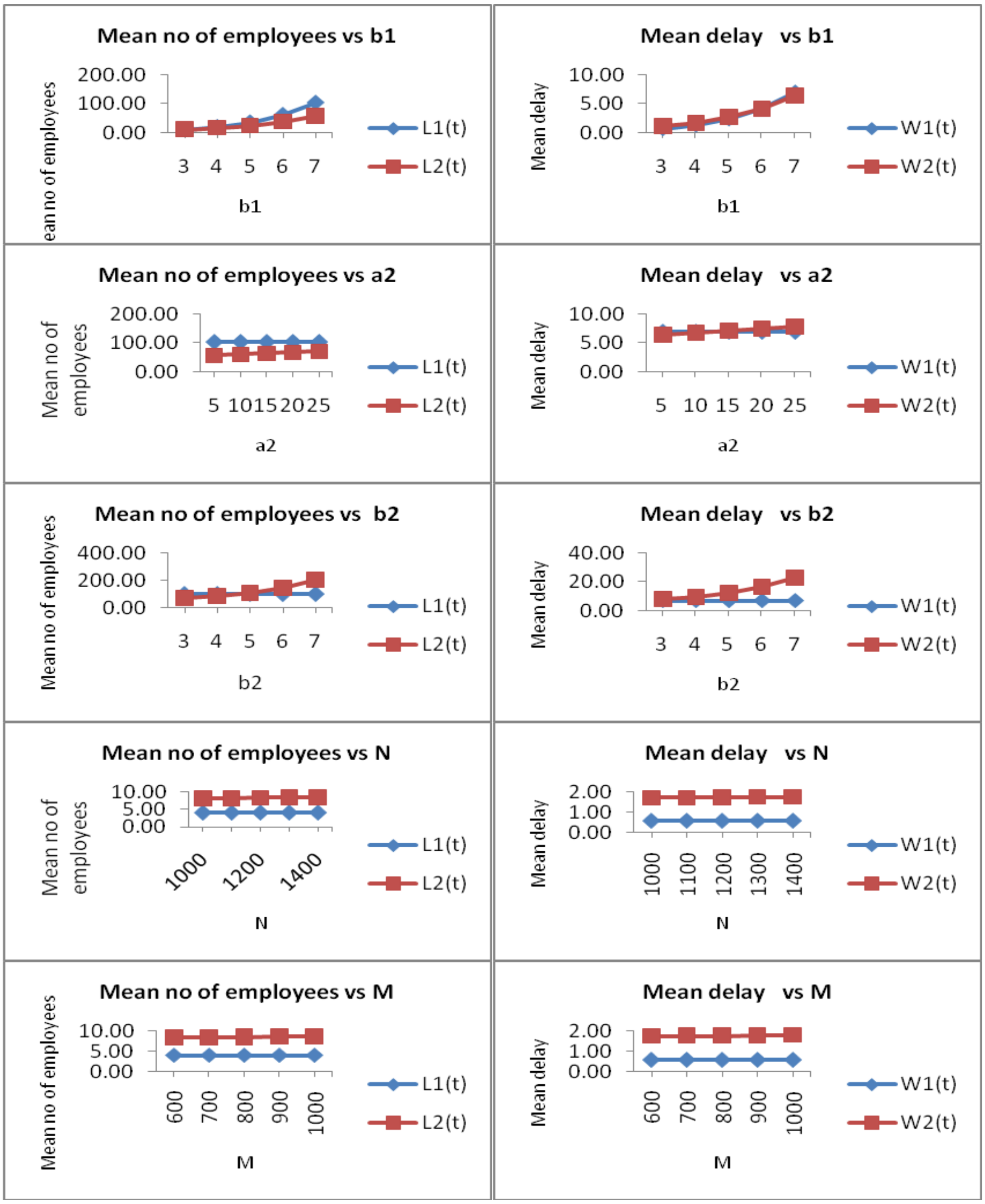

Figure 2b: *Relation between the parameters and performance measures*

When the leaving rate (β) of an employee in grade I increases from 4 to 8, the variance of the number of employees in grade I and grade II decreases from 2.7189 to 2.0589 and 9.0456 to 8.0974, respectively, while the coefficient of variation of the number of employees in both grades I and II increases from 0.6065 to 0.6969 and 0.3325 to 0.3514, respectively, when other parameters remain constant.

When the leaving rate (γ) of an employee in grade II increases from 5 to 9, the variance of the number of employees in grade I is unaffected, while in grade II it decreases from 8.0974 to 4.6108. When other parameters are held constant, the coefficient of variation of the number of employees in grade I is unaffected, while in grade II it increases from 0.3514 to 0.4657. When the recruitment rate parameter (a_1) of employees in grade I was changed from 5 to 25, the variance of the number of employees in grade I and grade II increased from 2.0589 to 10.2944 and 4.6108 to 10.1021 respectively, while the coefficient of variation of the number of employees in both grade I and grade II decreased from 0.6969 to 0.3117 and 0.4657 to 0.3146 when the other parameters remained constant.

Table 2: *Values of V_1, V_2, CV_1 and CV_2 for different values of parameters.*

T	α	β	γ	a_1	b_1	a_2	b_2	N	M	V_1	V_2	CV_1	CV_2
1.5	3	4	5	5	3	5	3	1000	600	4.0180	8.1042	0.4989	0.3512
2	3	4	5	5	3	5	3	1000	600	7.4352	13.5421	0.3667	0.2717
2.5	3	4	5	5	3	5	3	1000	600	11.9497	22.0582	0.2893	0.2129
3	3	4	5	5	3	5	3	1000	600	17.5364	32.7960	0.2388	0.1746
3.5	3	4	5	5	3	5	3	1000	600	24.1946	45.6831	0.2033	0.1480
1.5	3	4	5	5	3	5	3	1000	600	4.0180	8.1042	0.4989	0.3512
1.5	4	4	5	5	3	5	3	1000	600	3.5804	8.4006	0.5285	0.3450
1.5	5	4	5	5	3	5	3	1000	600	3.2370	8.6498	0.5558	0.3400
1.5	6	4	5	5	3	5	3	1000	600	2.9553	8.8626	0.5817	0.3359
1.5	7	4	5	5	3	5	3	1000	600	2.7189	9.0456	0.6065	0.3325
1.5	7	5	5	5	3	5	3	1000	600	2.5174	8.7435	0.6303	0.3382
1.5	7	6	5	5	3	5	3	1000	600	2.3435	8.4927	0.6532	0.3431
1.5	7	7	5	5	3	5	3	1000	600	2.1921	8.2802	0.6754	0.3475
1.5	7	8	5	5	3	5	3	1000	600	2.0589	8.0974	0.6969	0.3514
1.5	7	8	6	5	3	5	3	1000	600	2.0589	6.5928	0.6969	0.3895
1.5	7	8	7	5	3	5	3	1000	600	2.0589	5.7212	0.6969	0.4181
1.5	7	8	8	5	3	5	3	1000	600	2.0589	5.0994	0.6969	0.4428
1.5	7	8	9	5	3	5	3	1000	600	2.0589	4.6108	0.6969	0.4657
1.5	7	8	9	10	3	5	3	1000	600	4.1178	5.9837	0.4928	0.4088
1.5	7	8	9	15	3	5	3	1000	600	6.1767	7.3565	0.4024	0.3687
1.5	7	8	9	20	3	5	3	1000	600	8.2356	8.7293	0.3485	0.3385
1.5	7	8	9	25	3	5	3	1000	600	10.2944	10.1021	0.3117	0.3146
1.5	7	8	9	25	4	5	3	1000	600	19.7548	15.5521	0.2250	0.2536
1.5	7	8	9	25	5	5	3	1000	600	35.6026	24.0877	0.1676	0.2038
1.5	7	8	9	25	6	5	3	1000	600	61.6965	37.3243	0.1273	0.1637
1.5	7	8	9	25	7	5	3	1000	600	104.0989	57.6922	0.0980	0.1317
1.5	7	8	9	25	7	10	3	1000	600	104.0989	60.9278	0.0980	0.1281
1.5	7	8	9	25	7	15	3	1000	600	104.0989	64.1634	0.0980	0.1248
1.5	7	8	9	25	7	20	3	1000	600	104.0989	67.3990	0.0980	0.1218
1.5	7	8	9	25	7	25	3	1000	600	104.0989	70.6346	0.0980	0.1190
1.5	7	8	9	25	7	25	4	1000	600	104.0989	84.7664	0.0980	0.1086
1.5	7	8	9	25	7	25	5	1000	600	104.0989	107.9303	0.0980	0.0963
1.5	7	8	9	25	7	25	6	1000	600	104.0989	145.3699	0.0980	0.0829
1.5	7	8	9	25	7	25	7	1000	600	104.0989	205.2306	0.0980	0.0698
1.5	3	4	5	5	3	10	3	1000	600	4.0180	8.1042	0.4989	0.3512
1.5	3	4	5	5	3	10	3	1100	600	4.0208	8.1830	0.4987	0.3495
1.5	3	4	5	5	3	10	3	1200	600	4.0235	8.2617	0.4985	0.3479
1.5	3	4	5	5	3	10	3	1300	600	4.0263	8.3405	0.4984	0.3462
1.5	3	4	5	5	3	10	3	1400	600	4.0290	8.4193	0.4982	0.3446
1.5	3	4	5	5	3	10	3	1400	700	4.0290	8.4746	0.4982	0.3435
1.5	3	4	5	5	3	10	3	1400	800	4.0290	8.5298	0.4982	0.3424
1.5	3	4	5	5	3	10	3	1400	900	4.0290	8.5851	0.4982	0.3412
1.5	3	4	5	5	3	10	3	1400	1000	4.0290	8.6404	0.4982	0.3402

When the recruitment rate parameter (b_1) in grade I is changed from 3 to 7, the variance of the number of employees in grade I and grade II increases from 10.2944 to 104.0989 and 10.1021 to

57.6922, respectively, while the coefficient of variation of the number of employees in both grades I and II decreases from 0.3117 to 0.0980 and 0.3146 to 0.1317, respectively, when the other parameters remain constant.

When the recruitment rate parameter (a2) in grade II changes from 5 to 25, the variance of the number of employees in grade I does not change and in grade II it increases from 57.6922 to 70.6346, while the coefficient of variation of the number of employees in grade I does not change and in grade II it decreases from 0.1317 to 0.1190.

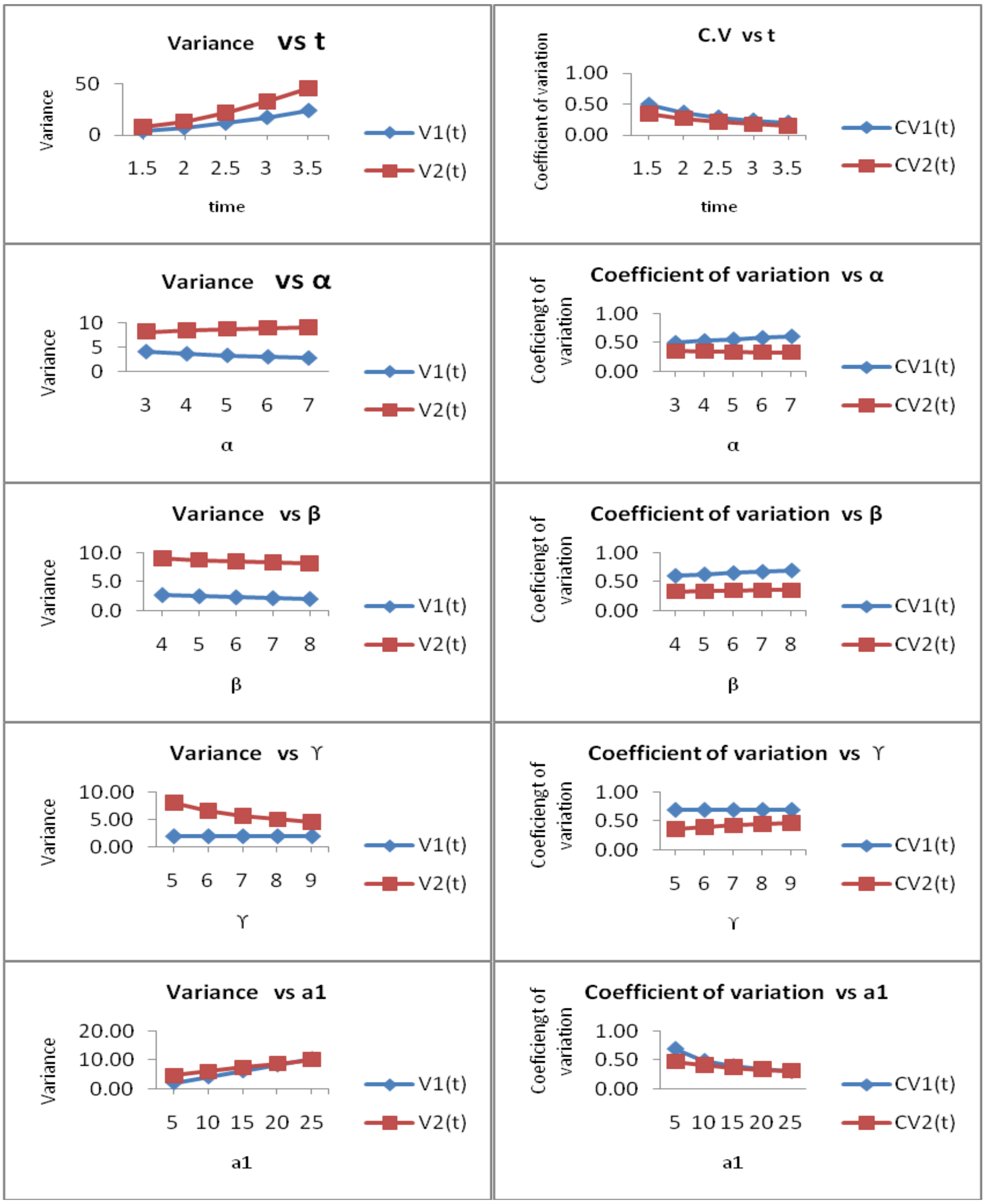

Figure 3a: *Relation between the parameters and performance measures.*

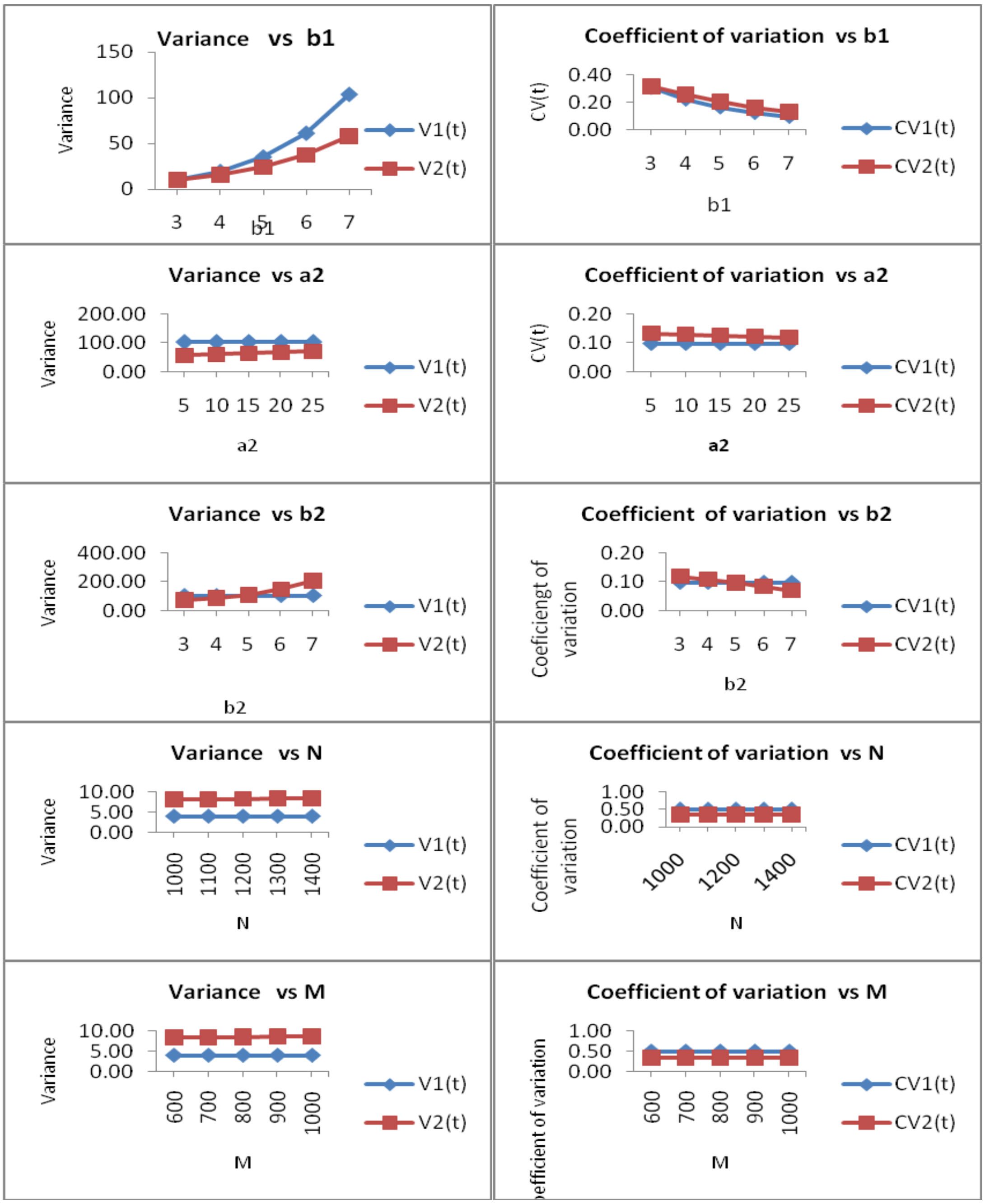

Figure 3b: *Relation between the parameters and performance measures.*

When the recruitment rate parameter (b2) of employees in grade II varies from 3 to 7, the variance of the number of employees in grade I is unaffected, and in grade II it increases from 70.6346 to 205.2306, while the coefficient of variation of the number of employees in grade I is unaffected, and in grade II it decreases from 0.1190 to 0.0698.

When other parameters were held constant, the initial number of employees in grade I (N) varied from 1000 to 1400, the variance of the number of employees in grade I and grade II increased from 4.0180 to 4.0290 and 8.1042 to 8.4193, respectively, and the coefficient of variation of the number of employees in grade I and grade II decreased from 0.4989 to 0.4982 and 0.3512 to 0.3446.

When other parameters were fixed, the variance of the number of employees in grade I was not influenced and in grade II it was increasing from 8.4193 to 8.6404, coefficient of variation of the number of employees in grade I was not influenced and in grade II it was decreasing from 0.3446 to 0.3402, when the primary number of employees in grade II (M) varies from 100 to 500.

V. Sensitivity analysis of the model

The model was sensitivity tested with respect to the value of time (t), recruitment rates $\lambda1(t)$ and $\lambda2(t)$, promotion rate parameter (α), and leaving parameters(β) and (γ) of both grade I and grade II, as well as all other parameters combined on the mean number of employees in grade I and grade II, mean duration of stay of an employee in grade I and grade II, and variance of the number of employees in grade I and grade II.

For different value of t,α,β,γ,a_1,b_1,a_2 and b_2 the mean number of employees in grade I and in grade II, mean duration of stay of an employee in grade I and in grade II, the variance of the number of employees in grade I and in grade II were computed and presented in Table 3a and 3b with variation of -15%,-10%,-5%,0%,5%,10% and 15% of the model parameters.

Time had a significant impact on the performance measurements (t). The mean number of employees, mean duration of stay of an employee, and variation of the number of employees in grade I and grade II increased when t increased from -15 % to 15%.

The mean number of employees, mean duration of stay of an employee, and variation of the number of employees in grade I decreased and in grade II it was increasing as the promotion rate parameter (α) increased from -15 % to 15%. The mean number of employees, mean duration of stay of an employee, and variation of the number of employees in grade I and grade II decreased as the leaving rate parameter (β) in grad-1 increased from -15 % to 15%.When the leaving rate parameter (γ) in grad-2 is increased from -15 % to 15%, the mean number of employees, mean duration of stay of

Table 3a:*The values of $L_1(t)$,$L_2(t)$,$W_1(t)$,$W_2(t)$,$V1(t)$ and $V_2(t)$ for different Values of t, α, β, γ, a_1, b_1, a_2 and b_2*

Para -meters	Performance Measure	-15%	-10%	-5%	0%	+5%	+10%	+15%
t=2	L1	5.2497	5.9333	6.6624	7.4356	8.2524	9.1123	10.0152
	L2	16.7277	18.7380	20.9764	23.4109	26.0219	28.7973	31.7300
	W1	0.7539	0.8499	0.9530	1.0629	1.1792	1.3019	1.4308
	W2	3.5231	3.9175	4.3407	4.7972	5.2901	5.8205	6.3878
	VI	5.2497	5.9333	6.6624	7.4356	8.2524	9.1123	10.0152
	V2	16.7276	18.7379	20.9763	23.4109	26.0219	28.7973	31.7300
α=3	L1	7.8716	7.7206	7.5754	7.4356	7.3010	7.1711	7.0457
	L2	23.0525	23.1764	23.2957	23.4109	23.5220	23.6292	23.7329
	W1	1.2022	1.1528	1.1065	1.0629	1.0218	0.9831	0.9466
	W2	4.7743	4.7801	4.7879	4.7972	4.8075	4.8187	4.8305
	VI	7.8716	7.7206	7.5754	7.4356	7.3010	7.1711	7.0457
	V2	23.0525	23.1764	23.2957	23.4109	23.5220	23.6292	23.7329
β=4	L1	8.0287	7.8206	7.6232	7.4356	7.2571	7.0870	6.9247
	L2	23.7293	23.6162	23.5103	23.4109	23.3172	23.2286	23.1447
	W1	1.2549	1.1854	1.1216	1.0629	1.0086	0.9585	0.9120
	W2	4.8301	4.8173	4.8064	4.7972	4.7895	4.7833	4.7784
	VI	8.0287	7.8206	7.6232	7.4356	7.2571	7.0870	6.9247
	V2	23.7293	23.6161	23.5103	23.4109	23.3171	23.2286	23.1446

Table 3b: *The values of $L_1(t)$,$L_2(t)$,$W_1(t)$,$W_2(t)$,$V1(t)$ and $V_2(t)$ for different Values of t, α, β, γ, a_1, b_1, a_2 and b_2*

Para -meters	Performance Measure	-15%	-10%	-5%	0%	+5%	+10%	+15%
γ=5	L1	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	L2	26.8975	25.5983	24.4462	23.4109	22.4709	21.6107	20.8184
	W1	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629
	W2	6.3982	5.7752	5.2484	4.7972	4.4064	4.0650	3.7647
	VI	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	V2	26.8974	25.5983	24.4461	23.4109	22.4709	21.6107	20.8184
a1=5	L1	6.3205	6.6922	7.0639	7.4356	7.8074	8.1791	8.5508
	L2	22.8696	23.0500	23.2305	23.4109	23.5913	23.7717	23.9521
	W1	0.9046	0.9572	1.0100	1.0629	1.1158	1.1688	1.2218
	W2	4.7704	4.7742	4.7834	4.7972	4.8146	4.8351	4.8582
	VI	6.3205	6.6922	7.0639	7.4356	7.8074	8.1791	8.5508
	V2	22.8696	23.0500	23.2305	23.4109	23.5913	23.7717	23.9521
b1=3	L1	4.7683	5.5453	6.4298	7.4356	8.5783	9.8752	11.3459
	L2	22.2169	22.5701	22.9665	23.4109	23.9089	24.4666	25.0909
	W1	0.6870	0.7953	0.9200	1.0629	1.2257	1.4108	1.6209
	W2	4.6951	4.7717	4.7796	4.7972	4.8525	4.9345	5.0407
	VI	4.7683	5.5453	6.4298	7.4356	8.5783	9.8752	11.3459
	V2	22.2169	22.5701	22.9664	23.4109	23.9089	24.4666	25.0909
a2=10	L1	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	L2	20.4589	21.4429	22.4269	23.4109	24.3948	25.3788	26.3628
	W1	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629
	W2	4.1923	4.3939	4.5955	4.7972	4.9988	5.2004	5.4021
	VI	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	V2	20.4589	21.4429	22.4269	23.4109	24.3948	25.3788	26.3628
b2=3	L1	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	L2	16.4841	18.5091	20.8066	23.4109	26.3602	29.6977	33.4714
	W1	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629
	W2	3.3778	3.7927	4.2635	4.7972	5.4015	6.0854	6.8587
	VI	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	V2	16.4841	18.5091	20.8066	23.4109	26.3602	29.6977	33.4714
N=1500	L1	4.0256	4.0276	4.0297	4.0318	4.0338	4.0359	4.0380
	L2	13.4547	13.5138	13.5729	13.6321	13.6912	13.7503	13.8094
	W1	0.5855	0.5858	0.5861	0.5864	0.5866	0.5869	0.5872
	W2	2.8212	2.8257	2.8307	2.8360	2.8418	2.8479	2.8543
	VI	4.0256	4.0276	4.0297	4.0318	4.0338	4.0359	4.0380
	V2	13.4537	13.5128	13.5719	13.6310	13.6901	13.7491	13.8082
M=500	L1	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	L2	23.4075	23.4086	23.4097	23.4109	23.4120	23.4131	23.4143
	W1	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629	1.0629
	W2	4.7969	4.7970	4.7971	4.7972	4.7973	4.7974	4.7975
	VI	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356	7.4356
	V2	23.4075	23.4086	23.4097	23.4109	23.4120	23.4131	23.4143

The mean number of employees, mean duration of stay of an employee, and variation of the number of employees in grade I and grade II increased when the recruitment rate parameter (a1) in grade I increased from -15 % to 15%. The mean number of employees, mean duration of stay of an employee, and variation of the number of employees in grade I and grade II increased when the recruitment rate parameter (b1) in grade I increased from -15 % to 15%.

When the recruitment rate parameter (a2) of employees in grade II goes from -15 % to 15%, the mean number of employees, the mean duration of stay of an employee, and the variance of the number of employees in grade I are unaffected, while they increase in grade II. When the recruitment rate parameter (b2) of employees in grade II is increased from -15 % to 15%, the mean number of employees, mean duration of stay of an employee, and variance of the number of employees in grade I are unaffected, but they increase in grade II.

V1.Comparative Studies of the models:

In this part, a comparison of the generated model with a model with homogeneous Poisson recruitment was shown. Table 4 shows the performance measures of both models for various values of t=1.6, 1.7, 1.8, 1.9, and 2.

Table 4: *Comparative study of models with Homogeneous and Non-Homogeneous Recruitments.*

t	Characteristics Measured	Non-Homogeneous recruitment	Homogeneous recruitment	Deference	Percentage of Variation
t=1.6	L1	2.1021	0.7348	1.3673	65.0445
	L2	4.7680	2.3187	2.4493	51.3695
	W1	0.3421	0.2017	0.1404	41.0406
	W2	1.1085	0.6329	0.4756	42.9048
	VI	2.1021	0.7348	1.3673	65.0445
	V2	4.7680	2.3187	2.4493	51.3695
t=1.7	L1	2.2347	0.7245	1.5102	67.5795
	L2	4.7067	1.9718	2.7349	58.1065
	W1	0.3575	0.2008	0.1567	43.8322
	W2	1.1500	0.6342	0.5158	44.8522
	VI	2.2347	0.7245	1.5102	67.5795
	V2	4.7067	1.9718	2.7349	58.1065
t=1.8	L1	2.3724	0.7193	1.6531	69.6805
	L2	4.7803	1.7598	3.0205	63.1864
	W1	0.3738	0.2003	0.1735	46.4152
	W2	1.2040	0.6611	0.5429	45.0914
	VI	2.3724	0.7193	1.6531	69.6805
	V2	4.7803	1.7598	3.0205	63.1864
t=1.9	L1	2.5127	0.7168	1.7959	71.4729
	L2	4.9365	1.6303	3.3062	66.9746
	W1	0.3906	0.2001	0.1905	48.7711
	W2	1.2579	0.6972	0.5607	44.5743
	VI	2.5127	0.7168	1.7959	71.4729
	V2	4.9365	1.6303	3.3062	66.9746
t=2.0	L1	2.6543	0.7155	1.9388	73.0437
	L2	5.1432	1.5513	3.5919	69.8378
	W1	0.4079	0.2000	0.2079	50.9684
	W2	1.3082	0.7320	0.5762	44.0453
	VI	2.6543	0.7155	1.9388	73.0437
	V2	5.1432	1.5513	3.5919	69.8378

Table 4 shows that the percentage variation of the performance measures between the two models increased as time progressed. The assumption of the Duane recruitment process was found to have a considerable impact on all of the manpower model's performance measures. Time has a substantial impact on system performance, and the proposed model can more correctly forecast system performance.

VII. Conclusion

The purpose of this work is to build and analyze a two-graded manpower model with direct recruitment in both grades for non-stationary recruitment. The Duane recruitment procedure was capable of identifying recruitments that were time-dependent. The model's characteristics were obtained explicitly to assist HR Managers in adopting optimal operating policies, such as the mean number of employees in each grade, the mean duration of stay of an employee in each grade, the variance of the number of employees in each grade, and the coefficient of variation of an employee in each grade in the organization. The model's sensitivity study revealed that the Duane recruitment process has a considerable impact on system performance indicators. A comparison of the suggested model with Poisson recruitment reveals that the proposed model predicts system properties more accurately. When the recruiting was done in a time-dependent manner, the performance measures could be anticipated more correctly and realistically using the evolving model. This model can also be expanded by factoring in cost considerations while determining the best values for the parameters, which will be done elsewhere.

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