Reliability and Sensitivity Analysis of Two Non-Identical Unit Standby System Subject to Pre-operation Random Inspection of Standby Unit

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Abstract

This paper examines the stochastic behavior of standby redundant system having two non-identical units. The system comprised of main unit and non-identical cold standby unit. When the main unit collapses, standby unit is exposed to operable conditions. Due to long-time and non-use of standby unit, though with small chances, it is observed that standby unit gets corrupt and becomes inoperable even in standby mode. Further, it demands repair/maintenance to make it worth-operating. Henceforth, it is considered to perform random inspection of standby unit to ensure that whether it is in operable condition or not. Inspection as well as repair both the tasks are performed by single repair facility. semi-Markov and regenerative processes are applied to derive expressions for the system performance indices. Profit function and bounds (upper/lower) for various costs involved are evaluated. Numerical study has been performed to illustrate the behavior of model developed. Sensitivity and relative sensitivity analysis has also been done for MTSF and steady-state availability.

Keywords: Reliability, Pre-operation random inspection, Cost-benefit analysis, Bounds, Sensitivity analysis

1. Introduction

Technological advances in recent decades have paved the way for numerous complicated and sophisticated systems. The ever increasing tech savvy inclination of consumers urges industries to introduce automation in their industrial process. Therefore, the need of hour is reduction in failures, availability and improvement in operational capacity of such systems. Redundancy is technique by which a system can be made highly reliable. Standby redundant systems have been used at a large scale in automation industry especially in computer and network, telecommunication and power systems. The two unit standby system and the various issues arising during the usage of such systems like switch over and activation time of standby unit, imperfect switching, random change of standby unit etc. have been addressed very extensively by several researchers. A standby system with switching device for activating standby unit and repaired failed unit for operations was investigated by Singh and Singh [1]. Mokaddis et al. [2] analyzed reliability models for standby system. Different working modes of the operative unit were taken into account. The perfect or imperfect switching of standby unit by assuming arbitrary distributions for failure and repair times were also studied.

Considering the activation time of standby unit, economic study of two-unit standby system was performed by Gupta et al. [3]. El-Said and El-Sherbeny [4] investigated profitability function for standby system, wherein the operative and standby unit interchanged randomly. Parashar and Taneja [5] analyzed stochastically hot standby PLC system. The study was carried out by collecting real data from various industries. Imperfect switching of standby unit as well as repairman patience time was studied by Rashad et al. [6]. A standby system with different failure types was discussed by Mahmoud and Mosherf [7].The preventive maintenance of online unit was also done when its operative time reaches to time t, subject to the availability of standby unit. Mathew et al. [8] analyzed two-unit working in parallel configuration casting plant system. Different kinds of failures were taken into the consideration. Jain and Rani [9] used Markov process to obtain availability characteristics for the standby system having switching failure and reboot delay. Manocha and Taneja [10] discussed two stages of repair for standby system. Jia et al. [11] compared perfect and imperfect switching policies for standby system. Barak et al. [12] investigated standby system, in which inspection of failed standby unit was conducted to confirm its reparability status. Wang et al. [13] investigated a warm standby system. The failures due to hardware and human errors were considered in their study and priority in use was given to main unit. Profit analysis was not done by the authors. El-Sherbeny et al. [14] discussed the idea of change between active unit and standby unit after random amount of time. Eventually, it can be concluded that certain technical issues that affect operational capacity of the system needs to be addressed as a prerequisite for standby units. Keeping this in view, the present article examines a two non-identical unit cold standby system, wherein standby unit may be inspected randomly to see as to whether it is worth useable or not. Sensitivity analysis with regard to MTSF and availability has also been done.The present paper is organised as follows.

System description and assumptions made to carry out the analysis are given in Section 2.Notations, different states and method used in the study are cited in Section 3, 4 and 5 respectively.In Section 6 stochastic model for the system (as defined in Section 2) is developed. Explicit expressions for different performance denoting characteristics of the system, profit and sensitivity function are derived in Section 7, 8 and 9 respectively. Numerical discussions are made in Section 10. Concluding remarks are stated in Section 11.

2. System Description and Assumptions

Proposed system consists of operative main unit and non-identical cold standby unit. Whenever, main unit get fail, the standby unit starts working and main unit goes for repair. There is a possibility that due to long-time non-use of standby unit in non-operative mode, it may be degraded and may become inoperable. The standby unit is inspected randomly to check either it can be made operable or it is inoperable due to degradation. Immediately the inoperable standby unit goes under repair/maintenance of the repairman. The repair process follows the first-come-first served (FCFS) rule. A single repair facility is considered for the system which takes cares of repair as well as inspection related activities. We use regenerative and semi-Markov process to obtain the various performance indicating characteristics of the system like Reliability, MTSF, point wise and transient availability, expected number of visits and time taken by repairman for repairing/inspecting the units. Finally these measures are used to formulate the profit and sensitivity function.The life time distribution of both the units is taken as exponential, whereas other time distribution are considered general. After each repair, unit is supposed to works like new one. The random variables used in developing stochastic model are independent.

3. Nomenclature

The notations for various rates/probabilities/pdf/states are:

λ/α: failure rate of main/standby unit
p/q: probability of operable/inoperable standby unit
p_1/q_1: probability of operable/inoperable standby unit after random inspection
W_{iF}/W_{iI}: P[repairman is engaged in regenerative state i for repair/inspection at instant t without switching to any other state]
®/ ©: symbol of Stieljes/ Laplace convolution.
E_0: Initial state of system
$g(t)(G(t))/g_1(t)(G_1(t))$: pdf (cdf) of repair time of main / standby unit
$h(t)(H(t))/i(t)(I(t))$: pdf (cdf) of time to/ time of inspection of standby unit
$q_{ij}(t)(q_{ij}^{(k)}(t))/Q_{ij}(t)(Q_{ij}^{(k)}(t))$: pdf/cdf of transition time from regenerative state i to j (or via non-regenerative state k).

Refer [5] for rest of the nomenclature used in the study

4. State of the System

The various states of the system at certain time point are described as:

State 0:(M_o, S)	State 1:(M_o, S_i)	State 2:(M_r, S_{wr})
State 3:(M_r, S_o)	State 4:(M_{wr}, S_I)	State 5:(M_o, S_r)
State 6:(M_{wr}, S_r)	State7:(M_R, S_{wr})	State 8:(M_{wr}, S_R)

where,

M_o: main unit is operative
S: standby unit
S_i: standby unit is under inspection
M_r: main unit is under repair
S_{wr}: standby unit is inline to get repaired
S_o: standby unit is operative
M_{wr}: main unit is waiting for repair
S_I: Inspection of standby unit is continued from last state
S_r: repair of standby unit is in progress
M_R: repair of main unit is in progress from last state
S_R: repair of standby unit is in progress from last state

5. Material and Methods

The time point at which system conditions are no longer relevant to system situation before to that time point are referred to as regenerative point, and the corresponding state is known to it as regenerative state otherwise non-regenerative state. In the model being discussed, when the repair/inspection is considered from previous state, the state is non-regenerative. The repair/inspection time distribution has been taken arbitrary; whereas the state where operation is continued from the previous state is the regenerative state as the failure time has been considered to follow exponential distribution which has the memory less property. Therefore the process is not purely Markov and hence semi-Markov (Branson and Shah[15]) process and regenerative process (Srinivasan and Gopalan [16]) have been used.

6. Stochastic Model

The transition between various states as described in Section 4 are shown in Fig.1. The state space is ξ=(0,1,2,3,4,5,6,7,8), where Ω=(2,6,7,8) and ϕ= (4) are failure and down state space respectively. By definition of regenerative process and assumptions made the sets ω=(0,1,2,3,5,6) and $\overline{\omega}$=(4,7,8) represents set of regenerative and non-regenerative states respectively. The transition densities

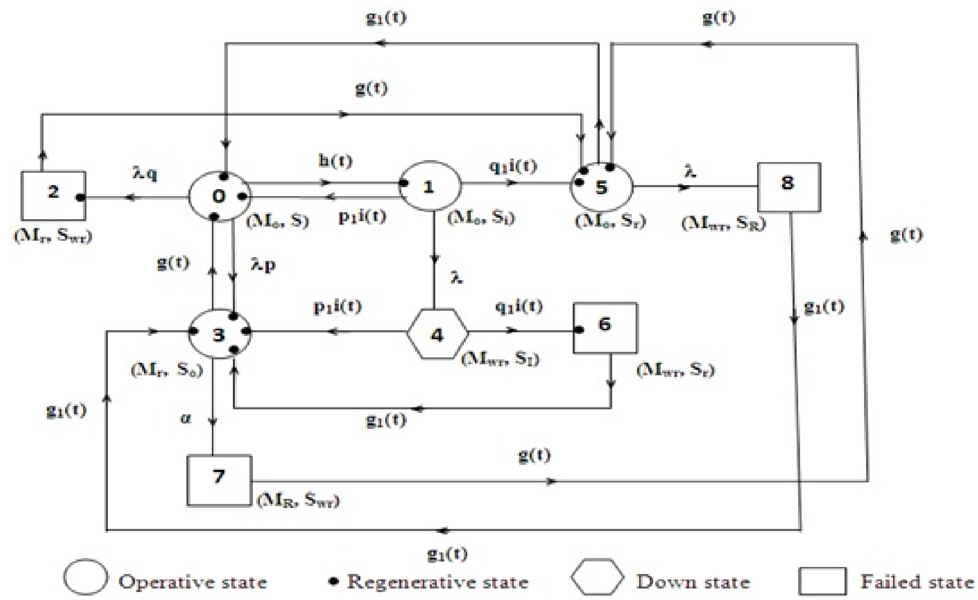

Figure 1: *State transition diagram*

from state i to j (or via k) are:

$$
\begin{aligned}
&q_{01}(t) = e^{-\lambda t}h(t), && q_{02}(t) = \lambda q e^{-\lambda t}\overline{H}(t), && q_{03}(t) = \lambda p e^{-\lambda t}\overline{H}(t) \\
&q_{10}(t) = p_1 e^{-\lambda t}i(t), && q_{13}^{(4)}(t) = p_1(\lambda e^{-\lambda t}©1)i(t), && q_{15}(t) = q_1 e^{-\lambda t}i(t) \\
&q_{16}^{(4)}(t) = q_1(\lambda e^{-\lambda t}©1)i(t), && q_{25}(t) = g(t), && q_{30}(t) = e^{-\alpha t}g(t) \\
&q_{37}(t) = \alpha e^{-\alpha t}\overline{G}(t), && q_{35}^{(7)}(t) = (\alpha e^{-\alpha t}©1)g(t), && q_{50}(t) = e^{-\lambda t}g_1(t) \\
&q_{53}^{(8)}(t) = (\lambda e^{-\lambda t}©1)g_1(t), && q_{58}(t) = \lambda e^{-\lambda t}\overline{G}_1(t), && q_{63}(t) = g_1(t)
\end{aligned} \tag{1}
$$

Mean sojourn time (μ_i) in state $i \in \omega$ is

$$
\begin{aligned}
&\mu_0 = \int_0^\infty e^{-\lambda t}\overline{H}(t)dt, && \mu_1 = \int_0^\infty e^{-\lambda t}\overline{I}(t)dt, && \mu_2 = \int_0^\infty \overline{G}(t)dt \\
&\mu_3 = \int_0^\infty e^{-\alpha t}\overline{G}(t)dt, && \mu_5 = \int_0^\infty e^{-\lambda t}\overline{G}_1(t)dt, && \mu_6 = \int_0^\infty \overline{G}_1(t)dt
\end{aligned} \tag{2}
$$

Further, defining

$$m_{ij} = E(q_{ij}(t)) = \int_0^\infty t q_{ij}(t)dt \tag{3}$$

and

$$m_{ij}^{(k)} = E(q_{ij}^{(k)}(t)) = \int_0^\infty t q_{ij}^{(k)}(t)dt \tag{4}$$

we have,

$$\begin{aligned} m_{01}+m_{02}+m_{03}&=\mu_0, & m_{10}+m_{15}+m_{13}^{(4)}+m_{16}^{(4)}&=K_1(say)\\ m_{25}&=\mu_2, & m_{30}+m_{37}&=\mu_3\\ m_{30}+m_{35}^{(7)}&=\mu_2, & m_{50}+m_{58}&=\mu_5\\ m_{50}+m_{53}^{(8)}&=\mu_6, & m_{63}&=\mu_6 \end{aligned} \tag{5}$$

7. System Performability Measures

7.1. System Reliability

If $mf_i(t)$ represents the cdf of time, taken by the system to transit from state $i, i \in \omega$ to a failed state, then from transition diagram we have

$$mf_0(t) = Q_{01}(t)®mf_1(t) + Q_{03}(t)®mf_3(t) + Q_{02}(t) \tag{6}$$

$$mf_1(t) = Q_{10}(t)®\ mf_0(t) + Q_{13}^{(4)}(t)®mf_3(t) + Q_{15}(t)®mf_5(t) + Q_{16}^{(4)}(t) \tag{7}$$

$$mf_3(t) = Q_{30}(t)®mf_0(t) + Q_{37}(t) \tag{8}$$

$$mf_5(t) = Q_{50}(t)®mf_0(t) + Q_{58}(t) \tag{9}$$

Making use of Laplace-Stieljes transformation for eqns. (6)-(9),the expressions obtained for $mf_0^{**}(s)$, reliability{R(t)} of the system and MTSF (mean time to system failure) are

$$mf_0^{**}(s) = L(s)/D(s) \tag{10}$$

$$R(t) = \mathcal{L}^{-1}\{1 - mf_0^{**}(s)/s\} \tag{11}$$

$$MTSF = \int_0^\infty R(t)dt = L/D \tag{12}$$

where

$$L(s) = Q_{01}^{**}(s)\{Q_{13}^{(4)**}(s)Q_{37}^{**}(s) + Q_{15}^{**}(s)Q_{58}^{**}(s) + Q_{16}^{(4)**}(s)\} + Q_{03}^{**}(s)Q_{37}^{**}(s) + Q_{02}^{**}(s) \tag{13}$$

$$D(s) = 1 - Q_{01}^{**}(s)\{Q_{10}^{**}(s) + Q_{13}^{(4)**}(s)Q_{30}^{**}(s) + Q_{15}^{**}(s)Q_{50}^{**}(s)\} - Q_{03}^{**}(s)Q_{30}^{**}(s) \tag{14}$$

$$L = \mu_0 + p_{01}K_1 + (p_{01}p_{13}^{(4)} + p_{03})\mu_3 + p_{01}p_{15}\mu_5 \tag{15}$$

$$D = 1 - p_{01}p_{10} - p_{01}p_{13}^{(4)}p_{30} - p_{01}p_{15}p_{50} - p_{03}p_{30} \tag{16}$$

7.2. System Availability

Let $W_i(t)$= P[system is in operative state $i, i \in \omega$, instead of transferring either to any state $j, j \in \omega$ or to itself via state $k, k \in \overline{\omega}$], Then

$$W_0(t) = e^{-\lambda t}\overline{H}(t) \tag{17}$$

$$W_1(t) = e^{-\lambda t}\overline{I}(t) \tag{18}$$

$$W_3(t) = e^{-\alpha t}\overline{G}(t) \tag{19}$$

$$W_5(t) = e^{-\lambda t}\overline{G}_1(t) \tag{20}$$

Defining $AV_i(t)$=P[system is operative at instant t | $E_0 = i,\ i \in \omega$]. Referring to contentions of regenerative process and from transition state diagram, the availabilities $AV_i(t)$ satisfies the relations

$$AV_0(t) = W_0(t) + q_{01}©AV_1(t) + q_{02}©AV_2(t) + q_{03}©AV_3(t) \tag{21}$$

$$AV_1(t) = W_1(t) + q_{10}©AV_0(t) + q_{13}^{(4)}©AV_3(t) + q_{16}^{(4)}©AV_6(t) + q_{15}©AV_5(t) \tag{22}$$

$$AV_2(t) = q_{25}©AV_5(t) \tag{23}$$

$$AV_3(t) = W_3(t) + q_{30}©AV_0(t) + q_{35}^{(7)}©AV_5(t) \tag{24}$$

$$AV_5(t) = W_5(t) + q_{50}©AV_0(t) + q_{53}^{(8)}©AV_3(t) \tag{25}$$

$$AV_6(t) = q_{63}©AV_3(t) \tag{26}$$

Using Laplace transformation and method of determinants for eqns. (21)-(26), we obtain

$$AV_0^*(s) = L_1(s)/D_1(s) \tag{27}$$

The system's transient and steady-state availability are

$$AV_0(t) = \mathcal{L}^{-1}\{L_1(s)/D_1(s)\} \tag{28}$$

$$AV_\infty = \lim_{t\to\infty} AV_0(t) = \lim_{s\to 0} sAV_0^*(s) = L_1/D_1 \tag{29}$$

where,

$$\begin{aligned} L_1(s) = &\{1 - q_{35}^{(7)*}(s)q_{53}^{(8)*}(s)\}\{W_0^*(s) + q_{01}^*(s)W_1^*(s)\} + \{q_{01}^*(s)q_{16}^{(4)*}(s)q_{63}^*(s) + q_{03}^*(s) \\ &+ q_{01}^*(s)q_{13}^{(4)*}(s)\}\{W_3^*(s) + q_{35}^{(7)*}(s)W_5^*(s)\} + \{q_{01}^*(s)q_{15}^*(s) \\ &+ q_{02}^*(s)q_{25}^*(s)\}\{q_{53}^{(8)*}(s)W_3^*(s) + W_5^*(s)\} \end{aligned} \tag{30}$$

$$\begin{aligned} D_1(s) = &\{1 - q_{35}^{(7)*}(s)q_{53}^{(8)*}(s)\}\{1 - q_{01}^*(s)q_{10}^*(s)\} - q_{02}^*(s)q_{25}^*(s)q_{50}^*(s) - q_{01}^*(s)q_{15}^*(s)q_{50}^*(s) \\ &- q_{30}^*(s)\{q_{03}^*(s) + q_{03}^*(s)q_{35}^{(7)*}(s)q_{50}^*(s) + q_{01}^*(s)q_{13}^{(4)*}(s) + q_{01}^*(s)q_{15}^*(s)q_{53}^{(8)*}(s) \\ &+ q_{02}^*(s)q_{25}^*(s)q_{53}^{(8)*}(s)\} - q_{01}^*(s)q_{16}^{(4)*}(s)q_{63}^*(s)\{q_{30}^*(s) + q_{35}^{(7)*}(s)q_{50}^*(s)\} \end{aligned} \tag{31}$$

$$\begin{aligned} L_1 = &(1 - p_{35}^{(7)}p_{53}^{(8)})(\mu_0 + p_{01}\mu_1) + \{p_{01}(p_{13}^{(4)} + p_{16}^{(4)} + p_{15}p_{53}^{(8)}) + p_{03} + p_{02}p_{53}^{(8)}\}\mu_3 + \{p_{01}(p_{13}^{(4)}p_{35}^{(7)} \\ &+ p_{16}^{(4)}p_{35}^{(7)} + p_{03}p_{35}^{(7)} + p_{15}) + p_{02}p_{25}\}\mu_5 \end{aligned} \tag{32}$$

$$\begin{aligned} D_1 = &(1 - p_{35}^{(7)}p_{53}^{(8)})(\mu_0 + p_{01}K_1 + p_{02}\mu_2) + (1 - p_{01}p_{10} - p_{01}p_{15}p_{50} - p_{02}p_{50})\mu_2 + \{(1 - p_{01}p_{10})p_{35}^{(7)} \\ &+ p_{02}p_{30} + p_{01}p_{15}p_{50} + p_{16}^{(4)}(1 - p_{35}^{(7)}p_{53}^{(8)})\}\mu_6 \end{aligned} \tag{33}$$

Employing the same procedure as discussed in Sub-section 7.2, other performability measures of the system are as follows:

7.3. Busy Period Analysis

7.3.1 Expected Time for Repairing the Failed Unit

Let $B_i(t)$=P[repairman is engaged in repair at instant t | $E_0 = i$, $i \in \omega$].The expected time taken by repairman in repairing the failed unit is

$$B_\infty = \lim_{t\to\infty} B_0(t) = \lim_{s\to 0} sB_0^*(s) = \lim_{s\to 0} s\{L_2(s)/D_1(s)\} = L_2/D_1 \tag{34}$$

where,

$$\begin{aligned} L_2(s) = &\{1 - q_{35}^{(7)*}(s)q_{53}^{(8)*}(s)\}\{q_{01}^*(s)q_{16}^{(4)*}(s)W_{6F}^*(s) + q_{02}^*(s)W_{2F}^*(s)\} + q_{01}^*(s)q_{15}^*(s)q_{53}^{(8)*}(s) \\ &+ \{q_{03}^*(s) + q_{01}^*(s)q_{16}^{(4)*}(s)q_{63}^*(s) + q_{01}^*(s)q_{13}^{(4)*}(s)\}\{W_{3F}^*(s) + q_{35}^{(7)*}(s)W_{5F}^*(s)\} \\ &+ q_{02}^*(s)q_{25}^*(s)\{q_{53}^{(8)*}(s)W_{3F}^*(s) + W_{5F}^*(s)\} \end{aligned} \tag{35}$$

$$\begin{aligned} L_2 = &p_{01}\{(p_{13}^{(4)} + p_{16}^{(4)})(\mu_2 + p_{35}^{(7)}\mu_6) + p_{15}p_{53}^{(8)} + (1 - p_{35}^{(7)}p_{53}^{(8)})\mu_6\} + p_{02}\{(1 - p_{35}^{(7)}p_{53}^{(8)})\mu_2 \\ &+ p_{53}^{(8)}\mu_2 + \mu_6\} + p_{03}(\mu_2 + p_{35}^{(7)}\mu_6) \end{aligned} \tag{36}$$

7.3.2 Expected Time for Inspection of the Standby Unit

Letting $I_i(t)$=P[repairman remains involved in inspection at time t | $E_0 = i, i \in \omega$].The expected time for which standby unit is under inspection, in steady-state is

$$I_\infty = \lim_{t\to\infty} I_0(t) = \lim_{s\to 0} sI_0^*(s) = \lim_{s\to 0} s\{L_3(s)/D_1(s)\} = L_3/D_1 \tag{37}$$

where,

$$L_3(s) = q_{01}^*(s)\{1 - q_{35}^{(7)*}(s)q_{53}^{(8)*}(s)\}W_{1I}^*(s) \tag{38}$$

$$L_3 = p_{01}\{1 - p_{35}^{(7)} p_{53}^{(8)}\}K_1 \tag{39}$$

7.4. Expected Number of Visits by the Repairman

If M(t) denotes the expected number of visits by repairman in the time interval (0,t] then $NV_i(t)$= E{M(t) | $E_0 = i, i \in \omega$}. In steady-state, the number of visits are

$$NV_\infty = \lim_{t\to\infty} NV_0(t) = \lim_{s\to 0} sNV_0^{**}(s) = \lim_{s\to 0} s\{L_4(s)/D_1(s)\} = L_4/D_1 \tag{40}$$

where,

$$L_4(s) = \{Q_{01}^{**}(s) + Q_{02}^{**}(s) + Q_{03}^{**}(s)\}\{1 - Q_{35}^{(7)**}(s)Q_{53}^{(8)**}(s)\} \tag{41}$$

$$L_4 = (1 - p_{35}^{(7)} p_{53}^{(8)}) \tag{42}$$

$D_1(s)$ and D_1 are specified in eqns. (31) and (33) respectively. Now, derived indexes are used to perform cost-benefit analysis in the succeeding section.

8. Cost-Benefit Analysis

As we know, the profit for any manufacturing system is the difference of expected revenue and expected recurring cost.Utilizing eqns.(29), (34), (37) and (40), the profit function for the defined system, in steady-state, is

$$P_\infty = (R_0 AV_\infty) - (C_B B_\infty + C_I I_\infty + C_V V_\infty) \tag{43}$$

where,R_0=Revenue generated per unit time
C_B/C_I= Recurring cost per unit time for repairing/inspecting the units
C_V= Recurring cost at per visit of repairman
For the system to be profitable, the eq.(43) is used to obtain the bounds for revenue/cost(s), which are shown in Table 1.

Table 1: *Bounds for revenue and various cost(s)*

Revenue/Cost	Bound	Value
R_0	Lower	$(C_B B_\infty + C_I I_\infty + C_V V_\infty)/AV_\infty$
C_B	Upper	$(R_0 AV_\infty - C_I I_\infty - C_V V_\infty)/B_\infty$
C_I	Upper	$(R_0 AV_\infty - C_B B_\infty - C_V V_\infty)/I_\infty$
C_V	Upper	$(R_0 AV_\infty - C_B B_\infty - C_I I_\infty)/V_\infty$

9. Sensitivity and Relative Sensitivity Analysis

Sensitivity analysis is performed to find out how the variation in incoming variable affects the specific outgoing variable under certain specific conditions. Since, there is significance difference between the values of incoming variables, so to compare their effects on outgoing variables, relative sensitivity function is used. Relative sensitivity function is defined as percentage change that results from the percentage change in one of the variable. The sensitivity and relative sensitivity functions for MTSF and availability (AV_∞) are formulated as:

$$\pi_k = \frac{\partial MTSF}{\partial k} \tag{44}$$

$$\delta_k = \pi_k(\frac{k}{MTSF}) \tag{45}$$

$$\rho_k = \frac{\partial AV_\infty}{\partial k} \tag{46}$$

$$\tau_k = \rho_k(\frac{k}{AV_\infty}) \tag{47}$$

where $k = \lambda, \alpha, \beta, \beta_1, \gamma, \theta$.

10. Results and Discussion

In this section numerical analysis is done to illustrate the developed stochastic model.Input/Output variables are specified in the subsections 10.1 and 10.2 respectively for further discussions.

10.1. Input Variables

The repair time of main/standby unit, time to inspection and time for inspection of standby unit are supposed to be exponential with parameters β,β_1,θ and γ respectively. Then
$G(t) = 1 - exp(-\beta t)$,$G_1(t) = 1 - exp(-\beta_1 t)$, $H(t) = 1 - exp(-\theta t)$ and $I(t) = 1 - exp(-\gamma t)$.
Time (t) and various rates/cost(s) are our input variables and their values are taken as:
$\lambda = 0.001, \alpha = 0.008, p = 0.98, q = 0.02, p_1 = 0.95, q_1 = 0.05, \beta_1 = 0.85, \gamma = 10, \beta = 0.65, \theta = 0.004$
$R_0 = 40, C_B = 5000, C_I = 2000, C_V = 2000$.

10.2. Output Variables

Measures including reliability, MTSF, availability, profit and sensitivity functions are output variables as obtained in sections 7, 8 and 9 respectively. Variations in output variables caused by changes in input variables have been investigated and are discussed in the following subsections.

10.3. Trend of Reliability{R(t)} w.r.t. time(t) for varying λ

Taking the other parameter constant, as mentioned in subsection 10.1, the mathematical expressions for reliability {R(t)} of the system for varied λ are as follows:
For $\lambda = 0.001$

$$R(t) = 0.991602 + 2.14908 \times 10^{-10} e^{-10.001t} - 3.11 \times 10^{-7} e^{-0.85122t} - 1.81 \times 10^{-5} e^{-0.658973t} + 0.00841596 e^{-0.00382477t} \tag{48}$$

For $\lambda = 0.002$

$$R(t) = 0.983342 + 4.29906 \times 10^{-10} e^{-10.002t} - 6.23 \times 10^{-7} e^{-0.852221t} - 3.6 \times 10^{-5} e^{-0.659946t} + 0.016695 e^{-0.00385044t} \tag{49}$$

For $\lambda = 0.003$

$$R(t) = 0.975214 + 6.4499 \times 10^{-10} e^{-10.003t} - 9.37 \times 10^{-7} e^{-0.853222t} - 5.4 \times 10^{-5} e^{-0.660919t} + 0.0248404 e^{-0.00387604t} \quad (50)$$

Fig.2 shows the trends of system reliability {R(t)} for varied (t, λ). Clearly, it goes down with the rise in the values of variables t and λ respectively.

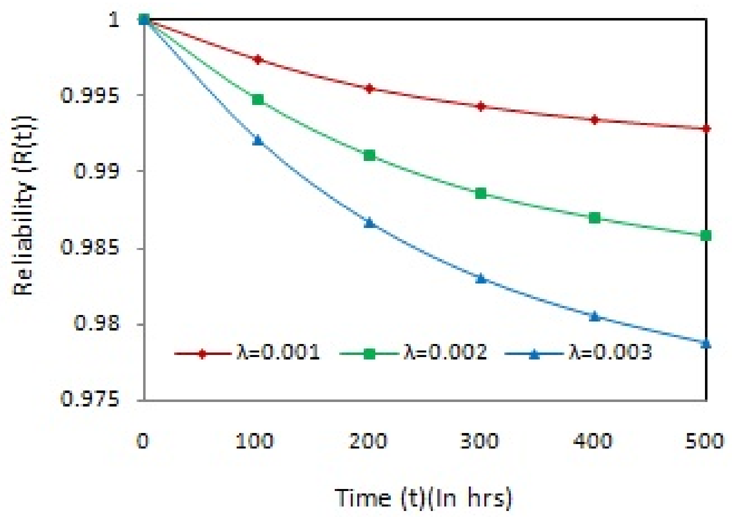

Figure 2: *Reliability {R(t)} w.r.t time (t)*

10.4. Trend of MTSF and Availability (AV_∞) for varying rates

The numerical values of MTSF and availability (AV_∞) are obtained for (λ, β) and (θ, γ) respectively. The other parameters are kept fixed as assumed in subsection 10.1. The results are tabulated as in Table 2 and 3 respectively. It is noted that,
(i) MTSF decreases as λ increases. However, it increases as β increases.
(ii) Availability (AV_∞) increases with the increase in both the parameters θ as well as γ.

Table 2: *MTSF w.r.t. λ for varied β*

λ	MTSF		
	β=0.55	β=0.65	β=0.75
0.0010	29215.21	31146.55	32740.61
0.0011	26563.96	28319.28	29768.06
0.0012	24354.58	25963.23	27290.93
0.0013	22485.11	23969.64	25194.91
0.0014	20882.71	22260.85	23398.31
0.0015	19493.96	20779.90	21841.26
0.0016	18278.80	19484.07	20478.85

Table 3: *AV_∞ w.r.t. θ for varied γ*

θ	AV_∞		
	γ=3	γ=5	γ=10
0.0020	0.9999343	0.9999346	0.9999348
0.0024	0.9999354	0.9999358	0.9999360
0.0028	0.9999364	0.9999369	0.9999371
0.0032	0.9999372	0.9999377	0.9999380
0.0036	0.9999379	0.9999384	0.9999387
0.0040	0.9999384	0.9999390	0.9999394

10.5. Trend of Profit function (P_∞) for varying rates/costs

The trend of profit function (P_∞) with respect to R_0 for varied β and C_B for varied R_0 is revealed by Fig.3 and Fig.4 respectively.Evidently,
(i)With the increase in R_0 and β, P_∞ increases.
(ii)With the increase in C_B, P_∞ decreases but increasing trend of P_∞ is observed with increase in R_0.

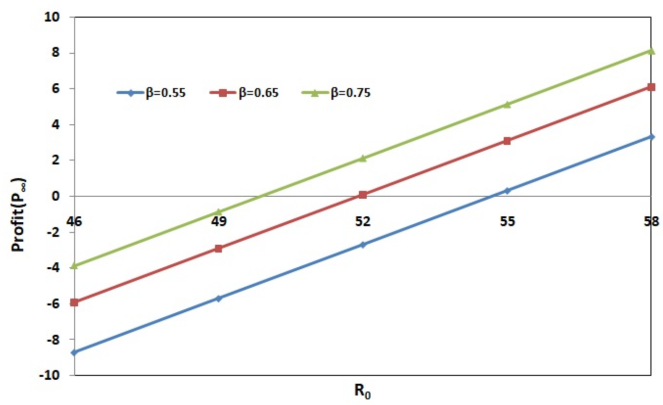

Figure 3: *P_∞ versus R_0 for varied β*

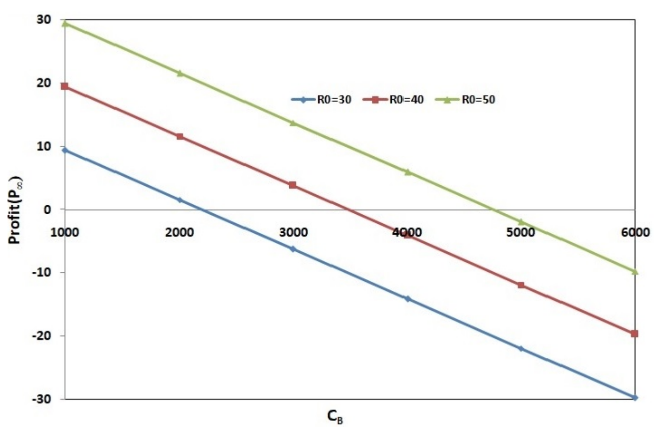

Figure 4: *P_∞ versus C_B for varied R_0*

Bearing economic viability of the system in mind, the bounds obtained for R_0 and C_B are shown in Table 4.

Table 4: *Bounds for revenue/cost*

Revenue/Cost	Varied Parameter	Bounds For Profitability($P_\infty > 0$)
R_0	β=0.55	R_0>54.69
	β=0.65	R_0>51.90
	β=0.75	R_0>49.86
C_B	R_0=30	C_B<2202.7
	R_0=40	C_B<3479.5
	R_0=50	C_B<4756.8

10.6. Numerical calculations for sensitivity analysis

Using the values of incoming variables (as considered in subsection 10.1) Table 5 and 6 represents the values of sensitivity and relative sensitivity functions (defined in section 9) for MTSF and AV_∞ respectively.

Table 5: *Sensitivity and Relative sensitivity of MTSF w.r.t. different rates*

Variable	MTSF	
(k)	$\pi_k=\frac{\partial MTSF}{\partial k}$	$\delta_k = \pi_k(\frac{k}{MTSF})$
λ	-8174507	-1.048
α	-685533	-0.352
β	8710.824	0.363
β_1	129.51	0.007
γ	0.567	0.0004
θ	-28955	-0.0074

Table 6: *Sensitivity and Relative sensitivity of availability (AV_∞) w.r.t. different rates*

Variable	Availability (AV_∞)	
(k)	$\rho_k=\frac{\partial A_\infty}{\partial k}$	$\tau_k = \rho_k(\frac{k}{AV_\infty})$
λ	0.0012	2.4×10^{-6}
α	-0.0056	-4.5×10^{-5}
β	0.0002	1.4×10^{-4}
β_1	-0.0001	-1.06×10^{-4}
γ	2.71×10^{-8}	1.4×10^{-7}
θ	0.0014	5.6×10^{-6}

Considering the absolute values of defined functions, Table 5 and Table 6 reveals that the MTSF is more sensitive with respect to failure rate of main unit λ whereas AV_∞ is impacted more by failure rate of standby unit α. However, the order of incoming variables in which they influence the MTSF and AV_∞ is:
MTSF: $\lambda > \alpha > \theta > \beta > \beta_1 > \gamma$.
$AV_\infty : \beta > \beta_1 > \alpha > \theta > \lambda > \gamma$.

11. Conclusion

This article proposes a probabilistic model for two non-identical units' standby system in which standby unit may be inspected randomly to ensure its operability. Various performability indices are derived. Keeping the cost factor in mind, bounds (lower/upper) for various costs are obtained to account for economic and budgetary constraints. The numerical study has been carried out for exponential case. Sensitivity analysis is performed for MTSF and steady-state availability of the system. The developed model is quite lucrative for any commercial/industrial establishment using such systems, in their production and operational commitments.

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Disclosure statement

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