

# COMPREHENSIVE OPTIMIZATION OF SIGNOMIALLY COMBINED-NUMERAL NONLINEARITY CODING PROBLEMS WITH FREE VARIABLES QUANTITY

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## Abstract

*Combined-numeral nonlinearity coding problem (CNNLCP) troubles concerning usual restrictions and empirical roles and constant then numeral variable quantity frequently appear in a production project, substance method business, and organization. Even though several optimize techniques need to be established for CNNLCP troubles, these techniques can hold signal relationships together with a particular variable quantity. Thus, this analysis intends a different approach used to explain a signal CNNLCP trouble and set free variable amount towards achieving an internationally optimum explanation. The signal CNNLCP trouble is initially converted into an individual with one certain variable quantity. However, the changed trouble is redeveloped as a curving combined-numeral system as the Convexness of the approaches and piecewise linearization systems. A comprehensive optimal signal CNNLCP trouble can ultimately be realized inside the acceptable inaccuracy. Algebraic models are also introduced to establish the effectiveness of the recommended approach.*

**Keywords:** Comprehensive Optimize, Combined-numeral nonlinearity coding, Set free variable quantity, Convexness.

## I. Introduction

Combined-numeral nonlinearity coding (CNNLCP) troubles concerning together constant and distinct variable quantity rise in several claims of a production project, substance method [12,14,26], for instance, combination and project of partings [1–4], no intricate isothermal apparatus webs [20], stage symmetry [28] and frame-conversation webs [29]. Biegler and Grossmann [7] demonstrate optimized procedures that have been affected in development techniques planning. They revealed that pattern and creation troubles had been controlled by nonlinearity coding and CNNLCP types. Floudas et al.. [13] indicated the investigation activity into comprehensive Optimize for 1998–2003, together with the determinist universal optimize improvements into CNNLCPs and connected products. Along with the expanding dependence on demonstrating optimized troubles in functional troubles, several hypothetical and algorithmic influences of CNNLCP have been planned. Though these troubles regularly consist of noncurved roles, the standard local optimize techniques cannot

be dealt with to ensure comprehensive optimality. Used to discuss the nonconvexities in CNNLCP troubles, the established procedures can be separated into binary attitudes.

The stochasticity processes consist of arbitrary factors in their pursuit and are dependent on an arithmetical dispute to demonstrate their merging. For example, Salcedo et al. [32] proposed an increased arbitrary examination process for explaining nonlinearity optimize troubles. Hussain and Al-Sultan [19] planned a fusion system for noncurved function minimization by applying the natural method to create examine instructions. Yiu et al. [39] established a Combined slope attitude shown on a virtual hardening process and slope-established system to explain multidimensional noncurved uninterrupted optimize troubles. The experimental method is a variation of stochasticity techniques, for example, the restriction analyses system [16]. The collection of all aspirant explanations that can be produced in each repetition must vary on the present repetition position and be changed by eliminating a subgroup of contestant explanations known as tabu. The meaning of which contestant results are tabu goes upon the changes that have got be there created among current repetition positions. While the tabu analyses are more efficient than virtual galvanizing, these stochasticity systems stated above cannot ensure discovering the universal optimum. Hence, the worth of the explanation is not confirmed. Likewise, the likelihood of finding the universal description reduces when the difficulty volume strengthens.

Determinist procedures in a typical analysis of optimizing methods [7,17,18], several determinist approaches for curved CNNLCP troubles have been evaluated. The processes contain area and constrained (CAC) [9,22,33], widespread binges decay(WBD) [15], outward estimate (OE) [10,11,31], continued reducing plane technique (CRPT)) [37],and simplified disjunctive coding (SDC) [21]. The CAC system can only get the universal explanation when each subproblem can be explained worldwide optimality. The WBD system, the OE system, and the CRPT system cannot explain CNNLCP troubles with noncurved restrictions since the troubles cannot develop a distinctive optimum in the resolution method. Lee and Grossmann [21] planned a resolution system for the SDC simulations that parallel distinct/permanent optimization troubles involving disconnections and nonlinearity inequities and reasoning proposals. The empirical roles and the restrictions in the GDP trouble are expected to be curved and constrained. Maranas and Floudas [25] required a process to produce curved estimators for universal geometric coding challenges via the hollow words' index conversion and straight dryness. Adjiman et al. [1,2] projected two worldwide optimize techniques, SMIN- $\alpha$  BB and GMIN- $\alpha$  BB, for noncurved CNNLCP established on the model of separate off-and-constrained and trust on Optimize or period-created changing-required updates to improve productivity. Even though one likely method to avoid noncurved ties in CNNLCP shows is a reformulation, for example, applying the index revolution to deal with the simplified symmetrical coding troubles.

In which a signal phrase  $x_1^\alpha x_2^\beta$  is assigned into an index term  $e^{a\ln x_1 + \beta \ln x_2}$  [12,14,26], the index alteration procedure can only be utilized to precisely certain variable quantity and is thus incapable of trading with noncurved CNNLCP difficulties with set free variable amount. Pörn et al. [30] announced separate Convexness approaches for converting noncurved CNNLCP troubles into curved issues and explaining them by a CNNLCP solver. They recommended an easy conversion,  $x + \tau = e^x$ , to deal with a free distinct varying. Introducing the converted effect into the earliest indication conditions will bring different signal periods, growing computational complications.

## II. Transformation of free variables

The mathematical formulation of a signomial CNNLCP problem with free variables considered in this study is expressed as follows:

$$\begin{aligned} & \text{Minimize } f(x, y) \\ & \text{Subjects to } g_i(x, y) \leq 0, \quad i = 1, 2, 3, \dots, I \end{aligned} \tag{1}$$

$$x = (x_1, x_2, \dots, x_p, x_{p+1}, \dots, x_n), x_i \leq x_i \leq x^i \tag{2}$$

$$y = (y_1, y_2, \dots, y_q, y_{q+1}, \dots, y_m), y_i \leq y_i \leq y^i \tag{3}$$

where  $x_i \in R^+$  for  $1 \leq i \leq p$ ,  $x_i$  are constrained set free variable quantity for  $1 + p \leq i \leq n$ ,  $y_j$  are +ve number /distinct variable quantity for  $1 \leq j \leq q$ ,  $y_j$  are constrained number /distinct variable quantity for  $q + 1 \leq j \leq m$ ,  $f(x, y)$  and  $g_i(x, y)$  are Combined-numeral signal roles,  $x_i$ , and  $x^i$  are more diminutive and more significant boundaries of the permanent variable quantity  $x_i$ , and  $y_j$  and  $y^j$  are more minor and more significant boundaries of the distinct variable quantity  $y_j$ , respectively.

Let  $x_i = x_i^+ - x_i^-, x_i^+ x_i^- \geq 0, \text{ for } i = p + 1, \dots, n, \tag{4}$

$$y_j = y_j^+ - y_j^-, y_j^+ y_j^- \geq 0, \text{ for } j = q + 1, \dots, m, \tag{5}$$

And nonlinearity relationships  $x_i^{\alpha_i}$  and  $y_j^{\beta_j}$  are expressed as

$$x_i^{\alpha_i} = (x_i^+)^{\alpha_i} + (-1)^{\alpha_i} (x_i^-)^{\alpha_i}, \alpha_i \in Z, \text{ for } i = p + 1, \dots, n, \tag{6}$$

$$y_j^{\beta_j} = (y_j^+)^{\beta_j} + (-1)^{\beta_j} (y_j^-)^{\beta_j}, \beta_j \in Z, \text{ for } j = q + 1, \dots, m, \tag{7}$$

If  $x_i^+ > 0$  and  $x_i^- = 0$ , then  $x_i$  is positive. Otherwise, if  $x_i^- > 0$  and  $x_i^+ = 0$ , then  $x_i$  is positive.

Remark 1

Let,  $x_i = x_i^+ - x_i^-, x_i^+ x_i^- \geq 0$ , and  $x_i^+$  and  $x_i^-$  the resulting inequities.

$$\begin{aligned} x_i^+ &\leq x_i^- \theta_i \\ x_i^- &\leq x_i^-(\theta_i - 1). \text{ where } \theta_i \in [0,1]. \end{aligned}$$

### I. Proposition

Let  $x_i^- \in R, 0 \leq x_i^+ \leq x_i^-, \lambda_i \in [0,1], \epsilon_0 \leq \bar{x}_i^+ \leq x_i^-, \epsilon_0 > 0$ ,

Then,  $x_i^+ = \bar{x}_i^+, \lambda_i \Rightarrow \begin{cases} (i) 0 \leq x_i^+ \leq x_i^-, \lambda_i \\ (ii) x_i^-(\lambda_i - 1) + \bar{x}_i^+ \leq x_i^+ \leq \bar{x}_i^+. \end{cases}$

Proof :

If  $x_i^+ = 0, \Rightarrow (i)$  is initiated, then  $\lambda_i = 0$ , hence  $\bar{x}_i^+ \lambda_i = 0$  then  $x_i^+ = \bar{x}_i^+ \lambda_i$

If  $x_i^+ > 0 \Rightarrow (ii)$  is initiated, then  $\lambda_i = 1$ , hence  $x_i^+ = \bar{x}_i^+$  then  $x_i^+ = \bar{x}_i^+ \lambda_i$

If  $\bar{x}_i^+ \lambda_i = 0 \Rightarrow \lambda_i = 0$  and  $(i)$  is initiated, hence  $x_i^+ = 0$  then  $x_i^+ = \bar{x}_i^+ \lambda_i$

If  $\bar{x}_i^+ \lambda_i > 0, \Rightarrow \lambda_i = 1$  and  $(ii)$  is initiated hence  $x_i^+ = \bar{x}_i^+$  and  $x_i^+ = \bar{x}_i^+ \lambda_i$

$x_i^+ = \bar{x}_i^+ \lambda_i$  is determined.

Now denote  $z^+$  and  $\tilde{z}^+$  as below :

$$z^+ = x_1^{\alpha_1} \dots x_p^{\alpha_p} (x_{p+1}^+)^{\alpha_{p+1}} \dots (x_n^+)^{\alpha_n} \text{ and}$$

$$\tilde{z}^+ = x_1^{\alpha_1} \dots x_p^{\alpha_p} (\tilde{x}_{p+1}^+)^{\alpha_{p+1}} \dots (\tilde{x}_n^+)^{\alpha_n}, \text{ where } \tilde{x}_i^+ \text{ are positive variables.}$$

From Proposition 1,

$$\begin{aligned} z^+ &= x_1^{\alpha_1} \dots x_p^{\alpha_p} (\tilde{x}_{p+1}^+ \lambda_{p+1})^{\alpha_{p+1}} \dots (\tilde{x}_n^+ \lambda_n)^{\alpha_n} \text{ and it is clear that} \\ z^+ &= \tilde{z}^+ \lambda_{p+1} \dots \lambda_n, \lambda_i \in [0,1] \end{aligned} \tag{8}$$

Remark 2

Let,  $\lambda, \lambda_i \in [0,1]$  for  $i = p + 1, \dots, n$ . then :

$$\lambda = \lambda_{p+1} \lambda_{p+2} \dots \lambda_n \Rightarrow \begin{cases} (i) \lambda \leq \lambda_i \text{ for } i = p + 1, \dots, n, \\ (ii) \lambda \geq \sum_{i=p+1}^n \lambda_i - n + p + 1 \end{cases}$$

By discussing Remark 2, Eq (8) becomes

$$z^+ = \bar{z}^+\lambda, \quad \lambda \in [0,1]. \tag{9}$$

From Proposition 1, Eq9 is equivalent to the following two linear inequalities.

- (i)  $0 \leq z^+ \leq \bar{z}^+\lambda,$
- (ii)  $\bar{z}^+ + \bar{z}^+(\lambda - 1) \leq z^+ \leq \bar{z}^+.$

$\lambda \in \{0,1\}, \bar{z}$  is an upper bound of  $z^+$

### III. Classification of curved relationships and curved relaxation approaches

Convexness policies used for signal periods are essential techniques for worldwide optimization efforts. Sun et al. [34] planned a Convexness technique used for international optimize efforts with unmodulated roles in various restricted situations. Wu et al.[38] established a more than typical Convexness, then coalification conversion was used to explain a standard worldwide optimize challenge and specific unmodulated estates. CNNLCP problem can be redeveloped with several Convexness techniques interested in a new curved Combined-numeral program resolvable to achieve an almost universal optimum. Björk et al. [8]planned a worldwide optimized system created on Convexness signal conditions. They examined that the correct selection of revolution for Convexness noncurved signal conditions strongly impacts the effectiveness of the optimized method. Tsai et al. [36] also recommended Convexness systems for the signal conditions with trio variable quantity. This analysis introduces general Convexness systems and laws to convert a CNNLCP problem into a curved Combined-numeral system.

#### I Proposition

Let ,  $f(x) = c \prod_{i=1}^n x_i^{\alpha_i}, x = (x_1, x_2, \dots \dots \dots x_n), c, x_i, \alpha_i \in R,$  for all  $i$  , is curved if  $c \leq 0, x_i \geq 0, \alpha_i \geq 0$  (for  $i = 1,2, \dots \dots \dots n$ )  $1 - \sum_i^n \alpha_i \geq 0$

Proof :

Let  $H_i(x)$  be the most crucial trivial of a Hessian matrix  $H(x)$  of  $f(x)$ . The determinant of  $H_i(x)$   $\det H_i(x) = (-1)^i \left( \prod_{j \in J_i} c \alpha_j x_j^{i \alpha_j - 2} \right) \left( \prod_{j \in J, j \neq i} x_j^{i \alpha_j} \right) \left( 1 - \sum_{j \in J_i} \alpha_j \right)$ .

Since,  $\det H_i(x) \geq 0$  when  $c \leq 0, x_i \geq 0, \alpha_i \geq 0$  for all  $i$  and  $1 - \sum_i^n \alpha_i \geq 0, H_i(x) i = 1,2, \dots \dots \dots n$ .

Corollary :

1 Let  $f(x) = c \prod_{i=1}^n x_i^{\alpha_i}, x = (x_1, x_2, \dots \dots \dots x_n), c, x_i, \alpha_i \in R,$  for all  $i$  , is curved if  $c \leq 0, x_i \geq 0, \alpha_i \geq 0$  (for  $i = 1,2, \dots \dots \dots n$ )

#### II Proposition

A nonlinearity relationship

$s = x_1^{\alpha_1} x_2^{\alpha_2} \dots \dots \dots x_n^{\alpha_n},$  where  $x_1, x_2, \dots \dots \dots x_n > 0, \alpha_i < 0$  (for  $i = 1,2,3 \dots \dots k$ ), and  $\alpha_i \geq 0$  ( $i = k + 1, k + 2, \dots \dots \dots n$ ),

- (i)  $s = \prod_{i=1}^k x_i^{\alpha_i} \prod_{i=k+1}^n z_i^{-\alpha_i},$
- (ii)  $z_i + L(-x_i^{-1}) \leq 0$  for  $i = k + 1, k + 2, \dots \dots \dots n$
- (ii)  $x_i^{-1} - z_i \leq 0 + L(-x_i^{-1}) \leq 0$  for  $i = k + 1, k + 2, \dots \dots \dots n$

Proof :

$$L(-x_i^{-1}) = -x_i^{-1}, z_i = x_i^{-1} \text{ for } i = k + 1, k + 2, \dots \dots n,$$

after (ii) and (iii) since  $z_i > 0$  and  $-\alpha_i \leq 0$  for  $i = k + 1, k + 2, \dots \dots n, s$  is then a curved period describing to corollary1.

the piecewise straight function  $L(f(x))$  for approaching the hollow role  $f(x)$  [27,35]. Splitting should be achieved to close the gap since a large enough reduction can be close to the earliest nonlinearity problem in any to define beforehand precision. Splitting programs for typical SOS Class 2 cases can be noticed, for example, in [5,6].

$\left| \frac{f(x)-L(f(x))}{f(x)} \right|$  is utilized to assess the inaccuracy in the straight calculation. Assume  $f(x)$  is an empirical task and  $x^*$  results from the converted system. In that case, the linearity makes non involve delicacy till  $\left| \frac{f(x^*)-L(f(x^*))}{f(x^*)} \right| \leq \varepsilon_2$ , where  $\varepsilon_2$  is the optimum acceptance. If  $g(x) < 0$  is a restriction and  $x^*$  is the result, then  $x^*$  is achievable if  $\left| \frac{f(x^*)-L(f(x^*))}{f(x^*)} \right| \leq \varepsilon_1$  and  $L(g(x^*)) < \varepsilon_1$  where  $\varepsilon_1$  is the feasibility acceptance.

### III Proposition

Let,  $s = -x_1^{\alpha_1} x_2^{\alpha_2} \dots \dots x_n^{\alpha_n}$  where  $x_1, x_2 \dots \dots x_n > 0, 0 \leq \alpha_1 \leq \alpha_2 \leq \dots \leq \alpha_k, , 0 \geq \alpha_{k+1} + \alpha_{k+2} \geq \dots \geq \alpha_n$  and  $\sum_{i=1}^r \alpha_i < 1$  for some most extensive numeral  $r$ , such that  $r \leq k$ ,

$$S = - \prod_{i=1}^r x_i^{\alpha_i} \prod_{i=r+1}^n z_i^{\beta}, \beta = \frac{1 - \sum_{i=1}^r \alpha_i}{n - r},$$

- $z_i + L\left(-x_i^{\frac{\alpha_i}{\beta}}\right) \leq 0$
- $-x_i^{\frac{\alpha_i}{\beta}} - z_i \leq 0$

Where  $L\left(-x_i^{\frac{\alpha_i}{\beta}}\right)$  is piecewise linearization function of a hollow period  $-x_i^{\frac{\alpha_i}{\beta}}$

Proof :

$$\left(-x_i^{\frac{\alpha_i}{\beta}}\right) = -x_i^{\frac{\alpha_i}{\beta}}, z_i = x_i^{\frac{\alpha_i}{\beta}} \text{ for } i = r + 1, r + 2, \dots \dots n,$$

Since  $\alpha_i > 0$  for  $i = 1, 2, \dots \dots r, z_i > 0$  for  $i = k + 1, k + 2, \dots \dots n, \beta > 0$  and  $\sum_{i=1}^r \alpha_i + (n - r)\beta = 1. s$

#### Remark 3

Let  $f(x) = x^\alpha$  for  $x > 0$  is curved at what time  $\alpha \leq 0$  or  $\alpha \geq 1$ .  $f(x)$  is hollow at what time  $0 \leq \alpha \leq 1$ .

#### Remark 4

Let  $y \in \{d_1, d_2 \dots \dots d_m\}$   $d_{j+1} > d_j > 0$  for  $j = 1, 2, 3, \dots \dots m - 1$   
 $y^\alpha = \sum_{j=1}^m d_j^\alpha u_j$ , where  $\sum_{j=1}^m u_j = 1$   $u_j \in \{0, 1\}$

#### Remark 5

Let  $s = u f(x)$  wherever  $f(x)$  is a straight serve is equal to the resulting straight variations:

- $\overline{f(x)}(u - 1) + f(x) \leq s \leq \overline{f(x)}(u - 1) + f(x).$
- $-\overline{f(x)}u \leq s \leq \overline{f(x)}u,$

Where  $u \in \{0, 1\}, s$  is an unobstructed in symbol flexible, and  $\overline{f(x)}$  is the greater duty-bound of  $f(x)$ ,

## IV. Examples

### I. Example

Minimize  $x_1^2 x_2^{-2} x_3 - 2x_2^{0.7} x_3^{0.2} + x_4 x_5^{-2} - 2x_1 - 4x_3$   
 Subject to  $x_1 + 6x_2 - x_3 - 5x_4 \leq 2,$  (10)

$$x_3^{1.5}x_4 + 0.5x_2 + 3x_1 \leq -10 \tag{11}$$

$$\begin{aligned} -x_1 - 0.5x_4 + x_5 &\leq 6, \\ -7 \leq x_1 \leq 5, \quad 1 \leq x_2 \leq 10, \quad 1 \leq x_3 \leq 5, \quad 2 \leq x_4 \leq 8, \quad 2 \leq x_5 \leq 9, \\ x_1, x_2, x_4, x_5 &\in R, \quad x_3 \in Z. \end{aligned} \tag{12}$$

By Remark 1,

$$\begin{aligned} \text{Minimize} \quad & x_1 = x_1^+ - x_1^-, \quad x_1^+, x_1^- \geq 0, \\ & (x_1^+)^2 x_2^{-2} x_3 + (x_1^-)^2 x_2^{-2} x_3 - 2x_2^{0.7} x_3^{0.2} + x_4 x_5^{-2} - 2x_1 - 4x_3 \\ \text{Subject to} \quad & x_1 = x_1^+ - x_1^-, \end{aligned} \tag{13}$$

$$x_1^+ \leq 5\theta_1, \tag{14}$$

$$x_1^- \leq 7(\theta_1 - 1), \tag{15}$$

$$x_1^+ - x_1^- + 6x_2 - x_3 - 5x_4 \leq 2 \tag{16}$$

$$x_3^{1.5}x_4 + 0.5x_2 + 3x_1^+ - 3x_1^- \leq -10 \tag{17}$$

$$\begin{aligned} -x_1^+ + x_1^- - 0.5x_4 + x_5 &\leq 6, \\ 0 \leq x_1^+ \leq 5, \quad 0 \leq x_1^- \leq 7, \quad 1 \leq x_2 \leq 10, \quad 1 \leq x_3 \leq 5, \quad 2 \leq x_4 \leq 8 \\ &, \quad 2 \leq x_5 \leq 9, \quad \theta_1 \in \{0,1\}, \quad x_2, x_4, x_5 \in R, \quad x_3 \in Z \end{aligned} \tag{18}$$

Now we familiarize two severely +ve variables  $\tilde{x}_1^+, \tilde{x}_1^-$  as follows:

$$0 \leq x_1^+ \leq 5\lambda_1, \tag{19}$$

$$\tilde{x}_1^+ + 5(\lambda_1 - 1) \leq \tilde{x}_1^+ \leq \tilde{x}_1^+ \tag{20}$$

$$0 \leq x_1^- \leq 7\lambda_2 \tag{21}$$

$$\tilde{x}_1^- + 7(\lambda_2 - 1) \leq \tilde{x}_1^- \leq \tilde{x}_1^- \tag{22}$$

For computer implementation,

$\tilde{x}_1^+, \tilde{x}_1^- \geq \varepsilon_0$  where  $\varepsilon_0 = 10^{-7}$  is a zero acceptance. The signomial times  $z_1^+ = (x_1^+)^2 x_2^{-2} x_3$  and  $z_1^- = (x_1^-)^2 x_2^{-2} x_3$ , respectively, where  $0 \leq z_1^+ \leq \bar{z}\lambda_1$ ,  $\tilde{z}_1^+ + \bar{z}(\lambda_1 - 1) \leq z_1^+ \leq \tilde{z}_1^+$ ,  $0 \leq z_1^- \leq \bar{z}\lambda_2$ ,  $\tilde{z}_1^- + \bar{z}(\lambda_2 - 1) \leq z_1^- \leq \tilde{z}_1^-$

. From Proposition 2, the nonlinearity term

$$-2x_2^{0.7} \cdot x_3^{0.2} \text{ is curved.}$$

The noncurved relationships  $x_3^{1.5}x_4$  and  $x_4x_5^{-2}$  can be changed into curved relations and  $z_4^{-1}x_5^{-2}$ , respectively, anywhere  $z_3 = x_3^{-1}$  and  $z_4 = x_4^{-1}$

According to Remark 4,

$$z_3 = x_3^{-1} \text{ can be linearized as } z_3 = u_1 + \frac{1}{2}u_2 + \frac{1}{3}u_3 + \frac{1}{4}u_4 + \frac{1}{5}u_5 \text{ where } x_3 = u_1 + 2u_2 + 3u_3 + 4u_4 + 5u_5.$$

- The noncurved relationships  $(\tilde{x}_1^+)^2 x_2^{-2} x_3$  and  $(\tilde{x}_1^-)^2 x_2^{-2} x_3$  can be transferred into curved relationships  $e^{2y_1^+ - 2y_2 + y_3}$  and  $e^{2y_1^- - 2y_2 + y_3}$ , respectively, where  $y_1^+ = \ln \tilde{x}_1^+$ ,  $y_1^- = \ln \tilde{x}_1^-$ ,  $y_2 = \ln x_2$  and  $y_3 = \ln x_3$ .

$$\begin{aligned} \text{Minimize} \quad & z_1^+ + z_1^- - 2x_2^{0.7} \cdot x_3^{0.2} + z_4^{-1}x_5^{-2} - 2x_1 - 4x_3 \\ \text{subject to} \quad & x_1 = x_1^+ - x_1^- \\ & (x_1^+ - x_1^- + 6x_2 - x_3 - 5x_4 \leq 2) - (\tilde{x}_1^- + 7(\lambda_2 - 1) \leq x_1^- \leq \tilde{x}_1^-) \\ & z_3^{-1.5}x_4^{-1} + 0.5x_2 + 3x_1^+ - 3x_1^- \leq -10, \\ & y_1^+ = L(\ln \tilde{x}_1^+), \quad y_1^- = L(\ln \tilde{x}_1^-), \quad y_2 = L \ln x_2, \\ & y_3 = u_1 \ln 1 + u_2 \ln 2 + u_3 \ln 3 + u_4 \ln 4 + u_5 \ln 5, \\ & 0 \leq z_1^+ \leq \bar{z}\lambda_1, \quad e^{2y_1^- - 2y_2 + y_3} + \bar{z}(\lambda_1 - 1) \leq z_1^+ \leq L(e^{2y_1^- - 2y_2 + y_3}), \\ & 0 \leq z_1^- \leq \bar{z}\lambda_2, \quad e^{2y_1^- - 2y_2 + y_3} + \bar{z}(\lambda_2 - 1) \leq z_1^- \leq L(e^{2y_1^- - 2y_2 + y_3}), \\ & x_3 = u_1 + 2u_2 + 3u_3 + 4u_4 + 5u_5, \end{aligned}$$

$$\begin{aligned}
 z_3 &= u_1 + \frac{1}{2}u_2 + \frac{1}{3}u_3 + \frac{1}{4}u_4 + \frac{1}{5}u_5 \\
 u_1 + u_2 + u_3 + u_4 + u_5 &= 1 \\
 x_4^{-1} - z_4 &\leq 0, z_4 + L(-x_4^{-1}) \leq 0, \\
 \epsilon_0 \leq \tilde{x}_1^+ &\leq 5, \epsilon_0 \leq \tilde{x}_1^- \leq 7, 1 \leq x_2 \leq 10, 1 \leq x_3 \leq 5, 2 \leq x_4 \leq 8, 2 \leq x_5 \leq 9, \\
 \theta_1, \lambda_1, \lambda_2 &\in \{0,1\}, u_1, u_1, u_1, u_1, u_1 \in \{0,1\}, \tilde{x}_1^+, \tilde{x}_1^-, x_2, x_4, x_5 \in R, x_3 \in Z
 \end{aligned}$$

The optimality acceptance and probability acceptance are within the prespecified error of 0.001. The universally optimal explanation found is  $(x_1, x_2, x_3, x_4, x_5) = (-5.353, 4.548, 1, 3.787, 2.541)$  along with the empirical cost is 2.803.

Table 1:

Number of variables in the reformulated model of each example				
	Quantity of originals variable quantity	Quantity of further constant variable quantity	Quantity of further second variable quantity	Quantity of originals variable quantity eradicated from construction
Example 1	5	206	206	1
Example 2	2	4	4	1
Example 3	4	201	201	1

II. Example

$$\begin{aligned}
 \text{Minimize} \quad & x_1^{0.5}x_2 + 3\ln x_1, \\
 \text{Subject} \quad & -x_1 + x_2 \leq 5, \\
 & x_1^{0.5} - x_2 \leq 6, \\
 & x_1 \in \{0.1, 0.5, 0.7, 1.2\}, -6 \leq x_2 \leq 4.
 \end{aligned}$$

The nonlinearity relationships  $x_1^{0.5}x_2, 3\ln x_1$  and  $x_1^{0.5}$  are nonconvex roles. By Remarks 4 and 5,

$$\begin{aligned}
 \text{Minimize} \quad & 0.1^{0.5}s_1 + 0.5^{0.5}s_2 + 0.7^{0.5}s_3 + 1.2^{0.5}s_4 + 3(u_1 \ln 0.1 + u_2 \ln 0.5 + u_3 \ln 0.7 + u_4 \ln 1.2) \\
 \text{Subject} \quad & -0.1u_1 - 0.5u_2 - 0.7u_3 - 1.2u_4 + x_2 \leq 5, \\
 & u_1 + u_2 + u_3 + u_4 = 1, \\
 & 0.1^{0.5}u_1 + 0.5^{0.5}u_2 + 0.7^{0.5}u_3 + 1.2^{0.5}u_4 - x_2 \leq 6, \\
 & -6u_i \leq s_i \leq 6u_i, 6(u_i - 1) + x_2 \leq s_i \leq 6(1 - u_i) + x_2, i = 1,2,3,4, s_1, s_2, s_3, s_4 \\
 & \text{are unrestricted in sign variables, } u_1, u_2, u_3, u_4 \in \{0,1\}, -6 \leq x_2 \leq 4.
 \end{aligned}$$

The converted sequencer can be resolved by LINGO [24] to find the universally optimum explanation  $(x_1, x_2) = (0.2, -5.753)$  and the empirical charge -8.705 contained by the optimality acceptance 0.001 as the probability acceptance 0.001.

III. Example

$$\begin{aligned}
 \text{Minimize} \quad & x_1x_4^3 - x_3 - 0.5x_1^2x_2^4 \\
 \text{Subject} \quad & x_1x_4^{1.5} - x_2 - x_2^{0.5}x_3^{0.4} \leq 4, \\
 & -x_1 - 2x_2 + x_3 \leq -2, \\
 & 0 \leq x_1 \leq 6, 1 \leq x_2 \leq 10, 1 \leq x_3 \leq 6, 20 \leq x_4 \leq 30, x_1, x_2, x_3, x_4 \in R
 \end{aligned}$$

The nonlinearity relationships  $x_1x_4^3, x_1^2x_2^4$  four and  $x_1x_4^{1.5}$  where  $x_1$  has zero-value smaller constrained Table 1 listing the quantity of variable quantity applied in the converted standard of Instance 3. Though the intended system needs the accumulation of variable another amount,

variable dual amount, and restrictions, it can prevent imprecision by requiring a little  $\epsilon > 0$  smaller bound for  $x_1$ . Explaining this system by the planned technique with LINGO [24], the universally optimal explanation achieved is smaller, particularly for  $x_1$ . Demonstrating this system by the intended design and LINGO [24], the universally optimum description achieved is  $(x_1, x_2, x_3, x_4) = (0, 4, 6, 20)$  and the empirical rate is  $-6$ . Still, explaining this system by really requiring  $x_1 \geq 0.001$ , the universally optimal explanation achieved is  $(x_1, x_2, x_3, x_4) = (0.001, 10, 6, 20)$ , and the empirical rate is 1.995.

## V. Conclusions

This analysis intends an optimized technique to discuss a signal CNNLCP difficulty and set the free variable quantity to achieve a comprehensive optimum. The free variable quantity practical trading techniques change over the variable amount and translate the analytical association among the variable quantity in a result period into a set of straight inequities, appropriately combined into the CNNLCP types. Several valuable instructions to essentially Convexness more than universal signal conditions in CNNLCP systems are also produced. Numerical illustrations are demonstrated to provide for the impacts of the recommended system.

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