Parallel System Analysis with Priority and Inspection Using Semi-Markov Approach

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Abstract

In this paper a parallel system has been discussed with the idea of priority to preventive maintenance over replacement. The system has two identical units and facility of inspection is given to the failed unit before repair/replacement. There is a single server who play four-in-one role of inspection, replacement, repair and preventive maintenance and comes immediately when required. Units are failed with constant rate whereas failure time is random. The distribution of time for repair activities is arbitrary and there rates follow exponential distribution. The random variable associate with different rates are stochastically independent. Mathematical expression for several reliability terms like MTSF, availability, busy period analysis for server , expected number of visits by the server and cost benefit are obtained by using semi-markov process and regenerative point technique. Graphs are drawn to find the effect of various parameters on MTSF, Availability and profit.

Keywords: parallel system, priority, preventive maintenance, replacement, inspection

I. Introduction

The world is moving day by day towards the smart technology. Advance development of technology has significantly increased cost and complexity of industrial systems. Thus it has become an essential to operate industrial systems with minimum down time in order to achieve optimized production, increase profit and to avoid the losses. Hence, the need for reliability modeling and analysis of complex industrial systems is inevitable. Reliability analysis of parallel systems has been broadly studied by many researchers because parallel configuration is more reliable then series. Dhillon and Viswanath [6] analyzed a parallel system with the common-cause failure. Sridharan and Kalyani [7] gives common-cause failure analysis of a two non-identical unit parallel system using GERT technique.

A parallel (2n-2) system was investigated by E.Papageorgious and G. Kokolakis [9] where two units start operation simultaneously and any one of them was replaced by one of (n-2) warm standby units upon failure. Reliability of parallel systems was studied based on multiple competing dependent failure process by Sanling and David [10]. This problem was formulated for

the conditioning on shock sets because not all shocks cause degradation. N. Sharma and J.P.Singh Joorel [8] investigate a two unit parallel system with inspection and preparation time for replacement. To improve the reliability PM and Inspection are widely used in modern engineering systems. PM can help to extend the life time of an equipment, increase the productivity and hence decrease unexpected maintenance spending. Inspection aims to find the defect of the system and type of the defect. Goel and Gupta [13] considered a two identical unit parallel system with the concept of PM, inspection and two types of repair. They assumed that the time to failure, commencement to PM and inspection are constant while repair and maintenance times are arbitrarily distributed. M.K.Kakkar and J.Bhatti [12] purposed a two dissimilar parallel unit framework under the presumption that the unit may also fail during the preventive maintenance (PM). Wang and Lin [5] found a methodology to optimize the non-periodic maintenance for a series-parallel system. Shruti [11] analyzed a stochastic model with two units subject to routine inspection, maintenance and replacement. In this research routine inspection is conducting over operative unit and after inspection either the unit is maintained or it failed. Repair and replacement of the unit is based upon guarantee period of the equipment. In many research priority concept is also used to make the system better in performance and hence more profitable. P. Kumar, A. Bharti, and A. Gupta [3] investigated and analyze a two unit parallel system in which priority was given to one unit over other. In this system priority unit was repairable and non-priority unit was non-repairable and preference to repair of priority unit was given over replacement of non-priority unit. R.Rathee and D.Pawar [4] introduced a reliability model of a parallel system in which priority to repair over replacement was given using maximum operation and repair times.A.Kumar and S.C.Malik [1] considered a computer system with two identical units and in each unit H/W and S/W components works together. Priority was given to PM of the unit over S/W replacement under certain assumptions. C.Aggarwal and N.Ahlawat [2] done the profit analysis of a standby system with priority to PM over repair by considering rest of the server between repairs.

In the present study a parallel system is investigate under some assumptions. Priority is given to PM over replacement with inspection. Inspection is done to find the type of failure. Semi-Markovian approach and regenerative-point-technique are used to obtain numerical expressions for various reliability terms such as MTSF, Availability, busy period analysis for server, expected number of visits by the server and cost benefit. Graphical interpretation is done to visualize the effect of several parameters (related to repair activities) on obtained reliability measurable terms.

For practical implication of system one of the example is a parallel compressor rack system. Jim Coats [14] gives the commercial and industrial applications of parallel compressor racks. There are several advantages of this system like low installation cost, capacity control, redundancy and maximum efficiency etc.

Figure 1: *Parallel compressor Rack*

II. Notations for System Model

S_i : *States of the sytem* $(i = 1,2,\ldots.,15)$
λ : Constant failure rate
$a/b/\propto_0$: repair/replacement/PM rate of the system
upm/UPM: Unit is under PM/continuosly under PM
wpm/WPM: Unit is waiting for PM/continue waiting for PM
FUi/FWi: Failed unit under inspection/waiting for inspection
FUI/FWI: Failed unit continuously under inspection/waiting for inspection
FUr/FUrp : Failed unit under repair/replacement
FUR/FURP: Failed unit continuously under repair/replacement
μ_i: *mean sojourn time in the state* S_i
m_{ij}: Contribution to mean sojourn time in state S_i when the system transits directly to state S_j
h(t)/f(t)/r(t)/g(t): pdf of the inspection/repair/replacement/PM time

III. System Description and Assumptions

A parallel system with two identical units is studied under some practical assumptions which are given below:

- Initially both the units are in operative condition
- Failure rate is constant
- Only one server operator is taken to do all repair activities
- Repair is perfect or units restore in initial condition after repair
- Post failure inspection is done to find unit is repairable or replaced by new
- Time taken for repair activities is arbitrary and there rates follow exponential distribution
- The random variable associate with different rates are stochastically independent

Table 1: *Description of the states*

States	Description
S_0	Both the units are in normal mode
S_1	One unit is operative and other is failed under inspection
S_2	Resume for PM

S_3	One is working and other is failed under replacement
S_4	One unit is continuously under inspection from previous state and other is waiting for inspection
S_5	One is working and other is failed under repair
S_6	One unit is continuously under inspection from previous state and other is waiting for PM
S_7	One unit is working and other under PM
S_8	One unit is continuously under replacement from previous state and other is waiting for inspection
S_9	One unit is under repair and other is continuously waiting for inspection from previous state
S_{10}	One unit is under replacement and other is continuously waiting for inspection from previous state
S_{11}	One unit is continuously under repair from previous state and other is waiting for inspection
S_{12}	One unit is continuously under repair from previous state and other is waiting for PM
S_{13}	One unit is under repair and other is continuously waiting for PM from previous state
S_{14}	One unit is under PM and other is continuously waiting for replacement from previous state
S_{15}	One unit is continuously under PM from previous state and other is waiting for inspection

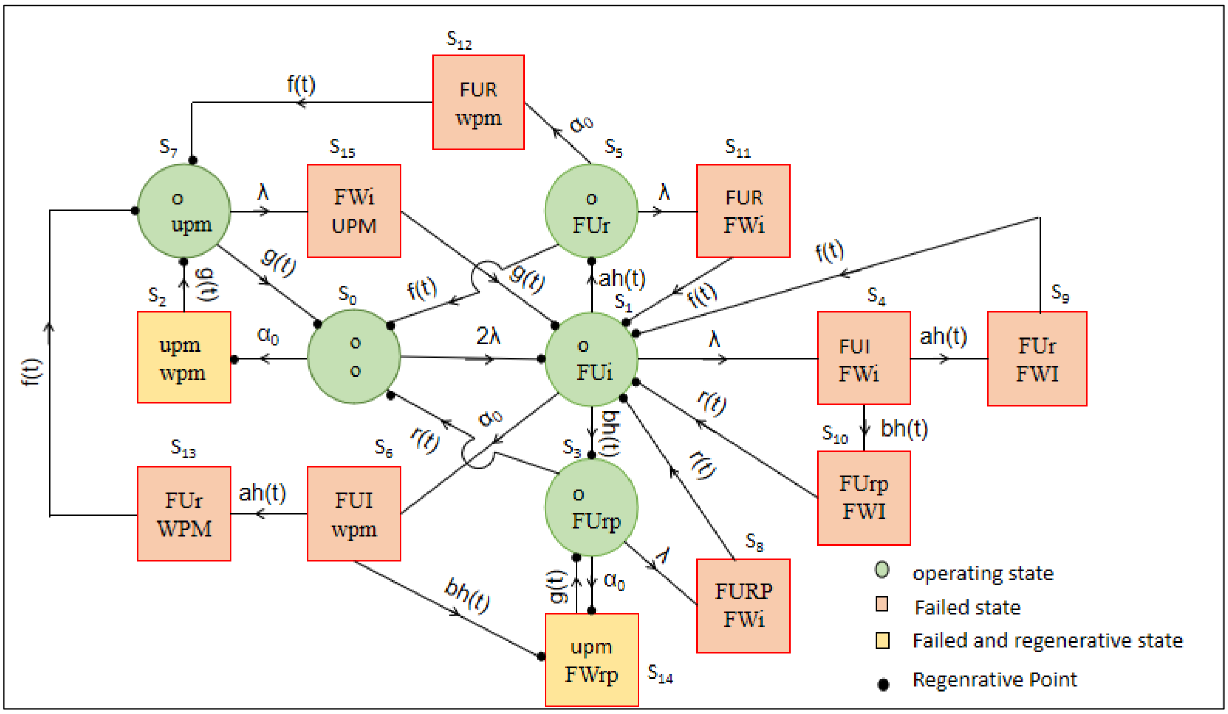

Figure 2: *Transition State Diagram*

IV. Formulation and Stochastic Analysis of the Model

I. Transition Probabilities & Mean Sojourn Times (μ_i)

Steady- state transition probabilities from regenerative state i to state j are given by the formula

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}\,(t)dt \tag{1}$$

Here $Q_{ij}(t)$ / $q_{ij}(t)$ are the c.d.f/p.d.f from state i to state j in (0,t) time.

$p_{01} = \frac{2\lambda}{2\lambda+\alpha_0}, \quad p_{02} = \frac{\alpha_0}{2\lambda+\alpha_0}$

$p_{13} = bh^*(\lambda+\propto_0),\ p_{14} = \frac{\lambda}{\lambda+\propto_0}(1-h^*(\lambda+\propto_0)),\ p_{15} = ah^*(\lambda+\propto_0),\ \ p_{16} = \frac{\propto_0}{\lambda+\propto_0}(1-h^*(\lambda+\propto_0))$

$p_{30} = r^*(\lambda+\propto_0),\ \ p_{38} = p_{31.8} = \frac{\lambda}{\lambda+\propto_0}(1-\ r^*(\lambda+\propto_0)),\ p_{3,14} = \frac{\propto_0}{\lambda+\propto_0}(1-r^*\ (\lambda+\propto_0))$

$p_{49} = p_{6,13} = a\ ,\ \ p_{4,10} = p_{6,14} = b$

$p_{50} = f^*(\lambda+\propto_0)\ \ ,\ p_{5,11} = p_{51.11} = \frac{\lambda}{\lambda+\propto_0}(1-\ f^*(\lambda+\propto_0))\ ,\ p_{5,12} = p_{57.12} = \frac{\propto_0}{\lambda+\propto_0}(1-\ f^*(\lambda+\propto_0))$

$p_{70} = g^*(\lambda)\ ,\ p_{7,15} = p_{71.15} = 1-g^*(\lambda)\ ,$

$p_{11.49} = \frac{\lambda a}{\lambda+\propto_0}(1-h^*(\lambda+\propto_0))\ ,p_{11.4,10} = \frac{\lambda b}{\lambda+\propto_0}(1-h^*(\lambda+\propto_0))\ ,$

$p_{1,14.6} = \frac{\propto_0 b}{\lambda+\propto_0}(1-h^*(\lambda+\propto_0))\ ,\ \ p_{17.6,13} = \frac{\propto_0 a}{\lambda+\propto_0}(1-\ h^*(\lambda+\propto_0))\ \ ,$

$p_{27} = p_{81} = p_{91} = p_{10,1} = p_{11,1} = p_{12,7} = p_{13,7} = p_{14,3} = p_{15,1} = 1$

It can verified that

$p_{01} + p_{02} = p_{13} + p_{15} + p_{11.49} + p_{11.4,10} + p_{17.6,13} + p_{1,14.6} =$
$p_{50} + p_{51.11} + p_{57.12} = p_{30} + p_{31.8} + p_{3,14} = p_{49} + p_{4,10} = p_{6,13} + p_{6,14} = p_{70} + p_{71.15} = p_{27} = p_{81} =$
$p_{91} = p_{10,1} = p_{11,1} = p_{12,7} = p_{13,7} = p_{14,3} = p_{15,1} = 1$

And $\mu_i'^s$ are given by the formula

$$\mu_i = E(t) = \int_0^\infty P(T>t)\,dt = \sum_j m_{ij} \tag{2}$$

and

$$m_{ij} = \frac{d[Q_{ij}^{**}(s)]}{ds}|s=0 \tag{3}$$

$\mu_0 = \dfrac{1}{2\lambda+\alpha_0}\ ,\mu_1 = \dfrac{1}{\lambda+\alpha_0}(1-h^*(\lambda+\alpha_0)),\mu_3 = \dfrac{1}{\lambda+\alpha_0}(1-r^*(\lambda+\alpha_0))$

$\mu_5 = \frac{1}{\lambda+\alpha_0}(1-f^*(\lambda+\alpha_0),\ \mu_7 = \frac{1}{\lambda}(1-g^*(\lambda))$

$\mu_1' = \ [\dfrac{1}{\lambda+\alpha_0} + \dfrac{\lambda b}{\alpha(\lambda+\alpha_0)} + \dfrac{1}{\gamma} + \dfrac{a}{\alpha}](1-h^*(\lambda+\alpha_0))$

$\mu_3' = \frac{(\beta+\lambda)}{\beta(\lambda+\alpha_0)}(1-f^*(\lambda+\alpha_0))\ ,\ \mu_5' = \frac{1}{\propto},\mu_7' = \frac{1}{\theta}$

II. Reliability & Mean Time to System Failure (MTSF)

Let $\Phi_i(t)$ be the cdf of first passing time from the state S_i to the state in which failure occur and we take absorbing state as the failed state. So, the expressions for $\Phi_i(t)$ from which MTSF of discussed system is obtained are given as

$$\Phi_i(t)\ =\ \sum_{i,j} Q_{ij}\,(t)\ Ⓢ\ \Phi_j(t)\ +\sum_{i,k} Q_{ik}\,(t) \tag{4}$$

Where j is the operating regenerative state to which the given regenerative state i can transit and k is the failed state to which state i can directly transit.

If we take LST of above relation (4) and solved them for $\Phi_0^{**}(s)$, we have

$$R^*(s) = \frac{1-\Phi^{**}(s)}{s} \tag{5}$$

The system reliability is obtained by taking Inverse Laplace transform of (5) and MTSF is given by the formula

$$\text{MTSF} = \lim_{s\to 0}\frac{1-\Phi^{**}(s)}{s} = \frac{\text{N}}{\text{D}} \tag{6}$$

Where,

$$N = \mu_0 + \mu_1 p_{01} + \mu_3 p_{01} p_{13} + \mu_5 p_{01} p_{15} \text{ and } D = 1 - p_{01} p_{13} p_{30} - p_{01} p_{15} p_{50} \tag{7}$$

III. Analysis of Availability

Let $A_i(t)$ be the probability of system working at time 't' w.r.t the condition that system goes to regenerative state S_i at t = 0. We have the relations for $A_i(t)$ as

$$A_i(t) = M_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © A_j(t) \tag{8}$$

Where j is any successive regenerative state to which the regenerative state i can transit through n transitions.

M_i (t) is the probability that the system in up state S_i up to the time t without visiting to any other regenerative state.

$$M_0(t) = e^{-(2\lambda+\propto_0)t} \tag{9}$$

$$M_1(t) = e^{-(\lambda+\propto_0)t}\overline{H(t)} \tag{10}$$

$$M_3(t) = e^{-(\lambda+\propto_0)t}\overline{R(t)} \tag{11}$$

$$M_5(t) = e^{-(\lambda+\propto_0)t}\overline{F(t)} \tag{12}$$

$$M_7(t) = e^{-(\lambda t)}\overline{G(t)} \tag{13}$$

Now, if we use LT of (8) and solved it for $A_0^*(s)$.We get the result for steady state availability as

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1} \tag{14}$$

Where

$$N_1 = \mu_0 A + (\mu_1 + \mu_5 p_{15})B + \mu_3 C + \mu_7 D \tag{15}$$

$$D_1 = (\mu_0 + \mu_2 p_{02})A + (\mu_1' + \mu_5' p_{15})B + \mu_3' C + \mu_7' D + \mu_{14} E \tag{16}$$

IV. Busy Period Analysis for Server

Let $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$be the probability of busy period of server during inspection, repair, replacement and PM at instant't' with the given condition that the system go to regenerative state Si at t=0. The recursive relations for $B_i^I(t), B_i^R(t), B_i^{Rp}(t), B_i^P(t)$ are as follows:

$$B_i^I(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © B_j^I(t) \tag{17}$$

$$B_i^R(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © \text{B}_j^R(t) \tag{18}$$

$$B_i^{Rp}(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © \text{B}_j^{Rp}(t) \tag{19}$$

$$B_i^P(t) = W_i(t) + \sum_{i,j} q_{ij}^{(n)}(t) © \text{B}_j^P(t) \tag{20}$$

Where j is any successive regenerative state to which the regenerative state i can transit through n transitions.

$W_i(t)$ is the probability of server busyness at state S_i due to repair activities at time t without making any transition to any other regenerative state or returning to the same via one or more non regenerative state.

Here,

$$W_1(t) = e^{-(\lambda+\propto_0)t}\overline{H(t)} + (\lambda e^{-(\lambda+\propto_0)t} © 1)\overline{H(t)} + (\alpha_0 e^{-(\lambda+\propto_0)t} © 1)\overline{H(t)} \tag{21}$$

$$W_5(t) = e^{-(\lambda+\propto_0)t}\overline{F(t)} + (\lambda e^{-(\lambda+\propto_0)t} © 1)\overline{F(t)} + (\propto_0 e^{-(\lambda+\propto_0)t} © 1)\overline{F(t)} \tag{22}$$

$$W_3(t) = e^{-(\lambda+\propto_0)t}\overline{R(t)} + (\lambda e^{-(\lambda+\propto_0)t} © 1)\overline{R(t)} \tag{23}$$

$$W_2(t) = \overline{G(t)} = W_{14}(t), W_7(t) = e^{-(\lambda)t}\overline{G(t)} + (\lambda e^{-(\lambda)t} © 1)\overline{G(t)}$$

Take LT of (17) to (20) and solving for $B_0^{I^*}(s), B_0^{R^*}(s), B_0^{Rp^*}(s), B_0^{P^*}(s)$ .The busy time in inspection, repair, replacement and preventive maintenance for server is given by

$$B_0^I(\infty) = \lim_{s\to 0} sB_0^{I^*}(s) = \frac{N_2}{D_1} \tag{24}$$

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R^*}(s) = \frac{N_3}{D_1} \tag{25}$$

$$B_0^{Rp}(\infty) = \lim_{s\to 0} sB_0^{Rp^*}(s) = \frac{N_4}{D_1} \tag{26}$$

$$B_0^P(\infty) = \lim_{s\to 0} sB_0^{P^*}(s) = \frac{N_5}{D_1} \tag{27}$$

Here,

$N_2 = W_1^*(0)B, N_3 = W_5^*(0)p_{15}B, N_4 = W_3^*(0)C$

$N_5 = W_2^*(0)p_{02}A + W_7^*(0)D + W_{14}^*(0)E$ and D_1 is mentioned above. (28)

V. Expected Number of Visits by The Server

Consider $I_0(t), R_0(t), Rp_0(t), Pm_0(t)$ as the expected number of visits make by the server for inspection, repair, replacement and PM in (0, t] .We have the following recursive relations for $I_0(t), R_0(t), Rp_0(t), Pm_0(t)$ are

$$I_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t) Ⓢ (C + I_j(t)) \tag{29}$$

$$R_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t) Ⓢ (C + R_j(t)) \tag{30}$$

$$Rp_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t) Ⓢ (C + Rp_j(t)) \tag{31}$$

$$Pm_i(t) = \sum_{i,j} Q_{ij}^{(n)}(t) Ⓢ (C + Pm_j(t)) \tag{32}$$

Where j is any successive regenerative state to which the regenerative state i can transit through n transitions and and C = 1 if j is the state where the server does the job afresh, otherwise C = 0.

Take LST of above equations solving for $I_0^{**}(s), R_0^{**}(s), Rp_0^{**}(s), Pm_0^{**}(s)$.The expected number of inspections, repairs, replacements, and preventive maintenance by the server is given by (per unit time)

$$I_0(\infty) = \lim_{s\to 0} sI_0^{**}(s) = \frac{N_6}{D_1} \tag{33}$$

$$R_0(\infty) = \lim_{s\to 0} sR_0^{**}(s) = \frac{N_7}{D_1} \tag{34}$$

$$Rp_0(\infty) = \lim_{s\to 0} sRp_0^{**}(s) = \frac{N_8}{D_1} \tag{35}$$

$$Pm_0(\infty) = \lim_{s\to 0} sPm_0^{**}(s) = \frac{N_9}{D_1} \tag{36}$$

Where,

$N_6 = B, N_7 = (p_{11.49} + p_{15} + p_{17.6,13})B,\ N_8 = (p_{11.4,10} + p_{1,14.6} + p_{13})B,$

$N_7 = p_{02}A + D + E$ and D_1 is already mentioned. (37)

Here A, B, C, D & E are

$$\mathrm{A} = (1 - \mathrm{p}_{3,14})(\mathrm{p}_{15}\mathrm{p}_{50} + \mathrm{p}_{70}(\mathrm{p}_{15}\mathrm{p}_{57.12} + \mathrm{p}_{17.6,13})) + (\mathrm{p}_{13} + \mathrm{p}_{1,14.6})\mathrm{p}_{30} \tag{38}$$

$$\mathrm{B} = (1 - \mathrm{p}_{3,14})(1 - \mathrm{p}_{02}\mathrm{p}_{70}) \tag{39}$$

$$\mathrm{C} = (\mathrm{p}_{13} + \mathrm{p}_{1,14.6})(1 - \mathrm{p}_{02}\mathrm{p}_{70}) \tag{40}$$

$$\mathrm{D} = (1 - \mathrm{p}_{3,14})(\mathrm{p}_{15} + \mathrm{p}_{17.6,13} - \mathrm{p}_{01}\mathrm{p}_{15}\mathrm{p}_{50} - \mathrm{p}_{15}\mathrm{p}_{51.11}) + \mathrm{p}_{02}\mathrm{p}_{30}(\mathrm{p}_{13} + \mathrm{p}_{1,14.6}) \tag{41}$$

$$\mathrm{E} = (\mathrm{p}_{13}\mathrm{p}_{3,14} + \mathrm{p}_{1,14.6})(1 - \mathrm{p}_{02}\mathrm{p}_{70}) \tag{42}$$

VI. Profit Analysis

In steady state the profit function of the system model can be obtained as

$$\mathrm{P} = \mathrm{k}_0\mathrm{A}_0 - \mathrm{k}_1\mathrm{B}_0^{\mathrm{I}} - \mathrm{k}_2\mathrm{B}_0^{\mathrm{R}} - \mathrm{k}_3\mathrm{B}_0^{\mathrm{Rp}} - \mathrm{k}_4\mathrm{B}_0^{\mathrm{Pm}} - \mathrm{k}_5\mathrm{I}_0 - \mathrm{k}_6\mathrm{R}_0 - \mathrm{k}_7\mathrm{Rp}_0 - \mathrm{k}_8\mathrm{Pm}_0 \tag{43}$$

Here,

P = Profit function of system model

k_0 = Revenue per unit up − time of the system
k_1, k_2, k_3, k_4 = Cost per unit time of the server when it is busy in inspection, repair, replacement, preventive maintenance
k_5, k_6, k_7, k_8 = Cost per unit time for inspection, repair, replacement, preventive maintenance

V. Analytical Study of the Model

In the present study concept of priority to PM over replacement with inspection is used. To see the applicability of this situation particular cases are taken for the included parameters like $F(t) = \propto e^{-\propto t}$, $r(t) = \beta e^{-\beta t}$, $h(t) = \gamma e^{-\gamma t}$, $g(t) = \theta e^{-\theta t}$. Results are obtained in form of tables and graphs by taking random values for the given parameters. On the basis of these cases numerical and graphical results are obtained which shows the effect of these parameters on MTSF, Availability and Profit function of the system. From the obtained results we conclude that PM does not effect the MTSF and as the failure rate increases the availability and the profit of the system is decreases. When the repair activities rate is increases then availability and profit is also increases. From the results we obtained that the system is highly profitable if we increase the PM rate at the very first stage when failure rate is very low but as the failure rate increases the system get more profit by enhancing the inspection rate. Tabular and graphical results are given below:

Table 2: *Values of MTSF w.r.t various parameters*

Failure rate	α=2.1,β=2,a=0.6,b=0.4, γ=1.3,α0=3	α= 4.1	β=5	γ= 3	α0= 3.1	a= 0.4, b= 0.6
0.1	0.33274	0.33275	0.33275	0.33279	0.32204	0.33274
0.2	0.33113	0.33119	0.33119	0.33130	0.32056	0.33113
0.3	0.32875	0.32887	0.32886	0.32908	0.31838	0.32875
0.4	0.32577	0.32596	0.32594	0.32627	0.31563	0.32577
0.5	0.32235	0.32260	0.32258	0.32302	0.31247	0.32234
0.6	0.31859	0.31890	0.31887	0.31943	0.30899	0.31858
0.7	0.31458	0.31495	0.31492	0.31558	0.30527	0.31457
0.8	0.31040	0.31082	0.31079	0.31153	0.30138	0.31039
0.9	0.30610	0.30657	0.30653	0.30736	0.29737	0.30609

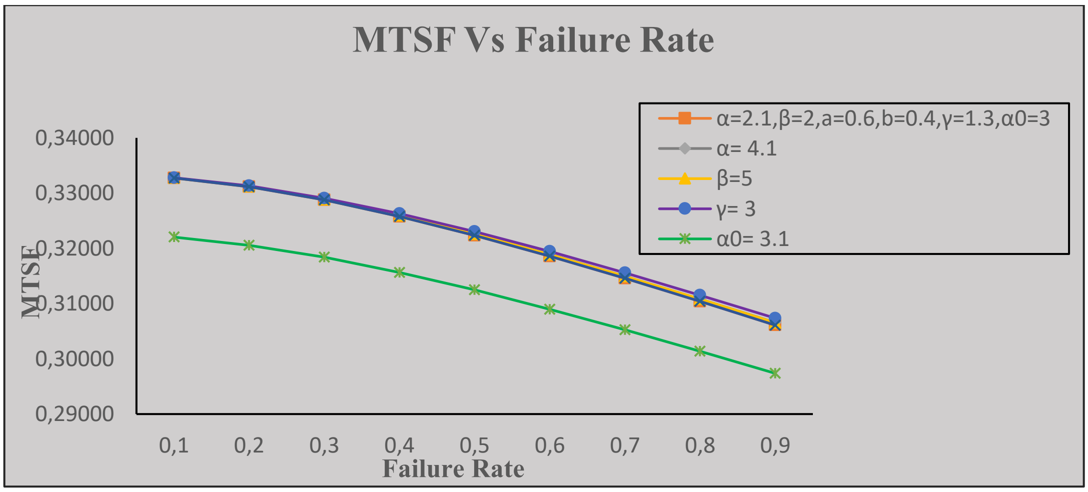

Figure 3: *MTSF VS Failure Rate*

Table 3: *Values of Availability w.r.t various parameters*

Failure Rate	α=2.1,β=2,a=0.6,b=0.4,γ=1.3, α0=3,θ=1.4	α=4.1	β=5	a=0.4,b=0.6	γ=3	α0=3.1	θ=2
0.1	0.53503	0.53911	0.54102	0.52635	0.55017	0.53213	0.56972
0.2	0.45652	0.47175	0.46481	0.45459	0.48943	0.45348	0.48158
0.3	0.42387	0.43952	0.43365	0.41815	0.46238	0.42072	0.45061
0.4	0.39668	0.41256	0.40718	0.38870	0.43924	0.39350	0.42408
0.5	0.37357	0.38955	0.38436	0.36427	0.41912	0.37041	0.40101
0.6	0.35358	0.36960	0.36443	0.34357	0.40138	0.35048	0.38067
0.7	0.33606	0.35206	0.34683	0.32572	0.38556	0.33303	0.36257
0.8	0.32052	0.33647	0.33114	0.31012	0.37131	0.31758	0.34631
0.9	0.30661	0.32248	0.31705	0.29630	0.35836	0.30376	0.33159

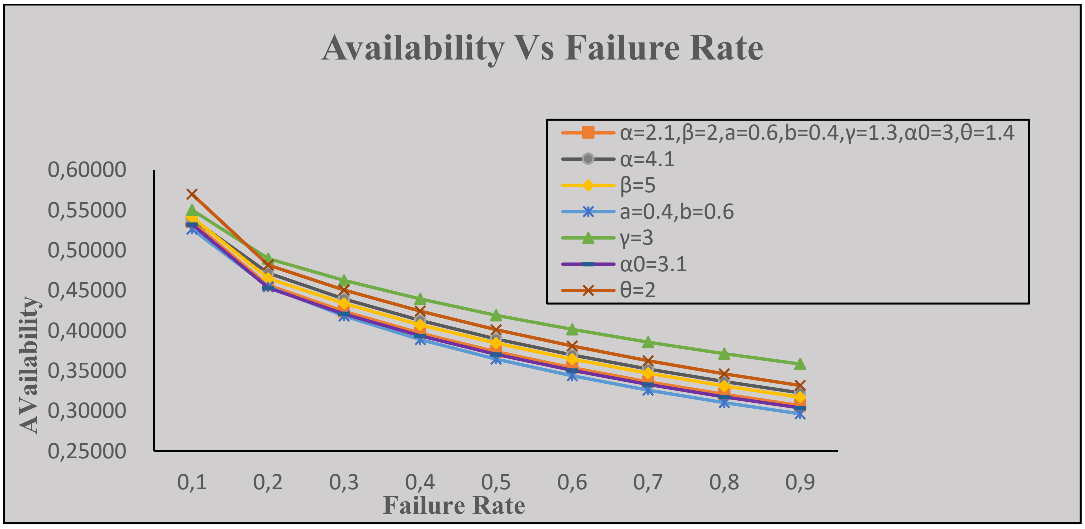

Figure 4: *Availability VS Failure Rate*

Table 4: *Values of Profit w.r.t various parameters*

Failure Rate	α=2.1,β=2,a=0.6,b=0.4,γ=1.3, α0=3,θ=1.4	α=4.1	β=5	a=0.4,b=0.6	γ=3	α0=3.1	θ=2
0.1	7355.86	7416.93	7463.24	7208.33	7569.95	7312.12	7759.37
0.2	6014.32	6239.56	6166.20	5957.81	6474.39	5970.56	6258.77
0.3	5391.47	5618.42	5571.51	5271.16	5911.14	5347.74	5648.91
0.4	4873.10	5098.06	5067.35	4713.77	5427.64	4830.27	5130.70
0.5	4433.01	4653.80	4633.45	4249.77	5006.14	4391.57	4683.55
0.6	4053.32	4268.64	4255.27	3855.69	4634.00	4013.52	4292.79
0.7	3721.35	3930.44	3922.08	3515.48	4301.91	3683.31	3947.67
0.8	3427.88	3630.28	3625.81	3217.79	4002.92	3391.63	3640.12
0.9	3166.00	3361.48	3360.25	2954.37	3731.65	3131.53	3363.95

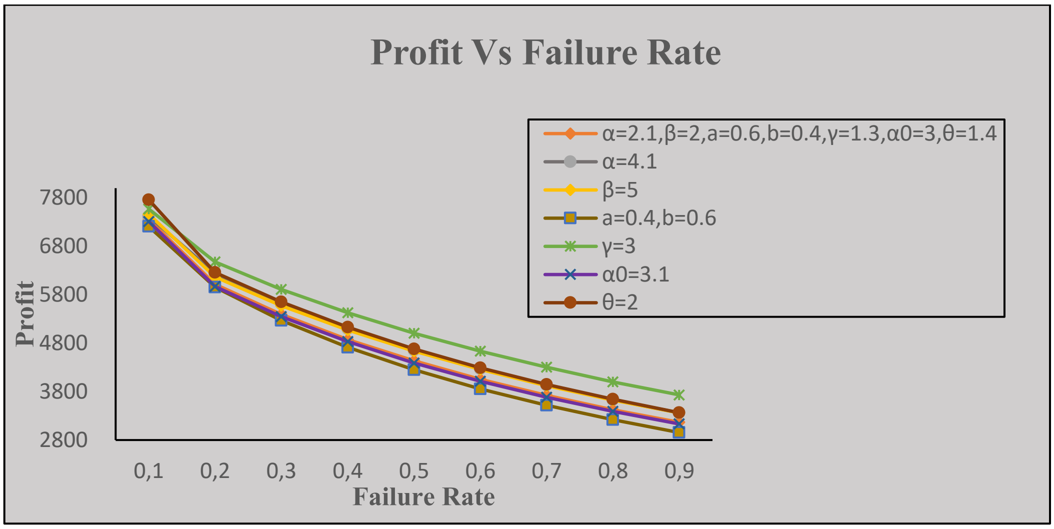

Figure 5: *Profit VS Failure Rate*

VI. Discussion

The performance of the system model for different values of failure and repair rates is shown by the table 2, table 3, and table 4 of MTSF, availability and profit analysis w.r.t. various parameters. Also, the behavior of the MTSF, availability and profit analysis depicted by figure 3, figure 4 and figure 5 respectively.

From the table 2 and figure 3 we conclude that the MTSF is decrease with the increase of the failure rate. If we fix all the parameters and change the value of repair rate α=4.1, replacement rate β=5 and inspection rate γ=3 one by one the MTSF is increase. And the MTSF is decrease if the preventive maintenance is increase by α_0 = 3.1 and the other parameters are fixed.

From the table 3 and figure 4 we analyze the availability of the system. By fixing all the parameters and changing in other parameter one by one we found that availability is increase in we change in α=4.1, replacement rate β=5, inspection rate γ=3 and preventive maintenance rate θ=2. The availability is decline with the increase of preventive maintenance by α_0 = 3.1 and inter change the repair and replacement rate (a=0.4 and b=0.6).

From the table 4 and figure 5 we conclude that profit of the function is decrease by increasing the preventive maintenance by α_0 = 3.1 and inter change the repair and replacement rate (a=0.4 and b=0.6) one by one with the fixed of all other parameters. If we change in α=4.1, replacement rate β=5, inspection rate γ=3 and preventive maintenance rate θ=2 found that profit of the system model is enhance as compared to the fixed values of all parameters.

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