Fuzzy Project Planning and Scheduling with Pentagonal Fuzzy Number

Adilakshmi Siripurapu[1,] Ravi Shankar Nowpada[2]

•

Dept. of Basic Science and Humanities, Vignan's Institute of Information Technology (A), Duvvada, Visakhapatnam, AP, India[1]
slakshmijagarapu@gmail.com

Dept. of Mathematics, Institute of Science, GITAM (Deemed to be University), Visakhapatnam, AP, India[2]
Drravi68@gmail.com

Abstract

In optimization approaches such as assignment issues, transportation problems, project schedules, artificial intelligence, data analysis, network flow analysis, an uncertain environment in organizational economics, and so on, ranking fuzzy numbers is essential. This paper introduces a new fuzzy ranking in Pentagonal fuzzy numbers. Each activity's duration is expressed as a Pentagonal fuzzy number in the project schedule. The new ranking function transforms every Pentagonal fuzzy number into a crisp number (normal number). We calculated the fuzzy critical path using a new algorithm. These approaches are illustrated with a numerical example.

Keywords: Activity duration, centroid, fuzzy ranking, fuzzy critical path, fuzzy number, project schedule.

1. Introduction

The critical path approach is one of the most important concepts in network analysis. It is used to solve project challenges by creating networks and establishing each activity's earliest start and earliest finish date. It is also a scheduling algorithm for a collection of project networks. It is also generally associated mostly with Program Evaluation and Review Technique (PERT).

Zadeh [12] introduced the existence of 'fuzzy logic', which considers inaccuracies and inconsistencies. Several academics have used fuzzy numbers in various forms, such as fuzzy triangular numbers, Trapezoidal fuzzy numbers, etc.

In many practical situations, the variables that define information uncertainty or vagueness are usually triangular or trapezoidal fuzzy numbers. Lee et al. [5] introduced Pentagonal fuzzy numbers and generalized the results of addition, subtraction, multiplication, and division based on Zadeh's extension principle. Pathinadhan et al. [6] proposed a new form of the non-normal generalized pentagonal fuzzy number, and some of its arithmetic operations, centroid, and median were discussed. Siji et al. [8] solved network problems with Pentagonal Intuitionistic fuzzy numbers using the ranking approach. Arokiamary et al. [1] determined the critical path analysis in a project network using the fuzzy TOPSIS method. Uthra et al. [11] defined a Generalized Intuitionistic Pentagonal fuzzy number and developed a new ranking formula.

Sahaya Sudha et al. [7] solved the fuzzy linear programming problem by applying the pentagonal fuzzy ranking function. Uma Maheswari et al. [10] introduced a simple approach for solving fuzzy transportation problems using fuzzy pentagonal numbers. Avishek Chakraborty et al. [2] studied interval-valued pentagonal fuzzy numbers, properties, ranking, and defuzzification. They solved the game problem by using a ranking function in fuzzy pentagonal numbers. Someshwar et al. [9] solved the linear programming problem using the pentagonal fuzzy ranking function. Das et al. [3] suggested a novel pentagonal neutrosophic technique for solving the linear programming problem.

2. Basic Definitions

In this section we look at a few definitions.

2.1 Fuzzy Set [12]

As stated in Zadeh's paper, the formalization of a fuzzy set is:

Let X be a space of points (objects), with a generic element of X denoted by x. Thus, $X = \{x\}$. A fuzzy set (class) A in X is characterized by a membership (characteristic function) function $\mu_A(x)$, which associates with each point in X a real number in the interval [0,1], with the value of $\mu_A(x)$ at x representing the "grade of membership" of x in A. When A set in the ordinary sense of the term, its membership function can take on only two values, 0 and 1, $\mu_A(x) = 1 \; or \; 0$ according to x does or does not belong to A.

2.2 Fuzzy Number [4]

It is a Fuzzy set of the following conditions:

- Convex fuzzy set
- Normalized fuzzy set.
- Its membership function is piece-wise continuous.
- It is defined in the real number.

Fuzzy numbers should be normalized and convex. Here the condition of normalization implies that the maximum membership value is 1.

2.3 Pentagonal Fuzzy Number (PFN) [6]

A pentagonal fuzzy number $\tilde{A}_p = (p_1, p_2, p_3, p_4, p_5)$ where p_1, p_2, p_3, p_4, p_5 real numbers and its membership function is defined by:

$$\mu_{\tilde{A}_p}(x) = \begin{cases} \frac{1}{2}\left(\frac{x-p_1}{p_2-p_1}\right); & p_1 \le x \le p_2 \\ \frac{1}{2}+\frac{1}{2}\left(\frac{x-p_2}{p_3-p_2}\right); & p_2 \le x \le p_3 \\ 1-\frac{1}{2}\left(\frac{x-p_3}{p_4-p_3}\right); & p_3 \le x \le p_4 \\ \frac{1}{2}\left(\frac{p_5-x}{p_5-p_4}\right); & p_4 \le x \le p_5 \\ 0; & otherwiise \end{cases}$$

The Pentagonal Fuzzy Number diagram is represented in Figure 1.

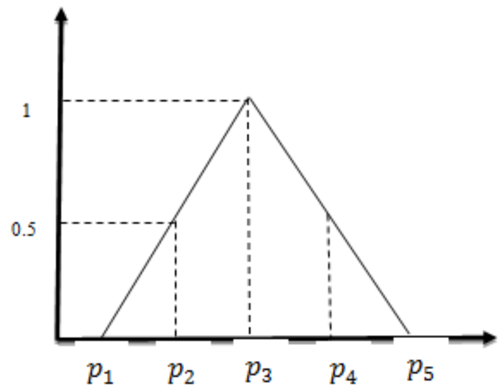

Figure 1: *Pentagonal Fuzzy Number*

2.4 Generalized Pentagonal Fuzzy Number (GPFN) [6]

The generalized pentagonal fuzzy number $\tilde{A}_p = (p_1, p_2, p_3, p_4, p_5;\ \omega)$ its membership function is expressed as;

$$\mu_{\tilde{A}_p}(x) = \begin{cases} \frac{\omega}{2}\left(\frac{x-p_1}{p_2-p_1}\right); & p_1 \le x \le p_2 \\ \frac{1}{2}+\frac{\omega}{2}\left(\frac{x-p_2}{p_3-p_2}\right); & p_2 \le x \le p_3 \\ 1-\frac{\omega}{2}\left(\frac{x-p_3}{p_4-p_3}\right); & p_3 \le x \le p_4 \\ \frac{\omega}{2}\left(\frac{p_5-x}{p_5-p_4}\right); & p_4 \le x \le p_5 \\ 0; & otherwiise \end{cases}$$

The generalized Pentagonal Fuzzy Number diagram is represented in Figure 2.

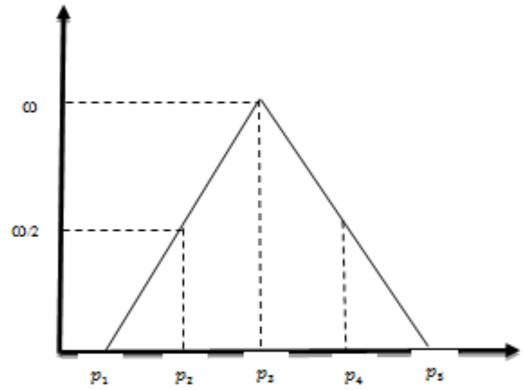

Figure 2: *Generalized Pentagonal Fuzzy Number*

3. Methodology

This section, proposed a new ranking function in Pentagonal fuzzy number.

Divide the pentagon into two triangles and one rectangle. Let G_1, G_2, and G_3 be the centroid of the three plane figures, respectively. The centroid of a fuzzy pentagon number is supposed to indicate the pentagon's balancing point (Figure 3).

The Proposal ranking in the Pentagonal fuzzy number diagram is represented in Figure 3.

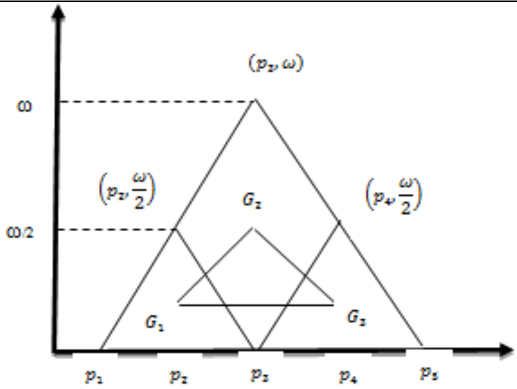

Figure 3: *Proposed Ranking Approach*

To determine the ranking of a generalized pentagon fuzzy number, the in centre of the centroid G_1, G_2, G_3 is used as a point of reference.

Consider the generalized pentagon fuzzy number $\tilde{A}_p = (p_1, p_2, p_3, p_4, p_5, \omega)$

The three plane figures' centroid is$G_1 = \left(\frac{p_1+p_2+p_3}{3}, \frac{\omega}{6}\right)$, $G_2 = \left(\frac{p_2+2p_3+p_4}{4}, \frac{\omega}{2}\right)$, $G_3 = \left(\frac{P_3+p_4+p_5}{3}, \frac{\omega}{6}\right)$ respectively.

The centroid of G_1, G_2 and G_3 is $\left(\frac{4P_1+7P_2+14p_3+7p_4+4P_5}{36}, \frac{5\omega}{36}\right)$.

$$G_{\tilde{A}_p}(x_0, y_0) = \left(\frac{4P_1 + 7P_2 + 14p_3 + 7p_4 + 4P_5}{36}, 5\omega/36\right)$$

The in centre of the centroid with Euclidean distance is;

$$\sqrt{x_0^2 + y_0^2}$$

Consider that the centroid's center with Euclidean distance is a new ranking function in the generalized Pentagonal fuzzy number.

Therefore, a new ranking in the Generalized Pentagonal fuzzy number is;

$$\Re(\tilde{A}_p) = \sqrt{x_0^2 + y_0^2}$$

3.1 Fuzzy Critical Path Analysis

The primary objective of the fuzzy critical path is to estimate the total project duration and assign start and finish dates to all project activities. This makes it possible to compare the actual progress to the estimated duration.

The following fuzzy factors should be known to prepare the project schedule.

(i) Project completion time
(ii) Each activity's earliest and latest times
(iii) Critical activities and the critical path
(iv) Float for each activity (i.e., the time required to complete a non-critical activity can be delayed without affecting the overall project completion time)

3.1.1 Notations

$FE\tilde{S}_{ij}$ = Earliest start of time of an activity (i, j)

$FL\tilde{S}_{ij}$ = Latest start time of an activity (i, j)
$FE\tilde{F}_{ij}$ = Earliest finish time of an activity (i, j)
$FL\tilde{F}_{ij}$ = Latest finish time of an activity(i, j)
$F\tilde{t}_{ij}$ = Total duration of an activity (i, j)
$FT\tilde{F}_{ij}$=Total float of an activity (i, j)

3.1.2 Algorithm for Fuzzy Critical Path

Step1: Construct a fuzzy project network with predecessor and successor events.
Step2: Express every activity time as PFN.
Step3: Transformed every PFN as a crisp number using a new ranking function.
Step4: Calculate earliest start time, $FE\tilde{S}_{ij} = \max_i\{FE\tilde{S}_{ij} + \tilde{t}_{ij}\}$, i = number of preceding nodes.
Step5: Calculate earliest finish time, $FE\tilde{F}_{ij} = FE\tilde{S}_i + \tilde{t}_{ij}$.
Step6: Calculate latest finish time $FL\tilde{F}_{ij} = \min_j\{FL\tilde{F}_{ij} - \tilde{t}_{ij}\}$, j = number of succeeding nodes
Step7: Calculate latest start time, $FL\tilde{S}_{ij} = FL\tilde{F}_{ij} - \tilde{t}_{ij}$
Step8: Calculate Total float, $FT\tilde{F}_{ij} = FL\tilde{F}_{ij} - FE\tilde{F}_{ij}$ *or* $FL\tilde{S}_{ij} - FE\tilde{S}_{ij}$

4 Application of Pentagonal fuzzy numbers in a project schedule

Consider the following fuzzy project network, in which the Pentagonal fuzzy number represents each activity. The fuzzy project network has seven nodes and nine activities. This example shows how to schedule a construction project using a project Network. My objective is to analyze the maximum path that is the essential Critical path for the construction process.

Table 1 represents the activities, their description and duration periods. Figure 4 represents the project network diagram.

Table 1: *Project Network Description*

Activity	Pentagonal fuzzy numbers
1→2	(1,2,3,4,5)
1→3	(6,7,8,9,10)
2→4	(11,12,13,14,15)
3→4	(16,17,18,19,20)
2→5	(21,22,23,24,25)
3→6	(26,27,28,29,30)
4→7	(31,32,33,34,35)
5→7	(36,37,38,39,40)
6→7	(41,42,43,44,45)

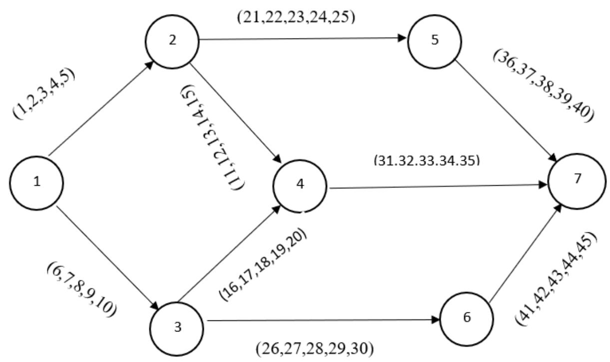

Figure 4: *Project Network*

4.1 Expected time of activities

Pentagonal fuzzy number transformed into activity duration by proposal ranking function. This activity duration is taken as the time between the nodes, and the fuzzy critical path is calculated by applying an algorithm. The expected time of activities is represented in Table 2, and the related diagram is represented in Figure 5.

Table 2: *Expected time of activities*

Activity $i \longrightarrow j$	Pentagonal fuzzy number	Expected time
1→2	(1,2,3,4,5)	3.0032
1→3	(6,7,8,9,10)	8.0012
2→4	(11,12,13,14,15)	13.0007
3→4	(16,17,18,19,20)	18.0005
2→5	(21,22,23,24,25)	23.0004
3→6	(26,27,28,29,30)	28.0003
4→7	(31,32,33,34,35)	33.0002
5→7	(36,37,38,39,40)	38.0002
6→7	(41,42,43,44,45)	43.0002

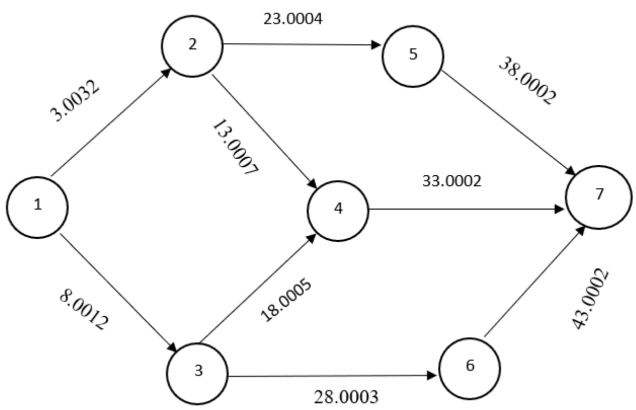

Figure 5: *Expected time of Activities*

4.2 Earliest, latest times and Total float of fuzzy activities

Computed Earliest, latest times and total float using formulas mentioned in procedure step 4, step 5, step6, step7 and step 8, respectively.

The earliest, latest and total float times of fuzzy activities represented in Table 3.

Table 3: *The earliest, latest times and total float of fuzzy activities with defuzzified values of PFN*

$i \longrightarrow j$	$F\tilde{t}_{ij}$	$FE\tilde{S}_{ij}$	$FE\tilde{F}_{ij}$	$FL\tilde{S}_{ij}$	$FL\tilde{F}_{ij}$	$FT\tilde{F}_{ij}$
1→2	3.0032	0	3.0032	14.9979	18.0011	14.9979
1→3	8.0012	0	8.0012	0	8.0012	0*
2→4	13.0007	3.0032	16.0039	33.0008	46.0015	29.9976
3→4	18.0005	8.0012	26.0017	28.001	46.0015	19.9998
2→5	23.0004	3.0032	26.0036	18.0011	41.0015	14.9979
3→6	28.0003	8.0012	36.0015	8.0012	36.0015	0*
4→7	33.0002	26.0017	59.0019	46.0015	79.0017	19.9998
5→7	38.0002	26.0036	64.0038	41.0015	79.0017	14.9979
6→7	43.0002	36.0015	79.0017	36.0015	79.0017	0*

4.3 Results

According to the fuzzy total float, the fuzzy critical activities are 1→3, 3→6, 6→7.
Therefore, the critical path of the fuzzy project network is 1→3→6→7.
Hence, the total duration of the project network is 79.0017≅79 days.

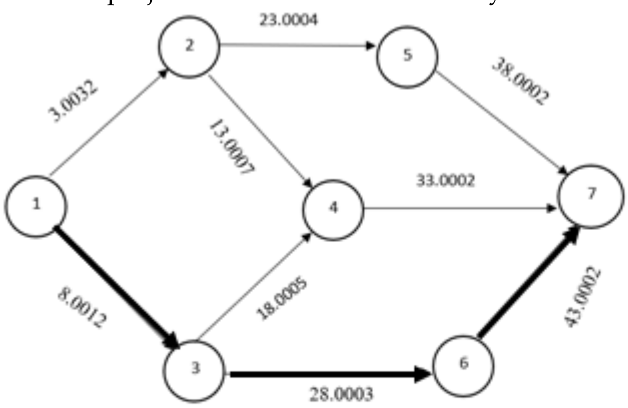

Figure 6: *Fuzzy Critical Path*

5. Conclusion

This paper introduced a new ranking function in Pentagonal fuzzy numbers. The proposed ranking function is derived from the centroid of PFN. In the network, every activity period is expressed by a PFN. The duration of every activity is transformed into the normal number or crisp number by a new ranking function. This normal number is considered the expected time of activity. The fuzzy critical path algorithm was used to identify the fuzzy critical path and project completion time. The proposal ranking can also be applied to more complex project networks in the real world. We can apply the ranking function of PFN to solve game problems and transportation problems.

References

[1] Arokiamary, A., & Jayapalan P. (2016) Fuzzy Critical Path Analysis in a Project Network Using Fuzzy Topsis Method, *International Journal of Mathematics And its Application*, Volume 4, Issue 4, 423-429.

[2] Avishek Chakraborty., Mondal, S. P., Alam, S., Ahmadian, A., Senu, N., De, D., & S. Salahshour. (2019). The fuzzy pentagonal number: its different representations, properties, ranking, defuzzification and application in-game problems. *Symmetry*, *11*(2), 248.

[3] Das, S. K., & Chakraborty, A. (2020). A new approach to evaluate linear programming problem in pentagonal neutrosophic environment. *Complex & intelligent systems*, *7*(1), 101-110.

[4] Kaufmann, A. and Madan M. Gupta (1986). Introduction to fuzzy arithmetic: theory and applications. *Van Nostrand Reinhold,* New York.

[5] Lee. B and Yong Sik Yun, The Pentagonal Fuzzy Numbers, *Journal of the Chungcheong Mathematical Society*, Volume 27, No. 2, May 2014.

[6] Pathinathan. T, Ponnivalvan. K, Ebinesar Mike. (2015). Different Types of Fuzzy Numbers and Certain Properties. *Journal of Computer and Mathematical Sciences*. 6. 631-651.

[7] Sahaya Sudha, S Vimalavirginmary, S Sathya (2017), A Novel approach for Solving Fuzzy Linear Programming Problem using Pentagonal Fuzzy Numbers, *International Journal of Advanced Research in Education & Technology,* Vol.4, Issue 1, ISSN: 2394-6814.

[8] Siji.S, K. Selva Kumari (2016). An Approach for solving Network Problem with Pentagonal Intuitionistic Fuzzy Numbers Using Ranking Technique, *Middle-East Journal of Scientific Research,* 24(9),2977-2980, ISSN:1990-9233.

[9] Someshwar Siddi & Reddy, Y. (2020). Solving Fuzzy LPP for Pentagonal Fuzzy Number Using Ranking Approach, *MuktShabd Journal*, and IX. 2674. ISSN: 2347-3150.

[10] Uma Maheswari & Kandasamy, Ganesan. (2018). Solving fully fuzzy transportation problem using pentagonal fuzzy numbers. *Journal of Physics: Conference Series*. 1000. 012014.

[11] Uthra, G., Thangavelu, K., & Shunmugapriya, S. (2017). Ranking Generalized Intuitionistic Pentagonal Fuzzy Number by Centroidal Approach, *International Journal of Mathematics and Its Applications*, Volume-5, Issue 4-D, 589-593, ISSN: 2347-1557.

[12] Zadeh L.A. (1965) Fuzzy sets, *Information and Control*, Volume 8, Issue 3, 338-353, ISSN: 0019-9958.