INVENTORY MODEL WITH EXPONENTIAL DETERIORATION AND SHORTAGES FOR SEVERAL LEVELS OF PRODUCTION

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Abstract

The EPQ models are mathematical models which represent the inventory situation in a production or manufacturing system. In production and manufacturing units, the EPQ model is extremely significant and also be utilized for scheduling the optimal operating policies of market yards, warehouses, godowns, etc. In this research study, we provide inventory model of economic production for deteriorating commodities at multiple levels, in which various production stages are mentioned as well as deterioration rates follow exponential distributions. After a specific period of time, it is feasible to swap production rates from one to another, which is advantageous by starting with a production of low rate, an enormous amount of manufacturing articles is avoided at the outset, resulting in lower holding costs. Variation in output level allows for customer happiness as well as potential profit. The goal of this study is to determine the best production time solution so as to reduce total cost of the entire cycle. Finally, numerical illustrations and parameter sensitivity analyses have been used to validate proposed inventory system's results.

Keywords: EPQ, optimal operating policies, Exponential distribution, multiple levels of production, cycle time.

1. Introduction

Inventory system plays a leading role in many real applications at certain places such as production processes, manufacturing units, transportation, market yards, ware houses, assembly lines etc. One of the vital sections of operations research is inventory management, which is used for determining the optimal operating policies for inventory management and control. Inventory models provide the basic frame work for analyzing several production systems. The inventory models are broadly categorized into two groups namely, (i) Economic Order Quantity models (EOQ models) and (ii) Economic Production Models (EPQ models). The EPQ models are more common in production and manufacturing processes, warehouses, etc. Recently much emphasis was given for analyzing EPQ models for deterioration items. Deterioration is a natural phenomenon of several commodities over time. For commodities like glassware, hardware, and steel, deterioration can be quite minor at times, causing deterioration to be taken into account when determining economic lot sizes. In general, some commodities decay at a faster pace than others, such as medicine, gasoline, strawberries, fish, blood, and food grains, which must be taken into account when determining the size of a production lot.

Damage, decay, spoiling, evaporation, and obsolescence are all examples of deterioration. In modern years, the issue of decaying inventory has gotten a lot of attention. The majority of studies on deteriorating inventory assumed a constant rate of deterioration. In general the exponential delivery is commonly used to describe a product in stock that deteriorates over interval. The rate of deterioration rises with age, therefore the longer an object is left unused, the faster it will fail. Many commodities decay in real life due to their inherent nature, such as fruits, vegetables, food items, seafood, agricultural products, textiles, chemicals, medicines, electronic components, cement, fertilisers, oils, gas, and so on, which are held in inventory at various locations.

The first economic quantity model was developed by Harris [1]. The inventory model of a decaying item at the end of a scarcity period was studied by Wagner and Whitin [2]. As a result, deterioration functions come in a variety of shapes and sizes, including constant and time-dependent functions. We used the Exponential as the function of deterioration in our proposed model. Berrotoni [3] explored that the leakage failure of both dry batteries and ethical drugs life expectancy may be described as an exponential distribution. In some circumstances, the deterioration rate rises with time. The longer an object is left unused, the faster it deteriorates. This study prompted Covert and Philip [4] to create an inventory model for deteriorating items with varying rates. It was made use of two variables the Weibull distribution will deteriorate as a time distribution.

Balki and Benkherouf [5] also proposed a model as such but with a stock-dependent and time-varying demand rate across a finite time horizon. Chang [6] improved previous model by accounting for profit in the inventory system. Additionally, Begam et al [7] devised an instantaneous replacement policy. They used a three-parameter Weibull distribution to time based model inventory deterioration rate. Begam et al [8] re-examined the previous model, ignoring scarcity and assuming demand to be a linear function of price. Rubbani et al [9] proposed an integrated methodology for deteriorating item pricing and inventory control. For decaying products, Sivasankari and Panayappan [10] suggested a production inventory model that considers two different degrees of output. Cardenas-Barron et al [11] projected substitute heuristic algorithm for a multi-product EPQ (Economic Production Quantity) a vendor-buyer cohesive model with JIT view point and a budget constraint. Sarkar et al. [12], in his investigation on EPQ model with rework in a manufacturing system of single-stage with scheduled backorders, and produced 3 different inventory models for 3 different density functions of distribution like Triangular , Uniform, and Beta. Cardenas-Barron et al [13] determined the ideal replenishment lot size and dispatch strategy for an EPQ inventory model with multiple deliveries and rework. When using a multi-shipment policy, Taleizadeh et al [14] presented study work addresses the problem of determining price for sale, lot size of replenishment, and shipments quantity for a model of economic quantity with rework for defective goods. Karthikeyan and Viji [15] modified this model by using the Exponential distribution for deterioration.

Determining the ideal quantity of production boxes for different periods as an aim in order to reduce total inventory costs. Lately, Viji and Karthikeyan [16] have established an inventory model of economic production quantity for constantly deteriorating products that takes into account three levels of manufacturing. Researchers have established an economic production inventory model for many levels of production with exponential distribution deterioration, demand is time dependent and continuous, and multiple rates of production are examined in this research. The following is a breakdown of the paper's structure. The assumptions as well as notations are presented in Section 2. The third section is dedicated to mathematical modelling. Section 4 includes a numerical example as well as a sensitivity analysis. The paper comes to a conclude with Section 5.

2. Assumptions

For developing the model the following assumptions are made:

- Multiple production rates are taken into account.
- The demand rate is continuous and linear which is $D(t) = \alpha + \beta t$ (1)
- The production system has a limited time horizon.
- Shortages are permitted, as are entire backlogs.
- The exponential distribution governs the time it takes for an item to deteriorate, which is
$$f(t) = \theta e^{-t\theta}, \theta > 0, t > 0 \quad (2)$$
- Consequently, the instantaneous rate of production is
$$h(t) = \frac{f(t)}{1-F(t)} = \theta, \theta > 0 \quad (3)$$
- Production rate (K), which is greater than demand rate(R).

The following notions are used to developing this model.

K is the production rate in units @ unit time.
R is the demand rate in units@ unit time.
The holding cost @ unit of time is denoted by the C_1
The shortages cost @ unit of time is denoted by C_2
The ordering cost @ unit of time is denoted by C_3
The production cost @unit of time is denoted by C_p
S is the shortage level.
Q is the optimum production quantity.
Q_1, Q_2, Q_3 and Q_4 are the maximum possible inventory level at time t_1, t_2, t_3 and t_4.
T is the total cycle length.

3. Mathematical formulation of the model

The following is a description of Figure 1.

Let's the production be assumed to begin at t = O and finishes at t = T. Let the rate of production be 'K' and the rate of demand be 'R' during the time interval [0, t1], where R is less than K. At time t = t1, the stock reaches a level Q1. Through the gaps of time [t_1, t_2], [t_2, t_3] and [t_3, t_4]. Let's call the rate of growth a1(K-R), a2(K-R), and a3(K-R), where a1, a2, and a3 are constants. At times t2, t3, and t4, the inventory level reaches levels Q2, Q3, and Q4, respectively. The product turns into technically superseded or else, customer taste changes during the decline time T. It is important to keep an eye on the product's stock levels. Due to demand, inventory levels begin to decline at a rate of R. To consume all units Q at the demand rate, it will take time T.

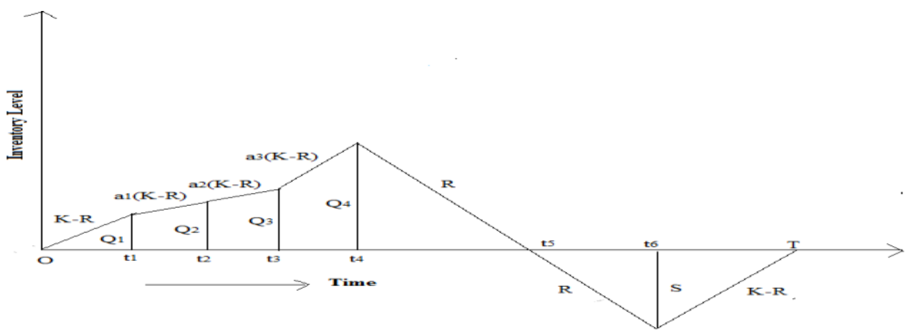

Figure1: Schematic diagram of the inventory level

The model governs distinguished equations as follows.

$$\frac{d}{dt}I(t) + \theta I(t) = (K - R) - (\alpha + \beta t), 0 \leq t \leq t_1 \quad (4)$$

$$\frac{d}{dt}I(t) + \theta I(t) = a_1(K - R) - (\alpha + \beta t), t_1 \leq t \leq t_2 \quad (5)$$

$$\frac{d}{dt}I(t) + \theta I(t) = a_2(K - R) - (\alpha + \beta t), t_2 \leq t \leq t_3 \quad (6)$$

$$\frac{d}{dt}I(t) + \theta I(t) = a_3(K - R) - (\alpha + \beta t), t_3 \leq t \leq t_4 \quad (7)$$

$$\frac{d}{dt}I(t) + \theta I(t) = -R, t_4 \leq t \leq t_5 \quad (8)$$

$$\frac{d}{dt}I(t) = -R, t_5 \leq t \leq t_6 \quad (9)$$

$$\frac{d}{dt}I(t) = (K - R), t_6 \leq t \leq T \quad (10)$$

I(0) = 0, I(t1) = Q1, I(t2) = Q2, I(t3) = Q3, I(t4) = Q4, I(t5) = 0, I(t6) = S, and I(T) = 0 are the initial conditions.

The solutions of equations (4) - (10), using the initial conditions, the on-hand inventory at time't' is calculated as follows:

$$I(t) = \frac{K - R}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right), 0 \leq t \leq t_1 \quad (11)$$

$$I(t) = \frac{a_1(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right), t_1 \leq t \leq t_2 \quad (12)$$

$$I(t) = \frac{a_2(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right), t_2 \leq t \leq t_3 \quad (13)$$

$$I(t) = \frac{a_3(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right), t_3 \leq t \leq t_4 \quad (14)$$

$$I(t) = \frac{-R}{\theta}\left(1 - e^{\theta(t_5 - t)}\right), t_4 \leq t \leq t_5 \quad (15)$$

$$I(t) = -R(t - t_5), t_5 \leq t \leq t_6 \quad (16)$$

$$I(t) = (K - R)(t - T), t_6 \leq t \leq T \quad (17)$$

Maximum inventories Q_1, Q_2, Q_3 and Q_4:

The maximum inventories are estimated using $I(t_1) = Q_1$, $I(t_2) = Q_2$, $I(t_3) = Q_3$, and $I(t_4) = Q_4$ during the times t_1, t_2, t_3, and t_4 and equations (11) – (14).

We have omitted the second and higher powers of in the $e^{-\theta t}$ for θ values

Therefore

$$Q_1 = \theta t_1\left(\frac{K - R}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right) \quad (18)$$

$$Q_2 = \theta t_2\left(\frac{a_1(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right) \quad (19)$$

$$Q_3 = \theta t_3\left(\frac{a_2(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right) \quad (20)$$

$$Q_4 = \theta t_4\left(\frac{a_3(K - R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right) \quad (21)$$

Shortage level S:
From equations (16) and (17) and using $I(t_6) = S$, we get,

$I(t_6) = S \Rightarrow I(t_6) = -R(t_6 - t_5) = S$ and

$I(t_6) = S \Rightarrow (K - R)(t_6 - T) = S$

Therefore $-R(t_6 - t_5) = (K - R)(t_6 - T)$

On simplification

Therefore

$$t_6 = \frac{R}{K}(t_5 - T) + T \tag{22}$$

As a result, total cost equals to the entirety of production, ordering, holding, deteriorating and shortage costs.

The overhead costs independently

(i) Production cost per unit time = R C_p
(ii) Ordering cost per unit time = C_3/T
(iii) Holding cost per unit time

$$= \frac{C_1}{T}\left(\int_0^{t_1} I(t)dt + \int_{t_1}^{t_2} I(t)dt + \int_{t_2}^{t_3} I(t)dt + \int_{t_3}^{t_4} I(t)dt + \int_{t_4}^{t_5} I(t)dt\right)$$

$$= \frac{C_1}{T}\left[\int_0^{t_1} \frac{K-R}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right)dt + \int_{t_1}^{t_2} \frac{a_1(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right)dt\right.$$

$$+ \int_{t_2}^{t_3} \frac{a_2(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right)dt + \int_{t_3}^{t_4} \frac{a_3(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\left(1 - e^{-\theta t}\right)dt$$

$$\left. + \int_{t_4}^{t_5} \frac{-R}{\theta}\left(1 - e^{\theta(t_5 - t)}\right)dt\right]$$

We have simplified the expansion of $e^{-\theta t}$ for small values of θ by ignoring the second and higher powers of in the expansion of $e^{-\theta t}$.

Holding cost per unit time

$$= \frac{C_1}{T}\left[(K-R)\left((1-a_1)\frac{t_1^2}{2} + (a_1 - a_2)\frac{t_2^2}{2} + (a_2 - a_3)\frac{t_3^2}{2} + a_3\frac{t_4^2}{2}\right)\right.$$

$$\left. -\left(\alpha - \frac{\beta}{\theta} + R\right)\frac{t_4^2}{2} - R\left(\frac{t_5^2}{2} - t_4 t_5\right) - \beta\frac{t_4^3}{3}\right] \tag{23}$$

Deteriorating cost per unit time

$$= \frac{C_P}{T}\left(\int_0^{t_1} h(t)I(t)dt + \int_{t_1}^{t_2} h(t)I(t)dt + \int_{t_2}^{t_3} h(t)I(t)dt + \int_{t_3}^{t_4} h(t)I(t)dt + \int_{t_4}^{t_5} h(t)I(t)dt\right)$$

, where h(t) = θ

$$= \frac{C_P}{T}\left[\int_0^{t_1} \theta\left(\frac{K-R}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right)\left(1-e^{-\theta t}\right)dt + \int_{t_1}^{t_2} \theta\left(\frac{a_1(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right)\left(1-e^{-\theta t}\right)dt\right.$$

$$+ \int_{t_2}^{t_3} \theta\left(\frac{a_2(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right)\left(1-e^{-\theta t}\right)dt + \int_{t_3}^{t_4} \theta\left(\frac{a_3(K-R)}{\theta} - \frac{\alpha}{\theta} - \frac{\beta t}{\theta} + \frac{\beta}{\theta^2}\right)\left(1-e^{-\theta t}\right)dt$$

$$\left. + \int_{t_4}^{t_5} \theta\left(\frac{-R}{\theta}\left(1-e^{\theta(t_5-t)}\right)\right)dt\right]$$

We have simplified the expansion of $e^{-\theta t}$ for small values of θ by ignoring the second and higher powers of in the expansion of $e^{-\theta t}$

Deteriorating cost per unit time

$$= \frac{C_P}{T}\left[\theta(K-R)\left((1-a_1)\frac{t_1^2}{2} + (a_1-a_2)\frac{t_2^2}{2} + (a_2-a_3)\frac{t_3^2}{2} + a_3\frac{t_4^2}{2}\right)\right.$$

$$\left. + (\beta - \theta\alpha - R\theta)\frac{t_4^2}{2} - R\theta\left(\frac{t_5^2}{2} - t_4 t_5\right) - \beta\theta\frac{t_4^3}{3}\right] \quad (24)$$

Shortage cost per unit time

$$= \frac{C_2}{T}\left[\int_{t_5}^{t_6} I(t)dt + \int_{t_6}^{T} I(t)dt\right]$$

$$= \frac{C_2}{T}\left[\int_{t_5}^{t_6} -R(t-t_5)dt + \int_{t_6}^{T} (K-R)(t-T)dt\right]$$

On simplification

$$\frac{C_2}{T}\left[R\left(\frac{T^2}{2} - \frac{t_5^2}{2} + t_5 t_6\right) - K\left(T t_6 - \frac{T^2}{2} - \frac{t_6^2}{2}\right)\right] \quad (25)$$

Substitute t_6 value in (25) from (22) and on simplification, we have

Shortage cost

$$= \frac{C_2}{T}\left[\frac{R^2 t_5^2}{K} + (K-R)T^2 + \frac{(2+K)RT^2}{2K} - \frac{KT^2}{2} - \frac{R^2 T t_5}{2} + RTt_5 - \frac{Rt_5^2}{2} - \frac{R^2T^2}{2K}\right] \quad (26)$$

Optimum quantity of the model:

$$Q = \int_0^{t_1} h(t)dt + \int_{t_1}^{t_2} h(t)dt + \int_{t_2}^{t_3} h(t)dt + \int_{t_3}^{t_4} h(t)dt + \int_{t_6}^{T} h(t)dt$$

$$Q = Q_1 + Q_2 + Q_3 + Q_4 + \theta(T - t_6)$$

On simplification

$$Q = Q_1 + Q_2 + Q_3 + Q_4 + \theta \frac{R}{K}(t_5 - T) \tag{27}$$

Therefore, total cost is the sum of the costs pertaining to Production, Setup, Holding, Deteriorating and Shortage.

$$TC(t_1, t_2, t_3, t_4, t_5, T) = RC_P + \frac{C_3}{T} + \frac{C_1}{T}\left[(K - R)\left((1 - a_1)\frac{t_1^2}{2} + (a_1 - a_2)\frac{t_2^2}{2}\right.\right.$$

$$\left.+(a_2 - a_3)\frac{t_3^2}{2} + a_3\frac{t_4^2}{2} - \left(\alpha - \frac{\beta}{\theta} + R\right)\frac{t_4^2}{2} - R\left(\frac{t_5^2}{2} - t_4 t_5\right) - \beta\frac{t_4^3}{3}\right]$$

$$+\frac{C_P}{T}\left[\theta(K - R)\left((1 - a_1)\frac{t_1^2}{2} + (a_1 - a_2)\frac{t_2^2}{2} + (a_2 - a_3)\frac{t_3^2}{2} + a_3\frac{t_4^2}{2}\right)\right.$$

$$\left.+ (\beta - \theta\alpha - R\theta)\frac{t_4^2}{2} - R\theta\left(\frac{t_5^2}{2} - t_4 t_5\right) - \beta\theta\frac{t_4^3}{3}\right]$$

$$+\frac{C_2}{T}\left[\frac{R^2 t_5^2}{K} + (K - R)T^2 + \frac{(2 + K)RT^2}{2K} - \frac{KT^2}{2} - \frac{R^2 T t_5}{2} + RT t_5 - \frac{R t_5^2}{2} - \frac{R^2 T^2}{2K}\right] \tag{28}$$

Let us consider that $t_1 = u\ t_5$, $t_2 = v\ t_5$, $t_3 = w\ t_5$ and $t_4 = x\ t_5$

Therefore total cost becomes from (28)

$$TC(t_5, T) = RC_P + \frac{C_3}{T} + \frac{C_1}{T}\left[(K - R)\left((1 - a_1)\frac{u^2 t_5^2}{2} + (a_1 - a_2)\frac{v^2 t_5^2}{2}\right.\right.$$

$$\left.+(a_2 - a_3)\frac{w^2 t_5^2}{2} + a_3\frac{x^2 t_5^2}{2} - \left(\alpha - \frac{\beta}{\theta} + R\right)\frac{x^2 t_5^2}{2} - R\left(\frac{t_5^2}{2} - x{t_5}^2\right) - \beta\frac{x^3 t_5^3}{3}\right]$$

$$+\frac{C_P}{T}\left[\theta(K - R)\left((1 - a_1)\frac{u^2 t_5^2}{2} + (a_1 - a_2)\frac{v^2 t_5^2}{2}\right.\right.$$

$$\left.+(a_2 - a_3)\frac{w^2 t_5^2}{2} + a_3\frac{x^2 t_5^2}{2} + (\beta - \theta\alpha - R\theta)\frac{x^2 t_5^2}{2} - R\theta\left(\frac{t_5^2}{2} - x{t_5}^2\right) - \beta\theta\frac{x^3 t_5^3}{3}\right]$$

$$+\frac{C_2}{T}\left[\frac{R^2 t_5^2}{K} + (K - R)T^2 + \frac{(2 + K)RT^2}{2K} - \frac{KT^2}{2} - \frac{R^2 T t_5}{2} + RT t_5 - \frac{R t_5^2}{2} - \frac{R^2 T^2}{2K}\right] \tag{29}$$

Since $TC(t_5, T)$ is minimum so that differentiating (29) with regard to t_5 and T likening to zero that is $\frac{\partial TC(t_5,T)}{\partial t_5} = 0$ and $\frac{\partial TC(t_5,T)}{\partial T} = 0$ also satisfy the condition $\left\{\left(\frac{\partial^2 TC(t_5,T)}{\partial {t_5}^2}\right)\left(\frac{\partial^2 TC(t_5,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC(t_5,T)}{\partial t_5 \partial T}\right)\right\} > 0$ and then solving to get t_5 and T.

MATHCAD is used to find the optimum solution to equation (29).

4. Numerical Illustration

The following numerical illustration analyses the above stated model by considering the values of the following. K= 1000, R = 500, C_1 = 5, C_2 = 0.5, C_3 = 50, $a_1 = a_2 = a_3 = 10$, $\theta = 2$, α = 0.2, $\beta = 2$, u = 0.2, v = 0.4, w=0.6 and x= 0.8. The optimum values are found as T_5 = 0.9 and T = 6.049, production cost = 25000, Holding cost = 198, setup cost = 8.333, deteriorating cost = 4.08, shortage cost = 8.825 and total cost = 364500.

Table 1 and 2 shows when there is an increase in rate of deterioration, the Cycle Time (T), Order Quantity (Q), and Total Cost (TC) increases. All increase as the rate of demand parameter increases.

Table 1: *Parameter variations on optimal values*

Parameters		Optimum values					
		t_1	t_2	t_3	t_4	t_5	T
C_1	**5.0**	0.18	0.37	0.56	0.74	0.893	6.049
	5.25	0.176	0.352	0.53	0.704	0.88	6.293
	5.50	0.17	0.339	0.51	0.679	0.848	6.682
	5.75	0.166	0.332	0.49	0.664	0.831	6.922
C_2	**0.5**	0.18	0.37	0.56	0.74	0.893	6.049
	0.525	0.517	1.033	1.55	2.066	1.051	8.27
	0.550	0.519	1.037	1.56	2.074	1.327	10.21
	0.575	0.526	1.053	1.58	2.105	1.501	11.1
C_3	**50**	0.18	0.37	0.56	0.74	0.893	6.049
	55	0.222	0.445	0.67	0.89	1.591	7.272
	60	0.516	1.033	1.55	2.066	2.282	8.52
	65	0.518	1.036	1.55	2.071	2.609	10.17
C_p	**50**	0.18	0.37	0.56	0.74	0.893	6.049
	55	0.203	0.407	0.61	0.813	1.017	6.476
	60	0.204	0.409	0.61	0.818	1.022	6.993
	65	0.205	0.41	0.62	0.821	1.026	7.462
θ	**2**	0.18	0.37	0.56	0.74	0.893	6.049
	2.1	0.22	0.44	0.66	0.879	1.098	7.621
	2.2	0.265	0.531	0.79	1.062	1.327	9.2
	2.3	0265	0531	0.80	1.062	1.327	9.2
α	**0.2**	0.18	0.37	0.56	0.74	0.893	6.049
	0.21	0.225	0.451	0.68	0.901	1.127	7.574
	0.22	0.248	0.496	0.74	0.992	1.440	10.59
	0.23	0.367	0.734	1.10	1.468	1.836	11.75
β	**2**	0.18	0.37	0.56	0.74	0.893	6.049
	2.1	0.308	0.617	0.93	1.233	1.241	8.768
	2.2	0.318	0.637	0.96	1.274	1.592	9.071
	2.3	0.354	0.708	1.06	1.416	1.770	8.48

Table 2: *Parameter variations on optimal values*

Parameters		Optimum values					
		Q_1	Q_2	Q_3	Q_4	Q	TC
	5.0	629.98	12610	18910	25210	57340	127000
	5.25	615.63	12320	18470	24630	56040	116300
	5.50	593.62	11880	17810	23750	54030	101400

		Optimum values					
Parameters		Q_1	Q_2	Q_3	Q_4	Q	TC
C_1	**5.75**	581.18	11630	17440	23250	52910	98420
C_2	**0.5**	629.98	12610	18910	25210	57340	127100
	0.525	1807	36160	54240	72310	61500	156300
	0.550	1814	36300	54440	72590	65100	170100
	0.575	1841	36840	55260	73680	67600	200700
C_3	**50**	629.98	12610	18910	25210	57340	127100
	55	778.32	15570	23360	31140	60850	156100
	60	1807	36150	54220	72290	63500	182400
	65	1812	36240	54360	72480	67900	215500
C_p	**50**	629.98	12610	18910	25210	57340	127100
	55	711.45	14230	21350	28470	60760	189100
	60	715.25	14310	21460	28620	62110	234700
	65	717.88	14360	21540	28720	65350	272700
θ	**2**	629.98	12610	18910	25210	57340	127100
	2.1	768.4	15370	23060	30750	64950	158100
	2.2	928.59	18580	27870	37160	72530	187100
	2.3	932.58	18780	27970	37260	78730	198100
α	**0.2**	629.98	12610	18910	25210	57340	127100
	0.21	788.49	15770	23660	31550	69770	152600
	0.22	867.58	17360	26040	34710	78970	172400
	0.23	1284	25700	38540	51390	84900	213100
β	**2**	629.98	12610	18910	25210	57340	127100
	2.1	1079	21580	32370	43160	58180	145700
	2.2	1114	22290	33430	44580	60020	152500
	2.3	1239	24790	37180	49570	62200	183700

5. Sensitivity analysis

The sensitivity analysis is carried out to effect the change in parameters -15% to 15%, the optimum values are varied to identify the relation between the parameters and optimum values of the production schedule are shown in Figure 2. The true solution with model parameters which are considered to be stationary at its value, in turn is the total cost function. It makes sense to investigate the sensitivity, or the effect of changing model parameters over a given optimum solution.

I. The optimal quantity (Q), production times t_1, t_2, t_3, and t_4, cycle time (T), maximum inventories Q_1, Q_2, Q_3, and Q_4, and total cost (TC) all increase when the value of the deteriorating parameter θ increases.

II. The optimal quantity (Q), production times t_1, t_2, t_3, and t_4, cycle time (T), maximum inventories Q_1, Q_2, Q_3, and Q_4, and total cost (TC) all increases when the value of ordering cost per unit (C3) increases.

III. The optimal quantity (Q), production times t_1, t_2, t_3, and t_4, maximum inventories Q_1, Q_2, Q_3, and Q_4, and total cost (TC) fall when the value of holding cost per unit (C1) decreases, but cycle time (T) increases.

IV. The optimal quantity (Q), production times t_1, t_2, t_3, and t_4, cycle time (T), maximum inventory Q_1, Q_2, Q_3, and Q_4, and total cost (TC) all increases when the value of shortage cost per unit (C2) increases.

V. The optimal quantity (Q), production times t_1, t_2, t_3, and t_4, cycle time (T), maximum inventory Q_1, Q_2, Q_3, and Q_4, and total cost (TC) increases as the value of demand parameters (α, β) increases.

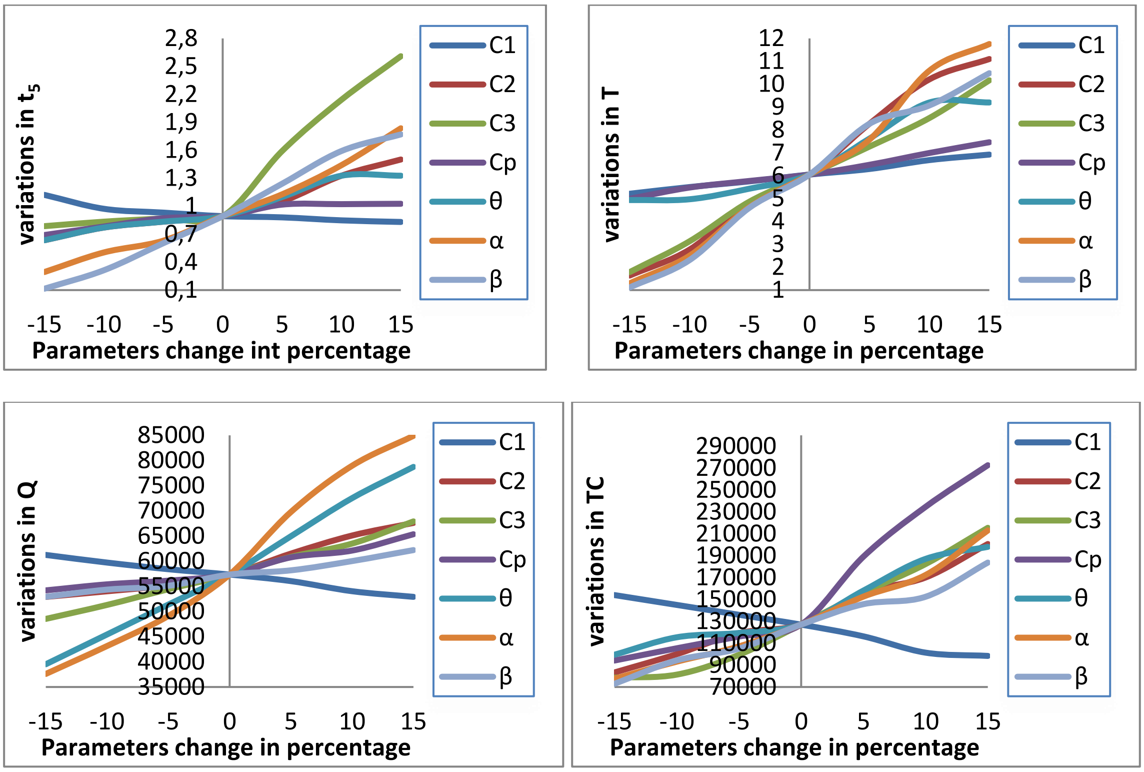

Figure 2: *Relationship between parameters and optimal values (t_5, T, Q, TC)*

6. Conclusion

The study explored an inventory model intended to deteriorating items that takes into account many levels of manufacturing. Researchers supposed that the rate of demand is reliant on time and which is linear and rate of deterioration is follows exponential distribution. The projected model is appropriate for the products introduced newly which have a consistent harmony up to a certain point in time. Such circumstances are beneficial, because, by starting at a modest production rates, a significant quantity of manufacturing goods will be avoided at the outset, resulting in a reduction in holding costs. As a result, we will receive customer happiness as well as possible profit. We developed a solution through mathematical model for this problem. Numerical illustration and sensitivity analysis are contributed to demonstrate the model. The suggested inventory model shall help manufacturers as well as retailers to indeed calculate the best order quantity, cycle time, and total inventory cost. This model is extended in a variety of ways for additional research, including demand with selling price, power demand, on-hand inventory demand etc.

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The Authors declare that there is no conflict of interest.

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