A NEW RANKING IN HEPTAGONAL FUZZY NUMBER AND ITS APPLICATION IN PROJECT SCHEDULING

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Abstract

Ranking fuzzy numbers is significant in optimization approaches such as assignment challenges, transportation problems, project schedules, artificial intelligence, data analysis, network flow analysis, an uncertain environment in organizational economics etc. This paper introduces a new fuzzy ranking in Heptagonal fuzzy numbers and arithmetic operations of Heptagonal fuzzy numbers defined. In the network, every activity duration is viewed by a Heptagonal fuzzy number. Every Heptagonal fuzzy number is transformed into a crisp number using the ranking function. By applying the traditional method, we calculate the fuzzy critical path. These procedures are illustrated with numerical examples and compared with existing ranking functions.

Keywords: Activity duration, centroid, fuzzy ranking, fuzzy critical path, heptagonal fuzzy number.

I. Introduction

One of the most significant concepts in network analysis is the critical path approach. It is utilized to resolve project problems by preparing the networks and determining the earliest date an activity may begin and be completed. It is also an algorithm for scheduling a collection of project networks. It is also frequently used in connection with the Program Evaluation and Review Technique (PERT).

Zadeh [10] introduced the 'fuzzy logic' concept considering inaccuracies and inconsistencies. Several academics have utilized various forms of fuzzy numbers to develop mathematical models over the last few decades. Examples for fuzzy numbers include Triangular fuzzy numbers, Trapezoidal fuzzy numbers, Pentagonal fuzzy numbers, etc.

In many practical situations, the variables that define information uncertainty or vagueness are usually Triangular or Trapezoidal fuzzy numbers. Chandrasekaran et al. [1] developed a new arithmetic operation in Heptagonal fuzzy numbers and solved the transportation problem in 2013. Rathi et al. [8] defined a new non-normal fuzzy number called the Heptagonal fuzzy number and arithmetic computations, suggested a parametric ranking strategy for ordering Heptagonal fuzzy numbers and employed the fuzzy assignment problem in 2014. The Heptagonal fuzzy number ranking is derived from the centroid of centroids and incentre of Heptagonal fuzzy numbers by Namarta et al. in 2017[7]. Developed a new ranking in Heptagonal fuzzy numbers and adapted it to transportation problems by Sahaya et al. [9]. Karthik et al. [4] in 2019 proposed linear and non-linear Heptagonal fuzzy numbers under uncertain environments and derived the Haar ranking technique for the Heptagonal fuzzy number. Malini [6] suggested a new ranking in Heptagonal fuzzy numbers and applied transportation problems. Hamildon et al. [3] determine the fuzzy critical path with normalized Heptagonal fuzzy data.

II. Preliminaries

In this section, we will look at a few key definitions.

I. Fuzzy Set [10]

As stated in Zadeh's paper, the formalization of a fuzzy set is:
Let X be a space of points (objects), with a generic element of X denoted by x. Thus, $X = \{x\}$. A fuzzy set (class) A in X is characterized by a membership (characteristic function) function $\mu_A(x)$, which associates with each point in X a real number in the interval [0,1], with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A. When A set in the ordinary sense of the term, its membership function can take on only two values, 0 and 1, $\mu_A(x) = 1 \; or \; 0$ according to x does or does not belong to A.

II. Fuzzy Number [5]

It is a Fuzzy set of the following conditions:

- Convex fuzzy set
- Normalized fuzzy set.
- Its membership function is piece-wise continuous.
- It is defined in the real number.

Fuzzy numbers should be normalized and convex. Here the condition of normalization implies that the maximum membership value is 1.

III. Heptagonal fuzzy number (HFN) [3]

A fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ is a normal Heptagonal fuzzy number, and its membership function is expressed as;

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{1}{2}\frac{(x-a_1)}{(a_2-a_1)}, & for \;\; a_1 \le x \le a_2 \\ 0.5, & for \;\; a_2 \le x \le a_3 \\ \frac{1}{2}+\frac{1}{2}\frac{(x-a_3)}{(a_4-a_3)}, & for \;\; a_3 \le x \le a_4 \\ \frac{1}{2}+\frac{1}{2}\frac{(a_5-x)}{(a_5-a_4)}, & for \; a_4 \le x \le a_5 \\ 0.5, & for \;\; a_5 \le x \le a_6 \\ \frac{1}{2}\frac{(a_7-x)}{(a_7-a_6)}, & for \;\; a_6 \le x \le a_7 \\ 0, & otherwise \end{cases}$$

The graphical depiction of normalized Heptagonal fuzzy number is represented in Figure 1.

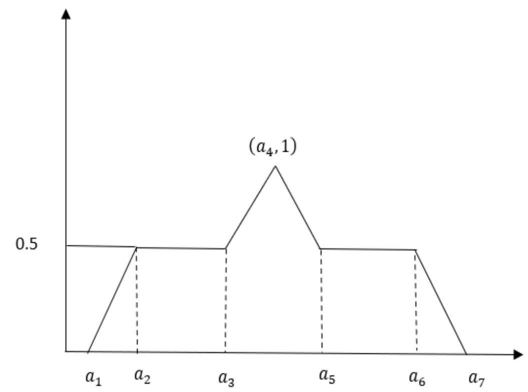

Figure 1: *Graphical representation of Heptagonal fuzzy number*

IV. Generalized Heptagonal fuzzy number (GHFN) [3]

A generalized Heptagonal fuzzy number is denoted by $\tilde{A}_{\tilde{H}} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, \omega)$ and its membership function is expressed as;

$$\mu_{\tilde{A}_{\tilde{H}}}(x) = \begin{cases} \frac{\omega}{2}\frac{(x-a_1)}{(a_2-a_1)}, & for \quad a_1 \le x \le a_2 \\ \frac{\omega}{2}, & for \; a_2 \le x \le a_3 \\ \frac{\omega}{2}+\frac{\omega}{2}\frac{(x-a_3)}{(a_4-a_3)}, & for \quad a_3 \le x \le a_4 \\ \frac{\omega}{2}+\frac{\omega}{2}\frac{(a_5-x)}{(a_5-a_4)}, & for \; a_4 \le x \le a_5 \\ \frac{\omega}{2}, & for \; a_5 \le x \le a_6 \\ \frac{\omega}{2}\frac{(a_7-x)}{(a_7-a_6)}, & for \; a_6 \le x \le a_7 \\ Otherwise, & 0 \end{cases}$$

The graphical depiction of generalized Heptagonal fuzzy number is represented in Figure 2.

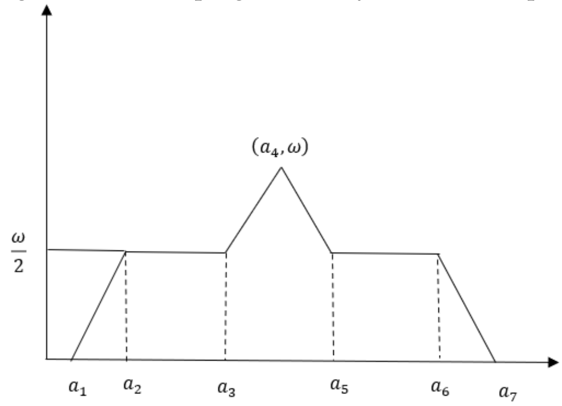

Figure 2: *Graphical representation of Generalized Heptagonal fuzzy number*

V. Arithmetic Operations of Heptagonal fuzzy number

According to Dubois [2], defined the arithmetic operation of Heptagonal fuzzy number.
Let $\tilde{A} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ and $\tilde{B} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7)$ be two Heptagonal fuzzy numbers then;

$k\tilde{A} = k(a_1, a_2, a_3, a_4, a_5, a_6, a_7) = (ka_1, ka_2, ka_3, ka_4, ka_5, ka_6, ka_7)$
$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7)$
$\tilde{A} \ominus \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4, a_5 - b_5, a_6 - b_6, a_7 - b_7)$
$\tilde{A} \otimes \tilde{B} = (a_1 * b_1, a_2 * b_2, a_3 * b_3, a_4 * b_4, a_5 * b_5, a_6 * b_6, a_7 * b_7)$
$\tilde{A} \oslash \tilde{B} = \left(\frac{a_1}{a_2}, \frac{a_2}{b_2}, \frac{a_3}{b_3}, \frac{a_4}{b_4}, \frac{a_5}{b_5}, \frac{a_6}{b_6}, \frac{a_7}{b_7}\right)$

Example:
Let $\tilde{A} = (3,6,9,12,15,18,21)$ and $\tilde{B} = (2,4,6,8,10,12,14)$ then
$\tilde{A} \oplus \tilde{B} = (5,10,15,20,25,30,35)$
$\tilde{A} \ominus \tilde{B} = (1,2,3,4,5,6,7)$
$\tilde{A} \otimes \tilde{B} = (6,24,54,96,150,216,294)$
$\tilde{A} \oslash \tilde{B} = (1.5, 1.5, 1.5,1.5,1.5,1.5,1.5)$
Remark: Some authors defined $\tilde{A} \ominus \tilde{B} = (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_3, a_6 - b_2, a_7 - b_1)$.
How is it possible?
Here I consider one example.
Let $\tilde{A} = (2,4,6,8,10,12,14)$ & $\tilde{B} = (2,4,6,8,10,12,14)$, Here both $\tilde{A}$ *and* $\tilde{B}$ are same HFNs.
Now $\tilde{A} \ominus \tilde{B} = (2 - 14, 4 - 12, 6 - 10, 8 - 8, 10 - 6, 12 - 4, 14 - 2)$
$= (-12, -8, -4,0,4,8,12)$.
It is a completely wrong output since both $\tilde{A}$ *and* $\tilde{B}$ are the same HFNs.
According to my definition;
$\tilde{A} \ominus \tilde{B} = (2 - 2, 4 - 4, 6 - 6, 8 - 8, 10 - 10, 12 - 12, 14 - 14) = (0,0,0,0,0,0,0)$

III. Existing Rankings

In this section, we explained existing ranking functions.

I. Existing Ranking1[7]

Namarta et al. suggested a ranking process for HFN prediction on the centroid of centroids and incentre of centroids. Their suggested order is as follows:

$$G(x_0, y_0) = \left(\frac{2a_1 + 7a_2 + 7a_3 + 22a_4 + 7a_5 + 7a_6 + 2a_7}{54}, \frac{11\omega}{54}\right)$$

II. Existing Ranking 2 [3]

Hamildon et al. identify the critical path of a project network with normal HFNs. In his approach,

$$\Re(\tilde{A}_{\tilde{H}}) = \frac{a_1 + a_2 + a_3 + 2a_4 + a_5 + a_6 + a_7}{8}$$

IV Proposal Ranking Function

We suggest an effective tool for calculating the rank of HFN. The proposal ranking in a HFN diagram is represented in Figure 3.

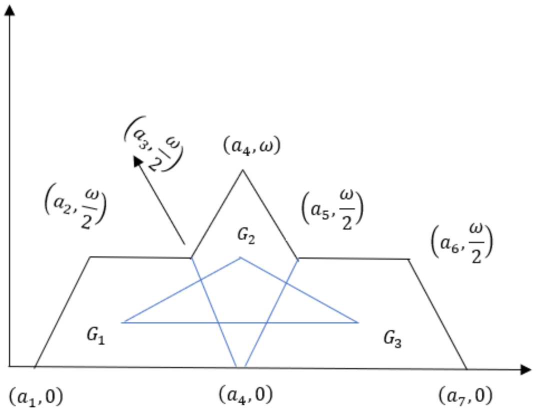

Figure 3: *Proposal ranking in GHFN*

In Figure 3, the heptagonal is divided into two trapezoidal and one rhombus. By applying the centroid formula of trapezoidal and rhombus, calculate the centroid of trapezoidal and rhombus, respectively. The circumcentre of the centroids of the HFN is taken into a balancing point of the Heptagon in Figure 7.3. The distance from the origin to the circumcentre of the centroids of this three-plane figure consider as a generalized HFN ranking function. Let the centroid of the three planar figures be G_1, G_2, G_3.

G_1 gives the centroid of the trapezoidal with vertices

$$(a_1,0),\left(a_2,\frac{\omega}{2}\right),\left(a_3,\frac{\omega}{2}\right)(a_4,0)$$

G_2 gives the centroid of the rhombus with vertices

$$(a_{4,}0),\left(a_3,\frac{\omega}{2}\right),(a_4,\omega),\left(a_5,\frac{\omega}{2}\right)$$

G_3 gives the centroid of the trapezoidal with vertices

$$(a_4,0),\left(a_5,\frac{\omega}{2}\right),\left(a_6,\frac{\omega}{2}\right),(a_7,0)$$

The centroid of these three planes is;

$$G_1=\left(\frac{a_1+a_2+a_3+a_4}{4},\frac{\omega}{4}\right),G_2=\left(\frac{a_3+a_4+a_5+a_4}{4},\frac{\omega}{2}\right),G_3=\left(\frac{a_4+a_5+a_6+a_7}{4},\frac{\omega}{4}\right)\text{ respectively.}$$

The circumcentre of G_1, G_2, and G_3 is

$$G_{\tilde{A}_{\bar{H}}}(x_0,y_0)=\left(\frac{a_1+a_2+2a_3+4a_4+2a_5+a_6+a_7}{12},\frac{\omega}{3}\right).$$

The generalized Heptagonal fuzzy number $\tilde{A}_{\bar{H}}=(a_1,a_2,a_3,a_4,a_5,a_6,a_{7,}\omega)$ new ranking function is;

$$\Re\left(\tilde{A}_{\bar{H}}\right)=\sqrt{x_0^2+y_0^2}$$

I. Ordering of a Heptagonal fuzzy number

Comparing fuzzy numbers using the ranking function $\Re: F(\mathcal{R})\longrightarrow\mathcal{R}$ is a successful approach**.** The ordering of two HFNs is described as follows;

- If $\Re\left(\tilde{A}_{\bar{H}}\right)>\Re\left(\tilde{B}_{\bar{H}}\right)\Longrightarrow\tilde{A}_{\bar{H}}>\tilde{B}_{\bar{H}}$
- If $\Re\left(\tilde{A}_{\bar{H}}\right)<\Re\left(\tilde{B}_{\bar{H}}\right)\Longrightarrow\tilde{A}_{\bar{H}}<\tilde{B}_{\bar{H}}$
- If $\Re\left(\tilde{A}_{\bar{H}}\right)=\Re\left(\tilde{B}_{\bar{H}}\right)\Longrightarrow\tilde{A}_{\bar{H}}=\tilde{B}_{\bar{H}}$

Here, we utilize three sets of Heptagonal fuzzy numbers. Analyze the ranking of 3 sets by proposal ranking function and existing ranking functions. The sets and the outcome obtained by the proposal and existing rankings are given in Table 1. Here I consider ω=1.

Table 1: *Analysis of ranking order by proposal ranking function*

HFN	The rank of HFN	Conclusion

	Set-1	
$\tilde{\boldsymbol{A}}$ =(1,2,3,4,5,6,7)	4.0138	$\mathcal{R}(\tilde{C}) > \mathcal{R}(\tilde{B}) > \mathcal{R}(\tilde{D}) > \mathcal{R}(\tilde{A})$ $\Rightarrow \tilde{C} > \tilde{B} > \tilde{D} > \tilde{A}$
$\tilde{\boldsymbol{B}}$ =(3,4,5,7,9,10,11)	7.0079	
$\tilde{\boldsymbol{C}}$ =(2,3,4,8,13,14,15)	8.3399	
$\tilde{\boldsymbol{D}}$ =(1,2,3,8,9,11,13)	6.9246	
	Set-2	
$\tilde{\boldsymbol{A}}$ =(6,8,9,10,11,12,13)	9.9334	$\mathcal{R}(\tilde{C}) > \mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{D}) > \mathcal{R}(\tilde{B})$ $\Rightarrow \tilde{C} > \tilde{A} > \tilde{D} > \tilde{B}$
$\tilde{\boldsymbol{B}} = (\mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}, \mathbf{7}, \mathbf{8}, \mathbf{9})$	6.0277	
$\tilde{\boldsymbol{C}} = (\mathbf{8}, \mathbf{9}, \mathbf{10}, \mathbf{11}, \mathbf{12}, \mathbf{13}, \mathbf{15})$	11.0983	
$\tilde{\boldsymbol{D}} = (\mathbf{5}, \mathbf{7}, \mathbf{8}, \mathbf{9}, \mathbf{10}, \mathbf{11}, \mathbf{12})$	8.9353	
	Set-3	
$\tilde{\boldsymbol{A}} = (\mathbf{5}, \mathbf{10}, \mathbf{15}, \mathbf{22}, \mathbf{23}, \mathbf{24}, \mathbf{25})$	19.0087	$\mathcal{R}(\tilde{A}) > \mathcal{R}(\tilde{B}) > \mathcal{R}(\tilde{C}) > \mathcal{R}(\tilde{D})$ $\Rightarrow \tilde{A} > \tilde{B} > \tilde{C} > \tilde{D}$
$\tilde{\boldsymbol{B}} = (\mathbf{4}, \mathbf{10}, \mathbf{12}, \mathbf{17}, \mathbf{18}, \mathbf{19}, \mathbf{21}$	15.1776	
$\tilde{\boldsymbol{C}} = (\mathbf{3}, \mathbf{10}, \mathbf{12}, \mathbf{13}, \mathbf{14}, \mathbf{16}, \mathbf{17})$	12.5133	
$\tilde{\boldsymbol{D}} = (\mathbf{3}, \mathbf{6}, \mathbf{8}, \mathbf{10}, \mathbf{11}, \mathbf{12}, \mathbf{13})$	9.3511	

The order of a HFN with the proposal and existing rankings are presented in Table 2.

Table 2: *Order of a HFN with the proposal and existing rankings*

Ranking function	Set-1	Set-2	Set-3
Namarta (2017)	$\tilde{C} > \tilde{B} > \tilde{D} > \tilde{A}$	$\tilde{C} > \tilde{A} > \tilde{D} > \tilde{B}$	$\tilde{A} > \tilde{B} > \tilde{C} > \tilde{D}$
Hamildon (2021)	$\tilde{C} > \tilde{B} > \tilde{D} > \tilde{A}$	$\tilde{C} > \tilde{A} > \tilde{D} > \tilde{B}$	$\tilde{A} > \tilde{B} > \tilde{C} > \tilde{D}$
Proposal ranking	$\tilde{C} > \tilde{B} > \tilde{D} > \tilde{A}$	$\tilde{C} > \tilde{A} > \tilde{D} > \tilde{B}$	$\tilde{A} > \tilde{B} > \tilde{C} > \tilde{D}$

V Application

This section performs an analytical example of the proposed fuzzy set CPM-based approach on an activity network. Think of a plant with 14 vertices and needs 21 primary activities, each activity connected by a direct link, like in the following graph (Figure 1.6). The fuzzy activity time is represented as a Heptagonal fuzzy number for every activity in Table 3 (All durations in days).

Table 3: *Project network with Heptagonal fuzzy number*

Activity	Heptagonal Fuzzy Number
1→2	(2,3,4,8,13,14,15)

1→3	(1,2,4,5,11,12,13)
1→4	(0,1,3,4,9,10,11)
1→5	(1,3,6,13,15,16,17)
1→6	(1,2,5,10,12,14,17)
2→7	(1,2,11,13,14,15,16)
3→7	(2,3,4,8,14,15,16)
3→10	(1,5,7,11,12,13,14)
4→8	(0,1,2,3,5,6,8)
4→9	(1,2,4,5,6,7,9)
5→9	(3,7,9,13,15,16,18)
5→13	(5,10,15,22,23,24,25)
6→14	(1,3,4,5,7,8,10)
7→11	(10,12,15,20,21,22,23)
7→12	(3,5,6,7,8,9,10)
8→12	(7,8,9,10,11,12,13)
9→13	(6,6,6,6,7,7,7)
10→14	(3,4,5,6,7,8,9)
11→14	(4,6,7,9,10,11,12)
12→14	(2,3,4,5,6,7,8)
13→14	(5,7,8,9,10,11,12)

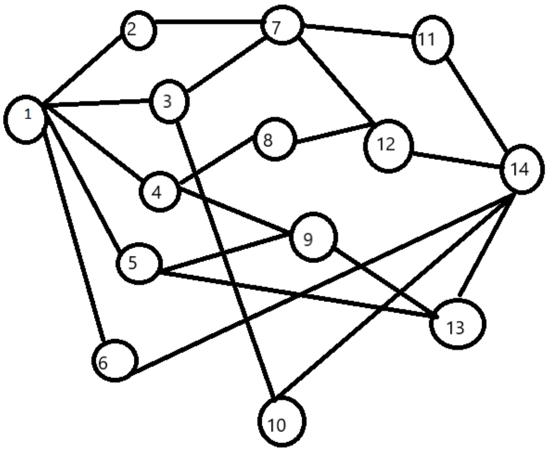

Figure 4: *Fuzzy project network*

I. Expected time of activities

Heptagonal fuzzy number transformed into an activity duration by proposal ranking function. This activity period is taken as the time within the nodes, and the fuzzy critical path is calculated by applying the conventional process. The activity period of the project network is represented in

Table 4, and Figure 5 represents the project network with defuzzified values of Heptagonal fuzzy numbers.

Table 4: *Activity duration with defuzzified value of HFN*

Activity	HFN	Activity period ($\tilde{t}_{ij}$)
1→2	(2,3,4,8,13,14,15)	8.3399
1→3	(1,2,4,5,11,12,13)	6.5085
1→4	(0,1,3,4,9,10,11)	5.1774
1→5	(1,3,6,13,15,16,17)	10.9217
1→6	(1,2,5,10,12,14,17)	9.0061
2→7	(1,2,11,13,14,15,16)	11.3382
3→7	(2,3,4,8,14,15,16)	8.6730
3→10	(1,5,7,11,12,13,14)	9.5891
4→8	(0,1,2,3,5,6,8)	3.4328
4→9	(1,2,4,5,6,7,9)	4.9279
5→9	(3,7,9,13,15,16,18)	12.0046
5→13	(5,10,15,22,23,24,25)	19.0029
6→14	(1,3,4,5,7,8,10)	5.3437
7→11	(10,12,15,20,21,22,23)	18.2530
7→12	(3,5,6,7,8,9,10)	6.9246
8→12	(7,8,9,10,11,12,13)	10.0055
9→13	(6,6,6,6,7,7,7)	6.3420
10→14	(3,4,5,6,7,8,9)	6.0092
11→14	(4,6,7,9,10,11,12)	8.5898
12→14	(2,3,4,5,6,7,8)	5.0110
13→14	(5,7,8,9,10,11,12)	8.9228

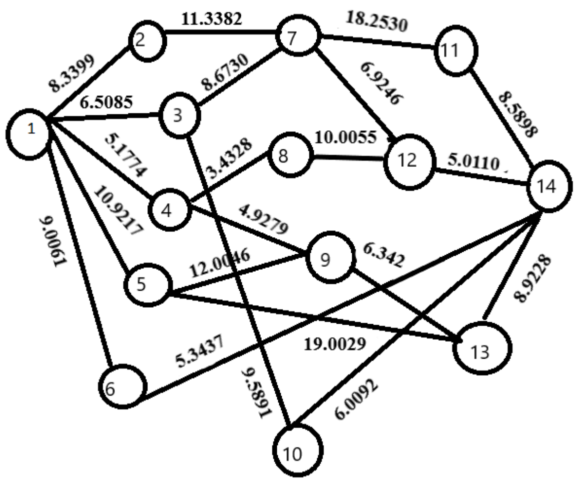

Figure 5: *Project network with defuzzified values of HFN*

II. Procedure for Fuzzy critical path method Based on Heptagonal fuzzy numbers

Step 1: Construct the network diagram of a given project.
Step 2: Represent every fuzzy activity time as a defuzzified value of the Heptagonal fuzzy number.
Step 3: Let $\tilde{E}_1 = (0,0,0,0,0,0)$ and calculate $\tilde{E}_i, i = 2,3, \dots \dots, n$ by using;

$$\tilde{E}_i = maximum\ end\ time\ of\ immediate\ predecessor + activity\ duration.$$

Step 4: Calculate the earliest finish time of an activity $i \rightarrow j$. That is earliest finish time is; $E\tilde{F}_{ij} = E\tilde{S}_{ij} + Fuzzy\ activity\ time$, where $E\tilde{S}_{ij} = \tilde{E}_i$.
Step 5: Let $\tilde{E}_n = \tilde{L}_n$ and calculate $\tilde{L}_i$, where $i = n-1, n-2, \dots \dots, 2,1$.

$$\tilde{L}_i = minimum\ end\ time\ of\ immediate\ successor -\ activity\ duration$$

Step 6: Calculate the latest start time of an activity $i \rightarrow j$. That is latest start time is;

$$L\tilde{S}_{ij} = L\tilde{F}_{ij} - Fuzzy\ activity\ time\text{=, where } L\tilde{F}_{ij} = \tilde{L}_j$$

Step 7: Calculate total float $T\tilde{F}_{ij} = L\tilde{F}_{ij} - E\tilde{F}_{ij}\ or\ L\tilde{S}_{ij} - E\tilde{S}_{ij}$.
Step 8: If $T\tilde{F}_{ij} = 0$, consider those activities as critical activities of a given project network.

III. Calculation of Earliest times

Node 1 is the starting node in the above network, and node 14 is the end node. Let $\tilde{E}_1 = 0$, and label node one as 0.

Iteration 1:
Node1 is the predecessor of node2.

$$\therefore \tilde{E}_2 = \tilde{E}_1 + \tilde{t}_{12} = 0 + 8.3399 = 8.3399$$

Iteration 2:
Node1 is the predecessor of node3.

$$\therefore \tilde{E}_3 = \tilde{E}_1 + \tilde{t}_{13} = 0 + 6.5085 = 6.5085$$

Iteration 3:
Node1 is the predecessor of node4.

$$\therefore \tilde{E}_4 = \tilde{E}_1 + \tilde{t}_{14} = 0 + 5.1774 = 5.1774$$

Iteration 4:
Node1 is the predecessor of node5.

$$\therefore \tilde{E}_5 = \tilde{E}_1 + \tilde{t}_{15} = 0 + 10.9217 = 10.9217$$

Iteration 5:
Node1 is the predecessor of node6.

$$\therefore \tilde{E}_6 = \tilde{E}_1 + \tilde{t}_{16} = 0 + 9.0061 = 9.0061$$

Iteration 6:
Node2 and node3 are predecessor of node 7.

$$\therefore \tilde{E}_7 = max\{\tilde{E}_2 + \tilde{t}_{27}, \tilde{E}_3 + \tilde{t}_{37}\} = \tilde{E}_2 + \tilde{t}_{27} = 8.3399 + 11.3382 = 19.6781$$

Iteration 7:
Node4 is the predecessor of node8.

$$\therefore \tilde{E}_8 = \tilde{E}_4 + \tilde{t}_{48} = 5.1774 + 3.4328 = 8.6102$$

Iteration 8:
Node4 and node5 are the predecessors of node9.

$$\therefore \tilde{E}_9 = max\{\tilde{E}_4 + \tilde{t}_{49}, \tilde{E}_5 + \tilde{t}_{59}\} = \tilde{E}_5 + \tilde{t}_{59} = 10.9217 + 12.0046 = 22.6343$$

Iteration 9:
Node3 is the predecessor of node10.

$$\therefore \tilde{E}_{10} = \tilde{E}_3 + \tilde{t}_{310} = 6.5085 + 9.5891 = 16.0976$$

Iteration 10:
Node7 is the predecessor of node11.

$$\therefore \tilde{E}_{11} = \tilde{E}_7 + \tilde{t}_{711} = 19.6781 + 18.2530 = 37.9311$$

Iteration 11:

Node7 and node8 are the predecessors of node 12.

$$\tilde{E}_{12} = max\{\tilde{E}_7 + \tilde{t}_{712}, \tilde{E}_8 + \tilde{t}_{812}\} = \tilde{E}_7 + \tilde{t}_{712} = 19.6781 + 6.9246 = 26.6027$$

Iteration 12:

Node5 and node 9 are predecessor of node 13.

$$\therefore \tilde{E}_{13} = max\{\tilde{E}_5 + \tilde{t}_{513}, \tilde{E}_9 + \tilde{t}_{913}\} = \tilde{E}_9 + \tilde{t}_{913} = 22.6343 + 6.3420 = 28.9763$$

Iteration 13:

Node6, node10, node11, node12, node13 are the predecessor of node14.

$$\therefore \tilde{E}_{14} = max\{\tilde{E}_6 + \tilde{t}_{614}, \tilde{E}_{10} + \tilde{t}_{1014}, \tilde{E}_{11} + \tilde{t}_{1114}, \tilde{E}_{12} + \tilde{t}_{1214}, \tilde{E}_{13} + \tilde{t}_{1314}\}$$
$$= \tilde{E}_{11} + \tilde{t}_{1114} = 37.9311 + 8.5898 = 46.5209$$

IV. Calculation of Latest times

In the above network, the end node is node14.

Let $\tilde{E}_{14} = \tilde{L}_{14}$ and label node14 as 46.5209.

Iteration1:

The successor of node 13 is node14.

$$\therefore \tilde{L}_{13} = \tilde{L}_{14} - \tilde{t}_{1314} = 46.5209 - 8.9228 = 37.5981$$

Iteration 2:

The successor of 12 is node14.

$$\therefore \tilde{L}_{12} = \tilde{L}_{14} - \tilde{t}_{1214} = 46.5209 - 5.0110 = 41.5099$$

Iteration 3:

The successor of 11 is node 14.

$$\therefore \tilde{L}_{11} = \tilde{L}_{14} - \tilde{t}_{1114} = 46.5209 - 8.5898 = 37.9311$$

Iteration 4:

The successor of 10 is node14.

$$\therefore \tilde{L}_{10} = \tilde{L}_{14} - \tilde{t}_{1014} = 46.5209 - 6.0092 = 40.5117$$

Iteration 5:

The successor of 9 is node13.

$$\therefore \tilde{L}_9 = \tilde{L}_{13} - \tilde{t}_{913} = 37.5981 - 6.3420 = 31.2561$$

Iteration 6:

The successor of node 8 is node 12.

$$\therefore \tilde{L}_8 = \tilde{L}_{12} - \tilde{t}_{812} = 41.5099 - 10.0085 = 31.5014$$

Iteration 7:

The successor of node 7 is node11 and node 12.

$$\therefore \tilde{L}_7 = min\{\tilde{L}_{11} - \tilde{t}_{711}, \tilde{L}_{12} - \tilde{t}_{712}\} = \tilde{L}_{11} - \tilde{t}_{711} = 37.9311 - 18.2530 = 19.6781$$

Iteration 8:

The successor of node 6 is node 14.

$$\therefore \tilde{L}_6 = \tilde{L}_{14} - \tilde{t}_{614} = 46.5209 - 5.3437 = 41.1772$$

Iteration 9:

The successor of node5 is node 9 and node 13.

$$\therefore \tilde{L}_5 = min\{\tilde{L}_9 - \tilde{t}_{59}, \tilde{L}_{13} - \tilde{t}_{513}\} = \tilde{L}_9 - \tilde{t}_{59} = 31.2561 - 12.0046 = 19.2515$$

Iteration 10:

The successor of node 4 is node8 and node9.

$$\therefore \tilde{L}_4 = min\{\tilde{L}_8 - \tilde{t}_{48}, \tilde{L}_9 - \tilde{t}_{49}\} = \tilde{L}_9 - \tilde{t}_{49} = 31.2561 - 4.9277 = 26.3284$$

Iteration 11:

The successor of node 3 is node7 and node 10.

$$\therefore \tilde{L}_3 = min\{\tilde{L}_7 - \tilde{t}_{37}, \tilde{L}_{10} - \tilde{t}_{310}\} = \tilde{L}_7 - \tilde{t}_{37} = 19.6781 - 8.6730 = 11.0051$$

Iteration 12:

The successor of node 2 is node7.

$$\therefore \tilde{L}_2 = \tilde{L}_7 - \tilde{t}_{27} = 19.6781 - 11.3382 = 8.3399$$

Iteration 13:

The successor of node1 is node2, node3, node4, node5 and node6.

$$\therefore \tilde{L}_1 = min\{\tilde{L}_2 - \tilde{t}_{12}, \tilde{L}_3 - \tilde{t}_{13}, \tilde{L}_4 - \tilde{t}_{14}, \tilde{L}_5 - \tilde{t}_{15}, \tilde{L}_6 - \tilde{t}_{16}\} = \tilde{L}_2 - \tilde{t}_{12} = 8.3399 - 8.3399 = 0$$

V. Calculation of Total float

Computed Earliest finish time, Latest start time and total float using formulas mentioned in procedure step 4, step 6, and step 7, respectively.

The earliest start and finish times, the latest start and finish times, total float of fuzzy activities are depicted in Table 5.

Table 5: ***The earliest, latest times and a total float of activities with defuzzified values***

Activity	$\tilde{t}_{ij}$	$E\tilde{S}_{ij}$	$E\tilde{F}_{ij}$	$L\tilde{S}_{ij}$	$L\tilde{F}_{ij}$	$T\tilde{F}_{ij}$
1→2	8.3399	0	8.3399	0	8.3399	0*
1→3	6.5085	0	6.5085	4.4966	11.0051	4.4966
1→4	5.1774	0	5.1774	21.151	26.3284	5.1774
1→5	10.9217	0	10.9217	8.3298	19.2515	7.6742
1→6	9.0061	0	9.0061	32.1711	41.1772	32.1711
2→7	11.3382	8.3399	19.6781	8.3399	19.6781	0*
3→7	8.6730	6.5085	15.1815	11.0051	19.6781	4.4966
3→10	9.5891	6.5085	16.0976	30.9226	40.5117	24.4141
4→8	3.4328	5.1774	8.6102	28.0686	31.5014	22.8912
4→9	4.9279	5.1774	10.1053	26.3282	31.2561	21.1508
5→9	12.0046	10.9217	22.9263	19.2515	31.2561	8.3298
5→13	19.0029	10.9217	29.9246	18.5952	37.5981	7.6735
6→14	5.3437	9.0061	14.3498	41.1772	46.5209	32.1711
7→11	18.2530	19.6781	37.9311	19.6781	37.9311	0*
7→12	6.9246	19.6781	26.6027	34.5853	41.5099	14.9072
8→12	10.0085	8.6102	18.6187	31.5014	41.5099	22.8912
9→13	6.342	22.6343	28.9763	31.2561	37.5981	8.6218
10→14	6.0092	16.0976	22.1068	40.5307	46.5399	24.4331
11→14	8.5898	37.9311	46.5209	37.9311	46.5209	0*
12→14	5.0110	26.6027	31.6137	41.5099	46.5209	14.9072
13→14	8.9228	28.9763	37.8991	37.5981	46.5209	8.6218

From the above table, the critical activities are 1→2, 2→7, 7→11, 11→14.

Therefore, the project critical path is 1→2→7→11→14.

As a result, the project will be completed in 46.5209≅ 46.5 days.

VI Results

Table 6 represents the fuzzy critical path and project completion time by proposal method and existing methods. The result graph is presented in Figure 6.

Table 6: ***Proposal method correlated with existing methods***

Ranking Method	Critical Path	Project completion time
Namarta (2017)	1→2→7→11	45.375
Hamildon (2021)	1→2→7→11	46.7853
Proposal method	1→2→7→11	46.5399

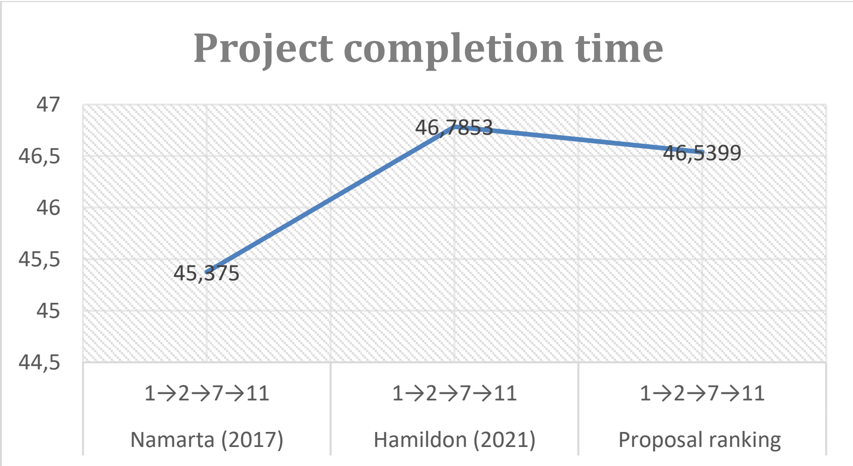

Figure 6: ***Proposal ranking results correlated with existing ranking results***

VII Conclusion

This paper introduced a new ranking function in Heptagonal fuzzy number. The proposed ranking function is derived from the centroid of HFN. In the network, every activity period is expressed by an HFN. The duration of every activity is transformed into the normal number or crisp number by a new ranking function. This normal number is considered as the expected time of activity. A conventional procedure identified the fuzzy critical path and project completion time. Numerous experiments have been conducted, and the results are correlated with some of the available ranking formulas. The attained results are similar to existing ranking results and the same critical path in all the methods. The proposal ranking can also be applied to more complex project networks in the real world. We can apply the ranking function of HFN to solve game problems and transportation problems.

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