SELECTION OF SINGLE SAMPLING PLANS BY VARIABLES BASED ON GENERALIZED BETA DISTRIBUTION

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Abstract

Statistical quality control (SQC) has wider applications in industries and production engineering. Product control, one of the two major categories of SQC, consists in procedures by which decisions are made on the disposition of one or more lots of finished items or materials produced by manufacturing industries. Sampling inspection by variables in product is the methodology that is employed for deciding about the disposition of a lot of individual units based on the observed measurements on a quality characteristic of randomly sampled units from the lot submitted for inspection. These procedures are defined under the assumption that the quality characteristic is measurable on a continuous scale and the functional form of the probability distribution must be known. Inspection procedures which have been developed based on the implicit assumption that the quality characteristic is distributed as normal with the related properties are found in the literature of sampling inspection procedures. The assumption of normality may not be realized often in practice and it becomes inevitable to investigate the properties of variable sampling plans based on non-normal distributions. In this paper a single sampling plan by variables is formulated and evaluated under the assumption that the quality characteristic is distributed according to a generalized beta distribution of first kind. Procedures are developed for determining the parameters of the proposed plan for specified requirements in terms of producer's and consumer's protection.

Key Words: Consumer's Quality Level, Generalized Beta Distribution, Normal Distribution, Operating Characteristic Function, Single Sampling Plan, Producer's Quality Level.

1. Introduction

Sampling inspection is an activity for taking decisions on one or more lots of finished products which have been submitted for inspection. The decision of either acceptance or rejection of the lots is usually taken by adopting suitable sampling inspection procedures, called sampling plans. Sampling plans are generally categorized into two types, namely, lot-by-lot sampling by attributes and lot-by-lot sampling by variables. In lot-by-lot inspection by attributes, one or more samples of items are drawn from a given lot of manufactured items; each item in the sample(s) is classified as conforming or nonconforming; and the decision of acceptance or rejection of the lot is made based on a specific rule. In lot-by-lot inspection by variables, one or more samples of items are drawn from a given lot; the measurement of a quality characteristic in each sampled item is recorded; and the decision of acceptance or rejection of the lot is made as a function of such measurements. The theory of inspection by variables is applicable when the quality characteristic of sampled items is measurable on a continuous scale and the functional form of the probability distribution is assumed to be known. A variables sampling is advantageous in the sense that it generates more information from each item inspected, requires small sample and provides same protection when compared to attributes sampling. See, [1] and [2].

On the basis of the implicit assumption that the quality characteristic is distributed according to normal with mean μ and standard deviation σ, the concept of variables sampling inspection has been studied by many researchers. Some of the early works on variables sampling inspection are seen in [3], [4], [5], [6] and [7]. Studies relating to sampling plans when the assumption of normality of the quality characteristic fails or the functional form of the underlying distribution deviates from normal or the form of the distribution is not known are also found in the literature of acceptance sampling. [8] – [23] are few references which deal with variables inspection using non-normal distributions.

The problem of designing single sampling plans by variables, when the quality characteristic, *X*, follows a normal distribution with mean μ and standard deviation σ, has been addressed in the past. See, [24]. In the industrial situations, quite often, the assumption of normality may not be valid or the quality characteristic may be distributed according to non-normal distributions. In such cases, the selection of variable sampling plans becomes complicated. However, the literature of acceptance sampling provides procedures for the designing of variables plans when the quality characteristic follows a probability distribution other than normal. A detailed survey on various works related to variable sampling plans with emphasis on non-normality is given in [11]. A computer-aided procedure has been developed in [25] for the identification of the appropriate distribution in designing sampling inspection plans by variables when the quality characteristics are defined by compositional proportions.

A generalized probability density function, termed as double bounded probability density function has been derived in [26]. It is also called a generalized beta distribution of first kind, in which the random variable *X* is defined within the range (0, 1). Practical applications of variables sampling plans using a generalized beta distribution can be visualized for bulk product inspection where the quality characteristics are the compositional proportions, such as proportion of binary mixtures of pharmaceutical powder, percentage of protein in milk powder, fatty acid composition of serum lipid fractions, etc. Sampling inspection plans for compositional fractions based on the beta distribution and the procedure for designing the plans to control the proportion nonconforming levels are discussed in [27].

In this paper, a study on single sampling plans by variables is formulated under the assumption that the quality characteristic is assumed to have a generalized beta distribution which would be appropriate in situations where the quality characteristics are compositional fractions. A procedure for determining the parameters of the proposed plan for specified requirements in terms of producer's and consumer's protection is also developed.

2. Single Sampling Inspection Plans by Variables

A single sampling inspection plan by variables is defined under the following assumptions:

(a) The quality characteristic, denoted by X, is measurable on a continuous scale and has a known form of probability distribution, represented by $F_X(x;\mu,\sigma)$, which is the distribution function of X with mean μ and variance σ^2.

(b) Each individual unit in a submitted lot has a one-sided specification, say, lower specification, L or upper specification, U. If, for a unit, $X > U$ (or $X < L$), the unit is classified as a non-conforming unit.

The operating procedure of a variable sampling plan is as follows:

Step 1: Draw a random sample of n units from a lot and observe the measurements, $x_1, x_2, \ldots, x_n$ of the quality characteristic, X.

Step 2: When σ is known, accept the lot if $\bar{x} + k\sigma \leq U$ (or $\bar{x} - k\sigma \geq L$); otherwise, reject the lot, where $\bar{x}$ is the sample mean.

When σ is unknown, accept the lot, if $\bar{x} + ks \leq U$ (or $\bar{x} - ks \geq L$); otherwise, reject the lot. Here, $s^2 = \frac{1}{n-1}\sum_{i:1}^{n}(x_i - \bar{x})^2$ is an unbiased estimate of σ^2.

Thus, a single sampling plan by variables is designated by two parameters, namely, the sample size, n, and the acceptability constant, k. When these parameters are known, the plan could be implemented. The explicit expressions for n and k can be derived by specifying two points on the operating characteristic curve of the plan, namely, $(p_1, 1-\alpha)$ and (p_2, β), where p_1 and p_2 are termed as producer's quality level (*PQL*) and the consumer's quality level (*CQL*), associated with the producer's risk, α and the consumer's risk, β, respectively. A sampling plan by variables is termed as a known σ or unknown σ plan according as σ is known or unknown.

3. Operating Characteristic Function

An important measure of performance of a variables sampling plan is its operating characteristic function, which is a function of the proportion, *p*, of non-conforming units, called incoming lot quality, and provides the probability, $P_a(p)$, of acceptance of a lot. The plot of $P_a(p)$ against *p* results in a curve, called operating characteristic (OC) curve. For a given upper specification limit, *U*, when σ is known, p and $P_a(p)$ are defined by

$$p = P(X > U|\mu) \tag{1}$$

and $P_a(p) = P(\bar{x} + k\sigma \leq U|\mu)$. (2)

PQL and *CQL*, using (1), are defined by

$$PQL = p_1 = P(X > U|\mu_1) \tag{3}$$

and $$CQL = p_2 = P(X > U|\mu_2), \tag{4}$$

where μ_1 and μ_2 are the means of the underlying distribution which results in *PQL* and *CQL*, respectively.

Assume that the random variable, *X*, is modeled by a two-parameter generalized beta distribution. The probability density function and the cumulative distribution function of the generalized beta distribution, according to [26], are respectively given by

$$f(x;a,b) = abx^{a+1}(1-x^a)^{b-1},\ 0 < x < 1 \tag{5}$$

and $$F(x) = 1-(1-x^a)^b, \tag{6}$$

where a > 0 and b > 0 are the shape parameters.

The moments about origin of the distribution are defined by

$$m_r = \frac{b\Gamma\left(1+\frac{r}{a}\right)\Gamma(b)}{\Gamma\left(1+b+\frac{r}{a}\right)},\ r = 1, 2, 3, 4, \cdots,$$

from which the measures such as mean, variance, skewness and kurtosis can be derived as $\mu = m_1$, $\sigma^2 = \mu_2$, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$ and $\beta_2 = \frac{\mu_4}{\mu_2^2}$, where $\mu_2 = m_2 - m_1^2$, $\mu_3 = m_3 - 3m_2m_1^2 + 2m_1^3$, and $\mu_3 = m_4 - 4m_3m_1 + 6m_2m_1^2 - 3m_1^4$.

From (1), (3) and (4), the lot quality levels, *p*, *PQL* and *CL* using standardized beta distribution are defined, respectively, by

$$p = P(T > K_p^*),$$

$$PQL = p_1 = P(T > K_{p_1}^*) \tag{7}$$

and $$CQL = p_2 = P(T > K_{p_2}^*), \tag{8}$$

where $T = \frac{X-\mu}{\sigma}$, $K_p^* = \frac{U-\mu \mid p}{\sigma}$, $K_{p_1}^* = \frac{U-\mu \mid p_1}{\sigma}$ and $K_{p_2}^* = \frac{U-\mu \mid p_2}{\sigma}$.

The producer's risk, α, and the consumer's risk, β, corresponding to *AQL* and *LQL* are, respectively, defined from (2) as

$$\alpha = P(rejecting\, the\, lot \mid \mu = \mu_1) = P(\bar{x} + k\sigma > U \mid \mu = \mu_1) \tag{9}$$

and $$1-\beta = P(rejecting\, the\, lot \mid \mu = \mu_2) = P(\bar{x} + k\sigma > U \mid \mu = \mu_2). \tag{10}$$

When σ is unknown, the estimate *s* is used in the decision criterion, and hence in the evaluation of α and β.

4. Designing Single Sampling Plans by Variables

In the industrial practice, the unknown standard deviation variables plans are more realistic than the known standard deviation variables plans. If the distribution is non-normal, the designing of unknown σ plans is rather complicated. Such problems introducing an expansion factor in terms of measures of skewness and kurtosis are addressed in [12], which also provides a methodology for determining the parameters of sampling plans by variables under the conditions of non-normal populations using the expansion factor. The procedures for the selection of unknown standard deviation sampling plans are provided in [23] giving protection to the producer and consumer under the assumption that the quality characteristics under study follow a Pareto distribution when the measures of skewness and /or kurtosis are specified.

4.1. Case of Unknown Sigma

The methodology proposed in [12] using the expansion factor will, now, be discussed for an unknown sigma plan by variables under the assumption of generalized beta distribution for the quality characteristic, *X*.

In the case of unknown sigma plan, the determination of n and k is usually based on the sampling distribution of $\bar{x}+ks$ or $\bar{x}-ks$. It is known that under the assumption of normal distribution, $\bar{x}$ and s are independent and distributed as normal. Therefore, $\bar{x}+ks$ and $\bar{x}-ks$ are normally distributed. Using these properties, formulae for finding the values of n and k can be obtained. The asymptotic distributions of $\bar{x}+ks$ and $\bar{x}-ks$ are shown to be normal having the means $\mu_y=\mu+k\sigma$ and $\mu_y=\mu-k\sigma$, respectively, and the common variance given by

$$\sigma_Y^2=\frac{\sigma^2}{n}\left[1+\frac{k^2}{4}(\beta_2-1)\pm k\beta_1\right], \tag{11}$$

where β_1 and β_2 represent the measures of skewness and kurtosis of the underlying distribution. Having defined $Z_p^*=\dfrac{U-\mu\,|\,p}{\sigma}$ and acceptance probability function for the case of unknown standard deviation as $P_a(p)=P_r[\bar{x}+k_U s\le U\,|\,p]=P_r(Y\le U\,|\,p)$, from [12], the expressions for Z_α, Z_β, Z^*_{PQL} and Z^*_{CQL} corresponding to *α*, *β*, *PQL* and *CQL*, respectively, are as given below:

$$Z_\alpha=\frac{U-(\mu\,|\,p_1+k_U\sigma)}{\sigma\sqrt{\dfrac{e_U}{n_U}}} \tag{12}$$

$$-Z_\beta=\frac{U-(\mu\,|\,p_2+k_U\sigma)}{\sigma\sqrt{\dfrac{e_U}{n_U}}} \tag{13}$$

$$Z^*_{PQL}=k_U+K_\alpha\sqrt{\frac{e_U}{n_U}} \tag{14}$$

$$Z^*_{CQL}=k_U-K_\beta\sqrt{\frac{e_U}{n_U}}\,. \tag{15}$$

Here, $e_U=1+\dfrac{k^2}{4}(\beta_2-1)+k\beta_1$ is the expansion factor, which can be used to obtain the known standard deviation plans. When the requirements are specified in terms of the points $(PQL,1-\alpha)$ and (CQL,β) on the OC curve such that $P_a(PQL)=1-\alpha$ and $P_a(CQL)=\beta$, the expressions for the plan parameters *n* and *k*, derived from (14) and (15), are as given below:

$$n_U=e_U\left[\frac{Z_\alpha+Z_\beta}{Z^*_{PQL}-Z^*_{LQL}}\right]^2 \tag{16}$$

and $$k_U=\frac{Z_\alpha Z^*_{CQL}+Z_\beta Z^*_{PQL}}{Z_\alpha+Z_\beta}. \tag{17}$$

In a similar way, when the lower specification limit, *L*, is specified, the expressions for n and k can be derived.

4.2. Numerical Illustration

Suppose that a set of measurements yields $\beta_1=0.0377$ and $\beta_2=2.0147$ It is desired to determine a variables sampling plan giving protection to the producer and the consumer in terms of $(PQL=0.01,\alpha=0.05)$ and $(CQL=0.06,\beta=0.10)$. For the given requirements, the values of *a* and *b* are found as *a* = 0.65 and *b* = 0.80. Associated with these values are the mean and standard deviation given by *M* = 0.5581 and *S* = 0.2545, respectively. Corresponding to *PQL* and *CQL*, the values of Z^*_{PQL} and Z^*_{CQL} are determined from (7) and (8) as 1.6717 and 1.4619, respectively. The normal deviates Z_α and Z_β are obtained as 1.645 and 1.282 by satisfying (9) and (10) for the specified sets of values of α and β. Substituting these values in (16) and (17), the parameters of the desired plan are determined as $n = 195$ and $k = 1.554$. The value of e_U is obtained as 1.67119. Thus, the parameters of a known standard deviation plan, are computed as $n'_U = \frac{n_U}{e_U} = 117$ and $k'_U = k_U = 1.554$.

4.3. Case of Known Sigma

The method of designing known sigma variables sampling plan under the assumption of Burr distribution utilizing the measures skewness and kurtosis is proposed in [15]. A similar procedure is developed here for the known sigma plan by variables when the underlying distribution is a two-parameter generalized beta distribution.

Let *M* and *S* be the mean and standard deviation of the two-parameter generalized beta distribution. Then, *PQL* and *CQL* are defined by

$$PQL = 1 - F(x_{PQL}; M, S) = (1 - x^a_{PQL})^b, a > 0, b > 0 \tag{18}$$

and
$$CQL = 1 - F(x_{CQL}; M, S) = (1 - x^a_{CQL})^b, a > 0, b > 0. \tag{19}$$

where
$$\frac{U - \mu|PQL}{\sigma} = \frac{x_{PQL} - M}{S} = Z^*_{PQL} \tag{20}$$

and
$$\frac{U - \mu|CQL}{\sigma} = \frac{x_{CQL} - M}{S} = Z^*_{CQL} \tag{21}$$

with Z^*_{PQL} and Z^*_{CQL} being the standardized values of x corresponding to *PQL* and *CQL*, respectively.

Assuming that the distribution of $\bar{x}$ is normal, α and β are defined as area under normal curve and are expressed by

$$\alpha = \int_{k_\alpha}^{\infty} \varphi(t)\,dt \tag{22}$$

and
$$1 - \beta = \int_{k_{1-\beta}}^{\infty} \varphi(t)\,dt, \tag{23}$$

where
$$\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}t^2}, \tag{24}$$

$$Z_\alpha = \sqrt{n}\,\frac{K - \mu_1}{\sigma} \tag{25}$$

and
$$Z_{1-\beta} = \sqrt{n}\,\frac{K - \mu_2}{\sigma}. \tag{26}$$

From equations (22) to (26), the expressions for *n* and *k* are, respectively, obtained as

$$n_U = \left(\frac{Z_\alpha + Z_\beta}{Z^*_{PQL} - Z^*_{CQL}}\right)^2 = \left(\frac{S[Z_\alpha + Z_\beta]}{x_{PQL} - x_{CQL}}\right)^2 \tag{27}$$

and
$$K = U - \frac{\sigma}{S}\left[\frac{Z_\alpha x_{CQL} + Z_\beta x_{PQL}}{Z_\alpha + Z_\beta} - M\right]. \tag{28}$$

If the acceptance criterion is written as $\bar{x} + k\sigma \le U$, according to [15], the expression for *k* is given by

$$k_U = \left[\frac{Z_\alpha x_{CQL} + Z_\beta x_{PQL}}{S(Z_\alpha + Z_\beta)} - \frac{M}{S}\right]. \tag{29}$$

5. Determination of n and k of a Variables Sampling Plan

The parameters of a sampling plan by variables can be derived from the generalized beta distribution when the third and fourth moments of the distribution of measurements are known or specified. It is known that the measures of skewness and kurtosis, specified by β_1 and β_2, of a generalized beta distribution are functions of the shape parameters a and b. Thus, for a specified values of β_1 and β_2, the values of a and b can be determined. In order to determine the required sampling plan by variables, the following procedure is followed:

Step 1: Specify β_1 and β_2.

Step 2: Specify the desired protection in terms of $(p_1, 1-\alpha)$ and (p_2, β).

Step 3: Choose the value a and b from Table 2 corresponding to the specified values of β_1 and β_2.

Step 4: For specified p_1 and p_2, determine x_{PQL} and x_{CQL} from $F_X(x)$, which is the cumulative distribution function of the generalized beta distribution, satisfying the equations (18) and (19), and obtain Z^*_{PQL} and Z^*_{CQL} from equations (20) and (21).

Step 5: For specified α and β, determine the normal deviates Z_α and Z_β, satisfying the equations (22) and (23).

Step 6: Determine the parameters n_σ and k_σ of the plan as n_U and k_U using equation (27) and (29).

Based on the procedure described, the parameters, n_σ and k_σ, of the sampling plans by variables for a wide range of values of PQL and CQL are obtained and given in Table 3 for various combination of values of a and b. The parameters provided in the table yield the maximum producer's risk of 5% and the maximum consumer's risk of 10%. To facilitate the computation of Z^*_{PQL} and Z^*_{CQL}, the mean, M, and standard deviation, S, are obtained for sets of values of a and b and provided in Table 1.

5.1. Numerical Illustration

It is desired to have a single sampling plan by variables when the set of measurements drawn from a generalized beta distribution has the measure of skewness and kurtosis specified as $\beta_1 = 0.0654$ and $\beta_2 = 2.1384$. Suppose that the desired protection against an upper specification limit is specified in terms of $(PQL = 0.01, \alpha = 0.05)$ and $(CQL = 0.06, \beta = 0.10)$.

Table 2 yields $a = 0.750, b = 0.50, M = 0.4156$ and $S = 0.2363$ associated with $\beta_1 = 0.0654$ and $\beta_2 = 2.1384$. The values of x_{PQL} and x_{CQL} are determined from (18) and (19) as 0.924 and 0.8458 for the specified $PQL = 0.01$ and $CQL = 0.04$. The standardized deviates Z^*_{PQL} and Z^*_{CQL} are obtained as 2.1512 and 1.8206, respectively, from (20) and (21). The values of Z_α and Z_β are determined as 1.645 and 1.282. On substitution of these values in (27) and (29), the parameters of the desired plan are determined as $n_\sigma = n_U = 78$, and $k_\sigma = 1.9654$. Table 3, when entered with the specified values of the quality levels, can be used to choose the parameters of the required plan corresponding to a = 0.75 and b = 0.50.

Table 1: Mean, M, and Standard Deviation, S of Generalized Beta Distribution

a	*b*								
	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	
0.500	0.3694	0.4063	0.4382	0.4661	0.4909	0.5132	0.5333	0.5517	M
	0.1738	0.1868	0.1969	0.205	0.2116	0.2168	0.2211	0.2246	S
0.550	0.3387	0.3759	0.4082	0.4367	0.4622	0.4852	0.506	0.5251	M
	0.1727	0.1871	0.1985	0.2076	0.2151	0.2212	0.2262	0.2303	S
0.600	0.3111	0.3484	0.381	0.4099	0.4359	0.4594	0.4808	0.5004	M
	0.1707	0.1863	0.1989	0.2091	0.2175	0.2244	0.2301	0.2349	S
0.650	0.2862	0.3234	0.3561	0.3853	0.4116	0.4356	0.4574	0.4775	M
	0.168	0.1848	0.1985	0.2097	0.2189	0.2266	0.233	0.2384	S
0.700	0.2638	0.3007	0.3334	0.3627	0.3893	0.4135	0.4357	0.4562	M
	0.1648	0.1827	0.1973	0.2094	0.2195	0.2279	0.235	0.241	S
0.750	0.2435	0.28	0.3126	0.3419	0.3687	0.3931	0.4156	0.4364	M
	0.1612	0.1801	0.1957	0.2086	0.2195	0.2286	0.2363	0.2429	S
0.800	0.2251	0.2611	0.2934	0.3228	0.3495	0.3741	0.3968	0.4178	M
	0.1573	0.1771	0.1936	0.2073	0.2189	0.2287	0.2371	0.2442	S
0.850	0.2084	0.2438	0.2758	0.305	0.3318	0.3565	0.3793	0.4005	M
	0.1533	0.1739	0.1911	0.2056	0.2179	0.2284	0.2373	0.245	S
0.900	0.1932	0.2279	0.2596	0.2886	0.3153	0.34	0.363	0.3843	M
	0.1492	0.1705	0.1884	0.2036	0.2165	0.2276	0.2371	0.2453	S
0.950	0.1793	0.2134	0.2446	0.2734	0.3	0.3247	0.3477	0.3691	M
	0.145	0.1669	0.1855	0.2013	0.2149	0.2265	0.2366	0.2452	S
1.000	0.1667	0.2	0.2308	0.2593	0.2857	0.3103	0.3333	0.3548	M
	0.1409	0.1633	0.1824	0.1988	0.213	0.2251	0.2357	0.2449	S
1.500	0.0852	0.1108	0.136	0.1604	0.1841	0.2068	0.2286	0.2494	M
	0.1029	0.1276	0.15	0.1701	0.1881	0.2042	0.2185	0.2313	S
2.000	0.0476	0.0667	0.0865	0.1068	0.127	0.147	0.1667	0.1859	M
	0.0753	0.0992	0.1219	0.1432	0.1628	0.1808	0.1972	0.2122	S
2.500	0.0284	0.0426	0.0583	0.0749	0.092	0.1095	0.127	0.1444	M
	0.0562	0.0782	0.1001	0.1212	0.1412	0.16	0.1775	0.1936	S
3.000	0.0179	0.0286	0.041	0.0547	0.0693	0.0844	0.1	0.1157	M
	0.043	0.0628	0.0833	0.1037	0.1235	0.1425	0.1604	0.1772	S

Table 1 (Continued)

a	*b*							
	0.600	0.650	0.700	0.750	0.800	0.850	0.900	
0.500	0.5686	0.5841	0.5985	0.6119	0.6243	0.6359	0.6468	M
	0.2274	0.2296	0.2314	0.2328	0.2339	0.2347	0.2352	S
0.550	0.5426	0.5588	0.5738	0.5877	0.6007	0.6129	0.6243	M
	0.2337	0.2365	0.2387	0.2405	0.2419	0.243	0.2439	S
0.600	0.5185	0.5352	0.5507	0.5652	0.5787	0.5914	0.6032	M
	0.2388	0.2421	0.2448	0.247	0.2488	0.2502	0.2513	S
0.650	0.496	0.5132	0.5292	0.5441	0.5581	0.5712	0.5835	M
	0.2429	0.2466	0.2498	0.2524	0.2545	0.2563	0.2577	S
0.700	0.4751	0.4927	0.5091	0.5244	0.5388	0.5523	0.565	M
	0.246	0.2503	0.2539	0.2569	0.2594	0.2615	0.2633	S
0.750	0.4556	0.4736	0.4903	0.506	0.5207	0.5345	0.5475	M
	0.2485	0.2532	0.2572	0.2607	0.2635	0.266	0.268	S
0.800	0.4374	0.4556	0.4726	0.4886	0.5036	0.5177	0.531	M
	0.2503	0.2555	0.2599	0.2637	0.267	0.2697	0.2721	S
0.850	0.4203	0.4387	0.456	0.4723	0.4875	0.5019	0.5155	M
	0.2515	0.2572	0.2621	0.2663	0.2698	0.2729	0.2755	S
0.900	0.4043	0.4229	0.4404	0.4569	0.4723	0.487	0.5008	M
	0.2524	0.2584	0.2637	0.2683	0.2722	0.2756	0.2785	S
0.950	0.3892	0.408	0.4257	0.4423	0.458	0.4728	0.4869	M
	0.2528	0.2593	0.2649	0.2698	0.2741	0.2778	0.281	S
1.000	0.375	0.3939	0.4118	0.4286	0.4444	0.4595	0.4737	M
	0.2528	0.2598	0.2658	0.2711	0.2756	0.2796	0.2831	S
1.500	0.2693	0.2884	0.3066	0.3239	0.3405	0.3564	0.3716	M
	0.2428	0.253	0.2622	0.2704	0.2777	0.2842	0.2901	S
2.000	0.2045	0.2227	0.2402	0.2571	0.2735	0.2893	0.3045	M
	0.2258	0.2382	0.2494	0.2596	0.2689	0.2773	0.2849	S
2.500	0.1616	0.1785	0.1951	0.2113	0.227	0.2423	0.2572	M
	0.2086	0.2223	0.2349	0.2465	0.2572	0.2669	0.2759	S
3.000	0.1315	0.1472	0.1627	0.178	0.1931	0.2078	0.2222	M
	0.1928	0.2074	0.221	0.2336	0.2452	0.2559	0.2658	S

Table 2: Measures, β_1 and β_2, of Skewness and Kurtosis Generalized Beta Distribution

a	*b*								
	0.200	0.250	0.300	0.350	0.400	0.450	0.500	0.550	
0.500	0.0676	0.0342	0.0133	0.0025	0.0001	0.0049	0.0157	0.0316	β_1
	2.4664	2.3723	2.3018	2.2505	2.2149	2.192	2.18	2.1771	β_2
0.550	0.1318	0.0804	0.0441	0.02	0.0059	0.0003	0.0017	0.0091	β_1
	2.5366	2.4147	2.3212	2.2503	2.1977	2.1601	2.135	2.1204	β_2
0.600	0.2138	0.1431	0.0904	0.0522	0.026	0.0096	0.0015	0.0003	β_1
	2.6341	2.4822	2.3641	2.2727	2.2026	2.1499	2.1115	2.085	β_2
0.650	0.3122	0.2205	0.1503	0.0974	0.0585	0.031	0.0131	0.0032	β_1
	2.7563	2.572	2.4276	2.3145	2.2262	2.1579	2.1059	2.0675	β_2
0.700	0.426	0.3114	0.2226	0.154	0.1018	0.0628	0.0348	0.016	β_1
	2.9013	2.6818	2.5094	2.3734	2.266	2.1815	2.1156	2.0651	β_2
0.750	0.5542	0.415	0.306	0.2209	0.1547	0.1037	0.0654	0.0374	β_1
	3.0679	2.8101	2.6076	2.4473	2.3199	2.2186	2.1384	2.0755	β_2
0.800	0.6965	0.5304	0.4	0.2972	0.2162	0.1528	0.1037	0.0664	β_1
	3.255	2.9558	2.721	2.5349	2.3864	2.2677	2.1726	2.0969	β_2
0.850	0.8523	0.6572	0.5036	0.3821	0.2857	0.2092	0.149	0.102	β_1
	3.4621	3.1178	2.8484	2.6349	2.4643	2.3273	2.2169	2.1282	β_2
0.900	1.0216	0.7949	0.6166	0.4751	0.3624	0.2723	0.2005	0.1436	β_1
	3.6887	3.2956	2.9889	2.7464	2.5525	2.3964	2.2702	2.1679	β_2
0.950	1.2042	0.9432	0.7384	0.5758	0.4458	0.3415	0.2577	0.1905	β_1
	3.9347	3.4886	3.142	2.8686	2.6502	2.4741	2.3314	2.2154	β_2
1.000	1.4	1.102	0.8687	0.6836	0.5355	0.4163	0.32	0.2422	β_1
	4.2	3.6964	3.3072	3.0009	2.7566	2.5598	2.4	2.2696	β_2
1.500	4.1297	3.2597	2.6111	2.1141	1.7247	1.4143	1.1633	0.9582	β_1
	7.9786	6.5824	5.5687	4.8073	4.2201	3.7575	3.3868	3.0856	β_2
2.000	8.531	6.5435	5.1512	4.1346	3.3675	2.7735	2.3038	1.9259	β_1
	14.2288	11.0692	8.9317	7.412	6.2892	5.4341	4.7668	4.2355	β_2
2.500	15.1716	11.1897	8.5727	6.7539	5.4343	4.4441	3.6804	3.0784	β_1
	23.8586	17.5151	13.5161	10.8234	8.9175	7.515	6.4505	5.622	β_2
3.000	24.8095	17.5104	13.0029	10.0171	7.9316	6.4136	5.2717	4.3895	β_1
	38.094	26.3968	19.5076	15.1041	12.1123	9.9818	8.4073	7.2085	β_2

Table 2 (Continued)

a	*b*							
	0.600	0.650	0.700	0.750	0.800	0.850	0.900	
0.500	0.052	0.0762	0.1037	0.1342	0.1672	0.2026	0.24	β_1
	2.182	2.1934	2.2107	2.2329	2.2595	2.2899	2.3237	β_2
0.550	0.0216	0.0386	0.0594	0.0835	0.1106	0.1403	0.1723	β_1
	2.1145	2.1162	2.1244	2.1382	2.1569	2.1798	2.2065	β_2
0.600	0.005	0.0148	0.029	0.0469	0.0682	0.0925	0.1193	β_1
	2.0686	2.0606	2.06	2.0656	2.0766	2.0924	2.1124	β_2
0.650	0	0.0026	0.0102	0.0221	0.0377	0.0566	0.0784	β_1
	2.0404	2.0229	2.0135	2.0111	2.0147	2.0236	2.0371	β_2
0.700	0.0048	0.0002	0.0013	0.0071	0.0171	0.0309	0.0478	β_1
	2.0272	2.0001	1.982	1.9717	1.9681	1.9702	1.9775	β_2
0.750	0.0182	0.0063	0.0007	0.0005	0.005	0.0136	0.0258	β_1
	2.0267	1.9899	1.9632	1.945	1.9342	1.9298	1.9309	β_2
0.800	0.0389	0.0196	0.0074	0.0012	0.0002	0.0037	0.0111	β_1
	2.0371	1.9906	1.9552	1.9292	1.9113	1.9003	1.8954	β_2
0.850	0.066	0.0393	0.0204	0.0081	0.0016	0.0001	0.0028	β_1
	2.0571	2.0006	1.9564	1.9226	1.8976	1.8802	1.8694	β_2
0.900	0.0989	0.0646	0.0389	0.0205	0.0085	0.0019	0	β_1
	2.0853	2.0188	1.9657	1.924	1.892	1.8682	1.8515	β_2
0.950	0.137	0.0948	0.0622	0.0377	0.0201	0.0085	0.002	β_1
	2.1209	2.0441	1.9821	1.9325	1.8934	1.8632	1.8407	β_2
1.000	0.1796	0.1295	0.0899	0.0592	0.036	0.0193	0.0082	β_1
	2.163	2.0758	2.0048	1.9471	1.9008	1.8642	1.836	β_2
1.500	0.7889	0.6482	0.5307	0.4322	0.3495	0.28	0.2217	β_1
	2.8381	2.6328	2.4614	2.3174	2.1959	2.0932	2.0062	β_2
2.000	1.6175	1.3629	1.1506	0.9721	0.8211	0.6927	0.5829	β_1
	3.8057	3.4531	3.1605	2.9155	2.7086	2.5329	2.3827	β_2
2.500	2.5949	2.2007	1.8751	1.6033	1.3743	1.1798	1.0136	β_1
	4.9637	4.4316	3.9953	3.633	3.3292	3.0721	2.8529	β_2
3.000	3.6928	3.1325	2.6749	2.2963	1.9795	1.7119	1.484	β_1
	6.273	5.528	4.9247	4.4289	4.0164	3.6697	3.3756	β_2

Table 3: Sample Size, *n*, and Acceptability Constant, *k*, of Single Sampling Plans by Variables Based on Generalized Beta Distribution Having a Maximum Producer's Risk of 5 Percent (α = 0.05) and a Maximum Consumer's Risk of 10 Percent (β = 0.10)

PQL	*CQL*	*b*	a								
			0.500	0.550	0.600	0.650	0.700	0.750	0.800	0.850	
0.005	0.01	0.200	244	209	180	155	135	118	104	91	*n*
			2.4205	2.5076	2.5939	2.6792	2.7637	2.8473	2.9298	3.0114	*k*
0.005	0.01	0.250	67	58	50	43	38	34	30	26	*n*
			2.2105	2.2836	2.3556	2.4264	2.4962	2.5648	2.6324	2.6988	*k*
0.005	0.02	0.300	89	77	67	59	51	45	40	36	*n*
			2.1311	2.2	2.2679	2.3346	2.4004	2.4651	2.5289	2.5916	*k*
0.006	0.03	0.350	70	60	53	47	41	36	33	29	*n*
			1.9813	2.041	2.0996	2.157	2.2134	2.2686	2.3228	2.376	*k*
0.007	0.04	0.400	63	55	49	43	38	34	30	27	*n*
			1.863	1.916	1.9678	2.0185	2.068	2.1165	2.1638	2.2102	*k*
0.008	0.05	0.450	61	54	48	42	38	34	30	27	*n*
			1.7649	1.8127	1.8593	1.9047	1.949	1.9922	2.0344	2.0756	*k*
0.01	0.04	0.500	142	125	111	98	88	78	70	64	*n*
			1.7406	1.7878	1.8338	1.8787	1.9226	1.9654	2.0073	2.0483	*k*
0.01	0.07	0.550	142	125	111	98	88	78	70	64	*n*
			1.7406	1.7878	1.8338	1.8787	1.9226	1.9654	2.0073	2.0483	*k*
0.02	0.09	0.600	75	67	60	54	48	44	40	36	*n*
			1.4919	1.5258	1.5584	1.5899	1.6203	1.6497	1.678	1.7054	*k*
0.02	0.03	0.650	2922	2585	2298	2052	1840	1656	1496	1356	*n*
			1.6059	1.6474	1.6878	1.7272	1.7657	1.8033	1.84	1.876	*k*
0.03	0.04	0.700	4520	4014	3582	3211	2889	2610	2366	2152	*n*
			1.5208	1.5578	1.5938	1.6288	1.6629	1.696	1.7283	1.7598	*k*
0.03	0.05	0.750	4520	4014	3582	3211	2889	2610	2366	2152	*n*
			1.5208	1.5578	1.5938	1.6288	1.6629	1.696	1.7283	1.7598	*k*
0.04	0.06	0.800	2002	1787	1603	1444	1305	1185	1079	986	*n*
			1.4037	1.4354	1.466	1.4957	1.5244	1.5523	1.5793	1.6056	*k*
0.04	0.08	0.850	634	567	510	460	417	379	346	317	*n*
			1.3458	1.3749	1.4029	1.4301	1.4563	1.4816	1.5061	1.5299	*k*
0.05	0.10	0.900	501	450	406	367	334	304	279	256	*n*
			1.2823	1.3085	1.3337	1.3579	1.3813	1.4038	1.4255	1.4464	*k*

Table 3 (Continued)

PQL	*CQL*	*b*	a							
			0.900	0.950	1.000	1.500	2.000	2.500	3.000	
0.005	0.01	0.200	81	72	64	24	12	7	4	*n*
			3.092	3.1716	3.2501	3.9699	4.5555	4.993	5.2813	*k*
0.005	0.01	0.250	23	21	19	8	4	2	1	*n*
			2.7642	2.8284	2.8915	3.4603	3.9127	4.2494	4.4768	*K*
0.005	0.02	0.300	32	29	26	10	5	3	2	*N*
			2.6534	2.7141	2.7739	3.3179	3.765	4.1177	4.3816	*K*
0.006	0.03	0.350	26	24	21	9	5	3	2	*n*
			2.4281	2.4792	2.5294	2.9786	3.3394	3.62	3.8287	*k*
0.007	0.04	0.400	25	22	20	9	5	3	2	*n*
			2.2555	2.2999	2.3433	2.7277	3.0319	3.2667	3.4418	*k*
0.008	0.05	0.450	25	22	20	9	5	3	2	*n*
			2.1157	2.155	2.1933	2.5298	2.793	2.9954	3.1469	*k*
0.01	0.04	0.500	58	52	48	21	12	7	5	*n*
			2.0883	2.1275	2.1657	2.5055	2.7778	2.9938	3.162	*k*
0.01	0.07	0.550	58	52	48	21	12	7	5	*n*
			2.0883	2.1275	2.1657	2.5055	2.7778	2.9938	3.162	*k*
0.02	0.09	0.600	33	30	28	14	8	5	4	*n*
			1.7319	1.7575	1.7822	1.9885	2.1343	2.2343	2.2987	*k*
0.02	0.03	0.650	1232	1124	1028	477	261	160	106	*n*
			1.9111	1.9455	1.9791	2.2801	2.5277	2.7323	2.9013	*k*
0.03	0.04	0.700	1963	1796	1648	788	443	278	189	*n*
			1.7904	1.8203	1.8494	2.1056	2.3092	2.4715	2.6002	*k*
0.03	0.05	0.750	1963	1796	1648	788	443	278	189	*n*
			1.7904	1.8203	1.8494	2.1056	2.3092	2.4715	2.6002	*k*
0.04	0.06	0.800	903	830	765	379	220	142	98	*n*
			1.631	1.6558	1.6798	1.8871	2.0463	2.169	2.2627	*k*
0.04	0.08	0.850	291	268	247	125	74	48	34	*n*
			1.5529	1.5751	1.5967	1.7808	1.919	2.023	2.1004	*k*
0.05	0.10	0.900	235	217	201	104	62	41	30	*n*
			1.4666	1.4861	1.5049	1.6621	1.7752	1.856	1.9121	*k*

6. Conclusion

The literature in statistical quality control provides various sampling inspection procedures which been developed based on the assumption that the quality characteristic under study follows a normal distribution. While such procedures are widely used in the industries, the departure from the assumption of normality or the violation of distributional assumptions are the major concern for the industrial practitioners as the decision that is made on the lot disposition in such situations would be inappropriate. Focusing on this vital aspect, in this paper, procedures for designing single sampling plans by variables are devised under the assumption that the quality characteristic is distributed according to a generalized beta distribution of first kind. The procedures and tables presented are appropriate for bulk inspection procedures where the quality characteristics are defined by compositional proportions.

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