DISCRETE-TIME WORKING VACATIONS QUEUE WITH IMPATIENT CLIENTS AND CONGESTION DEPENDENT SERVICE RATES

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Abstract

The current research article explores a finite capacity discrete-time multiple working vacations queue with impatient clients and congestion dependent service rates. An arriving client can choose either to enter the queue or balk with a certain probability. Due to impatience, he may renege after joining the queue as per geometric distribution. Rather than totally shutting down the service throughout the vacation period, the server functions with a different service rate. The times of services during regular service and during working vacation periods are considered to be geometrically distributed. The vacation periods are also presumed to be geometrically distributed. In addition, the service rates are considered to be dependent on the number of clients in the system during regular service period and during working vacation period. The model's steady-state probabilities are calculated using matrix approach and a recursive solution is also provided. The recursive solution is used for obtaining the corresponding continuous-time results. Various system performance metrics are presented. Finally, the numerical representation of the consequences of the model parameters on the performance metrics is furnished.

Keywords: Queue, discrete-time, working vacations, balking, reneging, congestion dependent service rates

1. Introduction

A discrete-time queueing model is one in which the time between two arrivals and the service times are discrete random variables. Such queueing models are more relevant than the continuous-time queuing models for designing and monitoring the efficacy of computer systems, communications network systems, industrial and production systems, traffic systems and healthcare systems. Furthermore, discrete-time analysis can be used to approximate a continuous-time system but this is not the case in reverse. Shizhong Zhou et al. [13] investigated a discrete-time queue with preferred customers and partial buffer sharing. Michiel De Muynck et al. [9] analysed a discrete-time queue with general service demands and phase-type service capacities. A discrete-time queue with three different strategies has been studied by Ivan Atencia et al. [7].

Individuals are always concerned while waiting for service, hence dissatisfaction is a most noteworthy characteristic in queueing systems. In reality, queues with impatient clients frequently occur where clients become irritated owing to lengthy waiting lines. Due to which, clients either balk (i.e., refuse to join the queue) or renege (i.e., abandon the queue without being served). In real-world congestion situations, performance examination of queueing systems with balking and reneging is advantageous since fresh managerial insights are gained. The importance of the aforementioned queueing systems can be seen in telecom companies, networking and telecommunication systems, production system, machinery operating systems, health emergency rooms, inventory management systems, etc. The amount of lost revenues can be estimated using the balking and reneging probabilities when deciding on the service rate of servers required in the service system to satisfy the needs that change over time. The concept of balking and reneging has been introduced by Haight [5] and Haight [6], respectively. Customers' balking and reneging behaviour in queueing theory were compared by Amit and Sonja [1]. Rakesh Kumar [10] conducted an economic analysis of a finite buffer multi server queuing model including balking, reneging and customer retention. The busy period study of a queuing model with balking and reneging was presented by Wang and Zhang [17]. G. S. Kuaban et al. [8] investigated a multi-server queuing model with balking and correlated reneging.

Working vacation (WV) models are the ones in which the server stays available and serves clients at a lower rate throughout the vacation period. If the queue is not empty at the vacation termination epoch, the server enters a regular service period with regular service rates; otherwise, the server returns to WV. Such type of working vacation policy is termed as multiple working vacations (MWV). This type of working vacation policy was introduced by Servi and Finn [12]. The discrete-time multiple working vacation queue with balking has been studied by Vijaya Laxmi et al. [14]. Vijaya Laxmi and Jyothsna [15] analyzed a finite buffer discrete-time batch service queue with multiple working vacations. A survey on working vacation queueing models has been presented by Chandrsekaran et al. [2]. Rama Devi et al. [11] analyzed an $M/M/1$ queue with working vacation, server failure and customer's impatience.

Queues with single server whose service rates are proportional to the queue size are accurate models for systems where the server's performance must be adjusted in accordance to the quantity of clients in the queue. In congestion dependent queueing systems, the server's service rate may be influenced by the availability of work in the system. The service rate for each client can be dynamically updated as a function of the number of clients in the system using congestion dependent services. Furthermore, queueing networks get benefited from queueing models with finite capacity and congestion dependent services, which often add to the complexity of these systems' solutions. For literature on congestion dependent queues, see [4, 16, 3] and the references therein.

The current paper considers a finite buffer discrete-time MWV queue with impatient clients and congestion dependent service rates. Service times are supposed to be geometrically distributed and congestion dependent throughout regular service and during working vacations. The inter-arrival times of clients and vacation times are both presumed to be geometrically distributed. The queue is analyzed under the late arrival system with delayed access (LAS-DA) and the steady-state system length distributions are obtained using matrix approach and a recursive solution is also provided. The results of the corresponding continuous-time queue are obatined from the recursive solution of the discrete-time queue. A few model performance metrics are developed using the steady-state probabilities. Finally, the parameter effect on the performance indices of the system is exhibited through some numerical results.

The remaining part of the paper is laid out in the following manner. In Section 2, the description of the model and steady-state probabilities are presented. The performance metrics of the model are displayed in Section 3. Section 4 depicts the impact of the model parameters on the performance metrics in the form of a table and graphs. The paper is concluded in Section 5.

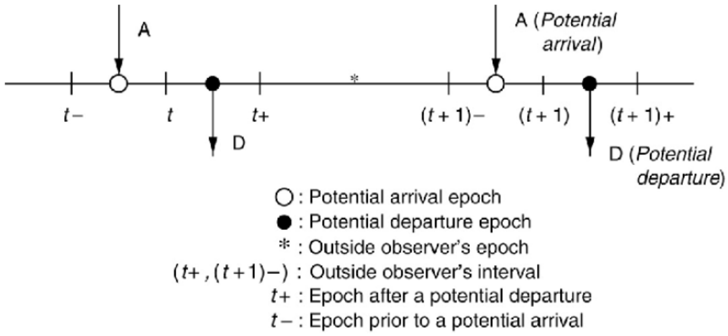

Figure 1: *Different time instants in late arrival system with delayed access (LAS-DA)*

2. Description of the model and steady-state probabilities

Under the late arrival system with delayed access (LAS-DA), we study a finite buffer discrete-time balking and reneging single server queue with multiple working vacations and congestion dependent service rates. Suppose that the time axis is divided into equal-length intervals with the duration of a slot being equal to one and are labeled as $0, 1, 2, \ldots, t, \ldots$. A possible arrival of a client occurs in $(t-, t)$ while a potential departure of the client occurs in $(t, t+)$. The different time instants in LAS-DA are displayed in Figure 1.

The system is presumed to have a finite capacity N. The inter-arrival times A of clients are independent and geometrically distributed with probability mass function (p.m.f.) $P(A = i) = \bar{\lambda}^{i-1}\lambda, i \geq 1, 0 < \lambda < 1$ where for $x\epsilon[0, 1]$, we denote $\bar{x} = 1 - x$. If a client arrives and discovers the system is busy, the client can choose to join the queue or balk. When the system size is n, let b_n indicate the probability that a client will join the queue for service or will balk with probability $\bar{b}_n$. Furthermore, we assume that $b_0 = 1, 0 < b_n < b_{n+1} \leq 1, 1 \leq n \leq N-1, b_N = 0$.

Each client will wait a specified amount of time T for service to commence after joining the queue. If it hasn't started by then, he'll become frustrated and exits the queue without being served. The impatient time T is geometrically distributed and independent with common p.m.f. $P(T = i) = \bar{\alpha}^{i-1}\alpha, i \geq 0, 0 < \alpha < 1$. As an impatient client's arrival and departure without service are unrelated, $r(n) = (n-1)\alpha, 1 \leq n \leq N$ can be used as the function of the average reneging rate of the client.

The clients are served on a first-come first-served (FCFS) discipline. The service times of clients S are geometrically distributed and independent with p.m.f. $P(S = i) = \bar{\mu}_n^{i-1}\mu_n, i \geq 1, 0 < \mu_n < 1$ when there are n clients in the system. The durations of service during a working vacation period S_v are geometrically distributed and independent with p.m.f. $P(S_v = i) = \bar{\eta}_n^{i-1}\eta_n, i \geq 1, 0 < \eta_n < 1$. When the server detects that the system is vacant, it follows multiple working vacation policy. Upon return of the server after a working vacation discovers that the system is vacant, another working vacation commences. Or else, the server initiates a regular service period. The vacation times V are geometrically distributed and independent with p.m.f. $P(V = i) = \bar{\phi}^{i-1}\phi, i \geq 0, 0 < \phi < 1$.

Let $P_n, 0 \leq n \leq N$ be the probability that the system has n clients when the server is on WV at steady-state and when the server is in regular service period $Q_n, 1 \leq n \leq N$ represents the probability that there are n clients in the system. The steady-state equations can be expressed as follows based on the one-step transition analysis.

$$
\begin{aligned}
P_0 &= \bar{\lambda} P_0 + h_1(\eta_1)P_1 + t_2(\eta_2)P_2 + h_1(\mu_1)Q_1 + t_2(\mu_2)Q_2, && (1)\\
P_n &= \bar{\phi}\left(g_n(\eta_n)P_n + f_{n-1}(\eta_{n-1})P_{n-1} + h_{n+1}(\eta_{n+1})P_{n+1} + t_{n+2}(\eta_{n+2})P_{n+2}\right), \\
&\quad 1 \le n \le N-2, && (2)\\
P_{N-1} &= \bar{\phi}\left(g_{N-1}(\eta_{N-1})P_{N-1} + f_{N-2}(\eta_{N-2})P_{N-2} + h_N(\eta_N)P_N\right), && (3)\\
P_N &= \bar{\phi}\left(g_N(\eta_N)P_N + f_{N-1}(\eta_{N-1})P_{N-1}\right), && (4)\\
Q_1 &= g_1(\mu_1)Q_1 + h_2(\mu_2)Q_2 + t_3(\mu_3)Q_3 + \phi\big(g_1(\eta_1)P_1 + \lambda P_0 + h_2(\eta_2)P_2 + t_3(\eta_3)P_3\big), && (5)\\
Q_n &= g_n(\mu_n)Q_n + h_{n+1}(\mu_{n+1})Q_{n+1} + t_{n+2}(\mu_{n+2})Q_{n+2} + f_{n-1}(\mu_{n-1})Q_{n-1} + \phi\big(g_n(\eta_n)P_n \\
&\quad + f_{n-1}(\eta_{n-1})P_{n-1} + h_{n+1}(\eta_{n+1})P_{n+1} + t_{n+2}(\eta_{n+2})P_{n+2}\big), \; 2 \le n \le N-2, && (6)
\end{aligned}
$$

$$
\begin{aligned}
Q_{N-1} &= g_{N-1}(\mu_{N-1})Q_{N-1} + h_N(\mu_N)Q_N + f_{N-2}(\mu_{N-2})Q_{N-2} + \phi\big(g_{N-1}(\eta_{N-1})P_{N-1} \\
&\quad + f_{N-2}(\eta_{N-2})P_{N-2} + h_N(\eta_N)P_N\big), && (7)\\
Q_N &= g_N(\mu_N)Q_N + f_{N-1}(\mu_{N-1})Q_{N-1} + \phi\big(g_N(\eta_N)P_N + f_{N-1}(\eta_{N-1})P_{N-1}\big), && (8)
\end{aligned}
$$

where

$$
\begin{aligned}
f_n(x) &= \begin{cases} \lambda, n = 0, \\ \lambda b_n \bar{x}\left(1-(n-1)\alpha)\right), \; 1 \le n \le N-1, \end{cases}\\
g_n(x) &= \begin{cases} (1-\lambda b_1)\bar{x} + \lambda b_1 x \;:\; n = 1, \\ (1-\lambda b_n)\bar{x}(1-(n-1)\alpha) + \lambda b_n \left(x(1-(n-1)\alpha) + \bar{x}(n-1)\alpha\right), \; 2 \le n \le N, \end{cases}\\
h_n(x) &= \begin{cases} (1-\lambda b_1)x, \; n = 1, \\ (1-\lambda b_n)\left(x(1-(n-1)\alpha) + \bar{x}(n-1)\alpha\right) + \lambda b_n x(n-1)\alpha, \; 2 \le n \le N, \end{cases}\\
t_n(x) &= (1-\lambda b_n)x(n-1)\alpha, \; 2 \le n \le N.
\end{aligned}
$$

Matrix Solution

To determine the steady-state probabilities, we propose a matrix solution. The infinitesimal generator of the Markov process can be represented as below using the lexicographical sequence for the states.

$$
\mathbf{Q} = \begin{pmatrix}
\mathbf{A}_0 & \mathbf{D}_0 & & & & & & & \\
\mathbf{B}_1 & \mathbf{A}_1 & \mathbf{D}_1 & & & & & & \\
\mathbf{C}_2 & \mathbf{B}_2 & \mathbf{A}_2 & \mathbf{D}_2 & & & & & \\
& \mathbf{C}_3 & \mathbf{B}_3 & \mathbf{A}_3 & \mathbf{D}_3 & & & & \\
& & \ddots & \ddots & \ddots & & & & \\
& & & & \mathbf{C}_{N-2} & \mathbf{B}_{N-2} & \mathbf{A}_{N-2} & \mathbf{D}_{N-2} & \\
& & & & & \mathbf{C}_{N-1} & \mathbf{B}_{N-1} & \mathbf{A}_{N-1} & \mathbf{D}_{N-1} \\
& & & & & & \mathbf{C}_N & \mathbf{B}_N & \mathbf{A}_N
\end{pmatrix},
$$

where

$$
\begin{aligned}
\mathbf{A}_0 &= \begin{pmatrix} -\lambda \end{pmatrix}; \quad \mathbf{D}_0 = \begin{pmatrix} \bar{\phi} f_0(\eta_0) & \lambda\phi \end{pmatrix};\\
\mathbf{B}_1 &= \begin{pmatrix} h_1(\eta_1) \\ h_1(\mu_1) \end{pmatrix}; \quad \mathbf{C}_2 = \begin{pmatrix} t_1(\eta_2) \\ t_1(\mu_2) \end{pmatrix};\\
\mathbf{A}_n &= \begin{pmatrix} \bar{\phi} g_n(\eta_n) - 1 & \phi g_n(\eta_n) \\ 0 & g_n(\mu_n) \end{pmatrix}, 1 \le n \le N;\\
\mathbf{B}_n &= \begin{pmatrix} \bar{\phi} h_n(\eta_n) & \phi h_n(\eta_n) \\ 0 & h_n(\mu_n) \end{pmatrix}, \; 2 \le n \le N;\\
\mathbf{C}_n &= \begin{pmatrix} \bar{\phi} t_n(\eta_n) & \phi t_n(\eta_n) \\ 0 & t_n(\mu_n) \end{pmatrix}, \; 3 \le n \le N.
\end{aligned}
$$

Let $\mathbf{\Pi} = (\mathbf{\Pi}_0, \mathbf{\Pi}_1, \ldots, \mathbf{\Pi}_{N-1}, \mathbf{\Pi}_N)$ be the vector of steady-state probabilities, where $\mathbf{\Pi}_0 = (P_0)$, $\mathbf{\Pi}_n = (P_n, Q_n)$, for $1 \leq n \leq N$. The equations at steady-state $\mathbf{\Pi Q} = \mathbf{0}$ can be expressed as

$$
\begin{aligned}
\mathbf{\Pi}_0\mathbf{A}_0 + \mathbf{\Pi}_1\mathbf{B}_1 + \mathbf{\Pi}_2\mathbf{C}_2 &= \mathbf{0}, && (9)\\
\mathbf{\Pi}_0\mathbf{D}_0 + \mathbf{\Pi}_1\mathbf{A}_1 + \mathbf{\Pi}_2\mathbf{B}_2 + \mathbf{\Pi}_3\mathbf{C}_3 &= \mathbf{0}, && (10)\\
\mathbf{\Pi}_{n-1}\mathbf{D}_{n-1} + \mathbf{\Pi}_n\mathbf{A}_n + \mathbf{\Pi}_{n+1}\mathbf{B}_{n+1} + \mathbf{\Pi}_{n+2}\mathbf{C}_{n+2} &= \mathbf{0},\ 2 \leq n \leq N-2, && (11)\\
\mathbf{\Pi}_{N-2}\mathbf{D}_{N-2} + \mathbf{\Pi}_{N-1}\mathbf{A}_{N-1} + \mathbf{\Pi}_N\mathbf{B}_N &= \mathbf{0}, && (12)\\
\mathbf{\Pi}_{N-1}\mathbf{D}_{N-1} + \mathbf{\Pi}_N\mathbf{A}_N &= \mathbf{0}. && (13)
\end{aligned}
$$

After recursive substitutions, equations (11) to (13) yields

$$
\mathbf{\Pi}_n = \mathbf{\Pi}_N\mathbf{M}_n\mathbf{D}_n^{-1}, 1 \leq n \leq N-1, \qquad (14)
$$

where

$$
\begin{aligned}
\mathbf{M}_{N-1} &= -\mathbf{A}_N,\\
\mathbf{M}_{N-2} &= -\mathbf{M}_{N-1}\mathbf{D}_{N-1}^{-1} - \mathbf{B}_N,\\
\mathbf{M}_{N-3} &= -\mathbf{M}_{N-2}\mathbf{D}_{N-2}^{-1}\mathbf{A}_{N-2} - \mathbf{M}_{N-1}\mathbf{D}_{N-1}^{-1}\mathbf{B}_{N-1} - \mathbf{C}_N,\\
\mathbf{M}_n &= -\mathbf{M}_{n+1}\mathbf{D}_{n+1}^{-1}\mathbf{A}_{n+1} - \mathbf{M}_{n+2}\mathbf{D}_{n+2}^{-1}\mathbf{B}_{n+2} - \mathbf{M}_{n+3}\mathbf{D}_{n+3}^{-1}\mathbf{C}_{n+3}, 1 \leq n \leq N-4.
\end{aligned}
$$

Assuming P_N to be known, equation (10) and(14) yields Q_N in P_N. From equations (14), $P_n (0 \leq n \leq N-1)$ and $Q_n, (1 \leq n \leq N-1)$ can be evaluated in terms of P_N. Finally, P_N is computed from the normalization condition $P_0 + \sum_{n=1}^{N} \mathbf{\Pi}_n\mathbf{e} = 1$, where $\mathbf{e}$ is a column vector with each component equal to one. The steady-state probabilities are calculated using a computer code.

Recursive Solution

In order to approximate the corresponding continuous-time results we have obtained the expressions of the steady-state probabilities using recursive method though the matrix method is easy to program and implement. Solving the equations (2) to (8) recursively and utilizing the normalization condition $\sum_{n=0}^{N} P_n + \sum_{n=1}^{N} Q_n = 1$, the explicit expressions of the steady-state probabilities are obtained as

$$
P_n = \psi_n \left(\sum_{n=0}^{N} \psi_n + \sum_{n=1}^{N} (\omega_n + k\varphi_n) \right)^{-1}, \ 0 \leq n \leq N,
$$

$$
Q_n = (\omega_n + k\varphi_n) \left(\sum_{n=0}^{N} \psi_n + \sum_{n=1}^{N} (\omega_n + k\varphi_n) \right)^{-1}, \ 1 \leq n \leq N,
$$

where

$$
\begin{aligned}
\psi_N &= 1,\\
\psi_{N-1} &= (1-\bar{\phi}g_N(\eta_N))/\bar{\phi}f_{N-1}(\eta_{N-1}),\\
\psi_{N-2} &= \left((1-\bar{\phi}g_{N-1}(\eta_{N-1}))\,\psi_{N-1}-\bar{\phi}h_N(\eta_N)\right)/\bar{\phi}f_{N-2}(\eta_{N-2}),\\
\psi_n &= \left((1-\bar{\phi}g_{n+1}(\eta_{n+1}))\,\psi_{n+1}-\bar{\phi}h_{n+2}(\eta_{n+2})\psi_{n+2}-\bar{\phi}t_{n+3}(\eta_{n+3})\psi_{n+3}\right)/\bar{\phi}f_n(\eta_n),\\
&\quad n=N-3,\ldots,0,\\
\varphi_N &= 1,\ \omega_N=0,\\
\varphi_{N-1} &= (1-g_N(\mu_N))\,/f_{N-1}(\mu_{N-1}),\\
\omega_{N-1} &= -\phi\left(g_N(\eta_N)+f_{N-1}(\eta_{N-1})\psi_{N-1}\right)/f_{N-1}(\mu_{N-1}),\\
\varphi_{N-2} &= \left((1-g_{N-1}(\mu_{N-1}))\varphi_{N-1}-h_N(\mu_N)\right)/f_{N-2}(\mu_{N-2}),\\
\varphi_n &= (1-g_{n+1}(\mu_{n+1}))\varphi_{n+1}-h_{n+2}(\mu_{n+2})\varphi_{n+2}-t_{n+3}(\mu_{n+3})\varphi_{n+3})\,/f_n(\mu_n),\\
&\quad n=N-3,\ldots,1,\\
\omega_{N-2} &= \big(\left((1-g_{N-1}(\mu_{N-1}))\omega_{N-1}-h_N(\mu_N)\omega_N\right)-\phi\big(g_{N-1}(\eta_{N-1})\psi_{N-1}\\
&\quad +f_{N-2}(\eta_{N-2})\psi_{N-2}+h_N(\eta_N)\psi_N\big)\big)/f_{N-2}(\mu_{N-2}),\\
\omega_n &= \big(\left((1-g_{n+1}(\mu_{n+1}))\omega_{n+1}-h_{n+2}(\mu_{n+2})\omega_{n+2}-t_{n+3}(\mu_{n+3})\omega_{n+3}\right)-\phi\big(g_{n+1}(\eta_{n+1})\psi_{n+1}\\
&\quad +f_n(\eta_n)\psi_n+h_{n+2}(\eta_{n+2})\psi_{n+2}+t_{n+3}(\eta_{n+3})\psi_{n+3}\big)\big)/f_n(\mu_n), n=N-3,\ldots,1,\\
k &= \left(\phi\big(g_1(\eta_1)\psi_1+\lambda\psi_0+h_2(\eta_2)\psi_2+t_3(\eta_3)\psi_3\big)-\big((1-g_1(\mu_1))\omega_1-h_2(\mu_2)\omega_2\right.\\
&\quad \left.-t_3(\mu_3)\omega_3\big)\right)/\big((1-g_1(\mu_1))\varphi_1-h_2(\mu_2)\varphi_2-t_3(\mu_3)\varphi_3\big).
\end{aligned}
$$

Remark: In continuous-time context, let β, ν_n, ϱ_n and ς represent the arrival rate, service rates during regular busy period, during WV period and vacation rate, respectively. Further, let the time axis be divided into equal length slots of length $\Delta > 0$, so that $\lambda = \beta\Delta$, $\mu_n = \nu_n\Delta$, $\eta_n = \varrho_n\Delta$ and $\phi = \varsigma\Delta$ where Δ is small enough. Now, the results of the corresponding continuous-time $M/M(n)/1/N/MWV$ queue with balking and reneging are obtained by substituting λ, μ_n, η_n and ϕ in the recursive solution.

3. Performance metrics

Let L_s denote the mean of the number of clients in the system and is expressed as

$$L_s = \sum_{n=1}^{N} n(P_n + Q_n).$$

During regular service and during WV, the busy probability of the server are denoted as p_b and p_v, respectively. The probabilities p_b and p_v are given by

$$p_b = \sum_{n=1}^{N} Q_n,\ p_v = \sum_{n=0}^{N} P_n.$$

The average balking rate (br), average reneging rate (rr) and the average rate of client loss due to impatience (lr) are given as

$$
\begin{aligned}
br &= \sum_{n=1}^{N} \lambda(1-b_n)(P_n+Q_n),\\
rr &= \sum_{n=1}^{N} (n-1)\alpha(P_n+Q_n),\\
lr &= br + rr.
\end{aligned}
$$

Table 1: *Performance metrics for different* η_m

η_m	0.054167	0.216667	0.4874999	0.866667	1.35417
L_s	2.075441	2.074633	2.073842	2.073065	2.072303
lr	0.545119	0.544945	0.544773	0.544603	0.544436
p_b	0.965786	0.965623	0.965460	0.965297	0.965134
p_v	0.034214	0.034376	0.034539	0.034702	0.034866

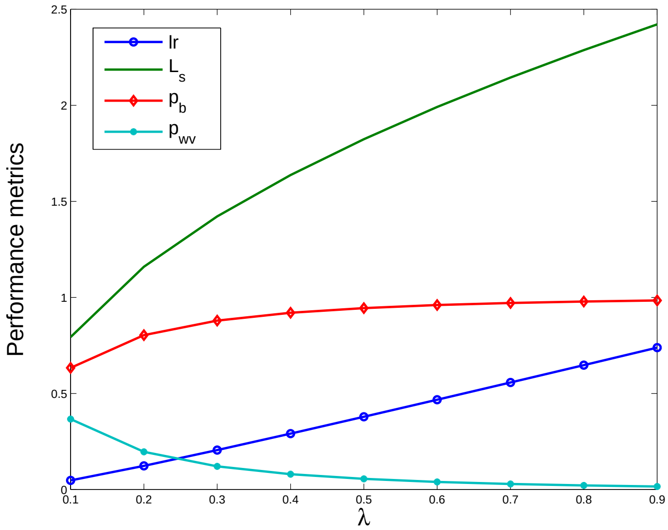

Figure 2: λ *versus performance metrics*

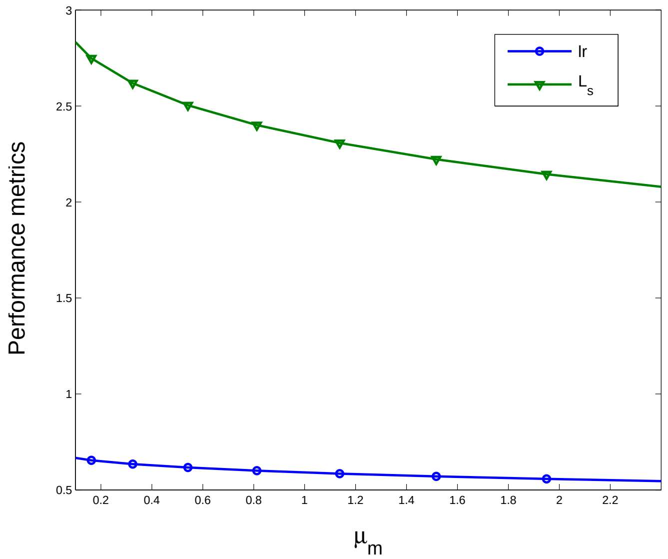

Figure 3: *Impact of* μ_m *on* lr *and* L_s

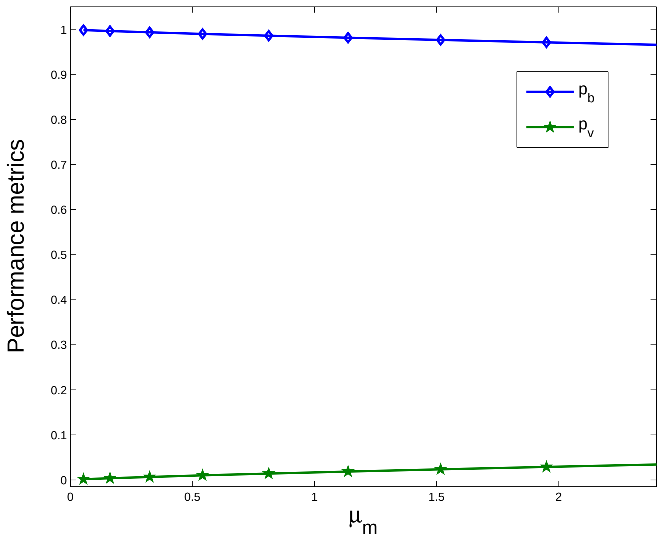

Figure 4: *μ_m versus p_b and p_v*

4. Numerical results

The influence of the system parameters on the model's various performance metrics is presented in this section. The capacity of the system is assumed as $N = 12$. The balking function is taken as $b_n = 1/(n+1)$, $1 \leq n \leq N-1$ with the assumption that $b_0 = 1$ and $b_N = 0$. The model parameters are arbitrarily chosen to be $\lambda = 0.7, \phi = 0.6, \alpha = 0.09$. The congestion-dependent service rates of the system are taken to be $\mu_n = 0.8n/N, \eta_n = 0.6n/N$ with means $\mu_m = 0.433333$ and $\eta_m = 0.325$, respectively.

The values of the performance metrics for various mean service rates during WV are presented in Table 1. The average number of clients in the system (L_s), average rate of losing a client (lr) and the probability of the server in regular service (p_b) - all show a diminishing trend as the mean service rate during WV grows. With an increase in η_m, the probability of the server being in WV (p_v) increases.

The arrival rate's impact on the different performance metrics of the model is displayed in Figure 2. From the figure it is evident that the performance metrics lr, L_s and p_b increase with the increase of λ while the performance metric p_v decreases with the increase of λ.

The influence of mean service rate during regular busy period μ_m on the performance measures L_s and lr is depicted in Figure 3. It is clearly apparent from the graph that both L_s and lr decrease with the increase of μ_m as intuitively expected.

Figure 4 presents the changes in p_b and p_v with the the increase of μ_m. With the increase of μ_m, the probability of the service being busy with regular service (p_b) falls but the probability of the server being busy in WV (p_v) increases.

5. Conclusions

The study of a finite buffer discrete-time congestion dependent queue with balking, reneging and multiple working vacations is presented in this paper. The stationary probabilities of the model are obtained using matrix method as well as recursive method. Different performance characteristics of the model such as average number of clients in the system, busy probability of the server during regular service, busy probability of the server during working vacations, average balking rate, average reneging rate and average rate of loosing a client are presented. A variety of numerical findings in the form of tables and graphs are used to demonstrate computational experiences. Future research can be focussed on the extension our findings to a $GI/Geo(n)/1/N$ queue with WV and impatient clients.

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