Sensitivity and Economic Analysis of an Insured System with Extended Conditional Warranty

Kajal Sachdeva

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Department of Mathematics, Maharshi Dayanand University, Rohtak, Haryana, India
kajal.rs.maths@mdurohtak.ac.in

Gulshan Taneja

•

Department of Mathematics, Maharshi Dayanand University, Rohtak, Haryana, India
drgtaneja@gmail.com

Amit Manocha*

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Department of Applied Sciences, TITS Bhiwani, Haryana, India
amitmanocha80@yahoo.com
*Corresponding Author

Abstract

Warranty and insurance are equally essential for a technological system to cover repair/replacement costs of all types of losses, i.e., natural wear/tear or unexpected external force/accidents. This paper examines the sensitivity and profitability of a stochastic model whose defects may cover under conditional warranty/insurance. The system user may extend the warranty period by paying an additional price. As a result, the system functions in normal warranty, extended warranty, and during non-warranty periods. If a system fault occurred is covered under warranty conditions, the manufacturer is responsible for all repair/replacement costs during normal/extended warranty which otherwise are paid by the insurance provider if covered under an insurance claim, or else, the user is responsible for the entire cost when coverage of fault neither falls in warranty conditions nor under the insurance policy. Using Markov and the regenerative process, various measures of system effectiveness associated with the profit of the user and the manufacturer are examined. Relative sensitivity analysis of the profit function and availability has been performed for all periods.

Keywords: Extended Conditional Warranty; Sensitivity Analysis; Profit; Insurance Cover

1. Introduction

Competitors add and offer new features to advertise their products in today's continuously expanding technological landscape. Offering a warranty on a system can be very beneficial to a company's growth. It relieves buyers' concerns, demonstrates the system's reliability, and is promotional. A warranty is a formal promise issued to the user for the free repair or system replacement if it fails. Researchers have focused their attention on warranty systems, policies, and warranty expense management in the past few decades [1–3, 6]. Generally, warranties cover the cost of failures that are defined in the contract at the time of purchased. Taneja [12] described the reliability analysis of a system with predetermined warranty conditions. Further, this work has extended to warranty period and non-warranty period [9–11]. Many systems have enormous

maintenance costs and are operated for long periods. Manufacturers are offering the option of extending the warranty period with an additional charge to avoid the cost of repair/replacement for an extensive period. Jack and Murthy [4] proposed the idea of employing a game-theoretic technique to determine the length and duration of an extended warranty based on the consumer's risk attitude. Padmanabhan and Rao [7] estimated the basic warranty time to be three years, a smart option for the increasing demand for long-term service contracts. Rinsaka and Sandoh [8] discussed an extended warranty in which the manufacturer replaces the system after the first failure and only performs minor repairs on subsequent failures. They also examined the optimal pricing for such an extended warranty.
However, financial protection is provided by insurance and warranty both against unpredictable damage or loss. There is a thin strip between them. While insurance protects against unintentional damage or loss, warranty protects against defective parts. Purchasing both at the same time is likely advantageous since it provides peace of mind in knowing that including insurance covers accidental damages, the warranty will cover faulty parts, effectively including the majority of faults/accidents that emerge in a technical system. Lutz and Padmanabhan [5] investigated the impact of impartial and independent insurance providers on manufacturer price strategy.
The cost analysis of an insured system with an extended conditional warranty is yet to be investigated. This paper proposes a model for a system with a conditional normal/extended warranty and long-term insurance which is structured as follows. Section 2 discusses the system's assumptions and description of the model. Section 3 describes the notations used in the analysis. Section 4 covers the system's stochastic modelling. The profitability measures and profit functions for the user, manufacturer, and insurance provider are drafted in Sections 5, 6, 7. Section 8 describes the sensitivity and relative sensitivity functions of availabilities and profit functions. Section 9 illustrates the above measurements using fixed parameter values that follow an exponential distribution. Section 10 concludes the study with interpretations.

2. Assumptions and System Characterizations

Following are the characterizations and suppositions used in the analysis of considered system:

1. The system consists of a single insured unit dividing its whole lifetime into three periods, i.e. normal warranty, extended warranty, and non-warranty periods.
2. The manufacturer or insurance provider inspects the failed system to assure
   (a) Whether system's flaws are covered by a warranty, an insurance claim, or neither.
   (b) Whether the system can be repaired or needs to be replaced.
3. In the case of normal and extended warranty periods, if an inspection indicates that
   (a) the defects are within the warranty domain, then the manufacturer bears the repair/replacement costs
   (b) the fault is covered by insurance, the repair/replacement costs are covered by the insurance company.
   (c) the defects are not covered by warranty or insurance, the user is responsible for all costs.
4. During the non-warranty period, the insurance provider or the user is solely responsible for repair/replacement costs as the case may be.
5. Transition time distributions have been taken as arbitrarily and the random variables that are involved are independent.

The system is depicted in Figure 1. The number of replacements, the availability, and the expected busy period are all calculated. The profit functions are assessed. The sensitivity analysis is also

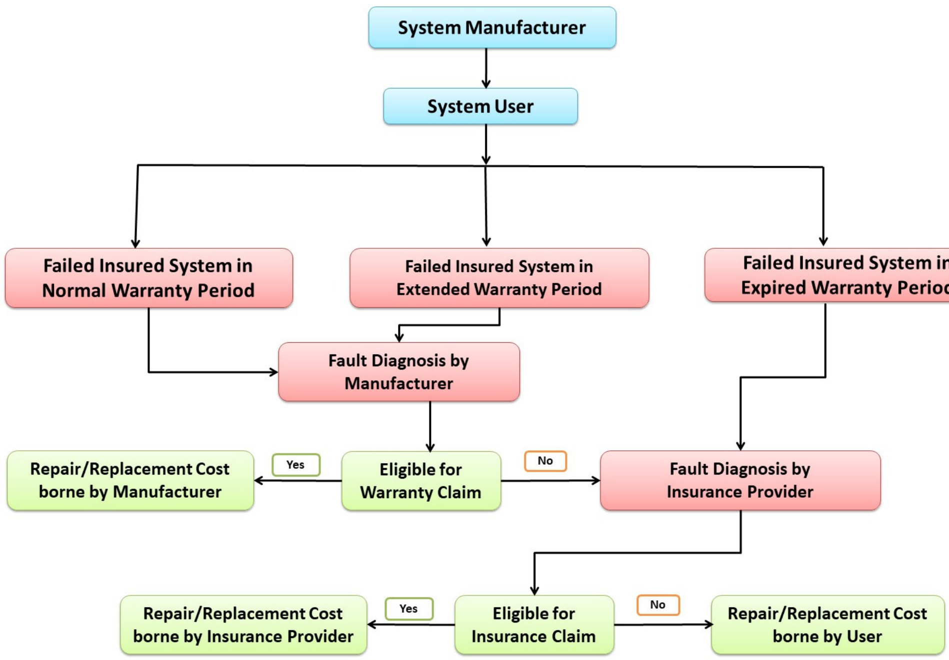

Figure 1: *System description*

performed for availability in three different time zones, as well as the manufacturer's and user's profit functions. Numerical estimates are based on exponential distributions. Various results are drawn about profitability and sensitivity.

3. Nomenclature

The following is the nomenclature for different probabilities/transition densities:

E_0	system state at time t=0
$p_w/\overline{p_w}$	probability that a fault is approved/ not approved under warranty conditions.
$r/\overline{r}$	probability that the fault is repairable or incurable, and that the system should be replaced.
$p_n/p_{et}/p_{ex}$	probability of a system failure within the normal/extended/non-warranty period.
$p_s/\overline{p_s}$	probability that the fault is covered or not covered under the provisions of the insurance policy.
$f_w(t)$	p.d.f. of failure time.
$i^m(t)/i^s(t)$	p.d.f. of the repairman's inspection time as contracted by the manufacturer/insurance provider.
$g^n(t)/g^{et}(t)/g^{ex}(t)$	p.d.f. of the repair time during normal/ extended/ expired warranty period
$h^n(t)/h^{et}(t)/h^{ex}(t)$	p.d.f. of the replacement time during normal/ extended/ expired warranty period
$Ak_i(t)$	probability that the system is operational at time t it is given that $E_0 = i$ during warranty period 'k=N/T/X'.

$IK_i(t)$ probability that the repairman of manufacturer or insurance company is busy in inspection at time t it is given that $E_0 = i$ during warranty period 'k'.

$Bk_i^m(t)(Bk_i^u(t))$ probability that the repairman of manufacturer is busy for repair/replacement when charges are borne by manufacturer or insurance provider (user) itself at time t it is given that $E_0 = i$ during warranty period 'k'.

$Rk_i^m(t)(Rk_i^u(t))$ expected number of replacement upto time t, when expenses are borne by manufacturer or insurance provider (user), given that $E_0 = i$ during warranty period 'k'.

The states of the system are specified by the following notations:

O_k operational unit in warranty period 'k'.

Fk_I failure unit under inspection by manufacturer in warranty period 'k'.

Fk_{IS} failure unit under inspection by insurance provider in warranty period 'k'.

$FM_{RN}(FM_{RPN})/FM_{RT}(FM_{RPT})$ failure unit under repair(replacement) in normal/extended warranty period, for which charges are to be paid by manufacturer.

FS_{Rk}/FS_{RPk} failure unit under repair/replacement in warranty period 'k', for which charges are to be paid by insurance provider.

FU_{Rk}/FU_{RPk} failed system under repair/replacement in warranty period 'k', for which expenses are to be borne by user itself.

where k stands for normal(N), extended(T), expired(X).

4. Stochastic Model

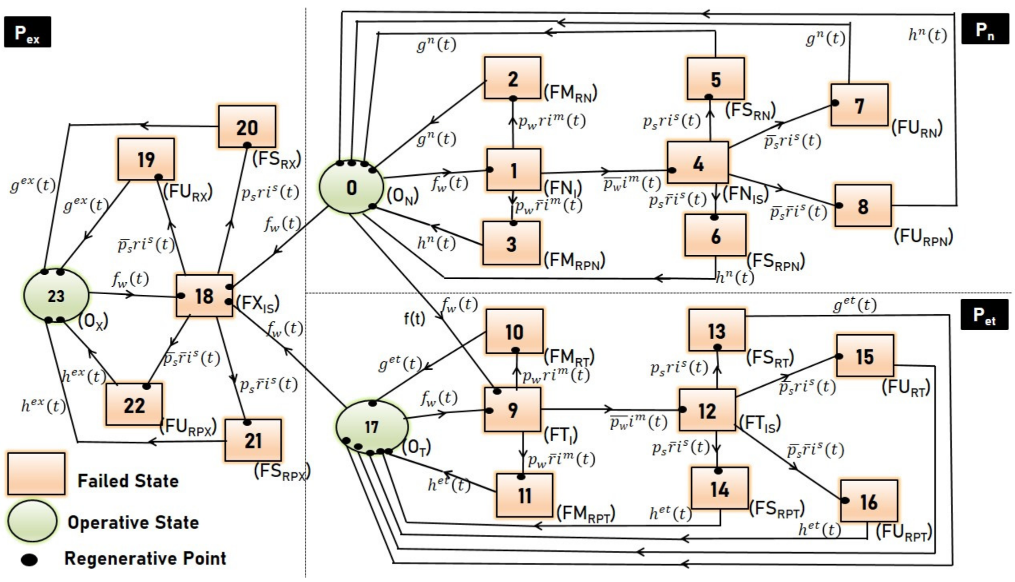

Figure 2: *State transition diagram*

Figure 2 illustrates the transition between several stages of the system. The state space is made up of the regenerative states, S={0, 1, 2, ..., 23}, where O={0, 17, 23} is operative state spaces

and F={1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22} is failed state space respectively. From figure 2, it may be observed that when system enters into a new state there is no continuation of inspection, repair and replacement from the previous state and hence at each time point, the process restarts probabilistically and thus the corresponding state where the system enters becomes the regenerative state. Further, it may also be observed that future state is independent of past and it depends only on present, thereby satisfying the Markov property. Therefore, the state transitions satisfy the Markov process and form the regenerative points. Thus, regenerative point technique is used to find various characteristics of the system.
The transition densities $q_{ij}(t)$ are:

$q_{01}(t) = f_w(t)$, $q_{12}(t) = p_w r i^m(t)$, $q_{13}(t) = p_w \bar{r} i^m(t)$ $q_{14}(t) = \overline{p_w} i^m(t)$,
$q_{45}(t) = p_s r i^s(t)$, $q_{46}(t) = p_s \bar{r} i^s(t)$, $q_{47}(t) = \overline{p_s} r i^s(t)$, $q_{48}(t) = \overline{p_s} \bar{r} i^s(t)$,
$q_{20}(t) = g^n(t)$ $q_{30}(t) = h^n(t)$, $q_{50}(t) = g^n(t)$, $q_{60}(t) = h^n(t)$
$q_{70}(t) = g^n(t)$, $q_{80}(t) = h^n(t)$ $q_{09}(t) = f_w(t)$, $q_{9,10}(t) = p_w r i^m(t)$,
$q_{9,11}(t) = p_w \bar{r} i^m(t)$ $q_{9,12}(t) = \overline{p_w} i^m(t)$, $q_{12,13}(t) = p_s r i^s(t)$, $q_{12,14}(t) = p_s \bar{r} i^s(t)$,
$q_{12,15}(t) = \overline{p_s} r i^s(t)$, $q_{12,16}(t) = \overline{p_s} \bar{r} i^s(t)$, $q_{10,17}(t) = g^{et}(t)$ $q_{11,17}(t) = h^{et}(t)$,
$q_{13,17}(t) = g^{et}(t)$, $q_{14,17}(t) = h^{et}(t)$ $q_{15,17}(t) = g^{et}(t)$, $q_{16,17}(t) = h^{et}(t)$
$q_{0,18}(t) = f_w(t)$, $q_{17,18}(t) = f_w(t)$, $q_{17,9}(t) = f_w(t)$ $q_{18,19}(t) = \overline{p_s} r i^s(t)$,
$q_{18,20}(t) = p_s r i^s(t)$, $q_{18,21}(t) = p_s \bar{r} i^s(t)$, $q_{18,22}(t) = \overline{p_s} \bar{r} i^s(t)$, $q_{19,23}(t) = g^{ex}(t)$
$q_{20,23}(t) = g^{ex}(t)$, $q_{21,23}(t) = h^{ex}(t)$ $q_{22,23}(t) = h^{ex}(t)$, $q_{23,18}(t) = f_w(t)$,

Mean sojourn time (μ_i) in state i, i $\in$ S is given as
μ_i=$\int_0^\infty t$ (corresponding p.d.f. of time for moving from i^{th} state) dt
Defining m_{ij}= $\int_0^\infty t q_{ij}(t) dt$, contribution to mean sourjoun time, we have

$$m_{01} = \int_0^\infty t q_{01}(t) dt = \int_0^\infty t f(t) dt = \mu_0$$

$$\begin{aligned} m_{12} + m_{13} + m_{14} &= \int_0^\infty t q_{12}(t) dt + \int_0^\infty t q_{13}(t) dt + \int_0^\infty t q_{14}(t) dt \\ &= \int_0^\infty t p_w r i^m(t) dt + \int_0^\infty t p_w \bar{r} i^m(t) dt + \int_0^\infty t \overline{p_w} i^m(t) dt \\ &= \int_0^\infty t i^m(t) dt \\ &= \mu_1 \end{aligned}$$

Similarly,

$m_{09} = m_{0,18} = \mu_0$; $m_{20} = \mu_2$
$m_{30} = \mu_3$ $m_{45} + m_{46} + m_{47} + m_{48} = \mu_4$;
$m_{50} = \mu_5$; $m_{60} = \mu_6$
$m_{70} = \mu_7$; $m_{80} = \mu_8$
$m_{9,10} + m_{9,11} + m_{9,12} = \mu_9$; $m_{10,17} = \mu_{10}$
$m_{11,17} = \mu_{11}$; $m_{12,13} + m_{12,14} + m_{12,15} + m_{12,16} = \mu_{12}$;
$m_{13,17} = \mu_{13}$ $m_{14,17} = \mu_{14}$;
$m_{15,17} = \mu_{15}$ $m_{16,17} = \mu_{16}$;
$m_{17,9} = m_{17,18} = \mu_{17}$ $m_{18,19} + m_{18,20} + m_{18,21} + m_{18,22} = \mu_{18}$;
$m_{19,23} = \mu_{19}$ $m_{20,23} = \mu_{20}$;
$m_{21,23} = \mu_{21}$ $m_{22,23} = \mu_{22}$;
$m_{23,18} = \mu_{23}$

In the following sections, several system profitability measures are achieved.

5. System Availability

1. **During Extended Warranty Period**

By definition of $AT_i(t)$, i=0, 9, 10, 11, 12, 13, 14, 15, 16, 17 (defined in Section 3) and the transitions that occurs during the extended warranty period, we have

$$\begin{aligned} AT_0(t) &= \overline{f_w(t)} + \int_0^t q_{09}(u) AT_9(t-u) du \\ &= M_0(t) + q_{09}(t) \odot AT_9(t) \end{aligned} \tag{1}$$

The term on L.H.S. of eq^n (1) denotes that the system is operational at time t given that $E_0 = 0$. The first term on R.H.S. indicates that the system will remain in state 0 rather than transitioning to another state.The second term denotes that the system transitions from state 0 to state 9 in time u<t and and then continues operational for t-u time from state 9 onwards.
Similarly the other recurrence relations are:

$$\begin{cases} AT_9(t) = q_{9,10}(t) \odot AT_{10}(t) + q_{9,11}(t) \odot AT_{11}(t) + q_{9,12}(t) \odot AT_{12}(t) \\ AT_{10}(t) = q_{10,17}(t) \odot AT_{17}(t) \\ AT_{11}(t) = q_{11,17}(t) \odot AT_{17}(t) \\ AT_{12}(t) = q_{12,13}(t) \odot AT_{13}(t) + q_{12,14}(t) \odot AT_{14}(t) + q_{12,15}(t) \odot AT_{15}(t) + q_{12,16}(t) \odot AT_{16}(t) \\ AT_{13}(t) = q_{13,17}(t) \odot AT_{17}(t) \\ AT_{14}(t) = q_{14,17}(t) \odot AT_{17}(t) \\ AT_{15}(t) = q_{15,17}(t) \odot AT_{17}(t) \\ AT_{16}(t) = q_{16,17}(t) \odot AT_{17}(t) \\ AT_{17}(t) = M_{17}(t) + q_{17,9}(t) \odot AT_9(t) \end{cases} \tag{2}$$

Solving eq^n(1)-(2) for $AT_0^*(s)$, where $AT_0^*(s)$ = L[$AT_0(t)$], we have

$$AT_0^*(s) = \frac{K_1(s)}{T_1(s)}$$

$$\begin{aligned} K_1(s) = & M_0^*(s) + q_{09}^*(s) q_{9,10}^*(s) q_{10,17}^*(s) M_{17}^*(s) + q_{09}^*(s) q_{9,11}^*(s) q_{11,17}^*(s) M_{17}^*(s) \\ & - q_{17,9}^*(s) q_{9,10}^*(s) q_{10,17}^*(s) M_0^*(s) - q_{17,9}^*(s) q_{9,11}^*(s) q_{11,17}^*(s) M_0^*(s) \\ & + q_{09}^*(s) q_{9,12}^*(s) q_{12,13}^*(s) q_{13,17}^*(s) M_{17}^*(s) + q_{09}^*(s) q_{9,12}^*(s) q_{12,14}^*(s) q_{14,17}^*(s) M_{17}^*(s) \\ & - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,13}^*(s) q_{13,17}^*(s) M_0^*(s) + q_{09}^*(s) q_{9,12}^*(s) q_{12,15}^*(s) q_{15,17}^*(s) M_{17}^*(s) \\ & - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,14}^*(s) q_{14,17}^*(s) M_0^*(s) + q_{09}^*(s) q_{9,12}^*(s) q_{12,16}^*(s) q_{16,17}^*(s) M_{17}^*(s) \\ & - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,15}^*(s) q_{15,17}^*(s) M_0^*(s) - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,16}^*(s) q_{16,17}^*(s) M_0^*(s) \\ & = NT_1^*(s)(say) \end{aligned} \tag{3}$$

$$\begin{aligned} T_1(s) = & 1 - q_{17,9}^*(s) q_{9,11}^*(s) q_{11,17}^*(s) - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,13}^*(s) q_{13,17}^*(s) \\ & - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,14}^*(s) q_{14,17}^*(s) - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,15}^*(s) q_{15,17}^*(s) \\ & - q_{17,9}^*(s) q_{9,12}^*(s) q_{12,16}^*(s) q_{16,17}^*(s) - q_{17,9}^*(s) q_{9,10}^*(s) q_{10,17}^*(s) \\ & = DT_1^*(s)(say) \end{aligned} \tag{4}$$

The system's steady state availability is evaluated using Abel's lemma as:

$$AT_0 = \lim_{s \to 0} s AT_0^*(s) = \frac{NT_1^*(0)}{DT_1^{*'}(0)} = \frac{NT_1}{DT_1} \tag{5}$$

Differentiating eq^n (4) w.r.t. s,

$$\begin{aligned}DT_1^{*'}(s) = {} & q_{11,6}^{*}{}'(s)(-q_{67}^*(s)q_{7,11}^*(s) - q_{68}^*(s)q_{8,11}^*(s) - q_{69}^*(s)q_{9,11}^*(s) \\ & - q_{6,10}^*(s)q_{10,11}^*(s)) - q_{67}^*{}'(s)q_{7,11}^*(s)q_{11,6}^*(s) - q_{68}^*{}'(s)q_{8,11}^*(s)q_{11,6}^*(s) \\ & - q_{69}^*{}'(s)q_{9,11}^*(s)q_{11,6}^*(s) - q_{6,10}^*{}'(s)q_{10,11}^*(s)q_{11,6}^*(s) \\ & - q_{7,11}^*{}'(s)q_{11,6}^*(s)q_{67}^*(s) - q_{8,11}^*{}'(s)q_{11,6}^*(s)q_{68}^*(s) \\ & - q_{9,11}^*{}'(s)q_{11,6}^*(s)q_{69}^*(s) - q_{10,11}^*{}'(s)q_{11,6}^*(s)q_{6,11}^*(s)\end{aligned} \tag{6}$$

Taking lim s $\to$ 0 in eq^n (3) and (6), we get

$$\begin{aligned}NT_1 = {} & \mu_{17}(p_w r + p_w\bar{r} + \overline{p_w}p_s r + \overline{p_w}p_s\bar{r} + \overline{p_w}\overline{p_s}r + \overline{p_w}\overline{p_s}\bar{r}) - \mu_0(p_w r \\ & + p_w\bar{r} + \overline{p_w}p_s r + \overline{p_w}p_s\bar{r} + \overline{p_w}\overline{p_s}r + \overline{p_w}\overline{p_s}\bar{r} - 1) \\ = {} & \mu_{17}\end{aligned} \tag{7}$$

$$\begin{aligned}DT_1 = {} & m_{9,12}(p_s r + p_s\bar{r} + \overline{p_s}r + \overline{p_s}\bar{r}) + m_{9,10} + m_{9,11} + \overline{p_w}\mu_{12} + p_w r m_{10,17} \\ & + p_w\bar{r}m_{11,17} + \overline{p_w}p_s r m_{13,17} + \overline{p_w}p_s\bar{r}m_{14,17} + \overline{p_w}\overline{p_s}r m_{15,17} + \overline{p_w}\overline{p_s}\bar{r}m_{16,17} \\ = {} & \mu_{17} + \mu_9 + \mu_{10}p_w r + \mu_{11}p_w\bar{r} + \mu_{12}\overline{p_w} + \mu_{13}\overline{p_w}p_s r + \mu_{14}\overline{p_w}p_s\bar{r} + \mu_{15}\overline{p_w}\overline{p_s}r \\ & + \mu_{16}\overline{p_w}\overline{p_s}\bar{r}\end{aligned} \tag{8}$$

2. **During Normal Warranty Period**
   Similarly, steady-state availabilities during normal warranty period are given as

$$AN_0 = \frac{NN_1}{DN_1}; \tag{9}$$

where

$$NN_1 = \mu_0,$$
$$DN_1 = \mu_0 + \mu_1 + p_w r\mu_2 + p_w\bar{r}\mu_3 + \overline{p_w}\mu_4 + \overline{p_w}p_s r\mu_5 + \overline{p_w}p_s\bar{r}\mu_6 + \overline{p_w}\overline{p_s}r\mu_7 + \overline{p_w}\overline{p_s}\bar{r}\mu_8$$

3. **During Non Warranty Period**
   Proceeding as above case, the steady-state availability during expired warranty period is:

$$AX_0 = \frac{NX_1}{DX_1} \tag{10}$$

where

$$NX_1 = \mu_{23}, \qquad DX_1 = \mu_{18} + \overline{p_s}r\mu_{19} + p_s r\mu_{20} + p_s\bar{r}\mu_{21} + \overline{p_s}\bar{r}\mu_{22}$$

6. Expected Busy Period and number of replacements

Using the definitions of Ik_i, Bk_i^m and Bk_i^u,i $\in$ S (defined in section 3) and the identical steps outlined in the preceding section, the expected time a repairman spends inspecting, repairing, or replacing a failed system in different warranty periods is given as:

$Ik_0 = \frac{Nk_2}{Dk_1}$; $\quad Bk_0^m = \frac{Nk_3}{Dk_1}$; $\quad Bk_0^u = \frac{Nk_4}{Dk_1}$; $\quad$ k=N, T, X.

where

$NN_2 = \mu_1 + \overline{p_w}\mu_4$; $\quad NT_2 = \mu_9 + \overline{p_w}\mu_{12}$; $\quad NX_2 = \mu_{18}$;
$NN_3 = p_w r\mu_2 + p_w\bar{r}\mu_3 + \overline{p_w}p_s r\mu_5 + \overline{p_w}p_s\bar{r}\mu_6$;
$NT_3 = p_w r\mu_{10} + p_w\bar{r}\mu_{11} + \overline{p_w}p_s r\mu_{13} + \overline{p_w}p_s\bar{r}\mu_{14}$; $\quad NX_3 = p_s r\mu_{20} + p_s\bar{r}\mu_{21}$;
$NN_4 = \overline{p_w}\overline{p_s}r\mu_7 + \overline{p_w}\overline{p_s}\bar{r}\mu_8$; $\quad NT_4 = \overline{p_w}\overline{p_s}r\mu_{15} + \overline{p_w}\overline{p_s}\bar{r}\mu_{16}$; $\quad NX_4 = \overline{p_s}r\mu_{19} + \overline{p_s}\bar{r}\mu_{22}$;

Furthermore, the expected number of replacements for three warranty periods in steady-state, according to the definitions of Rk_i^m and Rk_i^u,i ∈ S (specified in section 3), are:

$Rk_0^m = \frac{Nk_5}{Dk_1}$; $Rk_0^u = \frac{Nk_6}{Dk_1}$; k=N, T , X

where,

$NN_5 = p_w\bar{r} + \overline{p_w}p_s\bar{r}$; $NT_5 = p_w\bar{r} + \overline{p_w}p_s\bar{r}$; $NX_5 = p_s\bar{r}$;

$NN_6 = \overline{p_w p_s r}$; $NT_6 = \overline{p_w p_s r}$; $NX_6 = \overline{p_s r}$;

7. Cost-Benefit Analysis

The financial analysis aids both the maker and the consumer in identifying the variables that may result in long-term loss. In this section, we created profit functions to do a cost estimate. A profit function is a mathematical relationship between a system's total output and total expenditure. Thus, profit functions in steady-state are:

Profit to System and Insurance Provider

$$\begin{aligned} PM = {} & CP + EP - MP + SP - CM_1(p_n IN_0 + p_{et} IT_0) - CS_1(p_n IN_0 + p_{et} IT_0 + p_{ex} IX_0) \\ & - CM_2(p_n BN_0^m + p_{et} BT_0^m + p_{ex} BX_0^m) - CM_3(p_n RN_0^m + p_{et} RT_0^m + p_{ex} RX_0^m) \end{aligned} \tag{11}$$

Profit to System User

$$\begin{aligned} PU = {} & R_0(p_n AN_0 + p_{et} AT_0 + p_{ex} AX_0) - CU_2(p_n BN_0^u + p_{et} BT_0^u + p_{ex} BX_0^u) \\ & - CU_3(p_n RN_0^u + p_{et} RT_0^u + p_{ex} RX_0^u) - CP - EP - SP \end{aligned} \tag{12}$$

where,

CP= Expenses of purchasing the system
MP= Manufacturing cost of the system
EP= Expenses of extending the warranty period
SP= Expenses associated with insuring the system
R_0= Revenue generated by the system.
CM_1/CS_1= Expenses incurred by the manufacturer/insurance provider in hiring a repairman for inspection.
$CM_2(CU_2)$= Expenses incurred by the manufacturer or insurance company (user) in engaging a repairman for repair/replacement.
$CM_3(CU_3)$= Expenses incurred when a system is replaced, which are covered by the manufacturer or the insurance company (user).

All of the costs listed above are per unit time.

8. Sensitivity Analysis

Sensitivity analysis is an approach that examines whether a parameter has a high or low influence on the derived measures. Due to the wide range of numerical values for various parameters, relative sensitivity analysis is performed to compare the effects of various parameters. A relative sensitivity function is a standardized version of a sensitivity function.The sensitivity (Δ_{rk}, δ_{rs}) and relative sensitivity functions (z_{rk}, z_{rs}) for availabilities (AN_0, AT_0, AX_0) and profit functions (PM,PU) are defined using the $eq^n s$ (5), (9), (10), (11) and (12) and stated as follows:

$$\Delta_{rk} = \frac{\partial(Ak_0)}{\partial r}; \qquad z_{rk} = \frac{\Delta_{rk} r}{Ak_0} \qquad k = N, T, X \tag{13}$$

and

$$\delta_{rs} = \frac{\partial(Ps)}{\partial r}; \qquad z_{rs} = \frac{\delta_{rs} r}{Ps}; \qquad s = U, M \tag{14}$$

where r is the parameter

9. Results and Discussions

The system characteristics determined in the preceding sections 5-8 are illustrated numerically in this section. Assume that all of the distributions are exponentially distributed and their probability density functions are as follows:

$$\begin{cases} f_w(t) = \lambda_w e^{-\lambda_w t}, & i^m(t) = \gamma_m e^{-\gamma_m t}, & i^s(t) = \gamma_s e^{-\gamma_s t} \\ h^n(t) = \beta_n e^{-\beta_n t}, & h^{et}(t) = \beta_t e^{-\beta_t t}, & g^n(t) = \alpha_n e^{-\alpha_t t} \\ g^{et}(t) = \alpha_t e^{-\alpha_t t}, & g^{ex}(t) = \alpha_x e^{-\alpha_x t}, & h^{ex}(t) = \beta_x e^{-\beta_x t} \end{cases} \tag{15}$$

Consider the fixed value of parameters as

$$\begin{aligned} &p_w = 0.7, \overline{p_w} = 0.3, p_n = 0.2, p_{et} = 0.3, p_{ex} = 0.5, r = 0.7, \overline{r} = 0.3, p_s = 0.8, \overline{p_s} = 0.2, \\ &\lambda_w = 0.0005, \gamma_m = 1.5, \gamma_s = 1.2, \alpha_n = 0.5, \alpha_t = 0.4, \beta_n = 0.02, \beta_t = 0.02, \alpha_x = 0.25, \\ &\beta_x = 0.01, CP = 150, EP = 15, SP = 20, MP = 120, R_0 = 500, CM_1 = 80, CM_2 = 100, \\ &CM_3 = 15,000, CS_1 = 90, CU_2 = 120, CU_3 = 15,000. \end{aligned} \tag{16}$$

9.1. Variation in Profit Functions

The profit function has been graphically represented as a function of various parameters in this section. Figure 3 shows the change in manufacturer's profit (PM) versus p_s and λ_w. Profit falls

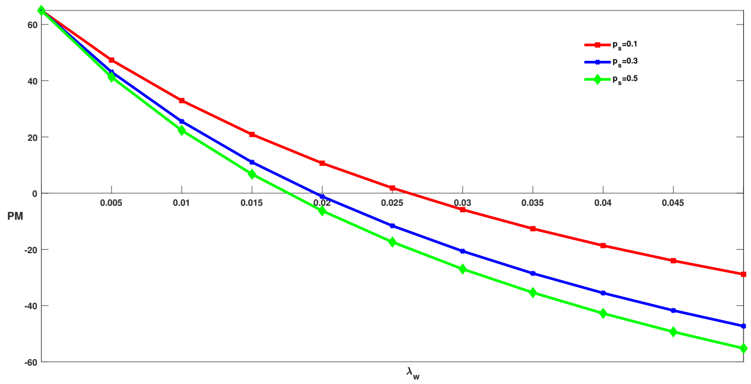

Figure 3: *Manufacturer's Profit (PM) for varied λ_w and p_s*

rapidly as λ_w and p_s rises.
The variation occured in user's profit (PU) due to changes in R_0 and $\overline{r}$ is depicted in Figure 4. Profit begins to rise as R_0 rises, and as $\overline{r}$ rises, profit declines. Figure 5 shows the decrease in profit differential (PU-PM) versus SP and MP. As SP rises, the profit margin narrows, whereas as MP rises, the profit margin widens. Lower/upper bounds for a system's profitability can also be determined, few of them mentioned are:

1. PM ≥ 0 if $\lambda_w \leq 0.018$ for $p_s = 0.5$.
2. PU ≥ 0 if $R_0 \geq 150$ for $\overline{r} = 0.1$.
3. PU $\geq$ PM if SP ≤ 132 for MP=100.

Lower/upper limits for other parameters can be interpreted in the same way.

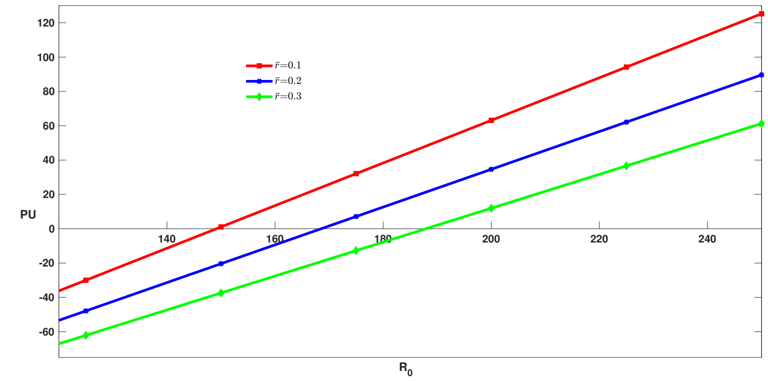

Figure 4: *User's Profit (PU) for varied R_0 and $\bar{r}$*

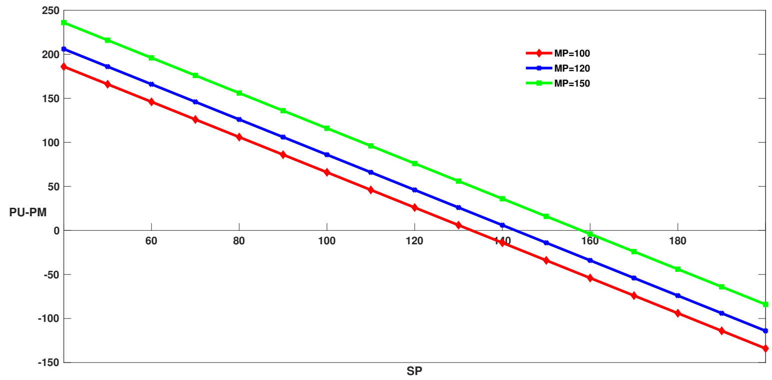

Figure 5: *Difference of Profit (PU-PM) for varied SP and MP*

9.2. Numerical Calculations for Sensitivity Analysis

We compute the sensitivity analysis for profit functions and availabilities in this section by treating all transition densities as exponential and having fixed parameter values as stated in eq^n (15) and eq^n (16) respectively.
The results for sensitivity and relative sensitivity functions for availabilities and profit functions (specified in section 8) are shown in Tables 1, 2 and 3 respectively. The absolute value of both the functions is considered for various conclusions.
It has been confirmed that

1. The availability of three alternative warranty times (AN_0,AT_0,AX_0) is substantially influenced by λ_w. Variation in γ_s and γ_m, on the other hand, appears to have the slightest effect.
2. The profit functions PM and PU are both quite sensitive to CP and R_0.
3. A relative change in PU and PM is caused by variations in CU_3 and CM_3.

Table 1: *Relative Sensitivity Analysis of Availabilities w.r.t. different rates*

Parameter (r)	Sensitivity Analysis $\Delta_r = \frac{\partial(A_0)}{\partial r}$	Relative Sensitivity Analysis $z_r = \frac{\Delta_r * r}{A_0}$
	Normal Warranty Period	
λ_w	-1982.8	-1
γ_m	$2.1842 * 10^{-4}$	$3.3047 * 10^{-4}$
γ_s	$1.0239 * 10^{-4}$	$1.2393 * 10^{-4}$
α_n	0.0014	$7.0606 * 10^{-4}$
β_n	0.3686	0.0074
	Extended Warranty Period	
λ_w	-17.3586	-0.0088
γ_m	$2.1835 * 10^{-4}$	$3.3042 * 10^{-4}$
γ_s	$1.0235 * 10^{-4}$	$1.2390 * 10^{-4}$
α_t	0.0021	$8.4742 * 10^{-4}$
β_t	0.3685	0.0074
	Expired Warranty Period	
λ_w	-32.53	-0.0165
α_x	0.0054	0.0014
β_x	1.4508	0.0148
γ_s	$3.3583 * 10^{-4}$	$4.0977 * 10^{-4}$

Table 2: *Relative Sensitivity of Manufacturer's Profit w.r.t. different rates/costs*

	Profit for System Manufacturer	
Parameter (r)	Sensitivity Analysis $\delta_r^m = \frac{\partial(P^m)}{\partial r}$	Relative Sensitivity Analysis $z_r^m = \frac{\delta_r^m * r}{P^m}$
λ_w	$-5.5312 * 10^3$	-0.0445
γ_m	0.0251	$6.0533 * 10^{-4}$
γ_s	0.0266	$5.1320 * 10^{-4}$
α_n	0.0253	$2.0338 * 10^{-4}$
β_n	6.7710	0.0022
α_t	-0.0014	$-9.0035 * 10^{-6}$
β_t	-0.2475	$-7.9585 * 10^{-5}$
α_x	0.2118	$8.5131 * 10^{-4}$
β_x	56.7230	0.0091
CM_1	$-2.2718 * 10-4$	$-2.9220 * 10^{-4}$
CS_1	$-4.3206 * 10-4$	$-6.2519 * 10^{-4}$
CM_2	-0.0081	-0.0130
CM_3	$-1.2890 * 10^{-4}$	-0.0311
CP	1	2.4117
EP	1	0.2412
MP	-1	-1.9293
SP	1	0.3216

Furthermore, the order in which input variables effect availabilities (A_0^n, A_0^{et}, A_0^{ex}) and profit functions (P^m,P^u) are

- Availability(AN_0): $\lambda_w > \beta_n > \alpha_n > \gamma_m > \gamma_s$.
- Availability(AT_0): $\lambda_w > \beta_t > \alpha_t > \gamma_m > \gamma_s$.
- Availability(AX_0): $\lambda_w > \beta_x > \alpha_x > \gamma_s$.
- Profit Function(P^m):$CP > MP > SP > EP > \lambda_w > CM_3 > CM_2 > \beta_x > \beta_n > \alpha_x > CS_1 > \gamma_m > \gamma_S > CM_1 > \alpha_n > \beta_t > \alpha_t$.
- Profit Function(P^u): $R_0 > CP > SP > EP > \lambda_w > \beta_x > \beta_t > \beta_n > \alpha_x > CU_3 > \gamma_s > \alpha_t > \gamma_m > \alpha_n > CU_2$.

Table 3: *Sensitivity and Relative Sensitivity of User's Profit w.r.t. different rates/costs*

	Profit for System User	
Parameter (r)	Sensitivity Analysis $\delta_r^u = \frac{\partial(P^u)}{\partial r}$	Relative Sensitivity Analysis $z_r^u = \frac{\delta_r^u * r}{P^u}$
λ_w	$-1.3446 * 10^5$	-0.0218
α_x	1.4179	0.0012
β_x	379.7890	0.0123
γ_s	0.1094	$4.2598 * 10^{-4}$
γ_m	0.0546	$2.6575 * 10^{-4}$
α_n	0.1396	$2.2649 * 10^{-4}$
β_n	37.3800	0.0024
α_t	0.3270	$4.2442 * 10^{-4}$
β_t	56.0506	0.0036
R_0	0.9874	1.6020
CU_2	-0.0019	$-7.3982 * 10^{-5}$
CU_3	$-1.9213 * 10^{-5}$	$-9.3514 * 10^{-4}$
CP	-1	-0.4867
EP	-1	-0.0487
SP	-1	-0.0649

10. Conclusion

The sensitivity and economic analysis of the insured system operating under normal warranty, extended warranty, and no warranty conditions were explored in this study. Various profitability indicators and profit functions for the user, manufacturer, and insurance provider have been drafted using Markov and regenerative techniques. After that, the measures are assessed using numerical calculation in which the transition density follows an exponential distribution. For system profitability, lower/upper bounds of the measures involved have been identified. The failure rate has significant influence on availability and profit, whereas the inspection rate has the least. Revenue and cost pricing also significantly impacts the system's profit. This research gives optimum analysis regarding benefits for the user, the manufacturer as well as the insurance provider.

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Disclosure statement

The authors declare that they have no conflict of interest.

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