Analysis of Triple-Unit System with Operational Priority

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Abstract

Reliability of three non-identical unit system is analyzed for various measures. Initially, main unit is operational, one is warm standby and other is cold standby. Single repair facility is present with the system. Operational priority is given to main unit over standby units. Failure times of all the components are exponentially distributed whereas repair time follows Weibull distribution. All the random variables are statistically independent. Semi-Markov process and regenerative point technique are used to analyze mean time to system failure, availability, busy period and expected number of visits by the server. System model's profit is analyzed for arbitrary values and are shown graphically.

Keywords: Reliability, Non-Identical, Standby, Priority, Semi-Markov, Regenerative Point.

I. Introduction

Unwavering property is the greatest amount of wanted trait of a system. One and all wishes to rehearse exceptionally robust systems in everyday life. Understanding high demand of highly reliable systems, numerous researchers studied various systems under various set of assumptions. Osaki and Asakura [1] discussed two-unit standby redundant system with repair and preventive maintenance, Murari and Goyal [2] studied a system model with three types of repair facilities, Gopalan and Nagarwalla [3] analyzed two-identical-unit cold standby system with repair and preventive maintenance, Dhillon and Yang [4] and Dhillon [5] done reliability and availability analysis of standby systems with common-cause failures and human errors. Goel et al. [6] discussed two-unit cold standby system with preventive maintenance and replacement of the duplicate unit, Kadyan et al. [7] stochastically analyzed non-identical units reliability models with priority and different failure modes, Kishan and Jain [8] studied two non-identical unit standby system with repair, inspection and post-repair under classical and Bayesian viewpoints, Kumar and Saini [9] analyzed single-unit system with preventive maintenance and Weibull distribution for failure and repair activities, Malik and Upma [10] analyzed profit of non-identical units system under preventive maintenance and replacement, Kumar et al. [11][12] analyzed warm standby non-identical units system with single server subject to priority and without priority for operation, Rathee et al. [13] studied two-unit parallel system subject to priority of repair of units over replacement. Kumar et al.

[14][15] analyzed profit of a warm standby non-identical unit system with single server performing in normal/ abnormal environment, also analyzed profit of the system with preventive maintenance, Ashok et al. [16] performed reliability analysis of a redundant system with 'FCFS' repair policy subject to weather conditions, Jain et al. [17] studied the reliability of a 1-out-of-2 system with standby and delayed service. Jain et al. [18] analyzed profit of a 1-out of 2 unit system with a standby unit and arrival time of server.

Every time, it is not feasible to meet the expenses of an identical unit in spare. Therefore, to keep the system operational, non-identical units might be taken as warm/ cold standby. When dealing with highly sensitive server systems in I.T. sector to keep data, we cannot rely on the systems of single unit or single unit in standby. In such cases, there is a need to keep two or more units in standby, therefore, priority is given to non-identical units in standby. Here, we developed and analyzed a reliability model of triple unit system.

II. System Assumptions

- The system comprises of three non-identical units.
- Initially, main unit (M) is operational, one unit (U_1) is warm standby and other unit (U_2) is cold standby.
- Single repair facility is present with the system.
- Operational priority is given to the main unit over the standby units.
- All unit works as new after repair.
- Failure times of the components are exponentially distributed whereas repair time follows Weibull distribution.

III. Method

Expressions for numerous reliability measures including mean time to system failure (MTSF), availability, busy period and expected number of visits by the server are evaluated using semi-Markov process and RPT. Profit of the system is analyzed for arbitrary values and represented graphically.

IV. Notations and Transition Diagram

Mo, U_1o, U_2o	:	Unit M, U_1, U_2 are operative.
U_1ws/U_2cs	:	U_1 is warm standby/ U_2 is cold standby.
Mur/MUR	:	M is under repair/ continuous repair.
U_1ur/U_1UR	:	U_1 is under repair/ continuous repair.
U_1wr/U_1WR	:	U_1 is waiting/ continuously waiting for repair.
U_2ur/U_2UR	:	U_2 is under repair/ continuous repair.
U_2wr/U_2WR	:	U_2 is waiting/ continuously waiting for repair.
$\lambda, \lambda_1, \lambda_2$	:	Failure rate of units M, U_1, U_2 respectively.
$f(t), r(t), r_1(t)$	:	p.d.f. of repair time of units M, U_1, U_2 respectively.
$F(t), R(t), R_1(t)$	:	c.d.f. of repair time of units M, U_1, U_2 respectively.
$q_{ij}(t)/Q_{ij}(t)$	:	p.d.f./ c.d.f. of earliest passing time from regenerative state i to j without passing through any other regenerative state in $(0, t]$.
$M_i(t)$	:	Probability that the system is initially up in regenerative state i at time t.
$W_i(t)$	:	Probability that the server is busy in regenerative state i at time t,.

m_{ij} : Contribution to mean sojourn time (μ_i) in regenerative state i when system transits directly to state j. $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$.

Ⓢ/©/©n : Symbol of Stieltjes/ Laplace/ Laplace 'n' times convolution.

*/** : Symbol of Laplace/ Laplace-Stieltjes transformation.

K_0 : Fixed revenue/ unit operational time of the system.

K_1 : Fixed cost/ unit busy period of the server.

K_2 : Fixed cost/ visit by the server.

P : Profit of the system.

The possible transition states are exhibited in figure 1.

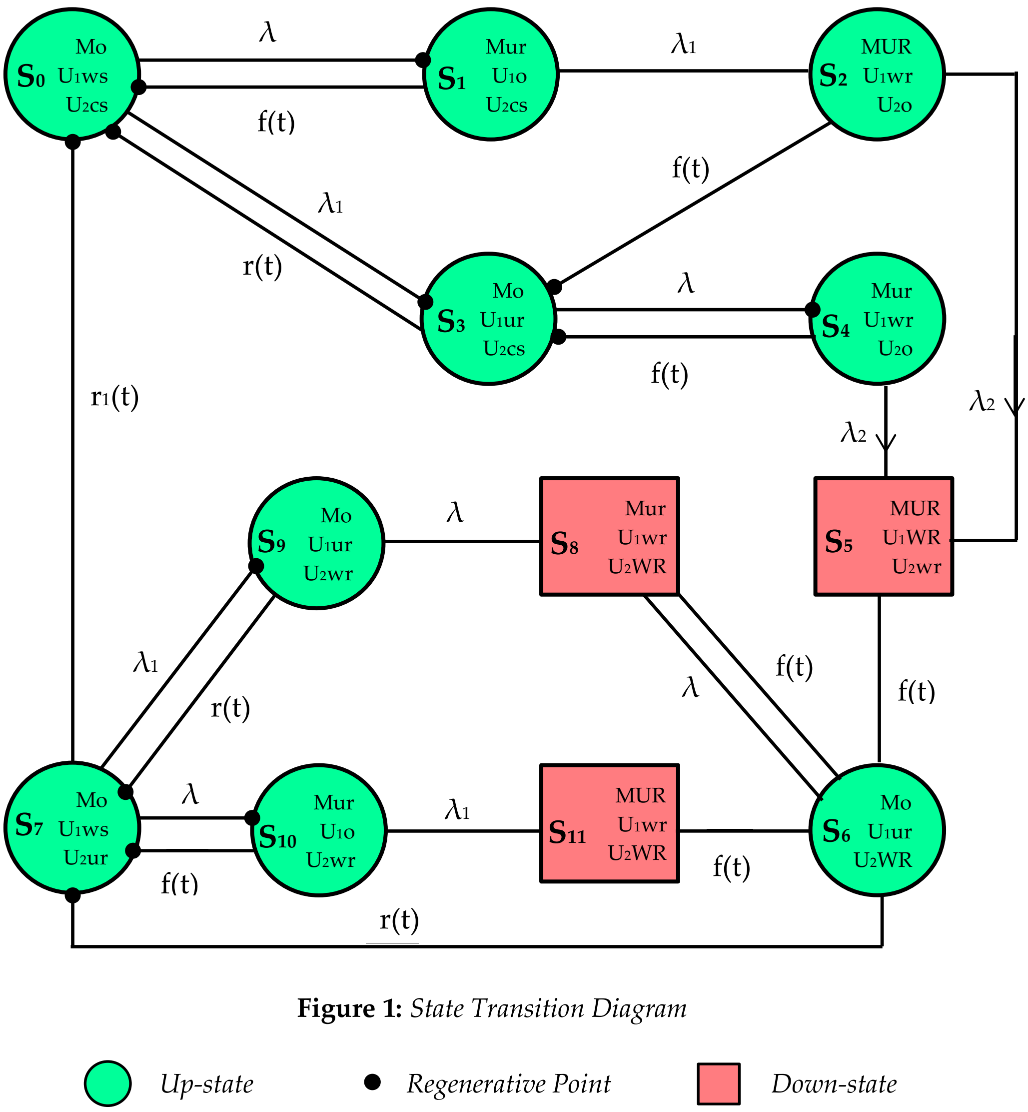

Figure 1: *State Transition Diagram*

V. Analysis of Reliability Measures

a) Transition Probabilities and Mean Sojourn Time

Simple probabilistic considerations for non-zero elements generate the expressions as

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)dt$$

We have $p_{01} = \frac{\lambda}{\lambda+\lambda_1}$, $p_{03} = \frac{\lambda_1}{\lambda_1+\lambda}$, $p_{10} = f^*(\lambda_1)$, $p_{12} = 1 - f^*(\lambda_1)$, $p_{23} = f^*(\lambda_2)$, $p_{25} = 1 - f^*(\lambda_2)$, $p_{30} =$

$r^*(\lambda)$, $p_{34} = 1 - r^*(\lambda)$, $p_{43} = f^*(\lambda_2)$, $p_{45} = 1 - f^*(\lambda_2)$, $p_{56} = p_{86} = p_{11,6} = 1$, $p_{67} = r^*(\lambda)$, $p_{68} = 1 - r^*(\lambda)$, $p_{70} = r_1^*(\lambda + \lambda_1)$, $p_{79} = \frac{\lambda_1}{\lambda+\lambda_1}[1 - r_1^*(\lambda + \lambda_1)]$, $p_{7,10} = \frac{\lambda}{\lambda+\lambda_1}[1 - r_1^*(\lambda + \lambda_1)$, $p_{97} = r^*(\lambda)$, $p_{98} = 1 - r^*(\lambda)$, $p_{10,7} = f^*(\lambda_1)$, $p_{10,11} = 1 - f^*(\lambda_1)$

It can be easy to verify that

$p_{01} + p_{03} = p_{10} + p_{12} = p_{23} + p_{25} = p_{30} + p_{34} = p_{43} + p_{45} = p_{56} = p_{67} + p_{68} = p_{70} + p_{79} + p_{7,10} = p_{86} = p_{97} + p_{98} = p_{10,7} + p_{10,11} = p_{11,6} = 1$

Now, μ_i in the state S_i are $\mu_i = \sum_j m_{ij}$ and $m_{ij} = \int t dQ_{ij}(t) = -q_{ij}^*(0)$

$\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{1}{\lambda_1}[1 - f^*(\lambda_1)]$, $\mu_3 = \frac{1}{\lambda}[1 - r^*(\lambda)]$, $\mu_4 = \frac{1}{\lambda_2}[1 - f^*(\lambda_2)]$, $\mu_7 = \frac{1}{\lambda+\lambda_1}[1 - r_1^*(\lambda + \lambda_1)]$, $\mu_9 = \frac{1}{\lambda}[1 - r^*(\lambda)]$, $\mu_{10} = \frac{1}{\lambda_1}[1 - f^*(\lambda_1)]$

b) Reliability and MTSF

Let cdf of earliest passage time from regenerative state i to a failed state is $\phi_i(t)$. The recursive relations for $\phi_i(t)$ are

$\phi_0(t) = Q_{01}(t) Ⓢ \phi_1(t) + Q_{03}(t) Ⓢ \phi_3(t)$

$\phi_1(t) = Q_{10}(t) Ⓢ \phi_0(t) + Q_{13.2}(t) Ⓢ \phi_3(t) + Q_{15.2}(t)$

$\phi_3(t) = Q_{30}(t) Ⓢ \phi_0(t) + Q_{34}(t) Ⓢ \phi_4(t)$

$\phi_4(t) = Q_{43}(t) Ⓢ \phi_3(t) + Q_{45}(t)$ … (1)

Taking LST of (1) and solving for $\phi^{**}(s)$, we have

$$R^*(s) = \frac{1-\phi^{**}(s)}{s} \quad \ldots (2)$$

The reliability of the system can be obtained by taking inverse LT of (2).

The MTSF is given by $\lim_{s\to 0} R^*(s)$. Thus,

$$MTSF = \frac{N_0}{D_0}, \text{ where}$$

$N_0 = (\mu_0 + \mu_1 p_{01})(1 - p_{43}p_{34}) + (\mu_3 + \mu_4 p_{34})(p_{01}p_{13.2} + p_{03})$ and $D_0 = (1 - p_{34}p_{43})(1 - p_{01}p_{10}) - p_{30}(p_{01}p_{13.2} + p_{03})$

c) Steady State Availability

Let $A_i(t)$ be the probability of the system to be operational at time 't' provided that the system arrived at regenerative state i at $t = 0$. The recursive relations for $A_i(t)$ are

$A_0(t) = M_0(t) + q_{01}(t)©A_1(t) + q_{03}(t)©A_3(t)$

$A_1(t) = M_1(t) + q_{10}(t)©A_0(t) + q_{13.2}(t)©A_3(t) + [q_{17.2,5,6}(t) + q_{17.2,5(6,8)^n}(t)]©A_7(t)$

$A_3(t) = M_3(t) + q_{30}(t)©A_0(t) + q_{34}(t)©A_4(t)$

$A_4(t) = M_4(t) + q_{43}(t)©A_3(t) + [q_{47.5,6}(t) + q_{47.5(6,8)^n}(t)]©A_7(t)$

$A_7(t) = M_7(t) + q_{70}(t)©A_0(t) + q_{79}(t)©A_9(t) + q_{7,10}(t)©A_{10}(t)$

$A_9(t) = M_9(t) + [q_{97}(t) + q_{97.(6,8)^n}(t)]©A_7(t)$

$A_{10}(t) = M_{10}(t) + [q_{10,7}(t) + q_{10,7.11,6}(t) + q_{10,7.11,(6,8)^n}(t)]©A_7(t)$ … (3)

where, $M_0(t) = e^{-(\lambda+\lambda_1)t}$, $M_1(t) = e^{-\lambda_1 t}\overline{F(t)}$, $M_3(t) = e^{-\lambda t}\overline{F(t)}$, $M_4(t) = e^{-\lambda_2 t}\overline{F(t)}$, $M_7(t) = e^{-(\lambda+\lambda_1)t}\overline{R_1(t)}$, $M_9(t) = e^{-\lambda t}\overline{R(t)}$, $M_{10}(t) = e^{-\lambda_1 t}\overline{F(t)}$

Taking LT of (3) and solving for $A^*(s)$. The steady state availability is given by

$$A(\infty) = \lim_{s\to 0} sA^*(t) = A = \frac{N_1}{D_1}, \text{ where}$$

$N_1 = [p_{70}\{(M_0 + M_1 p_{01})(1 - p_{34}p_{43}) + (M_3 + M_4 p_{34})(p_{03} + p_{01}p_{13.2})\} + D_0(M_7 + M_9 p_{79} + M_{10}p_{7,10})]$,

$D_1 = [p_{70}\{(\mu_0 + \mu_1 p_{01})(1 - p_{34}p_{43}) + (\mu_3 + \mu_4 p_{34})(p_{03} + p_{01}p_{13.2})\} + D_0(\mu_7 + \mu_9 p_{79} + \mu_{10}p_{7,10})]$ and D_0 is already specified.

d) Busy Period Analysis

Let $B_i(t)$ be the probability that the server is employed in restoring the unit at time 't' given that the system arrived at regenerative state i at $t = 0$. The recursive relations for $B_i(t)$ are

$B_0(t) = q_{01}(t)©B_1(t) + q_{03}(t)©B_3(t)$

$B_1(t) = W_1(t) + q_{10}(t)©B_0(t) + q_{13.2}(t)©B_3(t) + [q_{17.2,5,6}(t) + q_{17.2,5(6,8)^n}(t)]©B_7(t)$

$B_3(t) = W_3(t) + q_{30}(t)©B_0(t) + q_{34}(t)©B_4(t)$

$B_4(t) = W_4(t) + q_{43}(t)©B_3(t) + [q_{47.5,6}(t) + q_{47.5(6,8)^n}(t)]©B_7(t)$

$B_7(t) = W_7(t) + q_{70}(t)©B_0(t) + q_{79}(t)©B_9(t) + q_{7,10}(t)©B_{10}(t)$

$B_9(t) = W_9(t) + [q_{97}(t) + q_{97.(6,8)^n}(t)]©B_6(t)$

$B_{10}(t) = W_{10}(t) + [q_{10,7}(t) + q_{10,7.11,6}(t) + q_{10,7.11,(6,8)^n}(t)]©B_6(t)$ … (4)

where, $W_1(t) = e^{-\lambda_1 t}\overline{F(t)}$, $W_3(t) = e^{-\lambda t}\overline{F(t)}$, $W_4(t) = e^{-\lambda_2 t}\overline{F(t)}$, $W_7(t) = e^{-(\lambda+\lambda_1)t}\overline{R_1(t)}$, $W_9(t) = e^{-\lambda t}\overline{R(t)}$, $W_{10}(t) = e^{-\lambda_1 t}\overline{F(t)}$

Taking LT of (4) and solving for $B^*(s)$. The busy period of the server can be obtained as

$$B(\infty) = \lim_{s\to 0} s\,B^*(s) = B = \frac{N_2}{D_1}, \text{ where}$$

$N_2 = [p_{70}\{(W_1 p_{01})(1 - p_{34}p_{43}) + (W_3 + W_4 p_{34})(p_{03} + p_{01}p_{13.2})\} + D_0(W_7 + W_9 p_{79} + W_{10}p_{7,10})]$ and D_0, D_1 are previously specified.

e) Expected Number of Visits by the Server

Let expected number of visits by the server in *(0, t]* is $N_i(t)$, given that the system arrived at the regenerative state i at $t = 0$. The recursive relations for $N_i(t)$ are

$N_0(t) = Q_{01}(t)Ⓢ\{1 + N_1(t)\} + Q_{03}(t)Ⓢ\{1 + N_3(t)\}$

$N_1(t) = Q_{10}(t)Ⓢ N_0(t) + Q_{13.2}(t)Ⓢ N_3(t) + [Q_{17.2,5,6}(t) + Q_{17.2,5(6,8)^n}(t)]Ⓢ N_7(t)$

$N_3(t) = Q_{30}(t)Ⓢ N_0(t) + Q_{34}(t)Ⓢ N_4(t)$

$N_4(t) = Q_{43}(t)Ⓢ N_3(t) + [Q_{47.5,6}(t) + Q_{47.5(6,8)^n}(t)]Ⓢ N_7(t)$

$N_7(t) = Q_{70}(t)Ⓢ N_0(t) + Q_{79}(t)Ⓢ N_9(t) + Q_{7,10}(t)Ⓢ N_{10}(t)$

$N_9(t) = [Q_{97}(t) + Q_{97.(6,8)^n}(t)]Ⓢ N_6(t)$

$N_{10}(t) = [Q_{10,7}(t) + Q_{10,7.11,6}(t) + Q_{10,7.11,(6,8)^n}(t)]Ⓢ N_6(t)$ … (5)

Taking LT of (5) and solving for $N^{**}(s)$. The expected number of visits by the server can be obtained as

$$N(\infty) = \lim_{s\to 0} s\,N^{**}(s) = N = \frac{N_3}{D_1}, \text{ where}$$

$N_3 = p_{70}(1 - p_{34}p_{43})$ and D_1 is already specified.

VI. Profit Analysis

In steady state, system model's profit can be evaluated as $P = K_0 A - K_1 B - K_2 N$

VII. Particular Case

Let $f(t) = \alpha\eta t^{\eta-1}e^{-\alpha t^\eta}$, $r(t) = \alpha_1\eta t^{\eta-1}e^{-\alpha_1 t^\eta}$, $r_1(t) = \alpha_2\eta t^{\eta-1}e^{-\alpha_2 t^\eta}$ … (6)

are pdfs of Weibull distribution for repair time of units M, U_1 and U_2 respectively. Where α, α_1 and α_2 are different scale parameters and η is shape parameter.

On taking $\eta = 1$ in (6), Weibull distribution becomes exponential distribution. The transition probabilities p_{01} and p_{03} remains same whereas the remaining are

$p_{10} = \frac{\alpha}{\alpha+\lambda_1}$, $p_{12} = \frac{\lambda_1}{\alpha+\lambda_1}$, $p_{23} = \frac{\alpha}{\alpha+\lambda_2}$, $p_{25} = \frac{\lambda_2}{\alpha+\lambda_2}$, $p_{30} = \frac{\alpha_1}{\alpha_1+\lambda_2}$, $p_{34} = \frac{\lambda_2}{\alpha_1+\lambda_2}$, $p_{43} = \frac{\alpha}{\alpha+\lambda_2}$, $p_{45} = \frac{\lambda_2}{\alpha+\lambda_2}$, $p_{56} = p_{86} = p_{11,6} = 1$, $p_{67} = \frac{\alpha_1}{\lambda+\alpha_1}$, $p_{68} = \frac{\lambda}{\lambda+\alpha_1}$, $p_{70} = \frac{\alpha_2}{\alpha_2+\lambda+\lambda_1}$, $p_{79} = \frac{\lambda_2}{\alpha_2+\lambda+\lambda_1}$, $p_{70} = \frac{\lambda}{\alpha_2+\lambda+\lambda_1}$, $p_{97} = \frac{\alpha_1}{\alpha_1+\lambda}$, $p_{98} = \frac{\lambda}{\lambda+\alpha_1}$, $p_{10,7} = \frac{\alpha}{\alpha+\lambda_1}$, $p_{10,11} = \frac{\lambda_1}{\alpha+\lambda_1}$

Then μ_i are $\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$, $\mu_3 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha_1+\lambda_1)^{\frac{1}{\eta}}}$, $\mu_4 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_2)^{\frac{1}{\eta}}}$, $\mu_7 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\lambda_1+\alpha_2)^{\frac{1}{\eta}}}$, $\mu_9 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\alpha_1)^{\frac{1}{\eta}}}$, $\mu_{10} = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$

Similarly, $W_1 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$, $W_3 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha_1+\lambda_1)^{\frac{1}{\eta}}}$, $W_4 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_2)^{\frac{1}{\eta}}}$, $W_7 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\lambda_1+\alpha_2)^{\frac{1}{\eta}}}$, $W_9 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\alpha_1)^{\frac{1}{\eta}}}$, $W_{10} = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$

and $M_0 = \frac{1}{\lambda+\lambda_1}$, $M_1 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$, $M_3 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha_1+\lambda_1)^{\frac{1}{\eta}}}$, $M_4 = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_2)^{\frac{1}{\eta}}}$, $M_7 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\lambda_1+\alpha_2)^{\frac{1}{\eta}}}$, $M_9 = \frac{\Gamma(1+\frac{1}{\eta})}{(\lambda+\alpha_1)^{\frac{1}{\eta}}}$, $M_{10} = \frac{\Gamma(1+\frac{1}{\eta})}{(\alpha+\lambda_1)^{\frac{1}{\eta}}}$

i) For $\eta = 0.5$, the repair time distribution reduces to $f(t) = \frac{\alpha}{2\sqrt{t}}e^{-\alpha\sqrt{t}}$, $r(t) = \frac{\alpha_1}{2\sqrt{t}}e^{-\alpha_1\sqrt{t}}$, $r_1(t) = \frac{\alpha_2}{2\sqrt{t}}e^{-\alpha_2\sqrt{t}}$

As a result μ_i changes to $\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{2}{(\alpha+\lambda_1)^2}$, $\mu_3 = \frac{2}{(\alpha_1+\lambda_1)^2}$, $\mu_4 = \frac{2}{(\alpha+\lambda_2)^2}$, $\mu_7 = \frac{2}{(\lambda+\lambda_1+\alpha_2)^2}$, $\mu_9 = \frac{2}{(\lambda+\alpha_1)^2}$, $\mu_{10} = \frac{2}{(\alpha+\lambda_1)^2}$

Similarly, $M_0 = \frac{1}{\lambda+\lambda_1}$, $M_1 = \frac{2}{(\alpha+\lambda_1)^2}$, $M_3 = \frac{2}{(\alpha_1+\lambda_1)^2}$, $M_4 = \frac{2}{(\alpha+\lambda_2)^2}$, $M_7 = \frac{2}{(\lambda+\lambda_1+\alpha_2)^2}$, $M_9 = \frac{2}{(\lambda+\alpha_1)^2}$, $M_{10} = \frac{2}{(\alpha+\lambda_1)^2}$

and $W_1 = \frac{2}{(\alpha+\lambda_1)^2}$, $W_3 = \frac{2}{(\alpha_1+\lambda_1)^2}$, $W_4 = \frac{2}{(\alpha+\lambda_2)^2}$, $W_7 = \frac{2}{(\lambda+\lambda_1+\alpha_2)^2}$, $W_9 = \frac{2}{(\lambda+\alpha_1)^2}$, $W_{10} = \frac{2}{(\alpha+\lambda_1)^2}$

ii) For $\eta = 1$, the repair time distribution reduces to exponentials having pdf $f(t) = \alpha e^{-\alpha t}$, $r(t) = \alpha_1 e^{-\alpha_1 t}$, $r_1(t) = \alpha_2 e^{-\alpha_2 t}$

As a result μ_i changes to $\mu_0 = \frac{1}{\lambda+\lambda_1}$, $\mu_1 = \frac{1}{\alpha+\lambda_1}$, $\mu_3 = \frac{1}{\alpha_1+\lambda_1}$, $\mu_4 = \frac{1}{\alpha+\lambda_2}$, $\mu_7 = \frac{1}{\lambda+\lambda_1+\alpha_2}$, $\mu_9 = \frac{1}{\lambda+\alpha_1}$, $\mu_{10} = \frac{1}{\alpha+\lambda_1}$

Similarly, $M_0 = \frac{1}{\lambda+\lambda_1}$, $M_1 = \frac{1}{\alpha+\lambda_1}$, $M_3 = \frac{1}{\alpha_1+\lambda_1}$, $M_4 = \frac{1}{\alpha+\lambda_2}$, $M_7 = \frac{1}{\lambda+\lambda_1+\alpha_2}$, $M_9 = \frac{1}{\lambda+\alpha_1}$, $M_{10} = \frac{1}{\alpha+\lambda_1}$

and $W_1 = \frac{1}{\alpha+\lambda_1}$, $W_3 = \frac{1}{\alpha_1+\lambda_1}$, $W_4 = \frac{1}{\alpha+\lambda_2}$, $W_7 = \frac{1}{\lambda+\lambda_1+\alpha_2}$, $W_9 = \frac{1}{\lambda+\alpha_1}$, $W_{10} = \frac{1}{\alpha+\lambda_1}$

VIII. Graphical Presentation

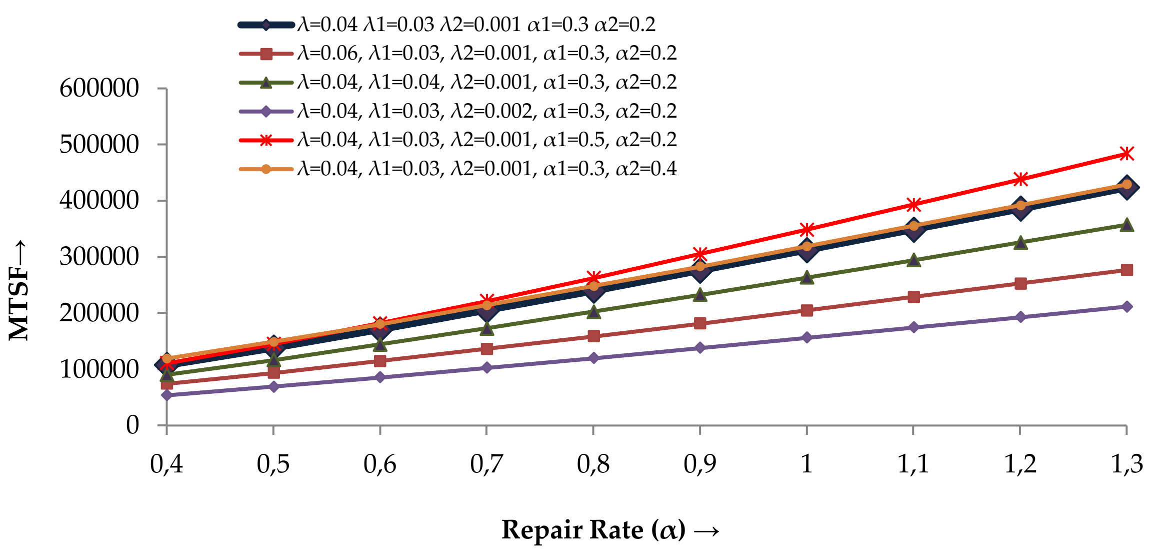

Figure 2: *MTSF vs Repair rate (η = 0.5)*

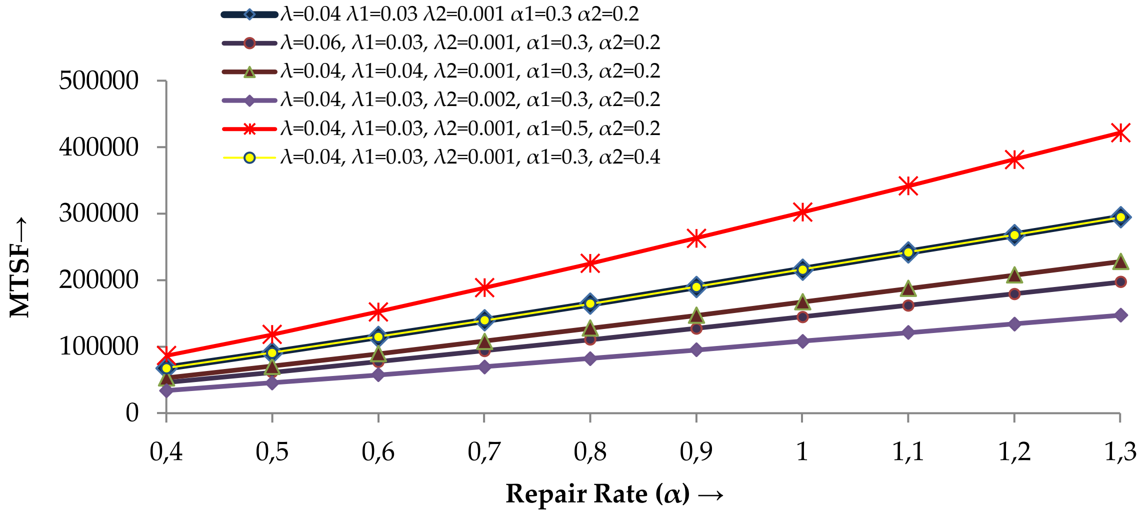

Figure 3: *MTSF vs Repair rate (η = 1)*

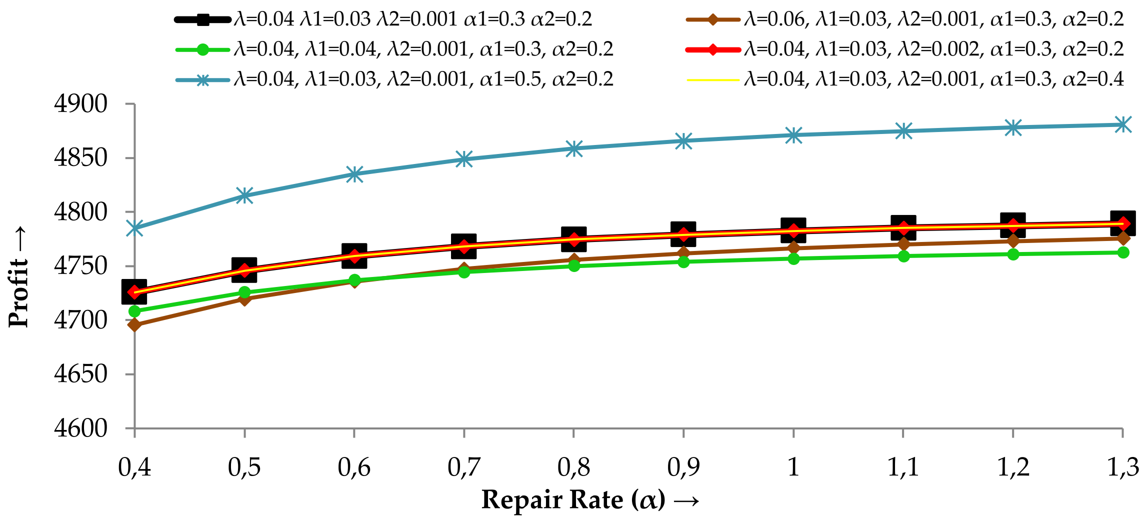

Figure 4: *Profit vs Repair rate (η = 0.5)*

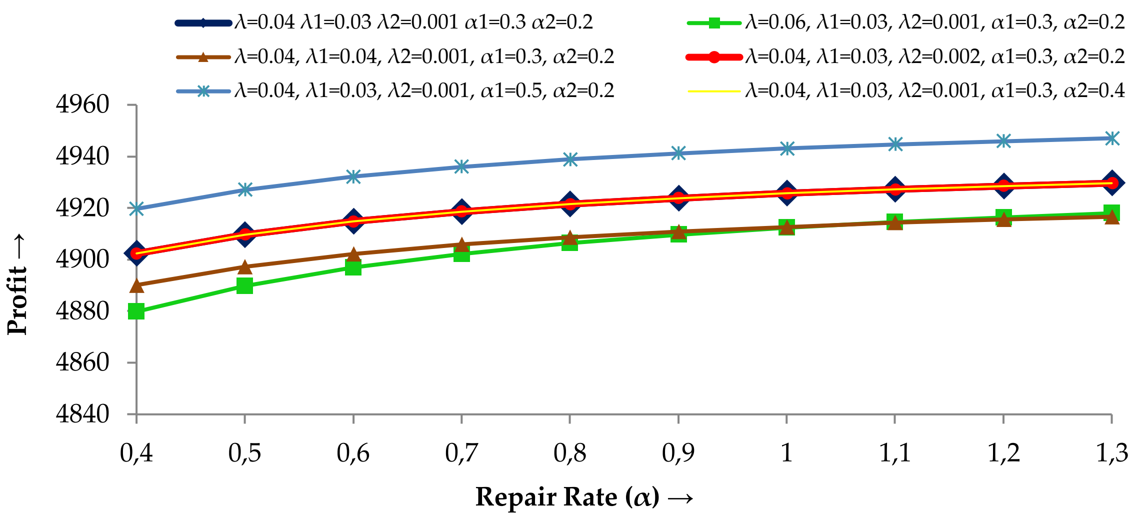

Figure 5: *Profit Vs Repair Rate (η = 1)*

IX. Conclusion

Figure 2 clearly indicate that for $\eta = 0.5$, MTSF is increasing with increasing repair rate of main unit (α) while figure 3 indicate comparatively less increment in MTSF for increasing α when $\eta = 1$. Therefore, we conclude that MTSF increases with increasing α. Figure 4 shows that for $\eta = 0.5$, system model's profit is increasing with increasing α whereas figure 5 shows relatively higher increase in profit for increasing α and α_1 (restoration rate of warm standby unit) when $\eta = 1$. Hence, profit of the system increases with increasing α and α_1 for constant η.

Present study concludes that the increasing restoration rate of the main unit increases MTSF whereas to make the system more profitable we should increase repair rate of both main as well as warm standby units.

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