AN UNIQUE OPTIMAL SOLUTION FOR TYPE – III TRIANGULAR INTUITIONSTIC FUZZY TRANSPORTATION ISSUE

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Abstract

In real-life decisions, usually we happen to suffer through different states of uncertainties. In order to counter these uncertainties, in this paper, the author formulated a transportation problem in which availability, demand and costs are mixed terms of real, triangular intuitionistic fuzzy numbers. In this paper, a simple method for solving type-3 intuitionistic fuzzy transportation is applied. So, the proposed method gives the optimal solution directly. The solution procedure is illustrated with the help of numerical examples.

Keywords: IFN, TIFN, IFTP of type - 3, Optimum Solution.

1. Introduction

The fuzzy set (FS) hypothesis was at first developed by [9] is useful from numerous points of view in various applications in different fields. The idea fuzzy numerical writing computer programs was created by Tanaka et al in 1947 outlining of fuzzy choice of [2]. Idea of Intuitionistic fuzzy sets (IFS's) recommended by [1] are chiefly valuable to manage numerous exemptions, disarray and ambiguities. The IFS's different the force of enrollment (MF) and the power of non-participation (NMF) of a component in the set. Uncertainties' assistance leader to concur the power of satisfaction, force of non-satisfaction and power of vulnerability for transfer and furthermore help to make determinant at stronghold degree of endorsement and non-endorsement for transportation cost (TC) in any transportation issue (TP). Also, without a doubt coming to dynamic issues IFS turned into an extreme technique which is for the most part closable. In like manner, its boss to use IFS diverged from FS to adapt to issues which our own dynamic or unworthiness. In

[6], investigate a general report on TP in fuzzy climate. Along these lines, IFS's are utilized by numerous creators for various ideal issues. [3] presented math activities of IFS's. Various analysts are likewise chipped away at and with IFS's. PSK method for solving mixed and type-4 intuitionistic fuzzy transportation issue (IFTP) was introduced by [8] which limit, requests and cost are availability, demand and costs are mixed terms of real, triangular intuitionistic fuzzy numbers and an algorithmic methodology for tackling IFTP was introduced [4]. By utilizing [8] paper in this article we address mathematical model. [7] Presented another positioning capacity utilizing centroid of centroids of IFN's.

In this article, we will acquaint another transportation strategy with acquire an ideal arrangement in an IFTP. For the new transportation method mathematical model is tackled. Plinth of article is managed: Section 2 quintessence goal, Section 3 gives Ranking capacity, Section 4 arrangements goal of IFTP of type-3 and computational system, region 5 consists Numerical model, at long last conclusion given in region 6.

2. Preliminaries

In this part a couple of essential definitions and math tasks are examined.

Intuitionistic Fuzzy Set (IFS): An IFS $\overset{\sim IFS}{A}$ in X an IFS is described as an object of following design

$$\overset{\sim IFS}{A} = \{ \langle x, \mu_{\underset{A}{\sim IFS}}(x), \nu_{\underset{A}{\sim IFS}}(x) \rangle : x \in X \}$$

where the functions$\mu_{\underset{A}{\sim IFS}} : X \to [0, 1]$ and $\nu_{\underset{A}{\sim IFS}}: X \to [0, 1]$ defines degree of Enrollment work and non-participation element$x \in X$, respectively and $0 \le \mu_{\tilde{A}^{IFS}}(x), \nu_{\tilde{A}^{IFS}}(x) \le 1$, for every $x \in X$.

Intuitionistic Fuzzy Numbers (IFN's): A subset of IFS, $\overset{\sim IFS}{A} = \{ \langle x, \mu_{\underset{A}{\sim IFS}}(x), \nu_{\underset{A}{\sim IFS}}(x) \rangle : x \in X \}$, of real line $\Re$ is called an IFN if the following holds:

(i) $\exists m \in \Re, \mu_{\tilde{A}^{IFS}}(m) = 1 \, and \, \nu_{\tilde{A}^{IFS}}(m) = 0$

(ii)$\mu_{\underset{A}{\sim IFS}} : \Re \to [0, 1]$is continuous and for every $x \in \Re, 0 \le \mu_{\tilde{A}^{IFS}}(x), \nu_{\tilde{A}^{IFS}}(x) \le 1$ holds.

Enrollment work and non-participation capacity of $\widetilde{A}^{IFS}$ is as follows,

$$\mu_{\underset{A}{\sim IFS}}(x) = \begin{cases} f_1(x), & x \in [m - \alpha_1, m) \\ 1, & x = m \\ h_1(x), & x \in (m, m + \beta_1] \\ 0, & otherwise \end{cases} \quad \text{and} \quad \nu_{\underset{A}{\sim IFS}}(x) = \begin{cases} 1, & x \in (-\infty, m - \alpha_2) \\ f_2(x), & x \in [m - \alpha_2, m) \\ 0, & x = m, x \in [m + \beta_2, \infty) \\ h_2(x), & x \in (m, m + \beta_2] \end{cases}$$

Where $f_i(x)$and $h_i(x)$; $i = 1,2$ are strictly increasing and decreasing functions in $[m - \alpha_i, m)$ and $(m, m - \beta_i]$ respectively. α_i and β_i are left and right spreads of $\mu_{\underset{A}{\sim IFS}}(x)$and $\nu_{\underset{A}{\sim IFS}}(x)$ respectively.

Triangular Intuitionistic Fuzzy Number (TIFN): A TIFN $\overset{\sim IFN}{A}$ is an IFS in $\Re$ with the following Enrollment function $\mu_{\underset{A}{\sim IFN}}$and non-participation capacity $\nu_{\underset{A}{\sim IFN}}$defined by

$$\mu_{\tilde{A}^{IFS}} = \begin{cases} 0, & x < a_1 \\ \frac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ 1, & x = a_2 \\ \frac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & otherwise \end{cases} \quad \text{and} \quad \nu_{\tilde{A}^{IFS}} = \begin{cases} 1, & x < a_1' \\ \frac{a_1' - x}{a_2 - a_1'}, & a_1' \le x \le a_2 \\ 0, & x = a_2 \\ \frac{x - a_2}{a_3' - a_2}, & a_2 \le x \le a_3' \\ 1, & otherwise \end{cases}$$

Where $a'_1 \le a_1 \le a_2 \le a_3 \le a'_3$.This TIFN is denoted by $\overset{\sim IFN}{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ in Figure 1.

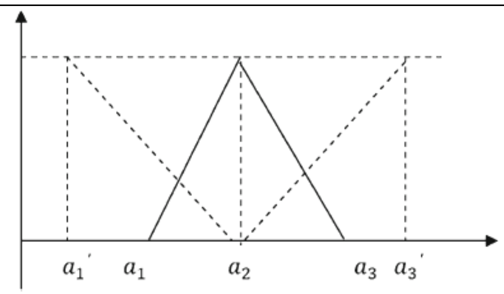

Figure 1: *Participation and non-enrollment elements of TIFN*

Arithmetic operations of TIFN:

For any two TIFN's $\overset{\sim IFN}{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ and $\overset{\sim IFN}{B} = (b_1, b_2, b_3; b'_1, b_2, b'_3)$, arithmetic operations are as follows,

(i) Addition: $\overset{\sim IFN}{A} \oplus \overset{\sim IFN}{B} = (a_1 + b_1, a_2 + b_{2,}, a_3 + b_3; a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3)$

(ii) Subtraction: $\overset{\sim IFN}{A} - \overset{\sim IFN}{B} = (a_1 - b_3, a_2 - b_{2,}, a_3 - b_1; a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1)$

(iii) Multiplication: $\overset{\sim IFN}{A} \otimes \overset{\sim IFN}{B} = (a_1 b_1, a_2 b_{2,}, a_3 b_3; a'_1 b'_1, a_2 b_2, a'_3 b'_3)$

(iv) Scalar multiplication: $k \times \tilde{A}^{IFN} = \begin{cases} (ka_1, ka_2, ka_3; ka'_1, ka_2, ka'_3), k \geq 0 \\ (ka_3, ka_2, ka_1; ka'_3, ka_2, ka'_1), k < 0 \end{cases}$

3. Ranking Function

Ranking function is taken from [7], i.e., the ranking function is defined [7], for Trapezoidal and triangular Intuitionistic fuzzy number as

$$R(\tilde{A}^{IFN}) = \left(\frac{a_1 + b_1 + 2(a_2 + b_3) + 5(a_3 + b_2) + (a_4 + b_4)}{18}\right)\left(\frac{4w_1 + 5w_2}{18}\right)$$

$$R(\tilde{A}^{IFN}) = \left(\frac{(a_1 + b_1) + 14a_2 + (a_4 + b_4)}{18}\right)\left(\frac{4w_1 + 5w_2}{18}\right)$$

Consider $w_1 = w_2 = 1$, we get ranking function is

$$R(\tilde{A}^{IFN}) = \left(\frac{(a_1 + b_1) + 14a_2 + (a_4 + b_4)}{36}\right)$$

Comparison of TIFN's: To differentiate TIFN's and each other, we need to rank them. A capacity like R: F(R) →R, which maps each TIFN's into real line, is called positioning capacity. Here, $F(\Re)$ means the arrangement of all TIFN's.

By using ranking function "R", TIFN's can be compared. Let $\overset{\sim IFN}{A} = (a_1, a_2, a_3; a'_1, a_2, a'_3)$ and $\overset{\sim IFN}{B} = (b_1, b_2, b_3; b'_1, b_2, b'_3)$ are two TIFN's then $R\left(\tilde{A}^{IFN}\right) = \frac{a_1 + 14a_2 + a_3 + a'_1 + a'_3}{36}$ and $R\left(\tilde{B}^{IFN}\right) = \frac{b_1 + 14b_2 + b_3 + b'_1 + b'_3}{36}$ then the orders are defined as follows

(i) $\overset{\sim IFN}{A} > \overset{\sim IFN}{B}$ if $R\left(\overset{\sim IFN}{A}\right) > R\left(\overset{\sim IFN}{B}\right)$,

(ii) $\overset{\sim IFN}{A} < \overset{\sim IFN}{B}$ if $R\left(\overset{\sim IFN}{A}\right) < R\left(\overset{\sim IFN}{B}\right)$, and

(iii) $\overset{\sim IFN}{A} = \overset{\sim IFN}{B}$ if $R\left(\overset{\sim IFN}{A}\right) = R\left(\overset{\sim IFN}{B}\right)$

Ranking function R also holds the following properties:

(i) $R\left(\overset{\sim IFN}{A}\right) + R\left(\overset{\sim IFN}{B}\right) = R\left(\overset{\sim IFN}{A} + \overset{\sim IFN}{B}\right)$, (ii) $R\left(k\overset{\sim IFN}{A}\right) = k\,R\left(\overset{\sim IFN}{A}\right) \forall k \in \boldsymbol{R}$

4. Mathematical Formulation of Triangular Intuitionistic Fuzzy transportation problem (TIFTP) of Type - III and proposed method

4.1. TIFTP of type - III

Consider Examine a TP with 'm' vendors and 'n' insistent. c_{ij} is value of transiting one module of outcome from i^{th} vendor to j^{th} insistent.

$\tilde{a}_i^{IFN} = (a_1^i, a_2^i, a_3^i; a_1^{i'}, a_2^i, a_3^{i'})$ be IF extent at i^{th} vendor.

$\tilde{b}_j^{IFN} = (b_1^i, b_2^i, b_3^i; b_1^{i'}, b_2^i, b_3^{i'})$ be IF abundant at j^{th} insistent.

$\tilde{x}_{ij}^{IFN} = (x_1^{ij}, x_2^{ij}, x_3^{ij}; x_1^{i'j'}, x_2^{ij}, x_3^{i'j'})$ be IF quantity transformed from i^{th} vendor to j^{th} insistent

Then balanced IFTP of type - III is given by

$$Min\tilde{Z}^{IFN} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \times x_{ij}^{IFN}$$

$$s.t. \sum\nolimits_{j=1}^{n} \tilde{x}_{ij}^{IFN} = \tilde{a}_i^{IFN} \quad, i = 1, 2, \ldots, m$$

$$\sum\nolimits_{i=1}^{m} \tilde{x}_{ij}^{IFN} = \tilde{b}_j^{IFN} \quad, j = 1, 2, \ldots, n$$

$$\tilde{x}_{ij}^{IFN} \geq \tilde{0}; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$$

The TP is termed as type – III TIFTP having availability, demand and costs are mixed terms real, fuzzy and TIFN's. To find optimum solution TIFTP of type – III, we are using the following transportation strategy.

4.2. Proposed Transportation strategy (Used in [5])

Stage 1: In the given transportation problem calculate the differences between maximum and minimum cost for each row and column.
Stage 2: Find sum of row difference and column difference and denote row sum by R and column sum by C. Identify Maximum sum of row and column. Select maximum difference in row and column.
Stage 3: Choose cell having most minimal expense in row and column identified in stage 2.
Stage 4: Make a feasible assignment to cell picked in stage 5. Delete fulfilled row/column.
Stage 5: Repeat technique until all the designations has been made.
Stage 6: The Optimum solution and triangular intuitionistic optimum value is attained in step 5, is optimum solution $\{x_{ij}\}$ and triangular intuitionistic fuzzy optimum value is $\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} \otimes x_{ij}$.

5. Numerical Example

Example for TIFTP of type – III: In this province, subsist numerical example ([8]) is solved by using above transportation strategy. Consider the 3×3TIFTP of type – III

Table 1: *TIFTP of Type - III*

	D_1	D_2	D_3	Supply($\tilde{a}_i^{IFN}$)
O_1	(8,10,12; 6,10,14)	4	(10,15,20)	(4,6,8; 3,6,9)
O_2	3	(6,12,18)	(4,6,8; 2,6,10)	8
O_3	(4,8,12)	(3,4,5; 1,4,6)	6	(2,5,8)
Demand $\tilde{b}_j^{IFN}$	(3,4,5)	(2,6,10; 1,6,11)	9	

The above TIFTP of type – III table can be rewrite as

Table 2: *Modified TIFTP of Type - III*

	D_1	D_2	D_3	Supply $\tilde{a}_i^{IFN}$
O_1	(8,10,12; 6,10,14	(4,4,4; 4,4,4)	(10,15,20; 10,15,20)	(4,6,8; 3,6,9)
O_2	(3,3,3; 3,3,3)	(6,12,18; 6,12,18)	(4,6,8; 2,6,10)	(8,8,8; 8,8,8)
O_3	(4,8,12; 4,8,12	(3,4,5; 1,4,6)	(6,6,6; 6,6,6)	(2,5,8; 2,5,8)
Demand $\tilde{b}_j^{IFN}$	(3,4,5; 3,4,5)	(2,6,10; 1,6,11)	(9,9,9; 9,9,9)	

Here, we have

$$(4,6,8;3,6,9)\oplus(8,8,8;8,8,8)\oplus(2,5,8;2,5,8)=$$
$$(3,4,5;3,4,5)\oplus(2,6,10,1,6,11)\oplus(9,9,9;9,9,9)=(14,19,24;13,19,25).$$

Accordingly, problem is balanced. Beyond comparison, foregoing IFN's encounter ranking functional values of $\tilde{a}_i^{IFN}$'s, $\tilde{b}_j^{IFN}$'s and costs as under:

$$R(\tilde{a}_1^{IFN})=3, R(\tilde{a}_2^{IFN})=4, R(\tilde{a}_3^{IFN})=2.5,$$
$$R(\tilde{b}_1^{IFN})=2, R(\tilde{b}_2^{IFN})=3, R(\tilde{b}_3^{IFN})=4.5,$$
$$R(\tilde{c}_{11}^{IFN})=5, R(\tilde{c}_{12}^{IFN})=2, R(\tilde{c}_{13}^{IFN})=8,$$
$$R(\tilde{c}_{21}^{IFN})=2, R(\tilde{c}_{12}^{IFN})=6, R(\tilde{c}_{13}^{IFN})=3,$$
$$R(\tilde{c}_{31}^{IFN})=4, R(\tilde{c}_{32}^{IFN})=2, R(\tilde{c}_{13}^{IFN})=3.$$

Table 3: *Row and Column difference table*

	D_1	D_2	D_3	Supply ($\tilde{a}_i^{IFN}$)	Row Diff
O_1	(8,10,12; 6,10,14)	(4,4,4; 4,4,4)	(10,15,20; 10,15,20)	(4,6,8; 3,6,9)	6
O_2	(3,3,3; 3,3,3)	(6,12,18; 6,12,18)	(4,6,8; 2,6,10)	(8,8,8; 8,8,8)	4
O_3	(4,8,12; 4,8,12)	(3,4,5; 1,4,6)	(6,6,6; 6,6,6)	(2,5,8; 2,5,8)	2
Demand ($\tilde{b}_j^{IFN}$)	(3,4,5; 3,4,5)	(2,6,10; 1,6,11)	(9,9,9; 9,9,9)		$R=12$
Col. diff	3	4	5	$C=12$	

The problem given in Table 3, transformed in Table 4 by using the Stage 2 and assign first allocation using stage 4 of proposed method.

Table 4: *First Allocation Table*

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	***Supply*** $(\tilde{a}_i^{IFN})$	***Row diff***
$\boldsymbol{O_1}$	(8,10,12; 6,10,14)	(4,4,4; 4,4,4) $\boxed{(\mathbf{4,6,8;3,6,9})}$	(10,15,20; 10,15,20)	~~(4,6,8; 3,6,9)~~ **0**	**6**
$\boldsymbol{O_2}$	(3,3,3; 3,3,3)	(6,12,18; 6,12,18)	(4,6,8; 2,6,10)	(8,8,8; 8,8,8)	4
$\boldsymbol{O_3}$	(4,8,12; 4,8,12)	(3,4,5; 1,4,6	(6,6,6; 6,6,6)	(2,5,8; 2,5,8)	2
Demand $(\tilde{b}_j^{IFN})$	(3,4,5; 3,4,5)	~~(2,6,10; 1,6,11)~~ (-6,0,6; -8,0,8)	(9,9,9; 9,9,9)		$\boldsymbol{R = 12}$
Col. diff	3	4	5	$C = 12$	

Using stage 4 of proposed method remove S_1 from Table 4. New reduced shown in Table 5 again apply the proposed procedure.

Table 5: *New Reduced Table*

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	***Supply*** $(\tilde{a}_i^{IFN})$	***Row Diff***
$\boldsymbol{O_2}$	(3,3,3; 3,3,3)	(6,12,18; 6,12,18)	(4,6,8; 2,6,10)	(8,8,8; 8,8,8)	4
$\boldsymbol{O_3}$	(4,8,12; 4,8,12)	(3,4,5; 1,4,6)	(6,6,6; 6,6,6)	(2,5,8; 2,5,8)	2
Demand $(\tilde{b}_i^{IFN})$	(3,4,5; 3,4,5)	(−6,0,6; −8,0,8)	(9,9,9; 9,9,9)		$R = 6$
Col. diff	2	4	0	$\boldsymbol{C = 6}$	

Table 6: *Second Allocation table*

	$\boldsymbol{D_1}$	$\boldsymbol{D_2}$	$\boldsymbol{D_3}$	***Supply*** $(\tilde{a}_i^{IFN})$	***Row Diff***
$\boldsymbol{O_2}$	(3,3,3; 3,3,3)	(6,12,18; 6,12,18)	(4,6,8; 2,6,10)	(8,8,8; 8,8,8)	4
$\boldsymbol{O_3}$	(4,8,12; 4,8,12)	(3,4,5; 1,4,6) $\boxed{(\mathbf{-6,0,6;-8,0,8})}$	(6,6,6; 6,6,6)	~~(2,5,8; 2,5,8)~~ (−4,5,14; −6,5,16)	2
Demand $(\tilde{b}_i^{IFN})$	(3,4,5; 3,4,5)	~~(−6,0,6; −8,0,8)~~ **0**	(9,9,9; 9,9,9)		$R = 6$
Col. diff	2	4	0	$\boldsymbol{C = 6}$	

Again, applying Stage 5 of proposed strategy, all allocations are made as shown in Table 7.

Table 7: *Final allocation table*

	D_1	D_2	D_3	$Supply\ (\tilde{a}_i^{IFN})$
O_1		$(\mathbf{4},\mathbf{6},\mathbf{8};\ \mathbf{3},\mathbf{6},\mathbf{9})$		(4,6,8; 3,6,9)
O_2	$(\mathbf{3},\mathbf{4},\mathbf{5};\ \mathbf{3},\mathbf{4},\mathbf{5})$		$(\mathbf{3},\mathbf{4},\mathbf{5};\ \mathbf{3},\mathbf{4},\mathbf{5})$	(8,8,8; 8,8,8)
O_3		$(\mathbf{-6},\mathbf{0},\mathbf{6};\ \mathbf{-8},\mathbf{0},\mathbf{8})$	$(\mathbf{4},\mathbf{5},\mathbf{6};\ \mathbf{4},\mathbf{5},\mathbf{6})$	(2,5,8; 2,5,8)
$Demand\ (\tilde{b}_j^{IFN})$	(3,4,5; 3,4,5)	(2,6,10; 1,6,11)	(9,9,9; 9,9,9)	

IF optimum solution in terms of TIFN's:

$$\tilde{x}_{12}{}^{IFN} = (4,6,8;3,6,9), \tilde{x}_{21}{}^{IFN} = (3,4,5;3,4,5), \tilde{x}_{23}{}^{IFN} = (3,4,5;3,4,5),$$
$$\tilde{x}_{32}{}^{IFN} = (-6,0,6;-8,0,8), \tilde{x}_{33}{}^{IFN} = (4,5,6;4,5,6).$$

Hence, total TIFTP of type – 3 optimum cost = (41,90,139; 21,90,159).

6. Conclusion

In this article, a new method has evolved which provides the opportunity to find the optimal objective value of the TIFTP of Type 3in terms of mixed intuitionistic fuzzy numbers. Based on current examination, it very well may be presumed that it is much simple to apply proposed strategy when contrasted with existing techniques [7], for tracking down the ideal optimum solution of TIFTP of type – III. Consequently, it is smarter to utilize the proposed technique rather than existing strategies for tackling TIFTP of type – III.

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