# Fractional Multi-objective Capacitated Transportation Problem with Different Membership Functions

[1]Sheema Sadia, [2]Qazi Mazhar Ali, [3]Zainab Asim, [4,*]Ahteshamul Haq

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[1,2,4]Department of Statistics & Operations Research Aligarh Muslim University, Aligarh-202002
[3]Faculty of Commerce & Management, SGT University, Gurgaon, Haryana
[1]sadia.sheema63@gmail.com, [2]qaziali88@gmail.com, [3]asmizainab90@gmail.com,
[4,*]a.haq@myamu.ac.in *Corresponding author

**Abstract**

*This Fractional Transportation Problem arises when an enterprise has to face the issue of maintaining a good ratio of some critical parameters. These parameters are directly concerned with product(s) transportation from sources to destination. This paper considers a multi-objective Capacitated Transportation Problem with Fractional Objectives. A fuzzy goal programming approach with different membership functions is applied to generate a different set of solutions. We also use Chebyshev's Goal Programming for obtaining the solutions. Finally, a numerical illustration is provided to validate our proposed model.*

**Keywords:** Multi-objective programming, Quadratic membership function, Mixed constraints, Fuzzy normal membership function, Fuzzy Cauchy membership function, Fuzzy programming

## 1. Introduction

A transportation problem (TP) occurs when a product (or products) must be transported from multiple sources (also known as origin, supply, or capacity centres) to multiple sinks (also called destination, demand or requirement centres). The fundamental TP was devised by Alfred Hitchcock [9]. The TP with fractional objective function is known as a fractional transportation problem (FTP). Swarup [14] was the first to propose it. It is crucial in logistics, supply chain management, stock cutting problems, resource allocation problems, ship and plane routing problems, cargo loading problems, and inventory problems. The FTP arises when an enterprise faces the challenge of maintaining a good ratio between critical parameters. These parameters are directly concerned with transporting a product or products from their origin to their destination. Fractional programming can optimize actual/standard transportation costs or total return/total investment on machines delivered from factories to workshops. In linear fractional TPs with mixed constraints, Gupta *et al.* [5] presented a paradox. Gupta and Arora [6-7] and Liu [10] are two other authors written about FTPs. In general, real-world TPs are modelled with multiple, conflicting objectives.

Furthermore, combining all objective functions into a single overall utility function is difficult for the decision-maker. So it is better to formulate a multi-objective TP. The capacitated TP are the TPs with bounds on general availabilities at assets and general vacation spot requirements. It may benefit telecommunication networks, production-distribution systems, rail and concrete street systems.

The capacitated TPs have also been discussed by authors like Arora and Gupta [2] and Gupta and Bari [8]. Zadeh [15] first delivered the idea of a fuzzy set concept. Then Zimmermann [16] carried out the fuzzy set concept with a few suitable membership functions to resolve linear programming problems with numerous goal functions. Bit *et al.* [3] implemented a fuzzy programming approach with a linear membership function to resolve the multi-objective TP. El-Wahed [4] gave the idea of a fuzzy programming approach to determine the optimal compromise solution of a MOTP with a fuzzy membership function. Akkapeddi [1] discussed the quadratic membership for the multi-objective TP. Singh [13] worked on multiple objective fractional costs TP with bottleneck time and impurities. Sadia *et al.* [12] presented a fuzzy approach to obtain the solution of multi-objective capacitated FTP. Fuzzy normal and fuzzy Cauchy membership functions were used by Mon and Cheng [11]. Gupta *et al.* [17] discussed two stage transportation problem with the different types of fuzzy environments. Kamal *et al.* [18] described the parameters estimation and goodness of fit for the multi-objective transportation problem under type-2 trapezoidal fuzzy numbers. The purpose of using FTP is to make the problem more realistic. It can prove more beneficial for the decision-maker to consider the proportion of transporting cost due to the covered path and favoured path because the transportation cost may vary due to the travelled and favoured path. Likewise, the proportion of exact and standard transportation time and transporting damage cost due to covered path and favoured path are also measured.

In this paper, we have taken mixed constraints of MOCFTP with fractional type objectives. As it is a multi-objective problem and the objectives are conflicting in nature. So a compromise solution is obtained by using the fuzzy programming approach. We have tried to use three membership functions: quadratic, fuzzy normal, and fuzzy Cauchy. As far as our knowledge, these membership functions have never been used to deal with TPs. The rest of the paper is organized as follows: Assumptions, notations and formulation are discussed in Section 2. In Section 3, we have discussed the algorithm using a fuzzy optimization approach with different membership functions and Chebyshev's Goal Programming. In section 4, an example of the proposed method is illustrated. The conclusion is presented in section 5.

## 2. Assumptions, notations and formulation of the problem

We consider mixed constraints MOCFTP under the following notations

### 2.1 Notations

| | |
|---|---|
| $m$ | Number of origins |
| $n$ | Number of destinations |
| $a_i$ | Units of supply $(i = 1,2,\dots,m)$ |
| $b_j$ | Units of demands $j = 1,2,\dots,n$ |
| $c_{ij}$ | Unit transporting cost due to travelled route from the $i^{th}$ starting point to $j^{th}$ endpoint. |
| $r_{ij}$ | Unit transporting cost due to preferred route from the $i^{th}$ starting point to $j^{th}$ endpoint |
| $t_{ij}^a$ | Actual transportation time from the $i^{th}$ starting point to $j^{th}$ endpoint |
| $t_{ij}^s$ | Standard transportation time from the $i^{th}$ starting point to $j^{th}$ endpoint |
| $d_{ij}$ | Damage transporting cost due to travelled origin from the $i^{th}$ path to $j^{th}$ endpoint |
| $x_{ij}$ | Units transported from the $i^{th}$ starting point to $j^{th}$ endpoint |
| $l_{ij}$ | Minimum quantity transported from the $i^{th}$ starting point to $j^{th}$ endpoint |
| $s_{ij}$ | Maximum transported quantity from the $i^{th}$ starting point to $j^{th}$ endpoint |

## 2.2 Problem's statement

Consider a TP of fractional type objective function with m numbers of starting points having $a_i(i = 1,2,\dots,m)$ units of supply to be transported among n numbers of endpoints with $b_j(j = 1,2,\dots,n)$ units of demand. The problem is determining the best transportation schedule for transporting the available quantity of products to meet demand while minimizing total transportation costs, time, and damage charges.

Let $x_{ij}$ be the number of units transported from $i^{th}$ starting point to the $j^{th}$ endpoint. The mathematical model of the MOCFTP with mixed constraints can be expressed as:

$$\text{Minimize} f_1 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}x_{ij}}$$

$$\text{Minimize} f_2 = max\left\{\frac{t_{ij}^{a}\middle|x_{ij} > 0}{t_{ij}^{s}\middle|x_{ij} > 0}\right\}$$

$$\text{Minimize} f_3 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}x_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n} r_{ij}x_{ij}}$$

$$\text{subject to:} \sum_{i=1}^{m} x_{ij} \le a_i\,; \sum_{j=1}^{n} x_{ij} \ge b_j$$

$$l_{ij} \le x_{ij} \le s_{ij}; x_{ij} \ge 0$$

## 2.3 Interpretation of objectives function

1. The proportion of unit transporting cost $c_{ij}$ and $r_{ij}$ due to travelled path and a preferred route respectively.

2. The proportion of the actual transportation time $t_{ij}^{a}$ and a standard transportation time $t_{ij}^{s}$.

3. The proportion of unit transporting damage cost $d_{ij}$ (loss of quantity and quality transportation) and $r_{ij}$ due to the travelled and a preferred path, respectively.

# 3. Solution Approach for MOCFTP

## 3.1 Fuzzy Optimization Approach-Algorithm:

In order to solve the multiobjective fractional capacitated TP with mixed constraints, we use the following algorithm

**Step 1**:- Firstly, we will formulate the payoff matrix as:-

Payoff Matrix = $f_1$ $f_2$ $f_3$

$$\begin{matrix} x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ \vdots \\ x_{ij}^{(k)} \end{matrix} \begin{bmatrix} f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) \\ f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) \\ \vdots & \vdots & \vdots \\ f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) & f_1(x_{ij}^{(1)}) \end{bmatrix}$$

where, $x_{ij}^{(k)}; k = 1,2,\dots,K$ are the $k^{th}$ individual optimal solutions that optimize the $k^{th}$ objective.

**Step 2:-** We will apply the fuzzy approach with the following membership functions defined below:

**A.** ***Quadratic membership function:*** To derive the compromise solution of MOCFTP, we used fuzzy programming. The membership functions for the cost objective are: $f_{kl}$ and $f_{ku}$ be the achieved aspired level and the highest acceptance level of the $k^{th}$ objective function, respectively. The membership function of the $k^{th}$ objective function is represented as follows:

$$\mu_k(f_k) = q_{k1}f_k^2 + q_{k2}f_k + q_3$$

The membership values of the $k^{th}$ objective function at the aspiration level and the highest acceptable level is 1 and 0, respectively. We used the equations as:

$$\mu_k(f_{kl}) = q_{k1}f_{kl}^2 + q_{k2}f_{kl} + q_3 = 1$$

$$\mu_k(f_{ku}) = q_{k1}f_{ku}^2 + q_{k2}f_{ku} + q_3 = 0$$

The above linear system of equations $q_{k2}$ and $q_{k3}$ are expressed in terms of $q_{k1}$. Thus, the quadratic membership function for the $k^{th}$ objective function is used in the following form:

$$\mu_k^Q(F_k) = \frac{f_{ku}-f_k}{f_{ku}-f_{kl}} + q_{k1}f_k^2 - q_{k1}(f_{kl}+f_{ku})f_k + q_{k1}f_{lk}f_{lu}$$

**B.** ***Fuzzy Normal:*** The membership function for Fuzzy Normal will take the following form:

$$\mu_k^{FN}\{F_k\} = \begin{cases} 1i & ff_k \le f_{kl} \\ exp\left[-k\left(\frac{f_k-f_{kl}}{f_{ku}-f_{kl}}\right)^2\right] if f_{kl} < f_k < f_{ku} & \\ 0i & ff_k \ge f_{ku} and k \ge 0 \end{cases}$$

**C.** ***Fuzzy Cauchy:*** The membership function for Fuzzy Cauchy will take the following form:

$$\mu_k^{FC}\{F_k\} = \begin{cases} 1i & ff_k \le f_{kl} \\ \frac{1}{1+\alpha\left(\frac{f_k-f_{kl}}{f_{ku}-f_{kl}}\right)^\beta} if f_{kl} < f_k < f_{ku} & \\ 0i & ff_k \ge f_{ku} \end{cases}$$

$$\alpha \ge 0 \text{and} \beta \text{ispositiveeven}$$

**Step 3**: The MOCFTP with mixed constraints can now be converted into equivalent non-linear models for the above-defined membership functions as follow:

**A.** ***Quadratic Membership function***: The proposed model for MOCFTP with mixed constraints on applying quadratic membership function will be of the following form:

$$\text{Minimize}\lambda$$

$$\text{Subjectto}$$

$$\frac{f_{1u}-f_1}{f_{1u}-f_{1l}} + q_{11}f_1^2 - q_{11}(f_{11}+f_{1u})f_1 + q_{11}f_{1u}f_{1l} \le \lambda$$

$$\frac{f_{2u}-f_2}{f_{2u}-f_{2l}} + q_{21}f_2^2 - q_{21}(f_{2l}+f_{2u})f_2 + q_{21}f_{2u}f_{2l} \le \lambda$$

$$\frac{f_{3u}-f_k}{f_{3u}-f_{3l}} + q_{31}f_3^2 - q_{31}(f_{3l}+f_{3u})f_3 + q_{31}f_{3u}f_{3l} \le \lambda$$

$$\sum_{i=1}^{m} x_{ij}\{\le/=/\ge\}a_i; \sum_{j=1}^{n} x_{ij}\{\le/=/\ge\}b_j$$

$$0 \le x_{ij} \le s_{ij}; x_{ij} \ge 0; \lambda \ge 0$$

**B.** ***Fuzzy Normal Membership Function:*** The proposed model for MOCFTP with mixed constraints on applying fuzzy normal membership function will be of the form:

$$\text{Minimize } \lambda$$

$$\text{subject to:}$$

$$exp\left[-k\left(\frac{f_1-f_{1l}}{f_{1u}-f_{1l}}\right)^2\right]\le\lambda,\; exp\left[-k\left(\frac{f_2-f_{2l}}{f_{2u}-f_{2l}}\right)^2\right]\le\lambda$$

$$exp\left[-k\left(\frac{f_3-f_{3l}}{f_{3u}-f_{3l}}\right)^2\right]\le\lambda$$

$$\sum_{i=1}^{m}x_{ij}\le a_i;\sum_{j=1}^{n}x_{ij}\ge b_j\,;0\le x_{ij}\le s_{ij};x_{ij}\ge 0;\lambda\ge 0$$

We will solve it for $k$=1

**C.** ***Fuzzy Cauchy Membership Function*****:** The proposed model for MOCFTP with mixed constraints on applying fuzzy Cauchy membership function will be of the form:

$$\text{Minimize}\lambda$$

$$\text{Subjectto}\frac{1}{1+\alpha\left(\frac{f_1-f_{1l}}{f_{1u}-f_{1l}}\right)^{\beta}}\le\lambda$$

$$\frac{1}{1+\alpha\left(\frac{f_2-f_{2l}}{f_{2u}-f_{2l}}\right)^{\beta}}\le\lambda$$

$$\frac{1}{1+\alpha\left(\frac{f_3-f_{3l}}{f_{3u}-f_{3l}}\right)^{\beta}}\le\lambda$$

$$\textstyle\sum_{i=1}^{m}x_{ij}\le a_i;\sum_{j=1}^{n}x_{ij}\ge b_j\,;0\le x_{ij}\le s_{ij};x_{ij}\ge 0;\lambda\ge 0$$

**D.** ***Chebyshev's Goal Programming:*** Chebyshev's Goal Programming is considered a particular case of the weighted Goal Programming technique. It seeks a solution that minimizes the worst deviation from each objective. The mixed constraints of the MOCFTP using Chebyshev's Goal Programming will be represented as:

$$\text{Maximize}\lambda$$

$$\text{Subjectto}f_1+\lambda\le f_{1u}$$

$$f_2+\lambda\le f_{2u}$$

$$f_3+\lambda\le f_{3u}$$

$$\sum_{i=1}^{m}x_{ij}\{\le/=/\ge\}a_i;\sum_{j=1}^{n}x_{ij}\{\le/=/\ge\}b_j$$

$$0\le x_{ij}\le s_{ij};x_{ij}\ge 0;\lambda\ge 0$$

where, the worst deviation level ($\lambda$) and aspiration levels for the upper bound is $f_{iu}(i=1,2,3)$.

# 4. Numerical Illustration

A case study is discussed to demonstrate and utility of the approaches. The numerical problem of simulated data (Sadia *et al.* [12]) is presented. The discussed models are defined to solve the problem. We consider three starting points and three endpoints. The fractional transportation cost, time and damage charges (both quantity and quality damage) are represented in Table [1-3].

**Table 1:** Cost charges matrix

| | $b_1$ | $b_2$ | $b_3$ | Supply |
|---|---|---|---|---|
| $a_1$ | 5/3 | 7/4 | 15/13 | ≤ 12 |
| $a_2$ | 8/12 | 17/14 | 12/7 | = 15 |
| $a_3$ | 19/15 | 10/6 | 13/8 | ≥ 20 |
| **Demand** | ≥ 9 | = 13 | ≤ 21 | |

**Table 2:** Time charges matrix

| | $b_1$ | $b_2$ | $b_3$ | Supply |
|---|---|---|---|---|
| $a_1$ | 17/9 | 5/2 | 10/3 | **≤ 12** |
| $a_2$ | 1/2 | 11/4 | 6/5 | **= 15** |
| $a_3$ | 13/8 | 16/12 | 10/11 | **≥ 20** |
| **Demand** | **≥ 9** | **= 13** | **≤ 21** | |

**Table 3:** Damage charges matrix

| | $b_1$ | $b_2$ | $b_3$ | Supply |
|---|---|---|---|---|
| $a_1$ | 13/8 | 15/9 | 8/11 | **≤ 12** |
| $a_2$ | 11/15 | 14/6 | 19/7 | **= 15** |
| $a_3$ | 9/7 | 15/6 | 8/17 | **≥ 20** |
| **Demand** | **≥ 9** | **= 13** | **≤ 21** | |

The mixed constraints of the MOCFTP will be as follows:

$$\text{Min} f_1 = \frac{5x_{11}+7x_{12}+15x_{13}+8x_{21}+17x_{22}+12x_{23}+19x_{31}+10x_{32}+13x_{33}}{3x_{11}+4x_{12}+13x_{13}+12x_{21}+14x_{22}+7x_{23}+15x_{31}+6x_{32}+8x_{33}}$$

$$\text{Min} f_2 = \frac{13x_{11}+15x_{12}+8x_{13}+15x_{21}+14x_{22}+19x_{23}+9x_{31}+15x_{32}+8x_{33}}{8x_{11}+9x_{12}+11x_{13}+15x_{21}+6x_{22}+7x_{23}+7x_{31}+6x_{32}+17x_{33}}$$

$$\text{Min} f_3 = \frac{17x_{11}+5x_{12}+10x_{13}+x_{21}+11x_{22}+6x_{23}+13x_{31}+16x_{32}+10x_{33}}{9x_{11}+2x_{12}+3x_{13}+2x_{21}+4x_{22}+5x_{23}+8x_{31}+12x_{32}+11x_{33}}$$

$$\text{Subjectto} \sum_{j=1}^{3} x_{1j} \le 12; \sum_{j=1}^{3} x_{2j} \le 15; \sum_{j=1}^{3} x_{3j} \le 20$$

$$\sum_{j=1}^{3} x_{i1} \le 9; \sum_{j=1}^{3} x_{i2} \le 13; \sum_{j=1}^{3} x_{i3} \le 21 \; 0 \le x_{11} \le 6, 0 \le x_{12} \le 7, 0 \le x_{13} \le 13, 0 \le x_{21} \le 6,$$

$$0 \le x_{22} \le 2, 0 \le x_{23} \le 13, 0 \le x_{31} \le 4, 0 \le x_{32} \le 7, 0 \le x_{33} \le 14.$$

*A.* ***Different membership functions for fuzzy programming approach***

The payoff matrix for $[l_{ij} = 0]$ is obtained after solving the problem as a single objective (ignoring the other objectives) using the LINGO optimization software will be as follows:

Payoff Matrix = $f_1 \quad f_2 \quad f_3$

$$\begin{matrix} x_{ij}^{(1)} \\ x_{ij}^{(2)} \\ x_{ij}^{(3)} \end{matrix} \begin{bmatrix} 1.316832 & 1.16129 & 1.34472 \\ 1.37988 & 1.068410 & 1.79661 \\ 1.406433 & 1.170886 & 1.168285 \end{bmatrix}$$

$$f_{1u} = 1.406433, f_{1l} = 1.316832, f_{2u} = 1.170886,$$
$$f_{2l}^{2} = 1.068410, f_{3u} = 1.79661 \text{and } f_{3l} = 1.168285$$

Individual optimum solutions are obtained by solving the above problem separately for each objective using the optimizing software LINGO in Table 4.

**Table 4:** Individual optimum solution

| | Objectives | Cost | Damage | Time |
|---|---|---|---|---|
| | *Objective Values* | ***1.316832*** | ***1.068410*** | ***1.168285*** |
| **Allocations** | $x_{11}$ | 0 | 0 | 0 |
| | $x_{12}$ | 4 | 7 | 6 |
| | $x_{13}$ | 5 | 0 | 0 |
| | $x_{21}$ | 2 | 6 | 6 |
| | $x_{22}$ | 6 | 2 | 0 |
| | $x_{23}$ | 7 | 7 | 9 |
| | $x_{31}$ | 4 | 3 | 3 |
| | $x_{32}$ | 7 | 4 | 7 |
| | $x_{33}$ | 9 | 14 | 12 |

The compromise solution obtained for **Quadratic Membership Function** is as follows: $x_{11}^* = 0, x_{12}^* = 7, x_{13}^* = 2, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 4, x_{32}^* = 4, x_{33}^* = 12$

The optimal compromise solution obtained using the **Fuzzy normal Membership Function** will be as follows: $x_{11}^* = 0, x_{12}^* = 4, x_{13}^* = 1, x_{21}^* = 5, x_{22}^* = 2, x_{23}^* = 8, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9$

The crisp problem for fuzzy Cauchy has been obtained after setting $\alpha = 0.5$and$\beta = 2$. The compromise solution obtained for **Fuzzy Cauchy Membership Function** is as follows: $x_{11}^* = 4, x_{12}^* = 4, x_{13}^* = 4, x_{21}^* = 5, x_{22}^* = 2, x_{23}^* = 8, x_{31}^* = 4, x_{32}^* = 7, x_{33}^* = 9$

The compromise solution obtained for **Chebyshev's Goal Programming** is as follows: $x_{11}^* = 0, x_{12}^* = 7, x_{13}^* = 2, x_{21}^* = 6, x_{22}^* = 2, x_{23}^* = 7, x_{31}^* = 4, x_{32}^* = 4, x_{33}^* = 12$

## 5. Conclusion

This article represents the optimal compromise solution with mixed constraints for a multiobjective fractional capacitated TP. Fuzzy programming with three different membership functions viz. quadratic, fuzzy normal and fuzzy Cauchy is used to obtain a compromise solution using a fuzzy programming approach, and Chebyshev's Goal Programming is also discussed to solve the problem multiobjective fractional capacitated TP. Finally, a comparative study is done with the results obtained in the paper and the results from Sadia *et al.* [12]. The results are summarized in Table 5.

This paper proposes fuzzy programming models by applying different membership functions to solve multiobjective fractional capacitated TP. Table 5 also compares the results obtained through different procedures to obtain and compare their efficiency. The methods used in the paper can also be applied for transportation, assignment and transhipment problems.

**Table 5:** Compromise optimum solution

| | **Approach** | **Membership/ Methods** | **Objective Values** | | |
|---|---|---|---|---|---|
| | | | ***Cost*** | ***Damage Charges*** | ***Time*** |
| Sadia *et al.* [12] | Fuzzy Programming | Linear | 1.359296 | 1.238494 | 1.389058 |
| | | Exponential | **1.349030** | 1.229213 | 1.314935 |
| | | Hyperbolic | 1.359296 | 1.238494 | 1.389058 |
| | Goal Programming | Lexicographic, $D_1$distance | 1.353103 | 1.129854 | **1.237344** |
| Discussed method | Fuzzy Programming | Quadratic | 1.349869 | **1.097561** | 1.257840 |
| | | Fuzzy Normal | 1.371758 | 1.249359 | 1.264085 |
| | | Fuzzy Cauchy | 1.359296 | 1.238494 | 1.389058 |
| | Goal Programming | Chebyshev's | 1.349869 | **1.097561** | 1.257840 |

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