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A Two Non-Identical Unit Parallel System with Repair and Post Repair Policies of a Failed Unit and Correlated Lifetimes

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Abstract

The paper deals with the analysis of a system model consisting of two non-identical units arranged in a parallel configuration. If a unit fails it goes to repair. After its repair, the repaired unit is sent for post repair to complete its repair. A single repairman is always available with the system to repair a failed unit and for post repair of repaired unit. A post repaired unit always works as good as new. Failure time of both the units is assumed to be correlated random variables having their joint distribution as bivariate exponential (B.V.E.). The repair time distribution of both the units are taken as general with different c.d.fs whereas the post repair time distribution of both the units are taken as exponential with different parameters.

Keywords: Transition probabilities, mean sojourn time, bi-variate exponential distribution, regenerative point, reliability, MTSF, availability, expected busy period of repairman, net expected profit.

1. Introduction

Undoubtedly, various actual systems in the field of manufacture, shipment, computation etc. are factitious / unnatural and manufactured by human while there are various natural systems also exist which illustrates that all the systems are manufactured by human and nature and can be simple or complicated. Chopra and Ram [5] studied a two non-identical unit parallel system with two types of failure - common cause failure and partial failure. A repairman is not always available with the system to repair a failed unit i.e. whenever a unit fails, a repairman is called to visit the system and he takes some significant amount of time to reach at the system. This time is known as the waiting time for repairman and during this time the failed unit waits for repair. Pundir et al. [14] investigated a two non-identical unit parallel system where priority is given to first unit in repair. Chandra et al. [4] analysed two different system models in which one consist of two identical units in parallel whereas the other composed of two non-identical units in parallel. They have obtained the reliability characteristics by applying the Semi Markov Process and Regenerative Point Technique. In the above papers, the authors considered a two identical / non-identical unit parallel system. The concept of repair and post repair is not considered in all the above system models.

Goel et al. [6] studied a two unit redundant system in which one unit is operative and the other is a warm standby which replaces the operative failed unit instantaneously. After repair of the operative failed unit it is sent for inspection to decide whether the repaired unit is perfect or not. If the repaired unit is found to be imperfect, it is sent for post repair. Goel et al. [7] investigated a stochastic model of a two unit warm standby system with 'n' failure modes of each unit. Before starting the repair, the failed unit is examined for the type of fault, which decides the failure mode. This process takes a significant random amount of time. After repair, the unit is inspected to decide whether the repair is perfect or not. If the repair is found to be imperfect, the unit is sent for post repair. Agarwal and Mahajan [1] analysed a two-unit degrading system model with repair. After each repair, the unit is tested to see whether the repair meets certain pre-defined specifications. If it does, the unit is put to operation, otherwise it goes to post repair. Pandey et al. [13] discussed a two non-identical unit system with two types of repair, the internal and the external. The external repair is called only when the internal staff fails to do the job. In the case of external repair, there is a provision of inspection, wherein if the repair is found unsatisfactory, it is sent for post repair. Agnihotri et al. [2] analysed a system model consisting of two non-identical parallel units. They have assumed that the repaired unit goes for inspection to decide whether the repair is perfect or not. If the repair is found imperfect then it is sent for post repair. Agnihotri and Satsangi [3] analysed a system model consisting of two non-identical parallel units, in which the one unit gets the priority over the other for repair, inspection and post repair. Mokaddis et al. [12] investigated a two dissimilar unit cold standby redundant system with random interchange of the units. In this system it is assumed that the failure, repair, post repair, interchange of units and inspection times are stochastically independent random variables, each having an arbitrary distribution. The system is analysed by the Semi-Markov Process technique. In all the above system models, the concept of repair and post repair is used. After repair of the operative failed unit, it is sent for inspection to decide whether the repaired unit is perfect or not. If the repaired unit is found to be imperfect then the unit is sent for post repair. The above authors have assumed that time to failure of both the units are uncorrelated random variables.

Gupta and co-workers [8, 9] studied two unit complex / duplicate system by assuming different presumptions. In both these system models the failure and repair times are taken as correlated random variables. The joint distribution of failure and repair times is considered as bivariate exponential in both the models. Gupta and co-workers [10, 11] have also studied two unit active redundant systems by assuming different assumptions. Considering the lifetimes of the units as correlated random variable i.e. the joint distribution of lifetimes of the units is taken as bivariate exponential.

The objective of this paper is devoted to raise the idea of repair and post repair in two non-identical units parallel system assuming that the lifetimes of the units are correlated random variables having their joint distribution as bivariate exponential with joint p.d.f. as follows-

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right); \quad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

where, $I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$

is the modified Bessel function of type-I and order zero.

By using regenerative point technique, the following measures of system effectiveness are obtained-

i. Transient-state and steady-state transition probabilities.
ii. Mean sojourn time in various regenerative states.
iii. Reliability and mean time to system failure (MTSF).

iv. Point-wise and steady-state availabilities of the system as well as expected up time of the system during time interval (0, t).
v. The expected busy period of repairman in time interval (0, t).
vi. Net expected profit earned by the system in time interval (0, t) and in steady-state.

2. System Description and Assumptions

1. The system consists of two non-identical units- unit-1 and unit-2. Initially, both the units are operative in parallel configuration.
2. Each unit has two possible modes- Normal (N) and Total failure (F).
3. When a unit fails it goes to repair. After its repair, the repaired unit is sent for post repair.
4. A single repair facility is always available with the system to repair a failed unit and for post repair of repaired unit.
5. A post repaired unit always works as good as new.
6. The repair / post repair discipline is FCFS.
7. Failure time of both the units are assumed to be correlated random variables having their joint distribution as bivariate exponential (B.V.E.) with density function as follows-

$$f(x_1, x_2) = \alpha_1\alpha_2(1-r)e^{-\alpha_1 x_1 - \alpha_2 x_2} I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right); \quad x_1, x_2, \alpha_1, \alpha_2 > 0; \quad 0 \le r < 1$$

where, $$I_0(z) = \sum_{k=0}^{\infty} \frac{(z/2)^{2k}}{(k!)^2}$$

8. The repair time distribution of both the units are taken as general with different c.d.fs whereas the post repair time distribution of both the units are taken as exponential with different parameters.
9. The system failure occurs when both the units are in total failure mode.
10. A repaired unit always works as good as new.

3. Notations and States of the System

We define the following symbols for generating the various states of the system-

N_o^1, N_o^2:	Unit-1 and Unit-2 in normal (N) mode and operative.
F_r^1, F_r^2 :	Unit-1 and Unit-2 is in failure (F) mode and under repair.
F_{pr}^1, F_{pr}^2 :	Unit-1 and Unit-2 is in failure (F) mode and under post repair.
F_w^1, F_w^2 :	Unit-1 and Unit-2 is in failure (F) mode and waits for repair.

Considering the above symbols in view of assumptions stated in section-2, the possible states of the system are shown in the transition diagram represented by **Figure. 1**. It is to be noted that the epochs of transitions into the state S_3 from S_1, S_4 from S_1, S_5 from S_2 and S_6 from S_2 are non-regenerative, whereas all the other entrance epochs into the states of the systems are regenerative.

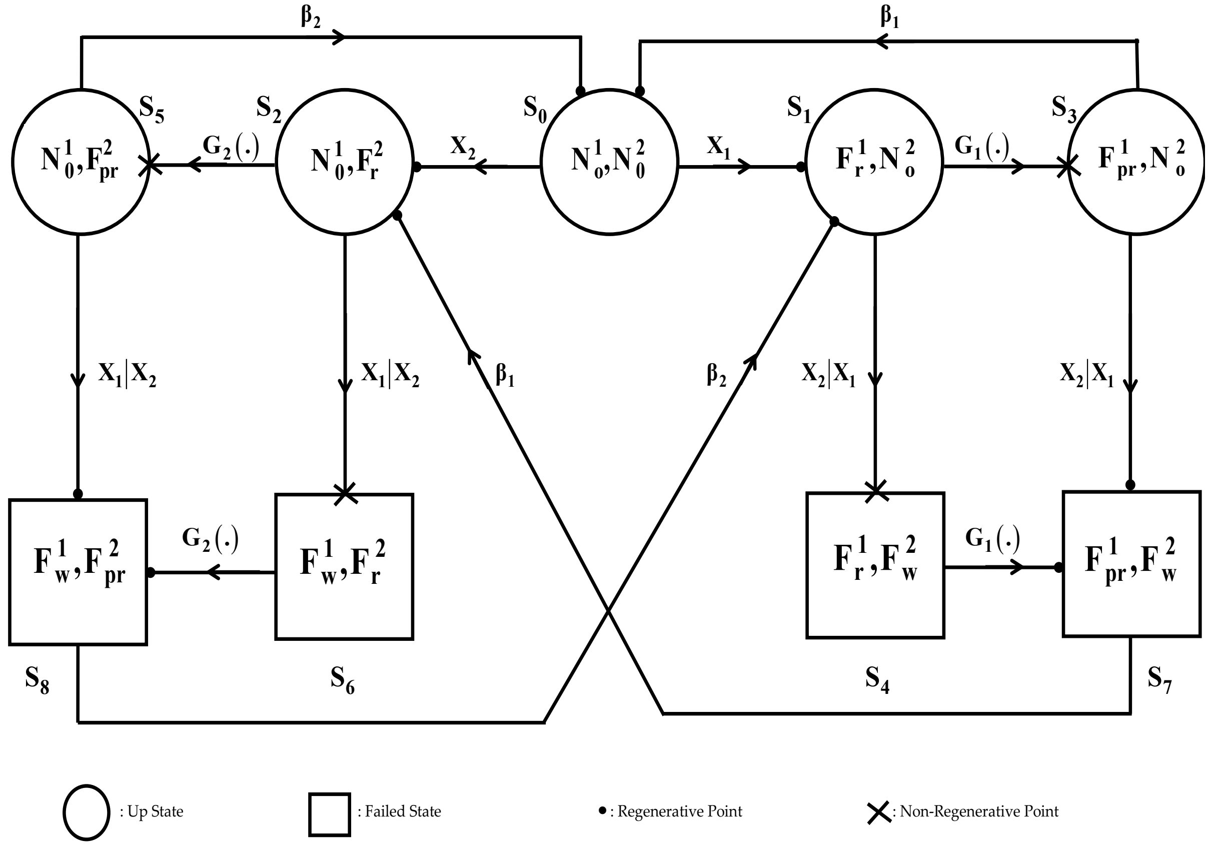

Figure 1: *Transition diagram*

The other notations used are defined as follows:

E	:	Set of regenerative states.
$X_i\left(i=1,2\right)$	:	Random variables representing the failure time of unit-1 in N-mode and unit-2 respectively for i = 1, 2.
$f\left(x_1,x_2\right)$	:	Joint p.d.f. of $\left(x_1,x_2\right)$.

$$f(x_1,x_2)=\alpha_1\alpha_2\left(1-r\right)e^{-\alpha_1 x_1-\alpha_2 x_2}\, I_0\left(2\sqrt{\alpha_1\alpha_2 r x_1 x_2}\right);$$

$$x_1,x_2,\alpha_1,\alpha_2>0\ ;\ 0\le r<1$$

where, $I_0\left(z\right)=\sum_{k=0}^{\infty}\frac{(z/2)^{2k}}{(k!)^2}$

$g_i\left(x\right)$	:	Marginal p.d.f. of $X_i=x$ $=\alpha_i\left(1-r_i\right)e^{-\alpha_i\left(1-r\right)x}$
$k_1\left(x_1\middle\vert X_2=x_2\right)$	:	Conditional p.d.f. of $X_1\vert X_2=x$. $=\alpha_1 e^{-\left(\alpha_1 x_1+\alpha_2 r x\right)}I_0\left(2\sqrt{\alpha_1\alpha_2 r x x_1}\right)$
$k_2\left(x_2\middle\vert X_1=x_1\right)$	:	Conditional p.d.f. of $X_2\vert X_1=x$. $=\alpha_2 e^{-\left(\alpha_2 x_2+\alpha_1 r x\right)}I_0\left(2\sqrt{\alpha_1\alpha_2 r x x_2}\right)$
$K_i\left(\cdot\vert x\right)$	:	Conditional c.d.f. of $X_i\vert X_j=x,\ i\ne j\ ;\ i,j=1,2$.
β_1,β_2	:	Constant post repair rate of unit-1 and unit-2 respectively.
$G_1\left(\cdot\right),G_2\left(\cdot\right)$	:	c.d.f. of repair time of unit-1 and unit-2 respectively.
$q_{ij}(\cdot),q_{ij}^{(k)}\left(\cdot\right)$	:	p.d.f. of transition time from state S_i to S_j and S_i to S_j via S_k.

$p_{ij}, p_{ij}^{(k)}$: Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k.

$p_{ij|x}, p_{ij|x}^{(k)}$: Steady-state transition probabilities from state S_i to S_j and S_i to S_j via S_k when it is known that the unit has worked for time x before its failure.

$*$: †Symbol for Laplace Transform i.e. $q_{ij}^*(s) = \int e^{-st} q_{ij}(t)\,dt$

~ : Symbol for Laplace Stieltjes Transform i.e. $\tilde{Q}_{ij}(s) = \int e^{-st} dQ_{ij}(t)$

© : Symbol for ordinary convolution i.e.

$$A(t)©B(t) = \int_0^t A(u)B(t-u)\,du$$

4. Transition Probabilities and Sojourn Times

Let $X(t)$ be the state of the system at epoch t, then $\{X(t); t \geq 0\}$ constitutes a continuous parametric Markov-Chain with state space $E = \{S_0 \text{ to } S_5\}$. The various measures of system effectiveness are obtained in terms of steady-state transition probabilities and mean sojourn times in various states. First we obtain the direct conditional and unconditional transition probabilities in terms of

$$\alpha_1' = \frac{\alpha_1}{\alpha_1 + \beta_2}, \quad \alpha_1'' = \frac{\alpha_1}{\alpha_1 + \lambda_2}, \quad \alpha_2' = \frac{\alpha_2}{\alpha_2 + \beta_1} \quad \text{and} \quad \alpha_2'' = \frac{\alpha_2}{\alpha_2 + \lambda_1}$$

as follows-

$$p_{01} = \int \alpha_1 (1-r) e^{-\{\alpha_1(1-r)+\alpha_2(1-r)\}t} dt = \frac{\alpha_1}{\alpha_1 + \alpha_2}$$

Similarly,

$$p_{02} = \frac{\alpha_2}{\alpha_1 + \alpha_2}, \qquad p_{72} = \int \beta_1 e^{-\beta_1 t} dt = 1, \qquad p_{81} = \int \beta_2 e^{-\beta_2 t} dt = 1$$

$$p_{10|x}^{(3)} = \int \beta_1 e^{-\beta_1 v} \bar{K}_2(v|x) \left\{ \int_0^v e^{\beta_1 u} dG_1(u) \right\} dv, \qquad p_{17|x}^{(3)} = \int e^{-\beta_1 v} \left\{ \int_0^v e^{\beta_1 u} dG_1(u) \right\} dK_2(v|x)$$

$$p_{17|x}^{(4)} = \int \bar{G}_1(u) dK_2(u|x), \qquad p_{20|x}^{(5)} = \int \beta_2 e^{-\beta_2 v} \bar{K}_1(v|x) \left\{ \int_0^v e^{\beta_2 u} dG_2(u) \right\} dv$$

$$p_{28|x}^{(5)} = \int e^{-\beta_2 v} \left\{ \int_0^v e^{\beta_2 u} dG_2(u) \right\} dK_1(v|x), \qquad p_{28|x}^{(6)} = \int \bar{G}_2(u) dK_1(u|x)$$

The unconditional transition probabilities with correlation coefficient from some of the above conditional transition probabilities can be obtained as follows:

$$p_{10}^{(3)} = \int p_{10|x}^{(3)} g_1(x)\,dx = \int p_{10|x}^{(3)} \{\alpha_1(1-r) e^{-\alpha_1(1-r)x}\}\,dx$$

Similarly,

$$p_{17}^{(3)} = \int p_{17|x}^{(3)} \{\alpha_1(1-r) e^{-\alpha_1(1-r)x}\}\,dx, \qquad p_{17}^{(4)} = \int p_{17|x}^{(4)} \{\alpha_1(1-r) e^{-\alpha_1(1-r)x}\}\,dx = p_{14}$$

$$p_{20}^{(5)} = \int p_{20|x}^{(5)} \{\alpha_2(1-r) e^{-\alpha_2(1-r)x}\}\,dx, \qquad p_{28}^{(5)} = \int p_{28|x}^{(5)} \{\alpha_2(1-r) e^{-\alpha_2(1-r)x}\}\,dx$$

$$p_{28}^{(6)} = \int p_{28|x}^{(6)} \{\alpha_2(1-r) e^{-\alpha_2(1-r)x}\}\,dx = p_{26}$$

It can be easily verified that,

$$p_{01} + p_{02} = 1, \quad p_{10}^{(3)} + p_{17}^{(3)} + p_{17}^{(4)} = 1, \quad p_{20}^{(5)} + p_{28}^{(5)} + p_{28}^{(6)} = 1, \quad p_{72} = p_{81} = 1 \qquad (1\text{-}5)$$

†The limits of integration are 0 to ∞ whenever they are not mentioned.

5. Mean Sojourn Times

The mean sojourn time ψ_i in state S_i is defined as the expected time taken by the system in state S_i before transiting into any other state. If random variable U_i denotes the sojourn time in state S_i then,

$$\psi_i = \int P\left[U_i > t\right]dt$$

Therefore, its values for various regenerative states are as follows-

$$\psi_0 = \int e^{-(\alpha_1+\alpha_2)(1-r)t}\,dt = \frac{1}{(\alpha_1+\alpha_2)(1-r)} \tag{6}$$

$$\psi_{1|x} = \int \bar{G}_1(t)\bar{K}_2(t\,|\,x)dt = \int \bar{G}_1(t)\left(\int_t^{\infty}\alpha_2 e^{-(\alpha_2 u+\alpha_1 rx)}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rxu)^j}{(j!)^2}du\right)dt$$

so that, $$\psi_1 = \int \psi_{1|x} g_1(x)dx = \int \psi_{1|x}\alpha_1(1-r)e^{-\alpha_1(1-r)x}dx \tag{7}$$

$$\psi_{2|x} = \int \bar{G}_2(t)\bar{K}_1(t\,|\,x)dt = \int \bar{G}_2(t)\left(\int_t^{\infty}\alpha_1 e^{-(\alpha_1 u+\alpha_2 rx)}\sum_{j=0}^{\infty}\frac{(\alpha_1\alpha_2 rxu)^j}{(j!)^2}du\right)dt$$

so that, $$\psi_2 = \int \psi_{2|x} g_2(x)dx = \int \psi_{2|x}\alpha_2(1-r)e^{-\alpha_2(1-r)x}dx \tag{8}$$

$$\psi_{3|x} = \int e^{-\beta_1 t}\bar{K}_2(t|x)dt$$

so that, $$\psi_3 = \int \psi_{3|x} g_1(x)dx = \frac{1}{\beta_1}\left\{1-\frac{\alpha_2'(1-r)}{\left(1-r\alpha_2'\right)}\right\} \tag{9}$$

$$\psi_4 = \int \bar{G}_1(t)dt \tag{10}$$

$$\psi_{5|x} = \int e^{-\beta_2 t}\bar{K}_1(t|x)dt$$

so that, $$\psi_5 = \int \psi_{5|x} g_2(x)dx = \frac{1}{\beta_2}\left\{1-\frac{\alpha_1'(1-r)}{\left(1-r\alpha_1'\right)}\right\} \tag{11}$$

$$\psi_6 = \int \bar{G}_2(t)dt \tag{12}$$

$$\psi_7 = \int e^{-\beta_1 t}dt = \frac{1}{\beta_1} \tag{13}$$

$$\psi_8 = \int e^{-\beta_2 t}dt = \frac{1}{\beta_2} \tag{14}$$

6. Analysis of Characteristics

6.1. Reliability and MTSF

Let $R_i(t)$ be the probability that the system operates during (0, t) given that at t = 0 system starts from $S_i \in E$. To obtain it we assume the failed states S_4, S_6, S_7 and S_8 as absorbing. By simple probabilistic arguments, the value of $R_0(t)$ in terms of its Laplace Transform (L.T.) is given by

$$R_0^*(s)=\frac{Z_0^*+q_{01}^*\left(Z_1^*+q_{13}^*Z_3^*\right)+q_{02}^*\left(Z_2^*+q_{25}^*Z_5^*\right)}{1-q_{01}^*q_{10}^{(3)*}-q_{02}^*q_{20}^{(5)*}} \tag{15}$$

We have omitted the argument 's' from $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. $Z_i^*(s)$; $i=0,1,2,3,5$ are the L. T. of

$$Z_0(t)=e^{-(\alpha_1+\alpha_2)(1-r)t}, \quad Z_1(t)=\bar{G}_1(t)\left\{\int_t^\infty \alpha_2 e^{-(\alpha_2 z+\alpha_1 rx)} I_0\left(2\sqrt{\alpha_1\alpha_2 rxz}\right)dz\right\} g_1(x)dx$$

$$Z_2(t)=\bar{G}_2(t)\left\{\int_t^\infty \alpha_1 e^{-(\alpha_1 z+\alpha_2 rx)} I_0\left(2\sqrt{\alpha_1\alpha_2 rxz}\right)dz\right\} g_2(x)dx$$

$$Z_3(t)=\int e^{-\beta_1 t}\left\{\int_t^\infty \alpha_2 e^{-(\alpha_2 z+\alpha_1 rx)} I_0\left(2\sqrt{\alpha_1\alpha_2 rxz}\right)dz\right\} g_1(x)dx$$

$$Z_5(t)=\int e^{-\beta_2 t}\left\{\int_t^\infty \alpha_1 e^{-(\alpha_1 z+\alpha_2 rx)} I_0\left(2\sqrt{\alpha_1\alpha_2 rxz}\right)dz\right\} g_2(x)dx$$

Taking the Inverse Laplace Transform of (15), one can get the reliability of the system when system initially starts from state S_0.

The MTSF is given by,

$$E(T_0)=\int R_0(t)dt=\lim_{s\to 0}R_0^*(s)=\frac{\psi_0+p_{01}\left(\psi_1+p_{13}\psi_3\right)+p_{02}\left(\psi_2+p_{25}\psi_5\right)}{1-p_{01}p_{10}^{(3)}-p_{02}p_{20}^{(5)}} \tag{16}$$

6.2. Availability Analysis

Let $A_i(t)$ be the probability that the system is up at epoch t, when initially it starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $A_0(t)$ in terms of its Laplace transforms i.e. $A_0^*(s)$ given as follows-

$$A_0^*(s)=\frac{N_1(s)}{D_1(s)} \tag{17}$$

where,

$$N_1(s)=\left(1-q_{17}^*q_{72}^*q_{28}^*q_{81}^*\right)Z_0^*+\left(q_{01}^*+q_{02}^*q_{28}^*q_{81}^*\right)\left(Z_1^*+q_{13}^*Z_3^*\right)+\left(q_{01}^*q_{17}^*q_{72}^*+q_{02}^*\right)\left(Z_2^*+q_{25}^*Z_5^*\right)$$

and

$$D_1(s)=1-q_{17}^*q_{72}^*q_{28}^*q_{81}^*-\left(q_{01}^*+q_{02}^*q_{28}^*q_{81}^*\right)q_{10}^{(3)*}-\left(q_{01}^*q_{17}^*q_{72}^*+q_{02}^*\right)q_{20}^{(5)*} \tag{18}$$

where, $A_i(t)$, i = 0,1,2,3,5 are same as given in section 6.1.

The steady-state availability of the system is given by

$$A_0=\lim_{t\to\infty}A_0(t)=\lim_{s\to 0}sA_0^*(s) \tag{19}$$

We observe that

$$D_1(0)=0$$

Therefore, by using L. Hospital's rule the steady state availability is given by

$$A_0=\lim_{s\to 0}\frac{N_1(s)}{D_1'(s)}=\frac{N_1}{D_1'} \tag{20}$$

where,

$$N_1=\left(1-p_{17}p_{28}\right)\psi_0+\left(p_{01}+p_{02}p_{28}\right)\left(\psi_1+p_{13}\psi_3\right)+\left(p_{01}p_{17}+p_{02}\right)\left(\psi_2+p_{25}\psi_5\right)$$

and

$$D_1' = (1-p_{17}p_{28})\psi_0 + \left(1-p_{02}p_{20}^{(5)}\right)\{(\psi_1+p_{13}\psi_3+p_{14}\psi_4)+p_{17}\psi_7\}$$
$$+\left(1-p_{01}p_{10}^{(3)}\right)\{(\psi_2+p_{25}\psi_5+p_{26}\psi_6)+p_{28}\psi_8\} \qquad (21)$$

The expected up time of the system in interval (0, t) is given by

$$\mu_{up}(t) = \int_0^t A_0(u)\,du$$

So that, $\mu^*_{up}(s) = \dfrac{A_0^*(s)}{s}$

(22)

6.3. Busy Period Analysis

Let $B_i^1(t)$, $B_i^2(t)$, $B_i^3(t)$ and $B_i^4(t)$ be the respective probabilities that the repairman is busy in the repair of unit-1 and unit-2 and in the post repair of unit-1 and unit-2 at epoch t, when initially the system starts operation from state $S_i \in E$. Using the regenerative point technique and the tools of L.T., one can obtain the values of above four probabilities in terms of their L.T. i.e. $B_i^{1*}(s)$, $B_i^{2*}(s)$, $B_i^{3*}(s)$ and $B_i^{4*}(s)$ as follows-

$$B_i^{1*}(s) = \frac{N_2(s)}{D_1(s)},\ B_i^{2*}(s) = \frac{N_3(s)}{D_1(s)},\ B_i^{3*}(s) = \frac{N_4(s)}{D_1(s)} \text{ and } B_i^{4*}(s) = \frac{N_5(s)}{D_1(s)} \qquad (23\text{-}26)$$

where,

$$N_2(s) = \left(q_{01}^* + q_{02}^* q_{28}^* q_{81}^*\right)\left(Z_1^* + q_{14}^* Z_4^*\right)$$
$$N_3(s) = \left(q_{01}^* q_{17}^* q_{72}^* + q_{02}^*\right)\left(Z_2^* + q_{26}^* Z_6^*\right)$$
$$N_4(s) = q_{13}^*\left(q_{01}^* + q_{02}^* q_{28}^* q_{81}^*\right)Z_3^* + \left(q_{01}^* q_{17}^* + q_{02}^* q_{28}^* q_{81}^* q_{17}^*\right)Z_7^*$$
$$N_5(s) = q_{25}^*\left(q_{01}^* q_{17}^* q_{72}^* + q_{02}^*\right)Z_5^* + \left(q_{01}^* q_{17}^* q_{72}^* q_{28}^* + q_{02}^* q_{28}^*\right)Z_8^*$$

and $D_1(s)$ is same as defined by the expression (18) of section 6.2.

Also Z_4^*, Z_6^*, Z_7^* and Z_8^* are the L. T. of

$$Z_4(t) = \bar{G}_1(t),\quad Z_6(t) = \bar{G}_2(t),\quad Z_7(t) = e^{-\beta_1 t},\quad Z_8(t) = e^{-\beta_2 t}$$

The steady state results for the above three probabilities are given by-

$$B_o^1 = \lim_{s\to 0} sB_0^*(s) = N_2/D_1',\quad B_o^2 = N_3/D_1',\quad B_o^3 = N_4/D_1' \text{ and } B_0^4 = N_5/D_1' \qquad (27\text{-}30)$$

$$N_2 = \left(1-p_{02}p_{20}^{(5)}\right)(\psi_1 + p_{14}\psi_4)$$
$$N_3 = \left(1-p_{01}p_{10}^{(3)}\right)(\psi_2 + p_{26}\psi_6)$$
$$N_4 = \left(1-p_{02}p_{20}^{(5)}\right)(p_{13}\psi_3 + p_{17}\psi_7)$$
$$N_5 = \left(1-p_{01}p_{10}^{(3)}\right)(p_{25}\psi_5 + p_{28}\psi_8)$$

and D_1' is same as given in the expression (21) of section 6.2.

The expected busy period in repair of unit-1 and unit-2 and post repair of unit-1 and unit-2 during time interval (0, t) are respectively given by-

$$\mu_b^1(t)=\int_0^t B_0^1(u)du, \quad \mu_b^2(t)=\int_0^t B_0^2(u)du, \quad \mu_b^3(t)=\int_0^t B_0^3(u)du \text{ and } \quad \mu_b^4(t)=\int_0^t B_0^4(u)du$$

So that,

$$\mu_b^{1*}(s)=B_0^{1*}(s)/s, \quad \mu_b^{2*}(s)=B_0^{2*}(s)/s, \quad \mu_b^{3*}(s)=B_0^{3*}(s)/s \quad \text{and} \quad \mu_b^{4*}(s)=B_0^{4*}(s)/s$$

(31-34)

6.4. Profit Function Analysis

The net expected total cost incurred in time interval (0, t) is given by

$P(t)=$ Expected total revenue in (0, t) - Expected cost of repair in (0, t)

$$=K_0\mu_{up}(t)-K_1\mu_b^1(t)-K_2\mu_b^2(t)-K_3\mu_b^3(t)-K_4\mu_b^4(t) \quad (35)$$

Where, K_0 is the revenue per- unit up time by the system during its operation. K_1, K_2, K_3 and K_4 are the amounts paid to the repairman per-unit of time when he is busy in the repair of unit-1 and unit-2 and in the post repair of unit-1 and unit-2 respectively.

The expected total profit incurred in unit interval of time is

$$P=K_0A_0-K_1B_0^1-K_2B_0^2-K_3B_0^3-K_4B_0^4 \quad (36)$$

7. Particular Case

When the time of satisfactory repair of unit-1 and unit-2 also follow exponential with p.d.fs as follows-

$$g_1(t)=\lambda_1 e^{-\lambda_1 t}, \qquad g_2(t)=\lambda_2 e^{-\lambda_2 t}$$

The Laplace Transform of above density function are as given below-

$$g_1^*(s)=\tilde{G}_1(s)=\frac{\lambda_1}{s+\lambda_1}, \qquad g_2^*(s)=\tilde{G}_2(s)=\frac{\lambda_2}{s+\lambda_2}$$

Here, $\tilde{G}_1(s)$ and $\tilde{G}_2(s)$ are the Laplace-Stieltjes Transforms of the c.d.fs $G_1(t)$ and $G_2(t)$ corresponding to the p.d.fs $g_1(t)$ and $g_2(t)$.

In view of above, the changed values of transition probabilities and mean sojourn times.

$$p_{10}^{(3)}=1-\frac{\lambda_1}{(\lambda_1-\beta_1)}\frac{\alpha_2'(1-r)}{(1-r\alpha_2')}+\frac{\beta_1}{(\lambda_1-\beta_1)}\frac{\alpha_2''(1-r)}{(1-r\alpha_2'')}, \qquad p_{17}^{(3)}=\frac{\lambda_1}{(\lambda_1-\beta_1)}\left[\frac{\alpha_2'(1-r)}{(1-r\alpha_2')}-\frac{\alpha_2''(1-r)}{(1-r\alpha_2'')}\right]$$

$$p_{17}^{(4)}=\frac{\alpha_2''(1-r)}{(1-r\alpha_2'')}, \qquad p_{20}^{(5)}=1-\frac{\lambda_2}{(\lambda_2-\beta_2)}\frac{\alpha_1'(1-r)}{(1-r\alpha_1')}+\frac{\beta_2}{(\lambda_2-\beta_2)}\frac{\alpha_1''(1-r)}{(1-r\alpha_1'')}$$

$$p_{28}^{(5)}=\frac{\lambda_2}{(\lambda_2-\beta_2)}\left[\frac{\alpha_1'(1-r)}{(1-r\alpha_1')}-\frac{\alpha_1''(1-r)}{(1-r\alpha_1'')}\right], \qquad p_{28}^{(6)}=\frac{\alpha_1''(1-r)}{(1-r\alpha_1'')}$$

$$\psi_1=\frac{1}{\alpha_2(1-r)+\lambda_1}, \qquad \psi_2=\frac{1}{\alpha_1(1-r)+\lambda_2}$$

$$\psi_4=\frac{1}{\lambda_1}, \qquad \psi_6=\frac{1}{\lambda_2}$$

8. Graphical Study of Behaviour and Conclusions

For a more clear view of the behaviour of system characteristics with respect to the various parameters involved, we plot curves for **MTSF** and **profit function** in **Fig. 2** and **Fig. 3** w.r.t. α_1 for three different values of correlation coefficient r =**0.16, 0.26** and **0.35** and two different values of repair parameter λ_1 =**0.41** and **0.5** while the other parameters are kept fixed as $\lambda_2 = 0.15$, $\alpha_2 = 0.05$, $\beta_1 = 0.35$ and $\beta_2 = 0.01$. From the curves of **Fig. 2,** we observe that **MTSF** increases uniformly as the values of r and λ_1 increase and it decreases with the increase in α_1. Further, to achieve **MTSF** at least **20** units we conclude from smooth curves that the value of α_1 must be less than **0.2, 0.15 and 0.11** respectively for $r = 0.16$, **0.26** , **0.35** when $\lambda_1 = 0.5$. Whereas from dotted curves we conclude that the value of α_1 must be less than **0.2, 0.16, 0.12** for $r = 0.16$, **0.26** , **0.35** when $\lambda_1 = 0.41$ to achieve at least **12** units of **MTSF**.

Similarly, **Fig. 3** reveals the variations in **profit (P)** w.r.t. α_1 for varying values of r and λ_1, when the values of other parameters are kept fixed as $\lambda_2 = 0.1$, $\alpha_2 = 0.05$, $\beta_1 = 0.35$, $\beta_2 = 0.3$, $K_0 = 800$, $K_1 = 600$, $K_2 = 500$, $K_3 = 400$ and $K_4 = 300$. Here also the same trends in respect of α_1, r and λ_1 are observed as in case of **MTSF**. Moreover, we conclude from the smooth curves that the system is profitable only if α_1 is less than **0.75, 0.5** and **0.37** respectively for $r = 0.25$, **0.35, 0.45** when $\lambda_1 = 0.5$. From dotted curves, we conclude that the system is profitable only if α_1 is less than **0.2, 0.15** and **0.12** respectively for $r = 0.25$, **0.35,** and **0.45** when $\lambda_1 = 0.4$.

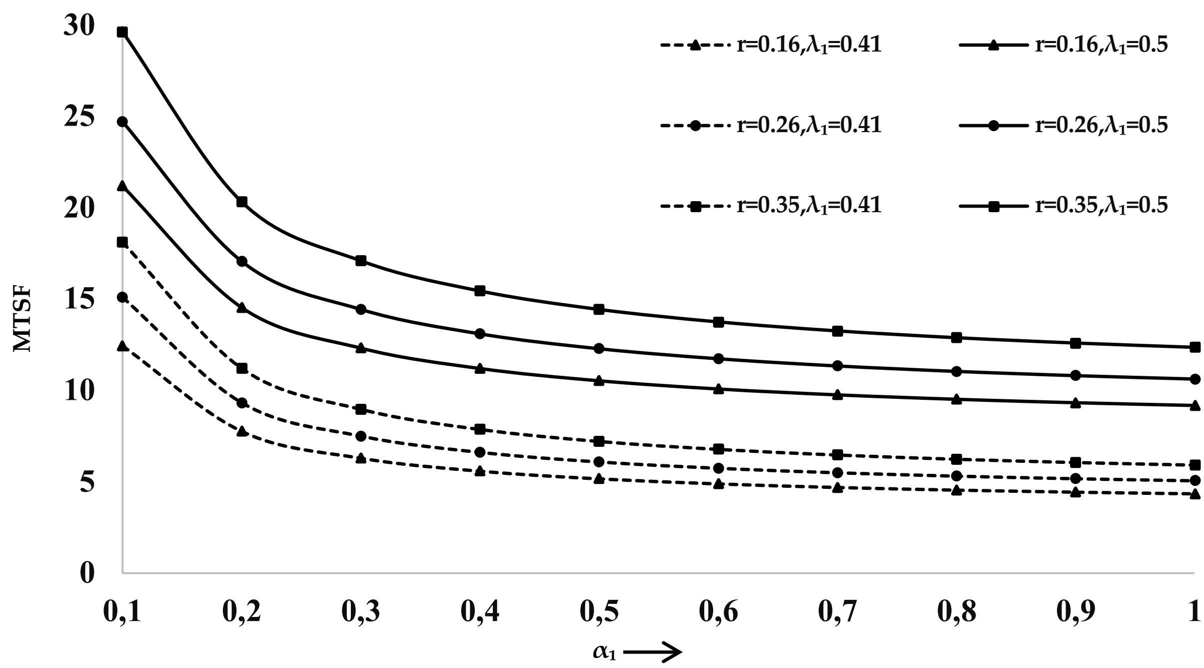

Figure 2: *Behaviour of MTSF w.r.t. α_1 for different values of r and λ_1*

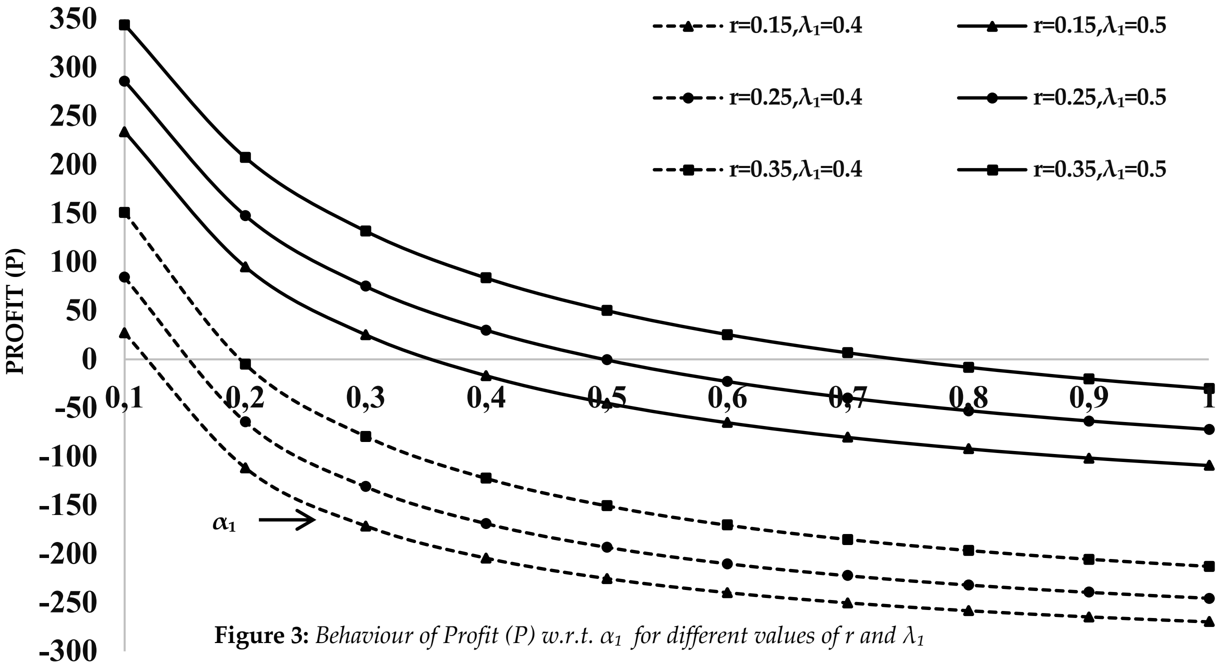

Figure 3: *Behaviour of Profit (P) w.r.t.* α_1 *for different values of r and* λ_1

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