RELIABILITY CRITERIA FOR DESIGNING LIFE TEST SAMPLING INSPECTION PLANS BASED ON LOMAX DISTRIBUTION

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Abstract

Acceptance sampling plays an important role in ensuring the quality of the products manufactured by the industrial production processes. Sampling inspection plans by attributes are adopted for taking decisions about the lots submitted for inspection. Such procedures are employed for sentencing individual lots or batches or lots in continuous stream. Reliability sampling is s specific inspection procedure which is used to decide whether the submitted lot or batch is acceptable or non-acceptable based on life tests. In reliability sampling, the lifetime of the items randomly drawn from the lot is considered as a random variable which follows a continuous probability distribution. In this paper, designing of single sampling plans for life tests is considered under the assumption that the lifetime random variable follows a Lomax distribution. Reliability criteria for designing life test plans when lot quality is evaluated in terms of mean life, median life, hazard rate and reliability life are proposed. Conversion factors for adapting acceptable quality levels to life and reliability testing under the assumption of Lomax distribution are determined and suitable illustrations are provided.

Keywords: Acceptable quality level, Consumer's risk, Lomax distribution, Operating characteristic function, Producer's risk, Reliability sampling, Single sampling plan.

1. Introduction

Sampling inspection is a product control strategy that decides whether a lot should be accepted or rejected based on the information obtained by the inspection of random sample(s) drawn from the submitted lot(s). Sampling inspection procedures are generally classified according to the nature of the quality characteristics, *viz*., measurable and non-measurable characteristics. When the quality characteristics are non-measurable, but are classified into go or no-go basis, such as good or bad, non-conforming or conforming, etc., the sampling inspection procedures are termed as attribute sampling. When the quality characteristics are measurable on a continuous scale, the corresponding sampling inspection procedures are called variables sampling, which are devised under the implicit assumption that the quality characteristic is a continuous random variable following a specific probability distribution. Reliability sampling plans, also termed as life test sampling plans, are operationally attributes sampling procedures, but involve lifetime of the components or items as a random variable which is distributed according to a specific continuous type probability distribution, such as the exponential, Weibull, lognormal, gamma distributions, etc. The lifetime of the components or items is observed by placing the sampled items under the test, called life test, which is defined as the process of evaluating the lifetime of the items through experiments. The literature in product control provides the importance of various continuous probability distributions like exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data in the studies relating to the design and evaluation of reliability sampling plans.

The earlier works, which laid the foundation for the expansion of several types of sampling plans, would include the theory of reliability sampling proposed and developed by various authors. One may refer [1] – [8] for the basic notions and terminologies of sampling inspection for life tests. The literature in statistical product control provides significant studies relating to the construction of life test sampling plans employing exponential, Weibull, lognormal and gamma distributions as well as several compound distributions for modeling lifetime data. A detailed account of the properties, methods of construction and performance of such plans is provided in [9] – [25], and the recent advances in the theory of life test sampling plans are discussed in [26] – [30].

Lomax distribution, introduced in [31], is a heavy-tailed probability distribution and is considered as Pareto Type II distribution. It has a wide range of applications in many fields which include business, economics, actuarial, medical and biological sciences. It has been proved to be much useful in reliability and life testing studies and in survival analysis. Properties of Lomax distribution and its extended form can be seen in [32] – [35]. In this paper, a specific life-test sampling plan is devised with reference to the life-time quality characteristic, which is modeled by Lomax distribution. A procedure for the selection of such plans indexed by acceptable and unacceptable mean life is evolved. Three different criteria for designing life-test plans when lot quality is evaluated in terms of mean life, hazard rate and reliability life are proposed. Factors for adapting acceptable quality level to life and reliability testing under the assumption of Lomax distribution are also illustrated.

2. Life Test Sampling Inspection Plans Based on Lomax Distribution

Sampling plans for life tests include a set of sampling procedures and rules for deciding whether to accept or reject a large number of items based on the sampled lifetime information about the items. Sampled items are tested for a set period of time under such plans. When all units are tested to failure, the standard plans can be used to compare the performance to the specified requirements, and the results can be used in an attribute sampling plan when the lifetimes are tested and the distributional assumption of the quality characteristics is fulfilled. Further, the number of failures which occur before a required time can be used with standard attributes plans in determining the disposition of the material. (See, [9]).

A typical life test sampling plan can be formulated in the following manner: Suppose, n items are placed for a life test and the experiment is stopped at a predetermined time, T. The number of failures occurred until the time point T is observed, and let it be d. The lot is accepted if d is less than or equal to the acceptance number, say, c; otherwise, it is rejected. Thus, the life test sampling plan is represented by n, the number of units on test, and the maximum allowable number of failures, c, called the acceptance number. Life tests, terminated before all units have failed, may be classified into two types, namely, failure terminated and time terminated. In a failure terminated life test, a given sample of n items is tested until the specified number of failures occurs and then the test is terminated. In time terminated life test, a given sample of n items is tested until a pre-assigned termination time, t, is reached and then the test is terminated.

Generally, these tests may be defined with reference to the specifications given in terms of one of the characteristics such as (i) the mean life, that is, the expected life of the product, (ii) the median life. (iii) the hazard rate, that is, the instantaneous failure rate at some specified time, t, and (iv) the reliable life, that is, the life beyond which some specified proportion of items in the lot will survive. One of the significant features of a life test plan is that it involves a random characteristic, called lifetime or time to failure, which can be more adequately described most often by skewed distributions. Application of continuous-type of distributions such as normal, exponential, Weibull, gamma and lognormal for lifetime variables in the studies concerned with the design and evaluation of life test sampling plans has been provided in the literature of sampling inspection, demonstrating the significant contributions of those distributions in the development of life test sampling plans. The Lomax distribution, one of the lifetime distributions, is now considered as the lifetime distribution for the design and evaluation of life-test sampling plan.

3. Lomax Distribution

Let T be a random variable representing the lifetime of the components. Assume that T follows Lomax distribution. The probability density function and the cumulative distribution function of T are, respectively, defined by

$$f(t;\theta,\lambda) = \frac{\lambda}{\theta}\left(1+\frac{t}{\theta}\right)^{-(\lambda+1)}, t > 0, \theta > 0, \lambda > 0 \tag{1}$$

and $$F(t;\theta,\lambda) = 1-\left(1+\frac{t}{\theta}\right)^{-\lambda}, t > 0, \theta > 0, \lambda > 0, \tag{2}$$

where λ and θ are the shape and scale parameters, respectively.

The mean life, the median life, the reliability function and hazard function for specified time t under Lomax distribution are. Respectively, given by

$$\mu = \frac{\theta}{\lambda-1}, \; for\ \lambda > 1, \tag{3}$$

$$\mu_d = \theta\left(\sqrt[\lambda]{2}-1\right), \tag{4}$$

$$R(t;\theta,\lambda) = \left(1+\frac{t}{\theta}\right)^{-\lambda}, t > 0, \theta > 0, \lambda > 0 \tag{5}$$

and $$Z(t;\theta,\lambda) = \frac{\lambda}{\theta}\left(1+\frac{t}{\theta}\right)^{-1}, t > 0, \theta > 0, \lambda > 0. \tag{6}$$

The reliability life is the life beyond which some specified proportion of items in the lot will survive. The reliability life associated with Lomax Distribution is defined and denoted by

$$\rho(t;\theta,\lambda) = \theta\left(R^{-1/\lambda}-1\right), \tag{7}$$

where $\underline{R}$ is the proportion of items surviving beyond life ρ. The proportion, p, of product failing before time t, is defined by the cumulative probability distribution of T and is expressed by

$$p = P(T \le t) = F(t;\theta,\lambda). \tag{8}$$

4. Application of Lomax Distribution in Reliability Sampling

The techniques for determining life test sampling plans based on Weibull distribution with mean life, hazard rate, and reliability life serving as reliability criteria for the submitted lots are discussed in [36] – [38]. The dimensionless quantities, *viz.*, $t/\mu\times100$, $tZ(t)\times100$ and $t/\rho\times100$, referred to as conversion factors, for determining the life test sampling plans under the reliability criteria are introduced in [39]. Analogous approaches are discussed, here, to construct the life test sampling plans using Lomax Distribution as the lifetime distribution for the lifetime quality characteristic.

The mean life criterion is determined by calculating the ratio $t/E(t)=t/\mu$, which is associated with the proportion, p, of products that fail before reaching the termination time t. Acceptable mean life and unacceptable mean life, which are associated with the producer's risk and the consumer's risk, are the two typically stipulated requirements when dealing with a life test sampling plan in practice for providing protection to the producer and the consumer, respectively. A desired sampling plan can be determined with the specification of these indices. The quality levels, corresponding to acceptable and unacceptable mean life, are defined by p_1 and p_2 with associated risks, where p_1 is the acceptable proportion of the lot failing before the specified time, t, and p_2 is the unacceptable proportion of the lot failing before the specified time, t. Based on mean life, median life, hazard rate, and reliability life as the criteria for life test plans under the Lomax Distribution, conversion factors are obtained. These conversion factors are used for deriving the plans satisfying the requirements.

When the test termination time is defined, the conversion factors can also be used to calculate the mean life, median life, hazard rate, and reliability life, and vice versa. The appropriate conversion factors in terms of percentages are computed and are provided in Table 1 through to 6. Numerical illustrations for demonstrating the use of tables for determining the operating characteristics of a given plan and finding the parameters of the single sampling plan satisfying the requirements in terms of acceptable quality level (acceptable mean life) and limiting quality level (unacceptable mean life) are provided in the following subsections.

4.1 Numerical Illustration

Under life testing experiments for ascertaining the reliability of components, an industrial practitioner desires to use a single sampling plan by attributes satisfying the requirements $(p_1 = 0.007, \alpha = 0.05)$ and $(p_2 = 0.05, \beta = 0.10)$. The past experimental results on the components produced by the industry have shown that the life time of the components follows Lomax distribution specified by the shape parameter $\lambda = 1.5$. For the specified requirements, the parameters of an optimum sampling plan is determined as n = 105 and c = 2 using the searching algorithm given in [40]. It is assumed to employ a test termination time of 250 hours and to count the number of failures over the span of 250 hours. Under the given conditions, the operating characteristics in terms of mean life are obtained using the operating characteristic function of the single sampling plan by attributes and provided in Table 7 along with the values of $k = t/\mu \times 100$ and $\mu = t/k \times 100$, where t = 250 hours.

4.2 Numerical Illustration

Suppose that a single sampling plan by attributes with parameters n and c is to be defined when the requirements are specified in terms of acceptable mean life of 200 hours and unacceptable mean life of 70 hours with the associated producer's and consumer's risks of 5% and 10%, respectively. Assume that the individual items are to be tested for 3 hours and that the lifetime of the items is distributed as Lomax Distribution with the shape parameter fixed as $\lambda = 2$. Then, at the specified levels, the values of k are determined as follows:

$$k_1 = t/\mu \times 100 = (3/200) \times 100 = 1.5$$

$$k_2 = t/\mu \times 100 = (3/70) \times 100 = 4.286$$

Entering Table 1 with these values, one obtains the proportions, $p_1 = 0.03$ and $p_2 = 0.08$, of product failing before the specified time t corresponding to the acceptable mean life and unacceptable mean life, respectively. The operating ratio, which is the measure of discriminating good and bad lots of items, is defined by OR = 0.08/ 0.03 = 2.67, corresponding to which a single sampling plan can be chosen from [9] as (n = 159, c = 8) or from [41] as (n =157, c = 8).

In a similar manner, while Tables 2 and 3 can be used to determine conversion factors so as to obtain the life test plans and the corresponding median life and hazard rate, Tables 4 through to 6 can be utilized for obtaining reliability life for the specified values of R, viz., 0.90, 0.95 and 0.99, respectively.

4.3 Numerical Illustration

The acceptable mean life under the life test sampling plans based on Lomax distribution can be determined using the ratio $k = t/\mu \times 100$ for any specified value of acceptable quality level, AQL, shown in [42]. When AQL is specified as 3 percent with 95 percent acceptance probability, the test termination time is given as t = 25 hours and the shape parameter is fixed as $\lambda = 1.5$, the average or expected mean life, μ, is determined as $\mu = t/k \times 100 = (25/1.026)\times 100 = 2436.6$ hours, which can be considered as an acceptable mean life. Accordingly, if a lot consisting of items which have the acceptable mean life specified at 2436.6 hours, the probability of acceptance of the lot would be 95%. Corresponding to the fixed value of AQL = 3 percent with 95 percent acceptance probability, the conversion factors for median life, hazard rate and reliability life criteria for the case in which the shape parameter is specified as $\lambda = 1.5$ under Lomax Distribution are given in the following table:

Criterion	Conversion Factor	Value of the Factor	AQL
Percent Nonconforming as per MIL - STD -105E	$p \times 100$	3	0.03
Mean Life	$k = t/\mu \times 100$	1.026	2436.6
Median Life	$k = t/\mu_d \times 100$	3.492284	715.9
Hazard Rate at 25 hours	$tz(t) \times 100$	3.015203	0.001206
Reliable Life (R = 0.90)	$t/\rho \times 100$	28.19134	88.7
Reliable Life (R = 0.95)	$t/\rho \times 100$	58.96959	42.4
Reliable Life (R = 0.99)	$t/\rho \times 100$	305.1403	8.2

It can be noted from the above table that when the proportion, p, of products that fail before reaching the termination time, i.e., t = 25 hours is specified as the acceptable level of 3 percent, the median life of the components is 715.9 hours, 90 percent of the components will survive beyond 88.7 hours, 95 percent of the components will survive beyond 42.4 hours and 99 percent of the components will survive beyond 8.2 hours.

Table 1. Values of $t/\mu \times 100$ Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	0.201817	0.336136	0.431968	0.503781	0.604234	0.671146
2	0.407337	0.677979	0.870847	1.015254	1.217073	1.351392
3	0.616667	1.025686	1.316821	1.534616	1.838731	2.040957
4	0.829918	1.379418	1.770079	2.062073	2.469426	2.740067
5	1.047205	1.739346	2.230818	2.597835	3.109387	3.448954
6	1.268648	2.105644	2.699241	3.142125	3.758848	4.167861
7	1.494373	2.478495	3.175561	3.695169	4.418056	4.897038
8	1.72451	2.858088	3.659998	4.257207	5.087262	5.636744
9	1.959193	3.244622	4.152782	4.828484	5.76673	6.38725
10	2.198566	3.638299	4.654149	5.409256	6.456732	7.148834
11	2.442774	4.039335	5.164347	5.999788	7.157552	7.921784
12	2.691971	4.447953	5.683634	6.600358	7.869484	8.706402
13	2.946319	4.864383	6.212277	7.211253	8.592833	9.502999
14	3.205984	5.288869	6.750557	7.832773	9.327917	10.3119
15	3.471141	5.721661	7.298763	8.465229	10.07507	11.13344
16	3.741973	6.163023	7.857198	9.108945	10.83462	11.96797
17	4.018672	6.613231	8.426179	9.76426	11.60695	12.81585
18	4.301438	7.072571	9.006036	10.43153	12.39241	13.67746
19	4.590479	7.541343	9.597112	11.11111	13.19139	14.5532

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
20	4.886016	8.01986	10.19977	11.8034	14.00431	15.44347
21	5.188277	8.508452	10.81438	12.50879	14.83158	16.3487
22	5.497506	9.007462	11.44134	13.2277	15.67364	17.26935
23	5.813954	9.51725	12.08106	13.96058	16.53095	18.20587
24	6.137887	10.03819	12.73397	14.70787	17.404	19.15874
25	6.469584	10.57069	13.40052	15.47005	18.29327	20.12848
50	18.52753	29.37005	36.44957	41.42136	47.92619	51.98421
60	27.03458	42.10079	51.60637	58.11389	66.40499	71.44177
70	40.50026	61.57216	74.22759	82.57418	92.79668	98.76032
80	65.59747	96.20089	113.1363	123.6068	135.5481	141.9952
90	132.7393	182.0794	204.5695	216.2277	226.7829	230.8869

Table 2. Values of $t / \mu_d \times 100$ Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	1.089282	1.144486	1.185111	1.216236	1.260759	1.291057
2	2.19855	2.308402	2.389184	2.451041	2.539475	2.599621
3	3.328382	3.492284	3.61272	3.704892	3.836589	3.92611
4	4.479376	4.696683	4.856241	4.978283	5.152561	5.270959
5	5.652156	5.922175	6.120285	6.271729	6.487866	6.634618
6	6.847369	7.169357	7.405412	7.58576	7.842995	8.01755
7	8.065691	8.438851	8.712204	8.920928	9.218458	9.42024
8	9.307824	9.731301	10.04127	10.27781	10.61479	10.84319
9	10.5745	11.04738	11.39323	11.65699	12.03252	12.28691
10	11.86648	12.38779	12.76873	13.0591	13.47224	13.75193
11	13.18456	13.75325	14.16847	14.48477	14.93453	15.23883
12	14.52958	15.14452	15.59314	15.93467	16.42001	16.74817
13	15.90238	16.56239	17.04349	17.40951	17.92931	18.28055
14	17.30389	18.00769	18.52026	18.90999	19.46309	19.8366
15	18.73505	19.48128	20.02428	20.43687	21.02205	21.41696
16	20.19683	20.98404	21.55635	21.99094	22.60689	23.02231
17	21.69028	22.51692	23.11736	23.57301	24.21838	24.65335
18	23.21647	24.08089	24.70821	25.18393	25.85728	26.3108
19	24.77653	25.67698	26.32984	26.82459	27.5244	27.99542
20	26.37166	27.30625	27.98323	28.49593	29.22058	29.708
21	28.00308	28.96982	29.66943	30.19889	30.94671	31.44936
22	29.6721	30.66887	31.3895	31.9345	32.70371	33.22037
23	31.38009	32.40461	33.14459	33.70382	34.49253	35.02192
24	33.12847	34.17833	34.93586	35.50793	36.31417	36.85492
25	34.91876	35.99138	36.76455	37.34802	38.16968	38.72038
50	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
60	145.9157	143.346	141.5829	140.2993	138.5568	137.4297
70	218.595	209.6426	203.6446	199.3517	193.6242	189.9814
80	354.054	327.5476	310.3914	298.4132	282.8268	273.1506
90	716.4437	619.9492	561.2397	522.0199	473.1921	444.1481

Table 3. Values of $tz(t)\times 100$ Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	1.001004	1.001674	1.002153	1.002513	1.003016	1.003352
2	2.004032	2.006727	2.008654	2.010101	2.01213	2.013483
3	3.00911	3.015203	3.019566	3.022844	3.027441	3.03051
4	4.016262	4.027152	4.034955	4.040821	4.049051	4.054551
5	5.025514	5.04262	5.054887	5.064113	5.077068	5.085729
6	6.036893	6.061658	6.079431	6.092805	6.111597	6.124167
7	7.050427	7.084316	7.108656	7.126985	7.152751	7.169998
8	8.066143	8.110646	8.142635	8.166739	8.200644	8.223352
9	9.084069	9.140701	9.181444	9.21216	9.255394	9.284369
10	10.10424	10.17454	10.22516	10.26334	10.31712	10.35319
11	11.12667	11.21221	11.27385	11.32038	11.38595	11.42995
12	12.15141	12.25377	12.3276	12.38337	12.46201	12.51481
13	13.17847	13.29929	13.3865	13.45242	13.54543	13.60792
14	14.20791	14.34883	14.45064	14.52763	14.63635	14.70944
15	15.23973	15.40243	15.52008	15.60911	15.73491	15.81953
16	16.27399	16.46018	16.59493	16.69697	16.84125	16.93836
17	17.31072	17.52214	17.67528	17.79133	17.95552	18.06611
18	18.34994	18.58836	18.76123	18.89230	19.07787	19.20295
19	19.39171	19.65893	19.85286	20.00000	20.20847	20.34908
20	20.43605	20.73392	20.95029	21.11456	21.34748	21.50467
21	21.483	21.81339	22.0536	22.23611	22.49505	22.66994
22	22.5326	22.89743	23.16293	23.36478	23.65138	23.84508
23	23.58491	23.98612	24.27836	24.50071	24.81664	25.03031
24	24.63995	25.07952	25.40002	25.64404	25.99101	26.22584
25	25.69776	26.17773	26.52803	26.79492	27.17469	27.43191
50	53.20635	55.50592	57.23373	58.57864	60.53543	61.88984
60	64.94378	68.56747	71.33223	73.5089	76.71379	78.95812
70	77.29026	82.77893	87.04709	90.45549	95.54978	99.17011
80	90.50676	98.70072	105.2368	110.5573	118.6736	124.5589
90	105.1888	117.6835	128.0528	136.7544	150.4732	160.7523

Table 4. Values of $t/\rho\times100$ when R = 0.90 Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	9.179481	9.238821	9.281348	9.313323	9.358197	9.388186
2	18.52739	18.6345	18.7112	18.76883	18.84968	18.90367
3	28.04859	28.19134	28.29348	28.37019	28.47773	28.54951
4	37.74813	37.91381	38.03227	38.12118	38.24575	38.32885
5	47.63126	47.80655	47.93179	48.02573	48.15727	48.24497
6	57.70344	57.87439	57.99643	58.08791	58.21594	58.30125
7	67.97035	68.12234	68.23074	68.31197	68.42555	68.50119
8	78.43793	78.5556	78.63946	78.70226	78.79002	78.84843
9	89.11233	89.1796	89.2275	89.26335	89.31342	89.34673
10	99.99997	99.99998	99.99998	99.99997	99.99998	99.99997
11	111.1076	111.0226	110.9622	110.917	110.8541	110.8122
12	122.4421	122.2536	122.1197	122.0197	121.8803	121.7877
13	134.0109	133.6993	133.4782	133.3132	133.0833	132.9307
14	145.8215	145.3665	145.0438	144.8031	144.468	144.2459
15	157.882	157.2619	156.8227	156.4952	156.0397	155.7378
16	170.2006	169.3929	168.8213	168.3955	167.8035	167.4114
17	182.786	181.7671	181.0466	180.5102	179.765	179.2718
18	195.6474	194.3922	193.5055	192.8458	191.93	191.3243
19	208.7942	207.2766	206.2055	205.4092	204.3044	203.5744
20	222.2365	220.4288	219.1543	218.2074	216.8947	216.0278
21	235.9846	233.8579	232.3599	231.2478	229.7072	228.6904
22	250.0496	247.5734	245.8309	244.5383	242.7488	241.5687
23	264.443	261.5851	259.5761	258.0868	256.0266	254.669
24	279.1768	275.9034	273.6046	271.9018	269.548	267.9981
25	294.2638	290.5392	287.9263	285.9922	283.3208	281.5631
50	842.7096	807.2465	783.1628	765.7495	742.2667	727.1703
60	1229.646	1157.156	1108.825	1074.341	1028.461	999.3483
70	1842.121	1692.333	1594.869	1526.535	1437.208	1381.488
80	2983.647	2644.116	2430.87	2285.098	2099.329	1986.27
90	6037.54	5004.519	4395.421	3997.365	3512.348	3229.714

Table 5. Values of $t/\rho \times 100$ when R = 0.95 Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	19.27196	19.32542	19.36365	19.39236	19.43257	19.4594
2	38.89754	38.97895	39.03713	39.08078	39.1419	39.18266
3	58.88693	58.96959	59.02862	59.07288	59.13483	59.17612
4	79.25073	79.3067	79.34663	79.37656	79.4184	79.4463
5	99.99998	99.99997	99.99998	99.99998	99.99998	99.99999
6	121.1461	121.0595	120.9978	120.9516	120.8871	120.8442
7	142.7011	142.4958	142.3496	142.2403	142.0876	141.9861
8	164.6774	164.3197	164.0653	163.8751	163.6098	163.4334
9	187.0879	186.5426	186.1551	185.8656	185.4619	185.1938
10	209.946	209.1762	208.6297	208.2216	207.6529	207.2754
11	233.266	232.233	231.5002	230.9533	230.1917	229.6865
12	257.0625	255.7255	254.778	254.0714	253.0879	252.436
13	281.3507	279.6673	278.4753	277.5869	276.3513	275.5327
14	306.1467	304.0722	302.6046	301.5115	299.9921	298.9862
15	331.4672	328.9547	327.1788	325.857	324.0209	322.8062
16	357.3296	354.3299	352.2115	350.6359	348.4488	347.0027
17	383.7522	380.2136	377.717	375.8613	373.2872	371.5865
18	410.7541	406.6223	403.71	401.5468	398.5482	396.5683
19	438.3553	433.5734	430.206	427.7065	424.2441	421.9597
20	466.5768	461.0847	457.221	454.3551	450.3881	447.7725
21	495.4405	489.1753	484.7719	481.5081	476.9936	474.0191
22	524.9694	517.8648	512.8765	509.1816	504.0749	500.7125
23	555.1877	547.174	541.5529	537.3925	531.6466	527.8662
24	586.1208	577.1245	570.8206	566.1584	559.7242	555.4942
25	617.7953	607.739	600.6999	595.4977	588.324	583.6111
50	1769.236	1688.568	1633.911	1594.456	1541.338	1507.245
60	2581.594	2420.495	2313.338	2237.011	2135.629	2071.404
70	3867.462	3539.96	3327.371	3178.576	2984.404	2863.486
80	6264.051	5530.865	5071.519	4758.068	4359.318	4117.05
90	12675.58	10468.27	9170.156	8323.379	7293.492	6694.403

Table 6. Values of $t/\rho\times 100$ when R = 0.99 Based on Lomax Distribution for Specified Values of λ

p%	Shape Parameter, λ					
	1.25	1.50	1.75	2.00	2.50	3.00
1	100.0001	100.0001	100.0001	100.0001	100.0001	100.0001
2	201.8351	201.698	201.6002	201.5269	201.4244	201.3561
3	305.5578	305.1403	304.8427	304.6198	304.3081	304.1006
4	411.2234	410.3754	409.7714	409.3192	408.6875	408.2672
5	518.8889	517.4535	516.4318	515.6675	514.6003	513.8907
6	628.6139	626.4268	624.8713	623.7084	622.0856	621.0071
7	740.4604	737.3495	735.1389	733.4872	731.1837	729.6538
8	854.4928	850.2781	847.2856	845.051	841.9366	839.8694
9	970.7784	965.2712	961.3644	958.4489	954.3879	951.6942
10	1089.387	1082.390	1077.43	1073.731	1068.583	1065.169
11	1210.392	1201.698	1195.541	1190.952	1184.568	1180.338
12	1333.869	1323.261	1315.755	1310.164	1302.392	1297.245
13	1459.898	1447.149	1438.135	1431.426	1422.105	1415.938
14	1588.561	1573.433	1562.747	1554.797	1543.761	1536.463
15	1719.947	1702.188	1689.656	1680.339	1667.413	1658.872
16	1854.144	1833.493	1818.933	1808.116	1793.119	1783.215
17	1991.248	1967.429	1950.651	1938.195	1920.938	1909.549
18	2131.358	2104.082	2084.887	2070.647	2050.931	2037.928
19	2274.578	2243.541	2221.721	2205.544	2183.162	2168.412
20	2421.016	2385.900	2361.235	2342.962	2317.699	2301.062
21	2570.786	2531.255	2503.517	2482.981	2454.611	2435.941
22	2724.009	2679.710	2648.658	2625.685	2593.972	2573.116
23	2880.808	2831.371	2796.752	2771.159	2735.856	2712.656
24	3041.316	2986.351	2947.900	2919.496	2880.344	2854.634
25	3205.672	3144.767	3102.206	3070.789	3027.518	2999.124
50	9180.371	8737.558	8438.036	8222.095	7931.735	7745.597
60	13395.61	12524.94	11946.82	11535.54	10989.96	10644.75
70	20067.84	18317.65	17183.60	16390.89	15357.76	14715.19
80	32503.47	28619.66	26190.94	24535.82	22433.07	21157.14
90	65772.19	54168.42	47357.61	42920.97	37532.35	34401.93

Table 7. Operating Characteristics of the Life Test Sampling Plan

Based on Mean Life Criterion
(n =105, c = 2 and λ = 1.5)

p	$P_a(p)$	$\lambda = 1.5$	
		$k = t/\mu \times 100$	$\mu = 250/k \times 100$
0.010	0.911201	0.336136	74374.66
0.015	0.790632	0.506334	49374.48
0.020	0.649366	0.677979	36874.30
0.025	0.510198	0.851089	29374.12
0.030	0.386710	1.025686	24373.94
0.035	0.284587	1.201788	20802.33
0.040	0.204328	1.379418	18123.58
0.045	0.143657	1.558597	16040.07
0.050	0.099187	1.739346	14373.22
0.055	0.067404	1.921687	13009.40
0.060	0.045163	2.105644	11872.85
0.070	0.019543	2.478495	10086.77
0.080	0.008106	2.858088	8747.105
0.090	0.003242	3.244622	7705.059
0.100	0.001256	3.638299	6871.342

5. Conclusion

A procedure for deriving single sampling plans for life tests is described under the condition that the lifetime quality characteristic is modeled by a Lomax distribution. The tables for the determining the sampling plans when lot quality is evaluated using four criteria, namely, mean life, median life, hazard rate and reliability life are constructed fixing a set of values of the shape parameter. Practitioners can generate the necessary sampling plans for other values of the shape parameter as per their requirements using the procedure described in this paper. Conversion factors are also included to assist in the calculation of the mean life, median life, and hazard rate at a certain test termination time and vice versa. The factors for adapting acceptable quality level as the index to mean life, median life, hazard rate and reliability life are also provided.

6. Acknowledgment

The authors are grateful to Bharathiar University, Coimbatore for providing necessary facilities to carry out this research work. The second author is indebted to the Department of Science and Technology, India for awarding the DST-INSPIRE Fellowship under which the present research has been carried out.

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