Reliability Estimation of a Serial System Subject to General and Gumbel-Hougaard Family Copula Repair Policies

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Abstract

Abstract: The dependability analysis of a hybrid series-parallel system with five subsystems A, B, C, D, and E is the subject of this research. Subsystem A has two active parallel units, whereas subsystem B has two out of four active units. Both units have a failure and repair time that is exponential. There are two states in the system under consideration: partial failure and complete failure. To assess the system's dependability, the system's first-order partial differential equations are constructed from the system transition diagram, resolved using the supplementary variables technique, and the reliability models are Laplace transformed. Failure times are assumed to follow an exponential distribution, whereas repair times are expected to follow a general distribution and a Gumbel-Hougaard family copula distribution. Reliability measurements of testing system effectiveness are derived and investigated, including reliability, availability, MTTF, sensitivity MTTF, and cost function. Tables and graphs show some of the most relevant findings.

Keywords: Reliability, estimation, availability, MTTF, Series system, Gumbel-Hougaard family copula

1. Introduction

Series-parallel systems are made up of multiple subsystems that are connected in a series. Each subsystem is made up of units or components that are connected in a parallel fashion. When all of the subsystems of a series-parallel system are operational, the system operates. Partially or completely failing such systems is possible. When any of the parallel units or components of the subsystem failed, it resulted in a partial failure of the system. This failure will not stop the system from working; rather, depending on the configuration of the components, the system will continue to operate at full or decreased capacity (see Ram and Manglik [29] and Ram et al. [30]). When any of the subsystems fails, the system's operation comes to a halt, resulting in total failure.

The majority of industrial and manufacturing systems are set up in a series-parallel fashion. Because of their widespread use in industrial and manufacturing settings, determining the reliability, availability, and profitability of series-parallel systems as industrial and manufacturing systems has become a more pressing concern. A good example can be found in a computer network (see Yusuf et al. [42], Rawal et al. [31] and Rawal et al. [32]). Feeding, crushing, refining, steam generation, evaporation, crystallization, fertilizer plant, sugar plant crystallization unit, and piston manufacturing factory are all instances of these systems.

Some studies consider reliability block diagrams to identify the role of cascading failures on the reliability of series–parallel systems (see Xie et al [35]) or consider optimal component grouping in series–parallel and parallel–series systems composed of k subsystems to improve reliability and availability of series–parallel systems (see Xie et. al.[36]). The reliability of such a system can be improved by determining the best time for performing PM and also finding the number of spare parts and facilities in single-item replacement and parallel systems to minimize the expected average cost per unit time (see Fallahnezhad and Najafian [7]), or by performing maintenance to the system components (see Chauhan and Malik [5]), or by performing maintenance to the system components (see Fallahnezhad and Najafian [7]). (see Khatab et al.[18]).

The importance of ensuring the survival of industrial and manufacturing systems, as well as their accompanying economies, through dependability, availability, and profit optimization has become critical to their expansion. The percentage of time that the system is available to users is known as system availability. When the system's dependability and availability are improved (increased), the accompanying income is improved as well. Crystallization system of a sugar plant, uncaser system of a brewing plant, thermal plant, two-wheeler automobile, cattle feed, and ice cream making unit of a milk plant are examples of such industrial and manufacturing systems (see Aggarwal et al. [2], Garg et al. [11] and Kumar and Mudgil [21]).

Studies on reliability and availability modeling, as well as performance evaluation of industrial and manufacturing systems, have been conducted, as indicated above. To determine the reliability and availability models of such systems, the governing differential difference equations are obtained and solved with Runge–Kutta fourth-order and using genetic algorithm method to analyze the system's reliability (see Aggarwal et al. [3], Garg et al. [12], Kadiyan et al. [17]) and making decisions using the developed model (see Aggarwal et al. [3], Garg et al (see Gupta and Tewari [14] and Fadi and Sibai [6])

Maintaining a high level of system reliability, availability, manufacturing output, and revenue generating requires proper maintenance planning. As a result, it's critical that the equipment is always available. Several approaches for studying behavioral studies (see Arvind et al. [4]) and maintenance planning and problem identification (see Khanduja et al. [16] and Xu et al. [37]) have been abandoned. Reliability, availability, mean time to failure, mean time between failures, and mean time to repair are all indicators of such performance.

The degree of identification of the most critical subsystem that results in low reliability, availability, and profit between the subsystems is critical in analyzing the performance of the system (see Kumar et al. [23] and Kumar and Lata [24]) / subsystems through reliability, availability, and generated profit, as well as the degree of identification of the most critical subsystem that results in low reliability, availability, and profit between the subsystems (see Freiheit et al. [8]). The ideal profit level, at which the profit is highest, can be identified using this mathematical approach (see Kumar and Tewari [22]).

Reliability, availability, and revenue enhancement literature Consider the options by categorizing the configurations' dependability and choosing the optimum structure that maximizes system reliability (see Peng et al. [28]). Particle swarm optimization (see Garg and Sharma [13]) and the concept of inherent availability (see Kaur et al. [19]) are two further ways for increasing reliability.

Few studies consider the application of redundancy application problem as a means of a maximizing the system reliability, availability and profit (see Mohammed et al. [27] and Zhu et al. [41]), identification and elimination of the most critical component with low reliability (see Yusuf et al. [39] and Yusuf [40]), by considering general repair as a way regaining the system to its former position before complete failure (see Yusuf et al. [38]), by employing human operator to avoid catastrophic breakdown (see Gahlot et al.[9],Gulati et al. [10],Lado and Singh [25], Lado et al. [26] and Singh and Ayagi [33]) or through the study of optimization allocation problem for a repairable series-parallel system having failure dependencies among the units of the system so as to reduce the number of repair teams are available for each subsystem (see Hu et al. [15]).

Existing literatures above either ignores the importance of repair policies on reliability, availability, mean time to failure and profit on both industrial growth, employment, increase in volume of business, etc. Most literatures laid emphasis of availability and performance evaluation of the systems alone without paying much attention to the impact of copula and general repair policies on reliability, availability, mean time to failure and generated revenue.

More sophisticated models of repairable series parallel systems should be developed to assist in reducing risk of a complete breakdown, operating costs, prolonging the overall reliability, availability, mean time to failure as well as generated revenue (profit). For this reason, this paper considered a series-parallel system consisting of five subsystem A, B, C, D and E. The performance of the system is studied using the supplementary variable technique and Laplace transforms. The various measures of reliability such as availability, reliability, mean time to system failure (MTTF), sensitivity for MTTF and cost analysis have been computed for various values of failure and repair rates.

The paper is organized as follows: Section 2 captures the description of the system, assumption and notations used for the study. Section 3 deals with the formulation and solution of mathematical model. Section 4 focuses on the analytical part of the study in which some particular cases are taken for discussion. The paper is concluded in section 5.

2. Notations, Assumptions, and Description of the System

2.1. Notations

t: Time variable on a time scale.

s: Laplace transform variable for all expressions

$\beta_1 / \beta_2 / \beta_3 / \beta_4 / \beta_5$: Failure rate of subsystem 1/ subsystem 2/ subsystem 3/ subsystem 4 and / subsystem 5 respectively.

$\phi(y) / \phi(z)$: Repair rate of subsystem 2 / subsystem 3.

$\mu_0(x) / \mu_0(y) / \mu_0(z) / \mu_0(m) / \mu_0(n)$: Repair rate for complete failed states of subsystem 1 / subsystem 2/ subsystem 3/ subsystem 4 and / subsystem 5 respectively.

$p_i(t)$: The probability that the system is in S_i state at instants for i =0 to 11

$\overline{P}(s)$: Laplace transformation of state transition probability $p(t)$

$P_i(x, t)$: The probability that a system is in state S_i for i=1..., the system under repair and elapse repair time is (x, t) with repair variable x and time variable t

$P_i(y, t)$: The probability that a system is in state S_i for i=1..., the system under repair and elapse repair time is (y, t) with repair variable y and time variable t

$P_i(z, t)$: The probability that a system is in state S_i for i=1…, the system under repair and elapse repair time is (z, t) with repair variable z and time variable t

$P_i(m, t)$: The probability that a system is in state S_i for i=1…, the system under repair and elapse repair time is (m, t) with repair variable m and time variable t

$P_i(n, t)$: The probability that a system is in state S_i for i=1…, the system under repair and elapse repair time is (n, t) with repair variable n and time variable t

$E_p(t)$: Expected profit during the time interval [0, t)

K_1, K_2: Revenue and service cost per unit time, respectively.

$\mu_0(x)$: The expression of joint probability according to Gumbel-Hougaard family copula definition

$$c_\theta\left(u_1(x), u_2(x)\right) = \exp\left(x^\theta + \left\{\log\phi(x)^\theta\right\}^{\frac{1}{\theta}}\right), \tag{1}$$

$1 \le \theta \le \infty$.

Where

$$\mu_1 = \phi(x) \tag{2}$$

and

$$u_2 = e^x \tag{3}$$

2.2 Assumptions

i. Firstly, it is assumed that all subsystems are in perfect operational state.

ii. Failure of any unit leads to insufficient performance of the system.

iii. Secondly, subsystem 1, subsystem 4, subsystem 5, one unit from subsystem 2 and at least three units from subsystem3 are compulsory for the system to operate.

iv. Thirdly, the system will not operate if any of the subsystems completely fail.

v. If a unit of the system failed, it can be tackled when it is in operation or failed state.

vi. All failure rates are constant and assumed to follow exponential distribution.

vii. General distribution is employed to repair partially failed states while Gumbel-Hougaard family copula distribution takes care of complete failed state.

viii. The repaired unit of the system is assumed to operate like new and no harm shows in the repair process.

ix. However, as soon as the failed unit gets repaired, it is ready to take the load for successful perform of the system.

2.3 Descriptions of the System

This system consists of five different subsystems. Subsystem 1/ Subsystem 4/ Subsystem 5: One unit whose failure led to complete failure of the system. Subsystem 2: Consists of two homogeneous units working under 1-out-of-2 policy in parallel configuration. If one unit fails, the system is partially operative and failed unit is assigned for repair. The failure of the second unit will automatically result in complete failure of the system. Subsystem 3: Consisting of four homogeneous units that work under 3-out-of-4 policy in parallel configuration. System experience partial failure if one unit fails and failed unit is assigned for repair system is operative while complete failure occurred if additional unit fails. General repair method is assigned for partially failed state and completely failed state is carried out by copula repair method.

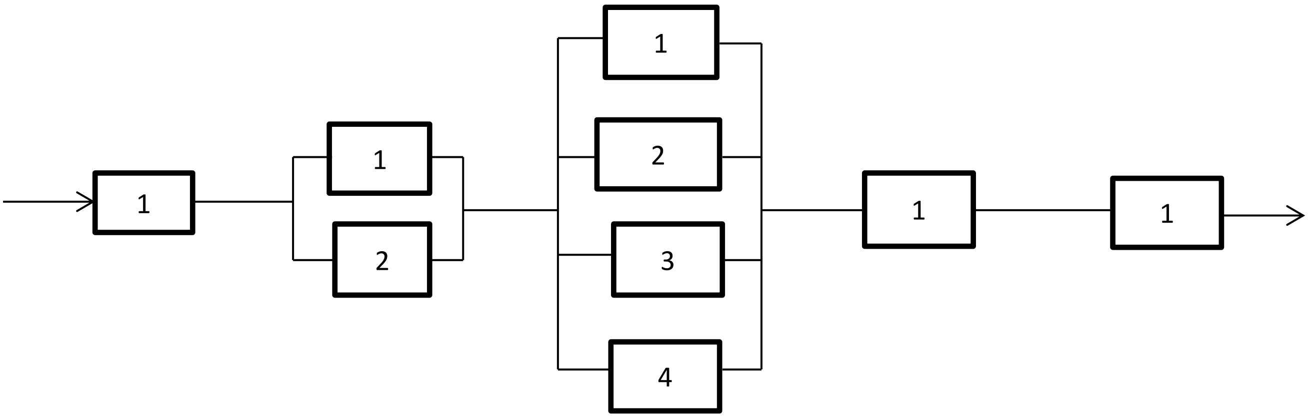

Figure 1: *System reliability block diagram*

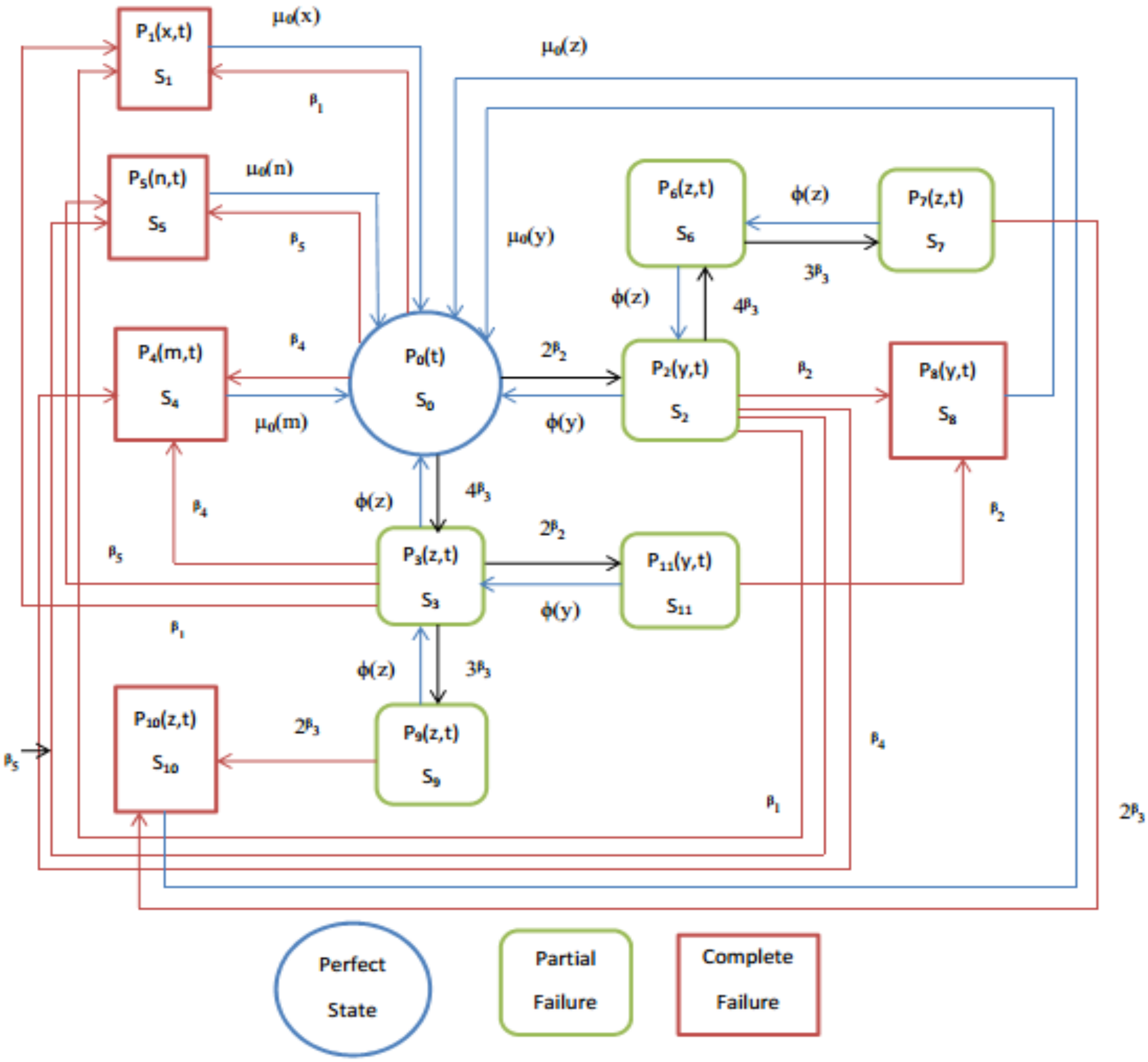

Figure 2: *Transition diagram of the System*

States Description

S_o: Good working state, all subsystems are adequate.

$S_1 / S_4 / S_5$: Completely failed states as such the system stopped working.

S_2 / S_{11}: One unit is operating and the second is on standby mode, system work in full capacity.

S_3 / S_6: Three units are operating while the remaining unit is on standby mode, system work in full capacity.

S_7 / S_9: One unit from subsystem 3 failed and is assigned for repair, system work partially.

S_8: Complete failed state due to the failure of second unit from subsystem 2.

S_{10}: Complete failed state due to the failure of second unit from subsystem 3.

3. Reliability Model Formulation

The resulting sets of partial differential equations are obtained through the transition diagram of the Mathematical model, by observing at probability of deliberations and connection of impacts.

$$\left(\frac{\partial}{\partial t}+2\beta_1+2\beta_2+4\beta_3+\beta_4+\beta_5\right)p_0(t)=\int_0^\infty \phi(y)\,p_2(y,t)dy+\int_0^\infty \phi(z)\,p_3(z,t)dz+\int_0^\infty \mu_0(x)\,p_1(x,t)dx+$$
$$\int_0^\infty \mu_0(y)\,p_8(y,t)dy+\int_0^\infty \mu_0(z)\,p_{10}(z,t)dz+\int_0^\infty \mu_0(m)\,p_4(m,t)dm+\int_0^\infty \mu_0(n)\,p_5(n,t)dn \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)p_1(x,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5+\phi(y)\right)p_2(y,t)=0 \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5+\phi(z)\right)p_3(z,t)=0 \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial m}+\mu_0(m)\right)p_4(m,t)=0 \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial n}+\mu_0(n)\right)p_5(n,t)=0 \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+3\beta_3+\phi(z)\right)p_6(z,t)=0 \tag{10}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\beta_3+\phi(z)\right)p_7(z,t)=0 \tag{11}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right)p_8(y,t)=0 \tag{12}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\beta_3+\phi(z)\right)p_9(z,t)=0 \tag{13}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(z)\right)p_{10}(z,t)=0 \tag{14}$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)p_{11}(y,t)=0 \tag{15}$$

3.1 Boundary and Initial Conditions

$$p_1(0,t)=\beta_1 p_0(t) \tag{14}$$

$$p_2(0,t)=2\beta_2 p_0(t) \tag{15}$$

$$p_3(0,t)=4\beta_3 p_0(0,t) \tag{16}$$

$$p_4(0,t)=\beta_4\left(p_0(t)+p_2(0,t)+p_3(0,t)\right) \tag{17}$$

$$p_5(0,t)=\beta_5\left(p_0(t)+p_2(0,t)+p_3(0,t)\right) \tag{18}$$

$$p_6(0,t)=4\beta_3 p_2(0,t) \tag{20}$$

$$p_7(0,t)=3\beta_3 p_6(0,t) \tag{21}$$

$$p_8(0,t)=\beta_2\left(p_2(0,t)+p_{11}(0,t)\right) \tag{22}$$

$$p_9(0,t)=3\beta_3 p_3(0,t) \tag{23}$$

$$p_{11}(0,t)=2\beta_3\left(p_7(0,t)+p_9(0,t)\right) \tag{24}$$

$$p_{11}(0,t)=2\beta_2 p_3(0,t) \tag{25}$$

All state transition probabilities are zero whenever t = 0 except $p_0(0)=1$.

3.2 Solution of Reliability Model

Captivating Laplace transformation of the equations (1) to (25) with the support of boundary conditions the following equations are obtained.

$$\left(s+\beta_1+2\beta_2+4\beta_3+\beta_4+\beta_5\right)\overline{p}_0(\mathrm{s})=1+\int_0^\infty \phi(y)\overline{p}_2(y,\mathrm{s})dy+\int_0^\infty \phi(z)\overline{p}_3(z,s)dz+\int_0^\infty \mu_0(x)\overline{p}_1(x,s)dx+$$

$$\int_0^\infty \mu_0(y)\overline{p}_8(y,s)dy+\int_0^\infty \mu_0(z)\overline{p}_{10}(z,s)dz+\int_0^\infty \mu_0(m)\overline{p}_4(m,s)dm+\int_0^\infty \mu_0(n)\overline{p}_5(n,s)dn \tag{26}$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{p}_1(x,s)=0 \tag{27}$$

$$\left(s+\frac{\partial}{\partial y}+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5+\phi(y)\right)\overline{p}_2(y,\mathrm{s})=0 \tag{28}$$

$$\left(s+\frac{\partial}{\partial z}+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5+\phi(z)\right)\overline{p}_3(z,\mathrm{s})=0 \tag{29}$$

$$\left(s+\frac{\partial}{\partial m}+\mu_0(m)\right)\overline{p}_4(m,s)=0 \tag{30}$$

$$\left(s+\frac{\partial}{\partial n}+\mu_0(n)\right)\overline{p}_5(n,s)=0 \tag{31}$$

$$\left(s+\frac{\partial}{\partial z}+3\beta_3+\phi(z)\right)\overline{p}_6(z,\mathrm{s})=0 \tag{32}$$

$$\left(s+\frac{\partial}{\partial z}+2\beta_3+\phi(z)\right)\overline{p}_7(z,\mathrm{s})=0 \tag{33}$$

$$\left(s+\frac{\partial}{\partial y}+\mu_0(y)\right)\overline{p}_8(y,s)=0 \tag{34}$$

$$\left(s+\frac{\partial}{\partial z}+2\beta_3+\phi(z)\right)\overline{p}_9(z,\mathrm{s})=0 \tag{35}$$

$$\left(s+\frac{\partial}{\partial z}+\mu_0(z)\right)\overline{p}_{10}(z,s)=0 \tag{36}$$

$$\left(s+\frac{\partial}{\partial y}+\beta_2+\phi(y)\right)\overline{p}_{11}(y,\mathrm{s})=0 \tag{37}$$

Boundary conditions

$$\overline{p}_1(0,s)=\beta_1\overline{p}_0(s) \tag{38}$$

$$\overline{p}_2(0,s)=2\beta_2\overline{p}_0(s) \tag{39}$$

$$\overline{p}_3(0,s)=4\beta_3\overline{p}_0(s) \tag{40}$$

$$\overline{p}_4(0,s)=\beta_4\left(\overline{p}_0(0)+\overline{p}_2(0,s)+\overline{p}_3(0,s)\right) \tag{41}$$

$$\overline{p}_5(0,s)=\beta_5\left(\overline{p}_0(0)+\overline{p}_2(0,s)+\overline{p}_3(0,s)\right) \tag{42}$$

$$\overline{p}_6(0,s)=4\beta_3\overline{p}_2(0,s) \tag{43}$$

$$\overline{p}_7(0,s)=3\beta_3\overline{p}_6(0,s) \tag{44}$$

$$\overline{p}_8(0,s)=\beta_2\left(\overline{p}_2(0,s)+\overline{p}_{11}(0,s)\right) \tag{45}$$

$$\overline{p}_9(0,s)=3\beta_3\overline{p}_3(0,s) \tag{46}$$

$$\overline{p}_{10}(0,s)=2\beta_3\left(\overline{p}_7(0,s)+\overline{p}_9(0,s)\right) \tag{47}$$

$$\bar{p}_{11}(0,s) = 2\beta_2 \bar{p}_3(0,s) \tag{49}$$

Outlining of equation (3.24) - (3.35) with the support of equation (3.36) to (3.46) the following result is obtained.

$$\bar{p}_0(s) = \frac{1}{D(s)} \tag{50}$$

$$\bar{p}_1(s) = \frac{\beta_1}{D(s)}\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{51}$$

$$\bar{p}_2(s) = \frac{2\beta_2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5)}{s+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5}\right\} \tag{52}$$

$$\bar{p}_3(s) = \frac{4\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5)}{s+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5}\right\} \tag{53}$$

$$\bar{p}_4(s) = \left(\frac{\beta_4+2\beta_2\beta_4+4\beta_3\beta_4}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{54}$$

$$\bar{p}_5(s) = \left(\frac{\beta_5+2\beta_2\beta_5+4\beta_4\beta_5}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{55}$$

$$\bar{p}_6(s) = \frac{8\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+3\beta_3)}{s+3\beta_3}\right\} \tag{56}$$

$$\bar{p}_7(s) = \frac{24\beta_2\beta_3^2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+2\beta_3)}{s+2\beta_3}\right\} \tag{57}$$

$$\bar{p}_8(s) = \left(\frac{2\beta_2^2+8\beta_2^2\beta_3}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{58}$$

$$\bar{p}_9(s) = \frac{12\beta_3^2}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+2\beta_3)}{s+2\beta_3}\right\} \tag{59}$$

$$\bar{p}_{10}(s) = \left(\frac{48\beta_2\beta_3^3+24\beta_3^3}{D(s)}\right)\left\{\frac{1-\bar{s}_{\mu_0}(s)}{s}\right\} \tag{60}$$

$$\bar{p}_{11}(s) = \frac{8\beta_2\beta_3}{D(s)}\left\{\frac{1-\bar{s}_{\phi}(s+\beta_2)}{s+\beta_2}\right\} \tag{61}$$

Where D(s) is well-defined as

$$D(s)=\left\{s+\beta_1+2\beta_2+4\beta_3+\beta_4+\beta_5-\left(\begin{array}{l}2\beta_2\bar{s}_\phi\left(s+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5\right)+\\4\beta_3\bar{s}_\phi\left(s+2\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5\right)+\\\left[\begin{array}{l}\beta_1+\left(2\beta_2^2+8\beta_2^2\beta_3\right)+\left(48\beta_2\beta_3^3+24\beta_3^3\right)+\\\left(\beta_4+2\beta_2\beta_4+4\beta_3\beta_4\right)+\left(\beta_5+2\beta_2\beta_5+4\beta_4\beta_5\right)\end{array}\right]\bar{s}_{\mu_0}(s)\end{array}\right)\right\} \quad (62)$$

(60)

If all Laplace transformations of the state transition probabilities that the system is operating is added together the result is obtained as:

$$\bar{p}_{up}(s)=\left[\bar{p}_0(s)+\bar{p}_2(s)+\bar{p}_3(s)+\bar{p}_6(s)+\bar{p}_7(s)+\bar{p}_9(s)+\bar{p}_{11}(s)\right] \quad (63)$$

Therefore,

$$\bar{p}_{up}(s)=\frac{1}{D(s)}\left\{\begin{array}{l}1+2\beta_2\left(\dfrac{1-\bar{s}_\phi\left(s+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5\right)}{s+\beta_1+\beta_2+4\beta_3+\beta_4+\beta_5}\right)+\\4\beta_3\left(\dfrac{1-\bar{s}_\phi\left(s+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5\right)}{s+\beta_1+2\beta_2+3\beta_3+\beta_4+\beta_5}\right)+\\8\beta_2\beta_3\left(\dfrac{1-\bar{s}_\phi\left(s+3\beta_3\right)}{s+3\beta_3}\right)+24\beta_2\beta_3^2\left(\dfrac{1-\bar{s}_\phi\left(s+2\beta_3\right)}{s+2\beta_3}\right)\\+12\beta_3^2\left(\dfrac{1-\bar{s}_\phi\left(s+2\beta_3\right)}{s+2\beta_3}\right)+8\beta_2\beta_3\left(\dfrac{1-\bar{s}_\phi\left(s+\beta_2\right)}{s+\beta_2}\right)\end{array}\right\} \quad (64)$$

On the other hand, the sum of all Laplace transformations of the state probabilities that the system fails can be summarized in the equation below;

$$\bar{p}_{down}(s)=1-\bar{p}_{up}(s)) \quad (65)$$

4. Analytical study of the model for particular cases

4.1 Availability analysis of the model for copula repair method

Supposing

$$S_{\mu_0}(s)=\bar{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s)=\frac{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}} \quad (66)$$

$$\bar{S}_\phi(s)=\frac{\phi}{s+\phi}, \quad (67)$$

and considering the same values of failure rates as $\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=0.02$, $\phi=\mu=x=y=z=m=n=1$ and repair rates as $\phi(y)=\phi(z)=1$ in equation (62), and carrying inverse Laplace transform, the expression obtained is availability function.

$$\bar{p}_{up}(s)=\left\{\begin{array}{l}-0.002531e^{-1.04000t}+0.025983e^{-2.79218t}\\-0.019633e^{-1.25934t}+0.999411e^{-0.00677t}\\-0.001770e^{-1.02000t}-0.001459e^{-0.06000t}\end{array}\right\} \quad (68)$$

Supposing different values of time variable t = 0, 1…10, units of time in equation (68), availability is computed (Table 1).

Table 1: *Availability computation using copula repair method*

Time	0	1	2	3	4	5	6	7	8	9	10
Availability	1.0000	0.9866	0.9837	0.9761	0.9724	0.9660	0.9595	0.9531	0.9467	0.9403	0.9339

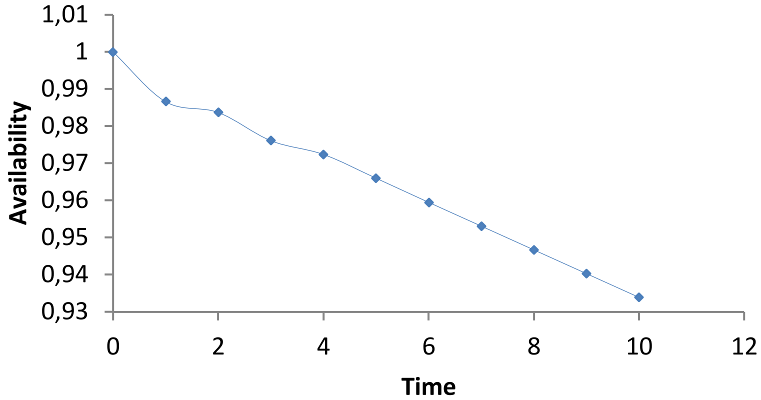

Figure 3: *Availability against time when copula repair method is used*

4.2 Availability analysis of the model for general repair method

Supposing, $\overline{S}_{\phi}(s)=\dfrac{\phi}{s+\phi}$, and considering the same values of failure rates as $\beta_1=\beta_2=\beta_3=\beta_4=\beta_5=0.02$, $\phi=\mu=x=y=z=m=n=1$ and repair rates as $\phi(y)=\phi(z)=1$ in equation (62), and carrying inverse Laplace transform, the expression obtained is availability function.

$$\overline{p}_{up}(s)=\left\{\begin{array}{l}-0.036975e^{-1.04000t}+0.001825e^{-1.02000t}\\+0.008761e^{-1.29593t}+0.068653e^{-1.03754t}\\+0.961175e^{-0.00651t}-0.003440e^{-0.06000t}\end{array}\right\} \tag{69}$$

Allowing different values of time variable t = 0, 1…10, units of time in equation (69), availability is computed (Table 2).

Table 2: *Availability computation using general repair method*

Time	0	1	2	3	4	5	6	7	8	9	10
Availability	1.0000	0.9680	0.9532	0.9441	0.9369	0.9305	0.9243	0.9183	0.9123	0.9064	0.9005

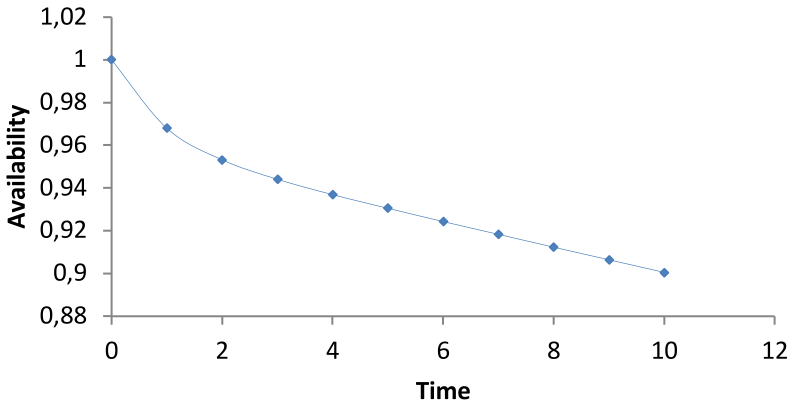

Figure 4: *Availability against time when general repair method is used*

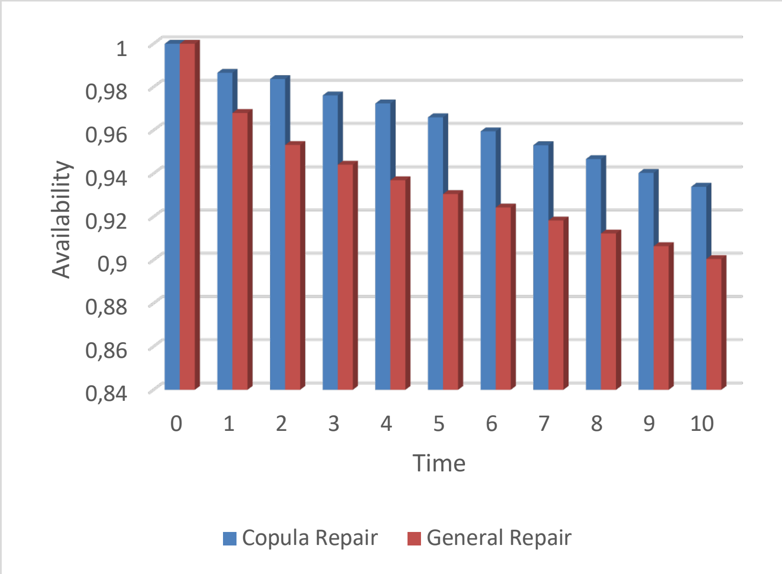

Figure 5: *Variation of Availability with to time under different repair policies*

4.3 Reliability Analysis of the model

Allowing all repair rates $\phi(y), \phi(z)\, and\, \mu_0$ in equation (62) to zero, considering the values of failure rates as $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0.02$ and captivating inverse Laplace transformation, the expression follows is reliability function.

$$R(t) = \begin{Bmatrix} -5.082323e^{-0.18000t} + 6e^{-0.16000t} + 0.026666e^{-0.06000t} \\ +0.035657e^{-0.04000t} + 0.020000e^{-0.02000t} \end{Bmatrix} \quad (70)$$

Taking different values of time variable t = 0, 1…10, units of time in equation (70), reliability is computed (Table 3).

Table 3: *Computation of reliability with respect to time*

Time	0	1	2	3	4	5	6	7	8	9	10
Reliability	1.0000	0.9467	0.8868	0.8237	0.7597	0.6967	0.6358	0.5779	0.5235	0.4729	0.4261

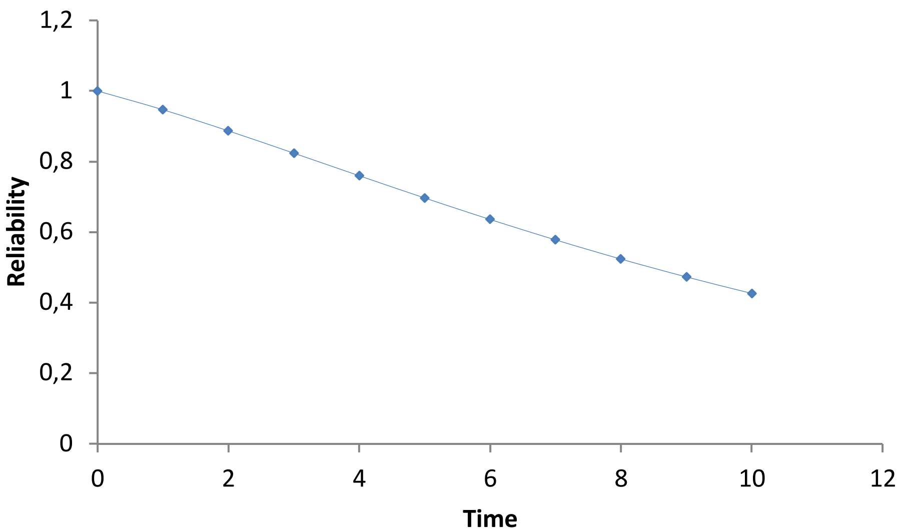

Figure 6: *Reliability function against time variable*

4.4 MTTF Analysis of the model

Letting all repairs to zero in equation (62) and the limit as s becoming close to zero, MTTF expression is obtained as:

$$MTTF = \lim_{s\to 0} \overline{p}_{up}(s) = \frac{1}{\beta_1 + 2\beta_2 + 4\beta_3 + \beta_4 + \beta_5} \left\{ \begin{array}{l} 1 + \dfrac{2\beta_2}{\beta_1 + \beta_2 + 4\beta_3 + \beta_4 + \beta_5} + \\ \dfrac{4\beta_3}{\beta_1 + 2\beta_2 + 3\beta_3 + \beta_4 + \beta_5} + \\ \dfrac{8\beta_2}{3} + 12\beta_2\beta_3 + 14\beta_3 \end{array} \right\} \tag{71}$$

Assuming $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0.02$ and varying $\beta_1, \beta_2, \beta_3, \beta_4, \beta_5$ one by one respectively as 0.01, 0.02...0.09 in equation (67), MTTF of the model is calculated with respect to failure rate (Table 4).

Table 4: *Computation of MTTF with respect to failure rate*

Failure rates	$MTTF\ \beta_1$	$MTTF\ \beta_2$	$MTTF\ \beta_3$	$MTTF\ \beta_4$	$MTTF\ \beta_5$
0.01	12.5772	12.5864	13.1197	12.5772	12.5772
0.02	11.6007	11.6007	11.6007	11.6007	11.6007
0.03	10.7579	10.8229	10.5096	10.7579	10.7579
0.04	10.0240	10.1850	9.6802	10.0240	10.0240
0.05	9.3795	9.6470	9.0273	9.3795	9.3795
0.06	8.8096	9.1835	8.4996	8.8096	8.8096
0.07	8.3024	8.7779	8.0643	8.3024	8.3024
0.08	7.8482	8.4184	7.6993	7.8482	7.8482
0.09	7.4394	8.0964	7.3887	7.4394	7.4394

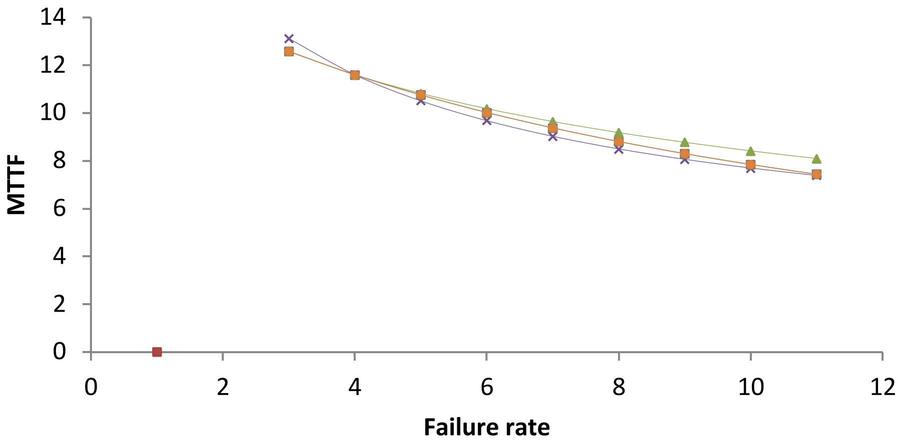

Figure 7: *MTTF against Failure rates*

4.5 Sensitivity Analysis of the model

Taking partial differential of the MTTF with respect to failure rates yield sensitivity of the model, there and then considering the failure rates as, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0.02$ in the partial differential function of the MTTF gives result as presented in Table 5.

Table 5: *Computation of sensitivity with respect to failure rate*

Failure rate	$\frac{\partial(MTTF)}{\beta_1}$	$\frac{\partial(MTTF)}{\beta_2}$	$\frac{\partial(MTTF)}{\beta_3}$	$\frac{\partial(MTTF)}{\beta_4}$	$\frac{\partial(MTTF)}{\beta_5}$
0.01	-105.3564	-112.4063	-183.4378	-105.3564	-105.3564
0.02	-90.4902	-86.7073	-126.5997	-90.4902	-90.4902
0.03	-78.4748	-69.9443	-94.1730	-78.4748	-78.4748
0.04	-68.6385	-58.2793	-73.0555	-68.6385	-68.6385
0.05	-60.4937	-49.7372	-58.3667	-60.4937	-60.4937
0.06	-53.6804	-43.2241	-47.7029	-53.6804	-53.6804
0.07	-47.9283	-38.0960	-39.7100	-47.9283	-47.9283
0.08	-43.0317	-33.9527	-33.5643	-43.0317	-43.0317
0.09	-38.8316	-30.5346	-28.7380	-38.8316	-38.8316

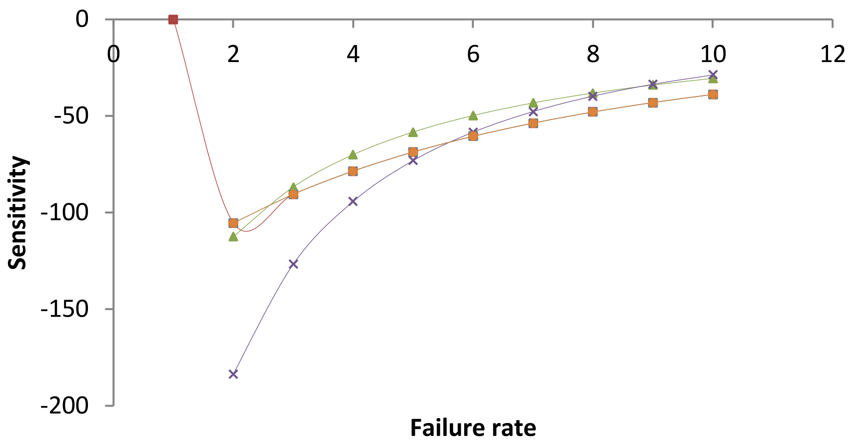

Figure 8: *Sensitivity against failure rates*

4.6 Profit Analysis of the model

4.6.1 Profit analysis using copula repair method:

Cost or profit investigation which is known as expected profit is done by integrating the $p_{up}(t)$ of the system, then multiplying the result by revenue per unit time (k_1) and eventually subtracting service cost per unit time (k_2) the relation that follows summaries the saying.

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{72}$$

Captivating fixed values of parameters of equation (62), equation (69) just arrive as

$$E_p(t) = k_1 \begin{Bmatrix} 0.002434e^{-1.04000t} - 0.009305e^{-2.79218t} + 0.015590e^{-1.25934t} - \\ 147.5751111e^{-0.00677t} + 0.001735e^{-1.02000t} + 0.001376e^{-1.06000t} + \\ 147.5632 \end{Bmatrix} - k_2(t) \tag{73}$$

Assuming K_1= 1and K_2= 0.1, 0.2..., 0.5, respectively and changing t = 0, 1, 2...10, units of time, the expected profit computations are done in the subsequent (Table 6).

Table 6: *Computation of profit within the limit (0, t], when copula repair is used*

Time	$E_p(t)$ K2=0.1	$E_p(t)$ K2=0.2	$E_p(t)$ K2=0.3	$E_p(t)$ K2=0.4	$E_p(t)$ K2=0.5
0	0	0	0	0	0
1	0.8899	0.7899	0.6899	0.5899	0.4899
2	1.7753	1.5753	1.3753	1.1753	0.9753
3	2.6566	2.3566	2.0566	1.7566	1.4566
4	3.5322	3.1322	2.7322	2.3322	1.9322
5	4.4015	3.9015	3.4015	2.9015	2.4015
6	5.2643	4.6643	4.0643	3.4643	2.8643
7	6.1207	5.4207	4.7207	4.0207	3.3207
8	6.9706	6.1706	5.3706	4.5706	3.7706
9	7.8141	6.9141	6.0141	5.1141	4.2141
10	8.6513	7.6513	6.6513	5.6513	4.6513

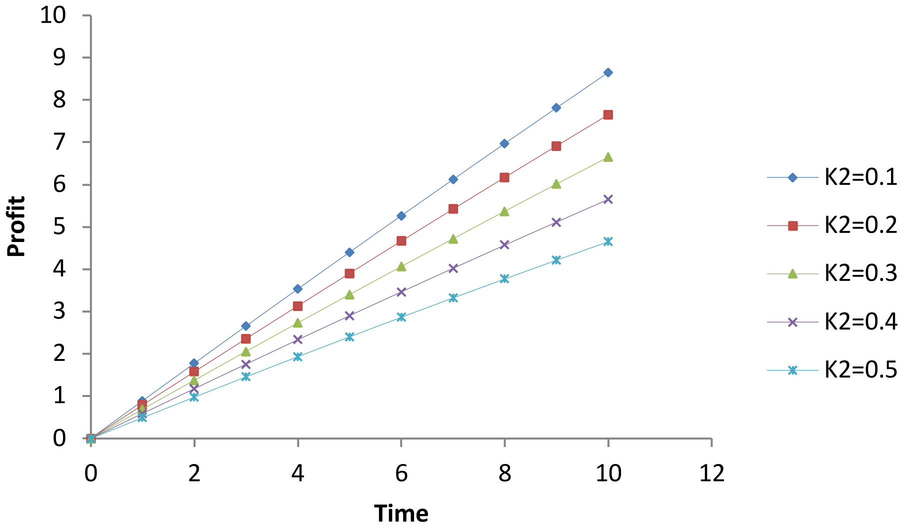

Figure 9: *Profit against time when copula repair method is used*

4.6.2 Profit inquiry using general repair method:

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \tag{74}$$

Captivating fixed values of parameters of equation (62), equation (71) just arrive as

$$E_p(t) = k_1 \begin{Bmatrix} 0.035553e^{-1.04000t} - 0.001789e^{-1.02000t} - 0.006760e^{-1.29593t} - \\ 0.066169e^{-1.03754t} - 147.527359e^{-0.00651t} + 0.003246e^{-1.06000t} + \\ 147.5632 \end{Bmatrix} - k_2(t) \tag{75}$$

Assuming K_1= 1and K_2= 0.1, 0.2..., 0.5, respectively and changing t = 0, 1, 2...10. Units of time, the expected profit computations are done in Table 7.

Table 7: *Computation of profit within the limit (0, t], when general repair is used*

Time	$E_p(t)$ K2=0.1	$E_p(t)$ K2=0.2	$E_p(t)$ K2=0.3	$E_p(t)$ K2=0.4	$E_p(t)$ K2=0.5
0	0	0	0	0	0
1	0.8816	0.7816	0.6816	0.5816	0.4816
2	1.7415	1.5415	1.3415	1.1415	0.9415
3	2.5899	2.2899	1.9899	1.6999	1.3899
4	3.4303	3.0303	2.6303	2.2303	1.8303
5	4.2641	3.7641	3.2641	2.7641	2.2641
6	5.0915	4.4915	3.8915	3.2915	2.6915
7	5.9129	5.2129	4.5129	3.8129	3.1129
8	6.7282	5.9282	5.1282	4.3282	3.5282
9	7.5376	6.6376	5.7376	4.8376	3.9376
10	8.3411	7.3411	6.3411	5.3411	4.3411

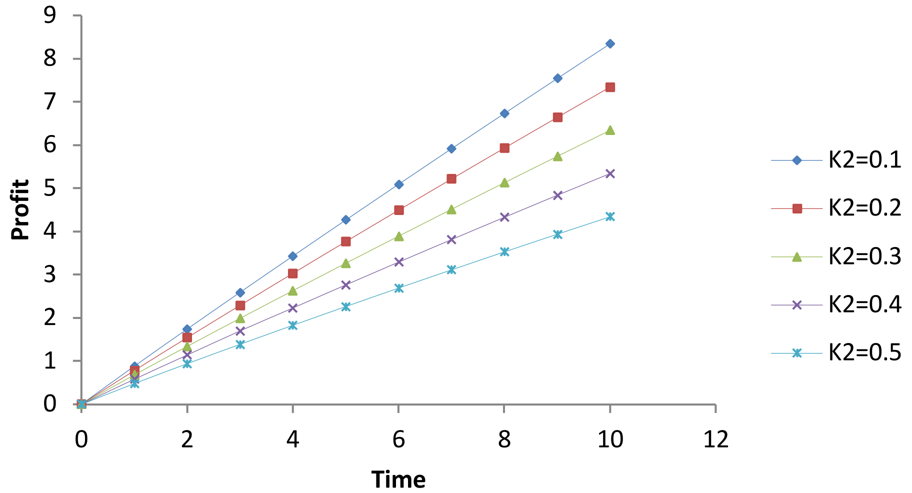

Figure 10: *Profit against time when general repair method is used*

5. Discussion and Concluding Remark

Table 1 and Figure 3 depicted the availability variation with respect of time. From Figure 3 it is clear that as failure rates increases than availability deceases when copula repair policy is employed. Similar observation can be Table 2 and Figure 4 presents the availability of system when the repair follows general distribution.

Table 3 and Figure 6 presents variation of reliability with respect to time. The reliability of system decreases with time when the failure rates increases. From Table 3 and Figure 6, it is enough to conclude that reliability has lower values in comparison with availability values in Tables 1 and 2.

Table 4 and Figure 7 depicts the mean-time-to-failure (MTTF) of the system with respect to variation in failure rates β_1, β_2, β_3, β_4 and β_5 respectively when other parameters are kept constant. The variation in MTTF corresponding to β_1, β_2, β_3, β_4 and β_5 are slightly closer. This analysis suggests that the failure rates β_1, β_2, β_3, β_4 and β_5 are more responsible for successful operation of the system. Table 6 and Figure 8 gives the information of the sensitivity analysis studied in this work. Table 5 and Figure 8 gives the information of the sensitivity analysis studied in section 6.5 of this work.

Table 6 and Figure 9 and Table 7 and Figure 10 depicted the profit variation with respect of time via two types of repair employing copula repair approach and general repair. From Table 6 and Figure 9 it is clear that as failure rates increases than profit deceases when copula repair policy is employed. Similarly, Table 7 and Figure 10 presents the profit of system when the repair follows general distribution. It is clear that as failure rates increases than profit deceases when general repair policy is employed.

Comparative analysis of copula and general repair in Table 1 and Table 2 are presented in Figure 5 which compare the two results of availability with respect to time under copula and general repair. It is evident from Table 1 and Table 2 and Figure 5 that availability of the system is better when the repair follows copula distribution.

Comparative analysis of copula and general repair in Table 6 and Table 7 which compare the results of profit with respect to time under copula and general repair when K_2= 0.5,04,0.3,0.2,0.1. It is evident from Table 6 and Table 7 when comparing the two procedures, repair policy of general and copula distributions, it appears that the predicted profit is larger when the repair policy is follows by copula distribution and lower when the repair follows general distribution. In both circumstances, the predicted profit is highest when the service cost is lowest and lowest when the service cost is highest. In conclusion, copula repair method yields better result, compared to general repair method; therefore, copula repair method is recommended for effective performance of the system.

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