QUEUING SYSTEM WITHOUT QUEUE AND DETERMINISTIC SERVICE TIME

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Abstract

There is a model of fault-counting data collected in the testing process of software development. In this model it is performed simulation based on the infinite server queueing model using the generated sample data of the fault detection time to visualize the efficiency of fault correction activities. In this model the thinning method using intensity functions of the delayed S-shaped and inflection S-shaped software reliability growth models [2], [3] to generate sample data of the fault detection time from the fault-counting data.

But this model does not allow to analyse such systems without dependence of input and service intensities. In this paper, we consider a queuing system model with an infinite number of servers and a deterministic service time. The input flow to the system is non-stationary Poisson. It is investigated analytically how the parameter of the Poisson distribution characterizing the number of customers in the system depends on the service time in the presence of a peak load determined by the variable intensity of the input flow. In numerical simulations it is shown how graphs of the Poisson distribution parameter depends on deterministic service time.

Keywords: Poisson flow, deterministic service time, queuing system with infinite servers.

Introduction

In [1] fault-counting data are collected in the testing process of software development. The authors perform simulation based on the infinite server queueing model using the generated sample data of the fault detection time to visualize the efficiency of fault correction activities. They apply the thinning method using intensity functions of the delayed S-shaped and inflection S-shaped software reliability growth models [2], [3] to generate sample data of the fault detection time from the fault-counting data. As a result the simulation based on the infinite server queueing model using the generated sample data of the fault detection time to visualize the efficiency of fault correction activities is performed in conditions of non-stationary intensity of the input flow. This approach makes it possible to raise the question of the behaviour of the queuing system in peak load mode.

However, the inclusion in the queuing model of the dependence between the intensities of the input flow and the service may be too restrictive. Indeed, if we assume that the customers service times are deterministic, then such a restriction no longer works. At the same time, a similar queuing system based on the infinite server queuing model is also found in other applications, for example, in the system of admission of visitors to services (for example, to a sports facility)

[4]. Moreover, the deterministic distribution of service time in combination with the admission of visitors at arbitrary times turns out to be a very convenient tool for attracting them to service. For this model, the question arises in what ways it is possible to reduce the number of customers in the system in peak load mode.In this paper, it is shown that the variation of the deterministic service time is a factor that can significantly affect the behaviour of the system in peak load mode.

I. Nonstationary Poisson model of a continuously functioning service system

The calculation of non-stationary queuing models is usually much more complicated than the calculation of stationary models. However, in many systems of everyday public services, it is usually necessary to deal with non-stationary systems. Therefore, it is necessary to build a non-stationary queuing model in such a way that its calculation would be quite simple and convenient.

In this paper, this can be achieved by assuming the determinism of the service time and the Poisson nature of the input non-stationary flow of customers. Consider the Poisson model of a queuing system in which customers form the following Poisson flow model. Each customer is in the system for the time α, then leaves the system. The moments of arrival of applications into the system form a Poisson flow with an intensity of $\lambda(t)$, $-\infty < t < \infty$. The peculiarity of this model is its non-stationarity and the possibility of including in it a group receipt of customers with different parameters of the Poisson distribution of their numbers.

Thus, this model adapts to the conditions of functioning of real service systems: continuously operating swimming pools, outdoor skating rinks, gyms, aerobics and fitness halls, ski bases. In the language of queuing theory, such a system can be interpreted as a system with a Poisson flow of customers having varying intensity, an infinite number of servers and a deterministic service time.

Here the function $\lambda(t)$ is assumed to be continuous at $0 \le t \le T - \alpha$,

$$\lambda(t) = 0,\ t < 0 \text{ or } T - \alpha \le t.$$

As an example of such a flow of moments when users come to the system, we can assume that these are visitors of a continuously working pool coming to free swimming. Then for a fixed time t, $0 \le t \le T$, the number of users who came to the pool has a Poisson distribution with the parameter

$$\Lambda(t) = \int_{t-\alpha}^{t} \lambda(\tau)d\tau,\ 0 \le t \le T. \tag{1}$$

This non-stationary queuing model can have numerous generalizations: multiphase systems and acyclic service networks, systems with multiple flows having different deterministic service times, etc. It can also be applied to the calculation of conveyor systems for processing parts.

II. Basic properties of the non-stationary model with deterministic service times

Let the function $\lambda(t)$ has a single extremum (maximum) at the point $t_* > a$. Consequently $\lambda(t)$ is non-decreasing in interval $0 \le t < t_*$ and non-increasing in interval $t_* < t \le T - a$. It follows from the formula (1) that the function $\Lambda(t)$ has maximum at point t^*, if the following equality takes place

$$\Lambda'(t) = \lambda(t^*) - \lambda(t^* - a) = 0. \tag{2}$$

Property 1. From the properties of the function $\lambda(t)$ follows the inequalities

$$t_* < t^* < t_* + a. \tag{3}$$

Thus, the maximum of the function $\Lambda(t)$ is shifted to the right relative to the maximum of the function $\lambda(t)$.
Property 2. From the formula (1) we have

$$\Lambda(t^*) \leq a\lambda(t_*). \tag{4}$$

Therefore, reducing the parameter a allows smoothing the peak of the function $\Lambda(t)$.
Property 3. If the following condition is true

$$2a = T, \int_0^{T-a} \lambda(\tau)d\tau \geq \lambda(t_*), \tag{5}$$

then the peak of the function $\Lambda(t)$ may be higher than the peak of the function $\lambda(t)$:

$$\Lambda(t^*) \geq \lambda(t_*). \tag{6}$$

Another condition of the formula (6) is

$$\lambda(t) \geq \lambda_*, \ 0 < t \leq T - a, \ a\lambda_* \geq \lambda(t_*). \tag{7}$$

III. Numerical experiments

All experiments will be made for the function

$$\lambda(t) = \frac{\exp\left(-(t-b)^2/\ (2c)\right)}{\sqrt{2\pi c}}, \ 0 < t < T - a, \ \lambda(t) = 0, \ t \leq 0 \text{ or } T - a \leq t$$

with different meanings of parameter a.
Figure 1 shows that an increase in parameter a causes the maximum of the function $\Lambda(t)$ to grow and shift to the right relative to the maximum of the function $\lambda(t)$.

IV. Conclusion

The results of an analytical study of the $\Lambda(t)$ function showed that parameter a, which characterizes the deterministic time of servicing customers in this system without a queue, is a convenient tool for smoothing the peak load characterized by the intensity of the input flow. The properties of the queuing model with the infinite number of servers and deterministic service time established in the work and computational experiments allow us to discover how service time affects the parameter of the Poisson distribution of the number of customers in the system. A decrease in this parameter leads to a smoothing of the peak in the intensity of the input flow, and an increase leads to an increase in this peak. Therefore, this study allows us to determine how to choose the service time in order to avoid peak loads in the system.

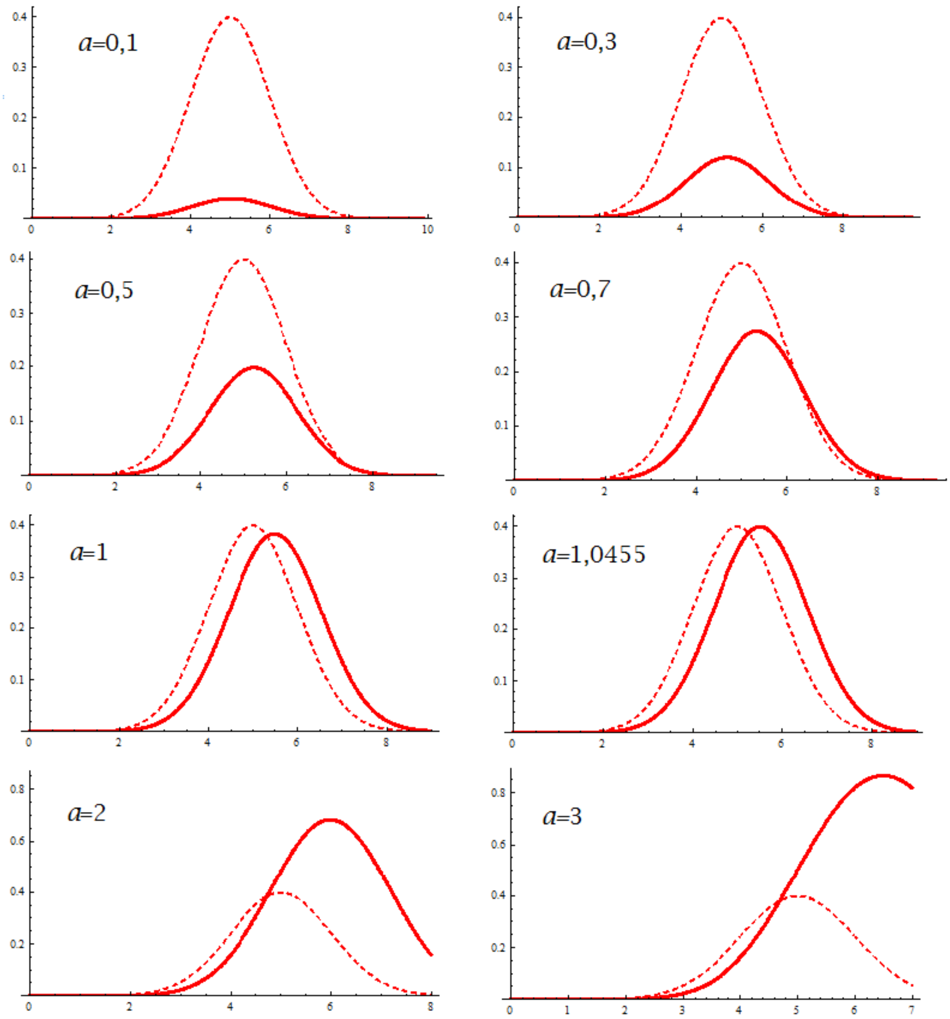

Fig. 1. Graphs of functions $\lambda(t)$ (dotted line), $\Lambda(t)$ (solid line) for $T = 10$, $b = 5$, $c = 1$.

V. Conclusion

The properties of the queuing model with the infinite number of servers and deterministic service time established in the work and computational experiments allow us to discover how service time affects the parameter of the Poisson distribution of the number of customers in the system. A decrease in this parameter leads to a smoothing of the peak in the intensity of the input flow, and an increase leads to an increase in this peak. Therefore, this study allows us to determine how to choose the service time in order to avoid peak loads in the system.

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