RELIABILITY ANALYSIS OF REVERSE OSMOSIS FILTRATION SYSTEM USING COPULA

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Abstract

In this study, the reliability metrics used to assess the strength of a three-subsystem reverse osmosis filtering system. The subsystems include sand filter, carbonated filter, and precision filter. Each subsystem is composed of active components that can operate in series parallel. The system of partial differential equations was built using the mnemonic rule and analytically solved. Other reliability variables that were investigated for determining system strength included availability, reliability, mean time to failure (MTTF), profit analysis, and sensitivity analysis. The Maple software was used to obtain numerical solutions. In addition, a graphical representation of the numerical results was provided to demonstrate the behaviors of reliability characteristics with regard to time and failure rate. The study could assist water treatment firms and their repairers in overcoming some of the challenges faced by repairers of specialized manufacturing and industrial systems working in harsh settings or contaminated environments unfit for human consumption.

Keywords: mnemonic, profit, sensitivity, reliability, repair, system, copula

I. Introduction

The removal of pollutants from drinking water caused by humans is a modern-day technical challenge. You'll learn about the detectable contamination of drinking water caused by anthropogenic (human-made) contaminants that is still present after a quick review of the treatment steps that municipal water goes through before it reaches your tap. Prescription medications, herbicides, and hormones have all been discovered in the drinking water systems of our respective countries. The engineering design method could be used to find solutions to a real-world problem (contaminated water) that could be dangerous to people's health. Water is perhaps the most important nutrient in our diets. In reality, a normal adult needs to drink around 2 liters (8 glasses) of water per day to replace the water lost through the epidermis, respiratory system, and urine. The Reliability, Availability, Maintainability, and Dependability Analysis of a Complex Reverse Osmosis Machine System in Water Purification was studied by Maihulla A. S. et al. [1] Al-Ghouti, M.A et al. [2] evaluated the recent developments and applications for municipal solid waste bottom and fly ashes. The Gumbel-Hougaard family copula was used to forecast the reliability and performance of a small serial solar photovoltaic system for rural use was carried out by Maihulla A.S. and Yusuf I. [3]. Evaluation of Ozone Pretreatment on Reverse Osmosis Flux Parameters for Surface Water Treatment, Ozone by Shalana L. et al. [4]. The study pertaining the Availability and reliability analysis of integrated reverse osmosis was carried out by safder U. et al. [5]. Calixto, Eduardo. [6] Discussed about Reliability, Availability, and Maintainability (RAM Analysis). The RAM analysis and availability optimization of thermal power plant water circulation system using PSO was carried out by Hanumant P. et al. [7]. The study that tackled the Reliability Assessment for Hybrid Systems of Advanced Treatment Units of Industrial Wastewater Reuse Using Combined Event Tree and Fuzzy Fault Tree Analyses. Was conducted by Farzad P. et al. [8]. Hajeeh, M. [11] & Chaudhuri, D. [9] conducted a research titled reliability and availability assessment of reverse osmosis Desalination. The feasibility and reliability of the Life Cycle Assessment for desalination, is a study carried out by Zhou, J. et al. [10]. Revas P. et al. [12] study the Real-Time Implementation of an Expert Model Predictive Controller in a Pilot-Scale Reverse Osmosis Plant for Brackish and Seawater Desalination. Goyal et al. [13] defined the most vulnerable aspect of serial processes such as evaporation systems in the sugar industry and water treatment plants. Using STP, this research deconstructs the efficiency indices of the power generation system. Simple probability theory concepts and the Markovian birth-death process were used to investigate the power system. As a Markov process progresses from one stage to the next, it becomes more complex.

M.F. Idrees [14] also works with the Performance Analysis and Treatment Technologies of a Reverse Osmosis Plan. C. Li, S. Besarati, and colleagues [15] A few years ago, I did research on reverse osmosis desalination using a low temperature supercritical organic Rankine cycle. In order to meet environmental and economic standards, any sophisticated reverse osmosis plant must have automation and reliability. S. Srivastava. [16]. S. Sadri [17] Created a computational model based on diffusion and convection transport mechanisms, as well as the concentration polarization concept, to predict RO membrane performance using a variety of feed water concentrations, feed flow rates, feed water pressures, membrane specifications, and feed water properties. Y. Li et al. [18] conducted a study of the concepts and categorization of membrane distillation, with an emphasis on the variables influencing it and ways to improving its efficacy. Experiments were conducted by E. O. Ezugbe et al. [19] Pure water and NaCl solutions ranging from 15 g/L to 300 g/L, as well as two distinct fiber types and architectures, were used. Vacuum membrane distillation (VMD) is a saltwater desalination technique. The pure water permeability and global heat transfer coefficient of the two systems were compared. The effects of hydrodynamics on global heat and mass transport coefficients are discussed. R. Tundis et al. [20] The chemical profile, antioxidant and anti-obesity properties of concentrated fractions obtained from micro-filtered OMW treated by direct contact membrane distillation were studied (DCMD). Using ultrahigh performance liquid chromatography, several phenols chosen as phytochemical markers were measured (UHPLC). To treat the pollutants found in olive mill wastewater (total organic carbon (TOC), dissolved organic carbon (DOC), total phosphorus (TP), total nitrogen (TN), and total polyphenols), a sequential Direct Contact Membrane Distillation (DCMD) and a Reverse Osmosis (RO) hybrid membrane system were used. The study was conducted by D. Teresa [21]. The influence of permeate flow and pressure on pollutant parameter removals was also studied. One of the biggest challenges that humanity must address in the twenty-first century is the scarcity of freshwater. P. Biniaz et al. [22] explored how an ecologically acceptable, cost-effective, and energy-efficient membrane distillation process might be developed (MD) process can reduce pollution caused by industrial and domestic wastes. Garud R. M. et al. [23] conducted a Short Review on Process and Applications of Reverse Osmosis. The Gumbel–Hougaard family copula was used to model the reliability and performance of a solar photovoltaic system was analyzed by Maihulla A. S. et al. [24]. Y.G. Lee et al. [25] created a model with five input factors (feed temperature, feed total dissolved solids (TDS), trans-membrane pressure (TMP), feed flow rate, and time) and two output parameters (permeate TDS and flow rate) to estimate the performance of a saltwater reverse osmosis (SWRO) desalination plant It was then used to simulate feed water temperature. D. Wirth [26] compared two hollow fiber module designs (inside/out and outside/in).

Our motivation for investigating the reverse osmosis filtration system derives from a severe problem that the water purification businesses are experiencing as a result of purification filter failure. And the slow progress in technological advancement in water purification as a result, as well as its relevance in the lives of people all over the world. Industry is working hard to keep up with the growing complexity of filtration systems. According to the paper's findings, Gumbel-Haugaard Family Copula analysis was performed to examine the filtration system's strength, efficiency, and performance improvement. Users will be able to serve the expense of medical care owing to un-pure water if the strength, efficiency, and performance of the filtration system are assessed. Protect yourself from aquatic pollution. The research is broken into five sections, one of which being the present introduction. Filtration modeling is discussed. Later in the second portion Section 4 contains a discussion and explanation of the results from the third section, which involves an analytical analysis of the system. Section 5 is devoted to the conclusion and ramifications of the findings.

II. Methods

I. Filtration

One of the most common methods for removing these materials is gravity filtration. Water containing solid impurities (e.g., precipitates after water softening) is passed through a porous material, usually sand and gravel layers, in this procedure. The force of gravity pushes the water through the medium. The gaps between the sand and gravel grains allow small water molecules to flow through. Precipitation-derived solids, on the other hand, become trapped in the pores and thus remain in the porous medium. The solid contaminants have been removed from the water that passes through the bottom of the filter.

II. Description of the model

A model with three series-parallel subsystems, A, B, and C, is shown in Figure 1. Two identical units work as 1-out-of-2 in subsystem A (sand filter), three identical units work as 1-out-of-3 in subsystem B (carbonated filter), and two parallel units work as 2-out-of-2 in subsystem C (precision filter). The two types of system failures are partial and complete failures. When a unit in a subsystem fails but the system continues to function, it is called partial failure, whereas total failure occurs when all of the subsystems fail. Copula is used to repair a system that has completely failed. In the system, there are eleven states: eight that are operational and three that are total failure states (see Figure2). The states are described briefly in Table 1.

III. Results

I. Formulation and Solution of Mathematical Model

The following set of difference-differential equations is related to the aforementioned mathematical model based on consideration likelihood and argument continuity.

$$\left(\frac{\partial}{\partial x}+\alpha_1+\alpha_2+2\alpha_3\right)P_0(t)=\int_0^\infty \beta_1 P_1(x,t)dx+\int_0^\infty \beta_2 P_2(y,t)dy+\int_0^\infty \psi(y)P_6(y,t)dy+\int_0^\infty \varphi(x)P_8(x,t)dx+\int_0^\infty \varphi(y)P_9(y,t)dy+\int_0^\infty \varphi(z)P_{10}(z,t)dz \tag{1}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\alpha_2+\beta_1\right)P_1(x,t)=0 \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_1+\alpha_2+\beta_2\right)P_2(y,t)=0 \quad (3)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_1+\alpha_2+\beta_2\right)P_3(y,t)=0 \quad (4)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_2+\beta_2\right)P_4(y,t)=0 \quad (5)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\alpha_2+\beta_2\right)P_5(y,t)=0 \quad (6)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\beta_1\right)P_6(x,t)=0 \quad (7)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\alpha_1+\beta_1\right)P_7(x,t)=0 \quad (8)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\varphi(x)\right)P_8(x,t)=0 \quad (9)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\varphi(y)\right)P_9(y,t)=0 \quad (10)$$
$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\varphi(z)\right)P_{10}(x,t)=0 \quad (11)$$

Boundary conditions

$$P_1(0,t)=\alpha_1P_0(t) \quad (12)$$
$$P_2(0,t)=\alpha_2P_0(t) \quad (13)$$
$$P_3(0,t)=\alpha_2^2P_0(t) \quad (14)$$
$$P_4(0,t)=\alpha_1\alpha_2P_0(t) \quad (15)$$
$$P_5(0,t)=\alpha_1\alpha_2^2P_0(t) \quad (16)$$
$$P_6(0,t)=\alpha_1\alpha_2P_0(t) \quad (17)$$
$$P_7(0,t)=\alpha_1\alpha_2^2P_0(t) \quad (18)$$
$$P_8(0,t)=\alpha_1^2(1+\alpha_1+\alpha_2^2)P_0(t) \quad (19)$$
$$P_9(0,t)=\alpha_2^3(1+\alpha_1)P_0(t) \quad (20)$$
$$P_{10}(0,t)=2\alpha_3P_0(t) \quad (21)$$

Initial condition $P_0(t)$ = 1 and other transition probability at t=0 are zero

Laplace transformation of (1) – (21)

$$(s+\alpha_1+\alpha_2+2\alpha_3)\bar{P}_0(s)=\int_0^\infty\beta_1\bar{P}_1(x,s)dx+\int_0^\infty\beta_2\bar{P}_2(y,s)dy+\int_0^\infty\psi(y)\bar{P}_6(y,s)dy+$$
$$\int_0^\infty\varphi(x)\bar{P}_8(x,s)dx+\int_0^\infty\varphi(y)\bar{P}_9(y,s)dy+\int_0^\infty\varphi(z)\bar{P}_{10}(z,s)dz \quad (22)$$

$$\left(s+\frac{\partial}{\partial x}+\alpha_1+\alpha_2+\beta_1\right)\bar{P}_1(x,s)=0 \quad (23)$$
$$\left(s+\frac{\partial}{\partial y}+\alpha_1+\alpha_2+\beta_2\right)\bar{P}_2(y,s)=0 \quad (24)$$
$$\left(s+\frac{\partial}{\partial y}+\alpha_1+\alpha_2+\beta_2\right)\bar{P}_3(y,s)=0 \quad (25)$$
$$\left(s+\frac{\partial}{\partial y}+\alpha_2+\beta_2\right)\bar{P}_4(y,s)=0 \quad (26)$$
$$\left(s+\frac{\partial}{\partial y}+\alpha_2+\beta_2\right)\bar{P}_5(y,s)=0 \quad (27)$$
$$\left(s+\frac{\partial}{\partial x}+\alpha_1+\beta_1\right)\bar{P}_6(x,s)=0 \quad (28)$$
$$\left(s+\frac{\partial}{\partial x}+\alpha_1+\beta_1\right)\bar{P}_7(x,s)=0 \quad (29)$$
$$\left(s+\frac{\partial}{\partial x}+\varphi(x)\right)\bar{P}_8(x,s)=0 \quad (30)$$
$$\left(s+\frac{\partial}{\partial y}+\varphi(y)\right)\bar{P}_9(y,s)=0 \quad (31)$$
$$\left(s+\frac{\partial}{\partial z}+\varphi(z)\right)\bar{P}_{10}(z,s)=0 \quad (32)$$

Laplace of the Boundary conditions

$$\bar{P}_1(0,s)=\alpha_1\bar{P}_0(s) \quad (33)$$
$$\bar{P}_2(0,s)=\alpha_2\bar{P}_0(s) \quad (34)$$
$$\bar{P}_3(0,s)=\alpha_2^2\bar{P}_0(s) \quad (35)$$
$$\bar{P}_4(0,s)=\alpha_1\alpha_2\bar{P}_0(s) \quad (36)$$
$$\bar{P}_5(0,s)=\alpha_1\alpha_2^2\bar{P}_0(s) \quad (37)$$
$$\bar{P}_6(0,s)=\alpha_1\alpha_2\bar{P}_0(s) \quad (38)$$
$$\bar{P}_7(0,s)=\alpha_1\alpha_2^2P_0(s) \quad (39)$$
$$\bar{P}_8(0,s)=\alpha_1^2(1+\alpha_1+\alpha_2^2)\bar{P}_0(s) \quad (40)$$

$\bar{P}_9(0,s) = \alpha_2^3(1+\alpha_1)\,\bar{P}_0(s)$ (41)

$\bar{P}_{10}(0,s) = 2\alpha_3\bar{P}_0(s)$ (42)

Solving equation (22) to (32) with the help of boundary condition (33) to (42) and applying the below shifting properties of Laplace:

$\int_0^\infty [e^{-sx}.\, e^{-\int_0^x f(x)dx}]dx = L\left\{\frac{1-\bar{S}_f(x)}{s}\right\} = \frac{1-\bar{S}_f(x)}{s}$ (43)

$\int_0^\infty [e^{-sx}.\, f(x)\, e^{-\int_0^x f(x)dx}]dx = L\{\bar{S}_f(x)\} = \bar{S}_f(s)$ (44)

And the identity; $\bar{P}_1(s) = \int_0^\infty \bar{P}_1(x,s)dx$ (45)

$\bar{P}_1(s) = \bar{P}_1(0,s)\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_3}\right\}$ (46)

$\bar{P}_2(s) = \bar{P}_2(0,s)\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_2}\right\}$ (47)

$\bar{P}_3(s) = \bar{P}_3(0,s)\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_2}\right\}$ (48)

$\bar{P}_4(s) = \bar{P}_4(0,s)\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_2)}{S+\alpha_2}\right\}$ (49)

$\bar{P}_5(s) = \bar{P}_5(0,s)\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1)}{S+\alpha_1}\right\}$ (50)

$\bar{P}_6(s) = \bar{P}_6(0,s)\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}$ (51)

$\bar{P}_7(s) = \bar{P}_7(0,s)\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}$ (52)

$\bar{P}_8(s) = \bar{P}_8(0,s)\left\{\frac{1-\bar{S}_{\varphi(x)}(S)}{S}\right\}$ (53)

$\bar{P}_9(s) = \bar{P}_9(0,s)\left\{\frac{1-\bar{S}_{\varphi(y)}(S)}{S}\right\}$ (54)

$\bar{P}_{10}(s) = \bar{P}_{10}(0,s)\left\{\frac{1-\bar{S}_{\varphi(z)}(S)}{S}\right\}$ (55)

Substituting the Laplace of the Boundary conditions i.e (33) to (42) into (43) to (55)

$\bar{P}_1(s) = \alpha_1\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_3}\right\}\bar{P}_0(s)$ (56)

$\bar{P}_2(s) = \alpha_2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_2}\right\}\bar{P}_0(s)$ (57)

$\bar{P}_3(s) = \alpha_2^2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_2}\right\}\bar{P}_0(s)$ (58)

$\bar{P}_4(s) = \alpha_1\alpha_2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_2)}{S+\alpha_2}\right\}\bar{P}_0(s)$ (59)

$\bar{P}_5(s) = \alpha_1\alpha_2^2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1)}{S+\alpha_1}\right\}\bar{P}_0(s)$ (60)

$\bar{P}_6(s) = \alpha_1\alpha_2\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}\bar{P}_0(s)$ (61)

$\bar{P}_7(s) = \alpha_1\alpha_2^2\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}\bar{P}_0(s)$ (62)

$\bar{P}_8(s) = \alpha_1^2(1+\alpha_1+\alpha_2^2)\left\{\frac{1-\bar{S}_{\varphi(x)}(S)}{S}\right\}\bar{P}_0(s)$ (63)

$\bar{P}_9(s) = \alpha_2^3(1+\alpha_1)\left\{\frac{1-\bar{S}_{\varphi(y)}(S)}{S}\right\}\bar{P}_0(s)$ (64)

$\bar{P}_{10}(s) = 2\alpha_3\left\{\frac{1-\bar{S}_{\varphi(z)}(S)}{S}\right\}\bar{P}_0(s)$ (65)

$(s+\alpha_1+\alpha_2+2\alpha_3)\bar{P}_0(s) = 1 + \beta_1\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2)\bar{P}_0(s) + \beta_2\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)\bar{P}_0(s) + [\varphi(x)\bar{S}_{\varphi(x)}(S)$
$+\varphi(y)\bar{S}_{\varphi(y)}(S) + \varphi(z)\bar{S}_{\varphi(z)}(S)]\,\bar{P}_0(s)$ (66)

$s+\alpha_1+\alpha_2+2\alpha_3 - [\beta_1\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2) + \beta_2\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2) + [\varphi(x)\bar{S}_{\varphi(x)}(S) + \varphi(y)\bar{S}_{\varphi(y)}(S) +$
$\varphi(z)\bar{S}_{\varphi(z)}(S)]]\bar{P}_0(s) = 1$ (67)

D(s) = $s+\alpha_1+\alpha_2+2\alpha_3 - [\beta_1\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2) + \beta_2\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2) + [\varphi(x)\bar{S}_{\varphi(x)}(S) +$
$\varphi(y)\bar{S}_{\varphi(y)}(S) + \varphi(z)\bar{S}_{\varphi(z)}(S)]]$ (68)

Since D(s)× $\bar{P}_0(s)$= 1 (69)

$\bar{P}_{up}(S) = \bar{P}_0(S) + \bar{P}_1(S) + \bar{P}_3(S) + \bar{P}_4(S) + \bar{P}_5(S) + \bar{P}_6(S) + \bar{P}_7(S)$ (70)

$\bar{P}_{up}(S) = [1 + \alpha_1\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_3}\right\} + \alpha_2^2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1+\alpha_2)}{S+\alpha_1+\alpha_2}\right\} + \alpha_1\alpha_2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_2)}{S+\alpha_2}\right\} + \alpha_1\alpha_2^2\left\{\frac{1-\bar{S}_{\beta_2}(S+\alpha_1)}{S+\alpha_1}\right\} +$

$$\alpha_1\alpha_2\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}+\alpha_1\alpha_2^2\left\{\frac{1-\bar{S}_{\beta_1}(S+\alpha_1)}{S+\alpha_1}\right\}]\bar{P}_0(s) \quad (71)$$

II. Availability Analysis

Setting all repairs to 1. i.e. $\varphi(x) = \varphi(y) = \varphi(z) = \beta_1 = \beta_2 = \beta_3 = 1$ (72)

$\bar{S}_{\phi}(S) = \frac{2.7183}{S+2.7183}$, $\frac{1-\bar{S}_{\phi}(S)}{S} = \frac{1}{S+\phi}$

Taking the values of different parameters as $\alpha_1 = 0.01, \alpha_1 = 0.02, \alpha_3 = 0.03, \alpha_4 = 0.04$ in (48) Then taking the inverse Laplace transform, we can obtain, the expression for availability as:

$$D(s) = S + 0.12- [\frac{0.03}{S+1.04} + \frac{0.03}{S+1.03} + 0.06 \,.\, (\frac{2.7183}{S+2.7183})\,] \quad (73)$$

$$\bar{P}_{up}(S) = [1 + \frac{0.03}{S+1.04} + \frac{0.03}{S+1.03} + \frac{0.0018}{S+1.03}] \quad (74)$$

Taking $S_{\alpha_0}(s) = \bar{S}_{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\varphi(x)\}^\theta]^{1/\theta}}$, $\bar{P}_{\phi}(s) = \frac{\phi}{s+\phi}$ *but* ϕ =1 and $\alpha_1 = 0.001$, $\alpha_2 = 0.002$, $\alpha_3 = 0.003$

And repair rates $\varphi(x) = \varphi(y) = \varphi(z) = \beta_1 = \beta_2 = \beta_3 = 1$ in equation (69), and applying the inverse Laplace transform to (69), the expression for system availability is

$$\begin{aligned}\bar{P}_{up}(t) = \{ & -0.00004904770570\,e^{-1.011000000\,t} + 0.001603099809\,e^{-2.722649460\,t} + (-0.006078301855 \\ & - 0.0014178109921)\,e^{(-1.014501644 - 0.00099381113161)\,t} + (-0.006078301855 \\ & + 0.0014178109921)\,e^{(-1.014501644 + 0.00099381113161)\,t} + 1.010602552\,e^{-0.02864725083\,t}\}\end{aligned} \quad (75)$$

Taking t = 0, 10,…,100, availability of the system is obtained and presented in Table 1 below:

Table 1: *Availability variance with respect to time*

Time in days	Availability
0	1.000000
10	0.997679
20	0.997531
30	0.997383
40	0.997235
50	0.997086
60	0.996939
70	0.996791
80	0.996643
90	0.996495
100	0.996347

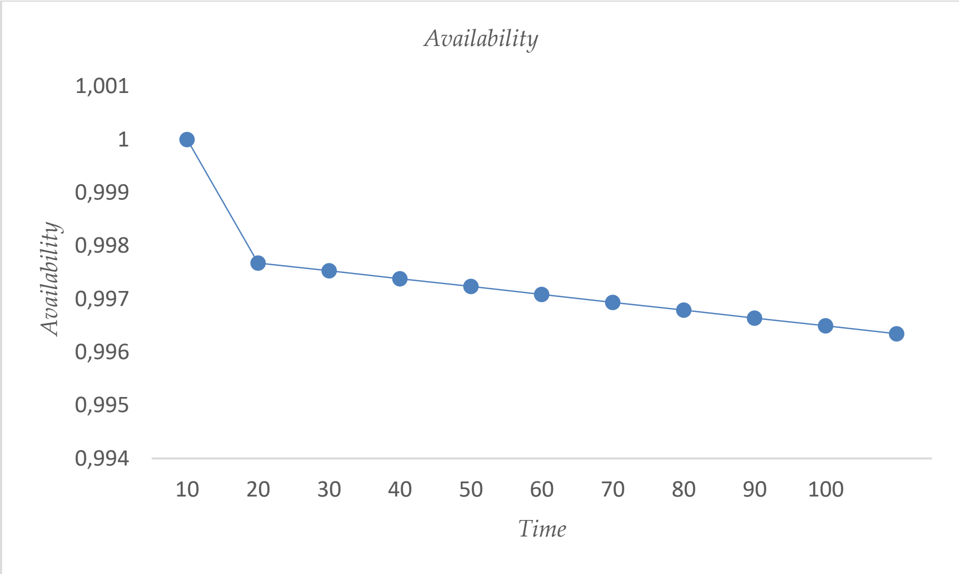


Figure 1: *Variation of availability with time*

III. Reliability Analysis

Letting all repair rates, $\varphi(x) = \varphi(y) = \varphi(z) = \beta_1 = \beta_2 = \beta_3 = 0$ in equation (70), and taking the values of failure rates and employing inverse Laplace transformation, the expression is reliability relation.

$$R(t) = \{ 0.002909090909\, e^{-0.011100000000\, t} + 0.2352941176\, e^{-0.016000000000\, t} + 0.3617967914\, e^{-0.033000000000\, t} + 0.4000000000\, e^{-0.013000000000\, t} \} \quad (76)$$

Using t = 0, 10...100 as time units in equation (72), reliability is determined and shown in Table 2 below.

Table 2: *Variation in Reliability as a Function of Time*

Time in Days	Reliability
0	1.000000
10	0.941981
20	0.887595
30	0.836601
40	0.788773
50	0.743903
60	0.701796
70	0.662272
80	0.625161
90	0.590308
100	0.557565

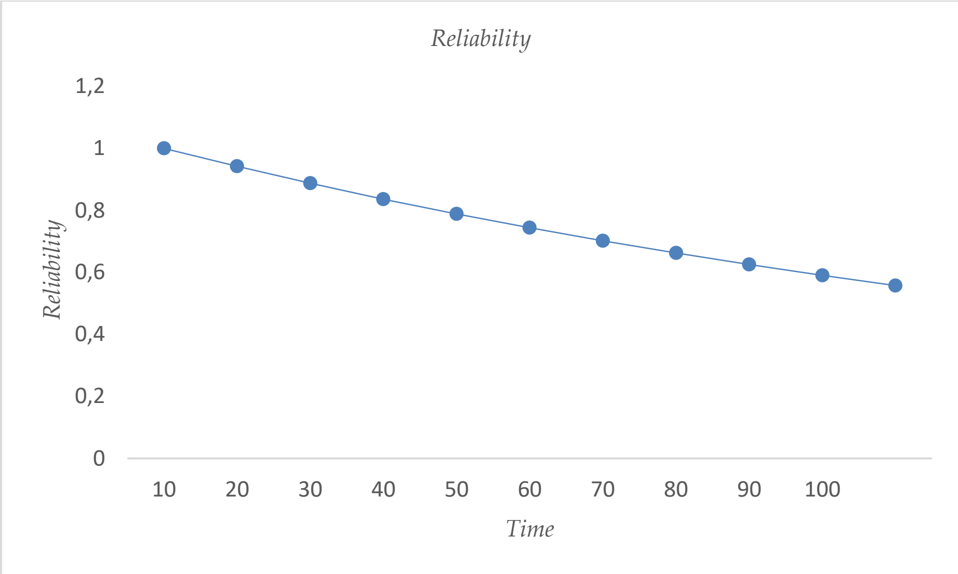


Figure 2. *Variation of reliability with time*

IV. Cost Analysis

The expression for the expected profit incurred in $[0,t)$

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{77}$$

Taking fixed values of parameters of equation (62), the subsequent equation (69) follows;

$$E_p(t) \;=\; E_p(t) = k_1 \begin{Bmatrix} -0.018340e^{-2.87072t} + 0.008479e^{-1.20122t} \\ +0.000094e^{-1.15618t} - 598.072776e^{-0.00016t} \\ +0.000183e^{-1.13000t} + 0.000237e^{-1.12000t} \\ +5985.0821 \end{Bmatrix} - k_2(t) \tag{78}$$

Table 3: *Expected profit as a function of time*

Time in days	$E_p(t)$					
0	0	0	0	0	0	0
10	3.97831	4.97831	5.97831	6.97831	7.97831	8.97831
20	7.95435	9.95435	11.95435	13.95435	15.95435	17.95435
30	11.92892	14.92892	17.92892	20.92892	23.92892	26.92892
40	15.90200	19.90200	23.90200	27.90200	31.90200	35.90200
50	19.87361	24.87361	29.87361	34.87361	39.87361	44.87361
60	23.84373	29.84373	35.84373	41.84373	47.84373	53.84373
70	27.81238	34.81238	41.81238	48.81238	55.81238	62.81238
80	31.77954	39.77954	47.77954	55.77954	63.77954	71.77954
90	35.74523	44.74523	53.74523	62.74523	71.74523	80.74523
100	39.70944	49.70944	59.70944	69.70944	79.70944	89.70944

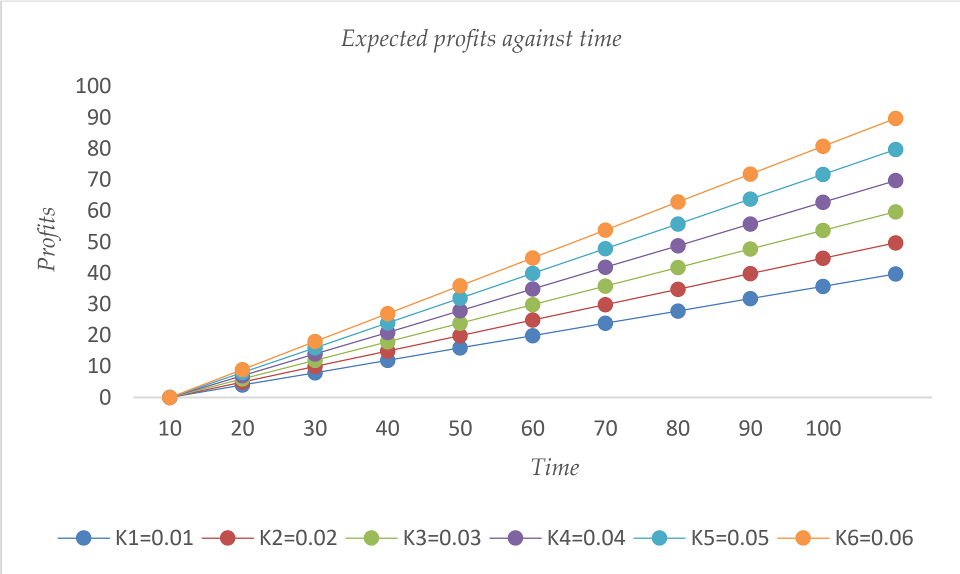


Figure 3: *Variation of profit with time*

V. Formulation and Analysis Mean Time to Failure

Setting repairs to zero in equation (70), the expression for MTTF is defined as follows:
Fixing $\alpha_1 = 0.001, \alpha_2 = 0.002, \alpha_3 = 0.003$ varying, failure rate in equation (66), MTTF is computed with respect to failure rate as presented in Table 3 below.

Table 4: *Variation of MTTF with failure rates* α_k

Failure rate	MTTF		
	(a)	(b)	(c)
0.001	222.8152	250.2918	401.6008
0.002	200.5337	222.8153	286.5720
0.003	182.3034	200.9009	222.8153
0.004	167.1114	183.0318	182.2731
0.005	154.2567	168.1965	154.2157
0.006	143.2384	155.6951	133.6447
0.007	133.6892	145.0273	117.9162
0.008	125.3336	135.8265	105.5002
0.009	117.9610	127.8176	95.44993

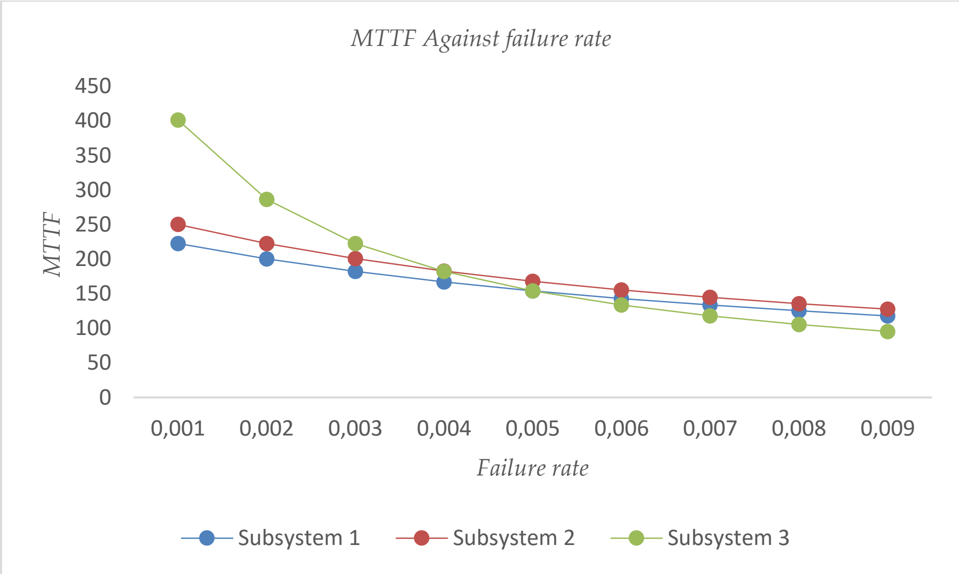


Figure 4: *MTTF as function of Failure rate*

VI. Sensitivity analysis corresponding to MTTF

The partial differentiation of MTTF with respect to the failure rate of the system can be used to investigate the system's sensitivity in MTTF. Using the set of parameters as $\alpha_1 = 0.001$, $\alpha_2 = 0.002$ $\alpha_3 = 0.003$, The MTTF sensitivity can be calculated in the partial differentiation of MTTF, as shown in the Table below and associated graphs in the Figure.

Table 5. *MTTF sensitivity as function of failure rate*

Failure rate	$\frac{\partial(MTTF)}{\alpha_1}$	$\frac{\partial(MTTF)}{\alpha_2}$	$\frac{\partial(MTTF)}{\alpha_3}$
0.001	-24757.25	-30952.89	-100000.61
0.002	-20053.37	-24386.44	-82020.57
0.003	-16573.03	-19689.49	-49563.88
0.004	-13925.95	-16214.28	-33163.29
0.005	-11865.90	-13571.10	-23737.80
0.006	-10231.31	-11514.08	-17826.70
0.007	-8912.61	-9881.903	-13877.30
0.008	-7833.35	-8565.144	-11108.57
0.009	-6938.88	-7487.473	-9092.820

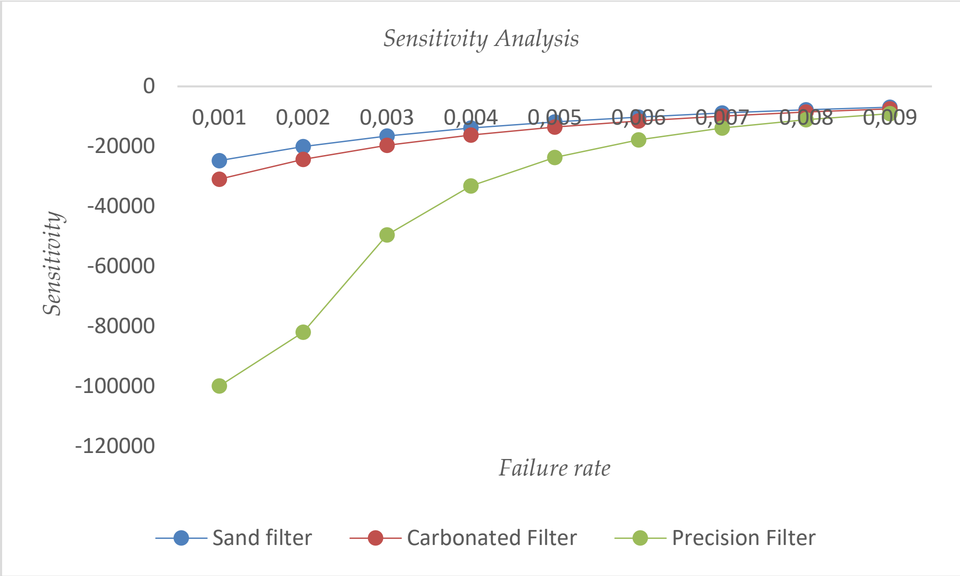


Figure 5: *Sensitivity analysis corresponding to mean time to failure (MTTF)*

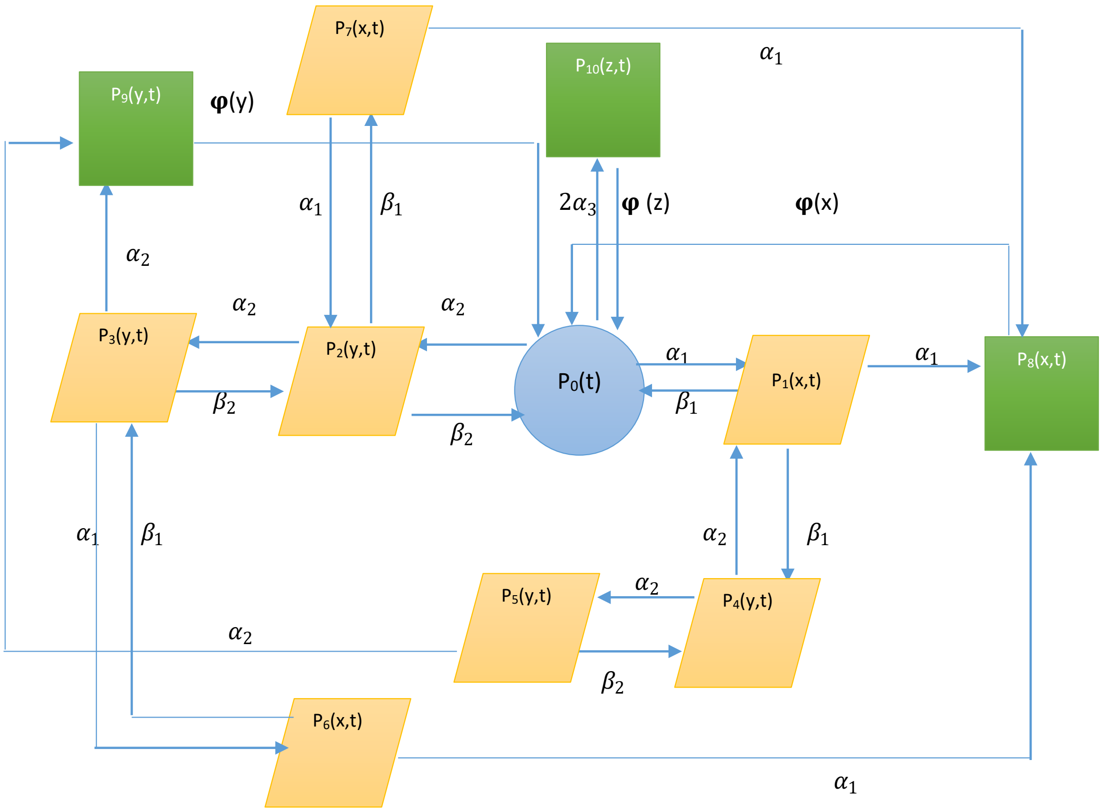
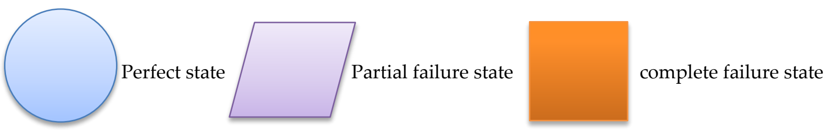


Figure 6: *Filtration system's transition diagram*



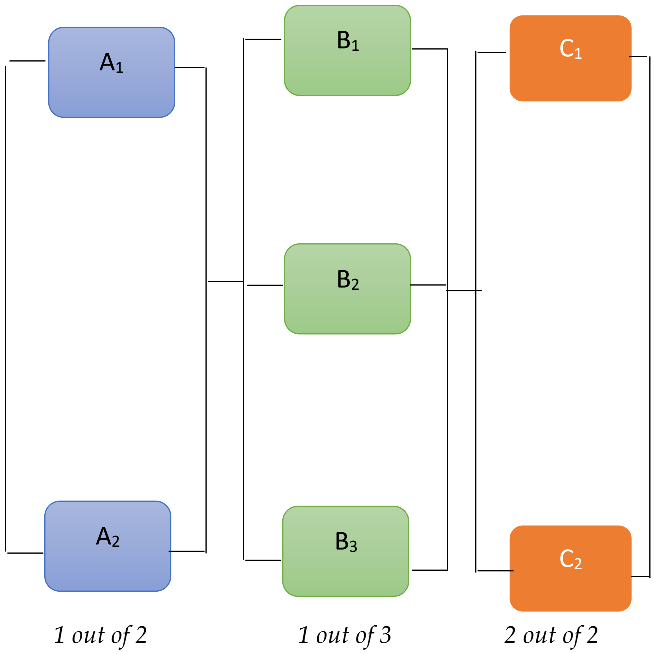


Figure 7: *Reliability block diagram of the filtration system*

VII. Description of the system

α_n: Failure rate for the subsystems. Where n=1,2,3.
β_q: Repair rate for the subsystems with incomplete failure Where q =1,2,3.
$\boldsymbol{\varphi}$(k) : Repair rate for the subsystems with complete failure. Where k= x, y, z.
P_0: Denote initial state where the system is working perfectly.
P_1: Denote state with an incomplete failure in subsystem 1 due to failure of first unit and copula repair is busy in repairing the failed unit.
P_2: Denote state with a complete failure in subsystem 2 due to failure of first unit and repair machine is busy in repairing the failed unit.
P_3: Denote state with a incomplete failure in subsystem 2
P_4: Denote state with an incomplete failure state due to failure of first unit in subsystem 1 and one unit from subsystem 2.
P_5: Denote state with an incomplete failure state due to failure of first unit in subsystem 1, first and second units from subsystem 2.
P_6: Denote state with an incomplete failure due to the failure of first and second units from subsystem 2, and one unit from subsystem 1.
P_7: Denote incomplete state of system due to failure of first unit from each of subsystems one and two
P_8: Denote complete failure state in subsystem 1.
P_9: Denote complete failure state in subsystem 2.
P_{10}: Denote complete failure state in subsystem 3.

IV. Discussion

I. Interpretation of the Result and Conclusion

Table1 and Figure 1 provide information how availability and time changes when failure rates are fixed at different values. When failure rates are fixed at lower values α_1 = 0.001, α_2= 0.002 and α_3 = 0.003. Table 1 shows the results, and Figure 1 shows the corresponding result, the system's availability decreases over time. And the decreases from the first point are greater than those from the second. This is due to the units in subsystem 1 having less redundancy. Figure 2 shows that the system is more reliable than it is available. However, this is due to a copula repair for a component that had completely failed. And the system will be assumed to be operational during the repair. The system's availability reduces with time and eventually stabilizes at zero over a sufficiently extended length of time. As a result, the graphical representation of the model shows that one may confidently describe the future behavior of a complex system at any time for any given set of parametric parameters. Figure 2 and table 2 indicates that incorporating copula considerably decreases system reliability with time. The graphical representation of the model shows that one may confidently forecast the future behavior of a complex system at any time for any given set of parametric variables. When the values of availability and reliability in Tables 1 and 2 are compared, it is clear that when repair is offered, the system performs far better than replacement.

Tables 4 and figure 4 yield the mean-time-to-failure (MTTF) of the system with respect to variation in the failure rates,α_1, α_2, and α_3 respectively when all other parameters are held constant Shown by color graphs (green, Pink, and ash) respectively. The system is also examined using the Gumbel-Hougaard family copula. According to the findings, incorporating copula considerably enhances system reliability.

Cost analysis of the system is done in the analytic part of the paper Figure 3 and table 3 shows the variation of cost-profit with variation in the values of parameters. Table 5 and figure 5 displayed the result of sensitivity analysis corresponding to mean time to failure (MTTF) with respect to the failure rates α_k from the table, MTTF sensitivity decreases with increase in α_k.

For the analysis of profit the following are used:

(a) Fixing $K_2 = 0.01$ varying time t from 0 to 100
(b) Fixing $K_2 = 0.02$ varying time t from 0 to 100
(c) Fixing $K_2 = 0.03$ varying time t from 0 to 100
(d) Fixing $K_2 = 0.04$ varying time t from 0 to 100
(e) Fixing $K_2 = 0.05$ varying time t from 0 to 100

Table 4 displayed the result of expected profit $E_p(t)$ with respect to K_2. From the table, it is evident that expected profit increases as K_2 decreases.

II. Conclusion

Due to a lack of data on the filtration system, the current paper developed a reliability modeling approach to investigate the filtration system's overall strength, efficiency, and performance. The reliability, availability, MTTF, and profit function of this paper can all be evaluated. We present a new filtration system model that includes three subsystems: a sand filter, an activated carbon filter, and a precision filter in this study.

The findings of the study suggest that reliability modeling can be used to investigate the strength, efficiency, and performance enhancement of a reverse osmosis (RO) filtering system. The system's strength, efficiency, and performance improvement are determined at this point. This study could be enhanced to incorporate a system with numerous subsystems and several repair machines to reduce repair facility congestion and handle problems using supplemental variable techniques. Among other things, the current effort will benefit water manufacturing and industrial uses that are hazardous to humans.

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