ESTIMATION AND TESTING PROCEDURES OF P(Y < X) FOR THE INVERSE DISTRIBUTIONS FAMILY UNDER TYPE-II CENSORING

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Abstract

We recommended an inverse distributions family. The challenge of estimating $R(t)$ and P in type-II censoring was measured to produce Uniformly Minimum Variance Unbiased Estimator (UMVUE) and Maximum Likelihood Estimator (MLE). The estimators have been created for $R(t)$ and P.Testing approaches for $R(t)$ and P under type-II censoring have been constructed for hypotheses associated with various parametric functions. The author provides an alternate method for generating these estimators.A comparative assessment of two estimating techniques has been conducted. The simulation technique has been used to assess the performance of estimators.

Keywords: Inverse distributions family; testing procedures; bootstrap sampling

1. Introduction

The reliability function describes the probability of a failure-free procedure until time t. The estimation of the stress-strength $[P(Y < X)]$ parameter , aimed at displaying system efficienc is one of the most important challenges in statistical inference, which can be applied to a wide variety of fields such as longevity mechanical system dependability , statistics, and bio-statistics. In reliability, the $P = P(Y < X)$ parameter , which define the lifetime for a specifi system, places the strength X against the stress Y. Several scholars have measured the problems of estimation of reliability functions under censoring.Lin et al. [13] illustrated the inverse gamma model's role in lifetime distribution.The inverse Weibull distribution produced a good fit discussed by Erto [11]. The inference reliability and $P(Y < X)$ of a scaled Burr distribution were calculated by Surles and Padgett [16].Yadav et al. [18] estimated the P (Y< X) of the inverse Weibull distribution With a progressive type-II censoring technique.Chaturvedi and Kumari [7] conducted a reliable Bayesian study of the generalized inverted family of distributions. Enis and Geisser [10] acquire an estimate of the likelihood that Y< X. Weerahandi and Johnson [17] investigated testing reliability in $P(Y < X)$ while X and Y are repeatedly distributed. Estimators of $P(Y < X)$ in the gamma model are explored by Constatine et al. [9].A comparative study for Burr distribution is presented in $R(t)$ and P estimated by Awad and Gharraf [1].Nigm and Amboeleneen [14] use progressive censoring to evaluate the parameters of the Inverse Weibull distribution. Chaudhary and Chauhan [6] performed estimation and test approaches for $P(Y < X)$ of the Weibull distribution with type- I and type-II censoring.Chaturvedi and Kumari [3] developed estimate and analysis processes for the reliability of a broad range of distributions.

We consider a family of inverse distributions, which is reflecte in this paper .The UMVUES and MLES of $R(t)$ and P are calculated using type-II censoring. A new approach for estimating the UMVUES and MLES of $R(t)$ was invented, in which the expression of $R(t)$ and P is not required. Initially, the estimators for $R(t)$ are generated using this method.The $R(t)$ derivative estimators are used to construct the p.d.f. at a certain point, and then determine P estimators.We calculated P by considering instances in which X and Y are similar distributions but have dissimilar values. We now have extended the findin to any distribution from the projected inverse distributions family where X and Y are members. The testing procedures are also being planned.A performance comparison performance of two estimating approaches was conducted. The simulation technique was used to examine the performance of estimators.

2. Inverse Distributions Family

Suppose a random variable $(r.v.)Y$ having pdf

$$f(y;\gamma,\beta,\mu)=\frac{\gamma^{\beta}G^{\beta-1}\left(y^{-1};\mu\right)G'\left(y^{-1};\mu\right)}{y^{2}\Gamma(\beta)}\exp\left(-\gamma G\left(y^{-1};\mu\right)\right)\quad y>0,\gamma>0,\beta>0 \tag{1}$$

Where, $G\left(y^{-1};\mu\right)$, is depend on μ and a function of y. Further more, $G\left(y^{-1};\mu\right)$ real-valued, rigorously reducing function of y with $G(\infty;\mu)=\infty$ and $G'\left(y^{-1};\mu\right)$ stances for the derivative of $G\left(y^{-1};\mu\right)$ by y^{-1}.Let β and μ are known and γ is unknown during this whole section.
The (1) demonstrates that the inverse distributions family can be transformed into the inverse distributions listed below as special cases:

1. If $G(y;\mu)=y^{p}, p>0,\beta>0$, we obtained the inverse generalized gamma distribution.
2. If $G(y;\mu)=y^{2},\beta=k+1$, $(\mathrm{k}=0)$, we achieved the inverse Rayleigh distribution.
3. If $G(y;\mu)=\log\left(1+\frac{y^{b}}{v^{b}}\right), b>0, v=1,\beta>1$, we achieved the inverse Burr distribution.
4. If $G(y;\mu)=\log\left(1+\frac{y^{b}}{v^{b}}\right), b=1, v>1,\beta>1$,we obtained the inverse Lomax distribution.
5. If $G(y;\mu)=\log\left(\frac{y}{a}\right)$ and $\beta=1$, we achieved the inverse Pareto distribution.
6. If $G(y;\mu)=y^{r}\exp(ay), r>0, a>0,\beta=1$, we obtained the inverse modifie Weibull distribution.
7. If $G(y;\mu)=\mu y+\frac{vy^{2}}{2},\alpha=\beta=1$,we obtained the inverse linear exponential distribution.
8. If $G(y;\mu)=\log y$, we achieved the inverse log-gamma distribution.

3. UMVUES of γ and Reliability Functions

We investigate estimation with censored type-II data. Suppose $Y_{(1)}\leq Y_{(2)}\leq\ldots\ldots\ldots\ldots\ldots Y_{(n)}$. Assume n objects are subjected to a test, when the firs r observations are noted, the test is stopped. Supposing, $0<r<n$, be the lifespans of leading r values. Noticeably , $(n-r)$ objects stay alive awaiting $Y_{(r)}$.

Lemma 1. Suppose $S_{r}=\sum_{i=1}^{r}G\left(y_{(i)}^{-1},\mu\right)+(n-r)G\left(y_{(r)}^{-1},\mu\right)$. Then, for the inverse distributions family, S_{r} is complete and sufficien indicated as (1). Additionally , the pdf of S_{r} is

$$k\left(s_{r};\mu\right)=\frac{\gamma^{r\beta}s_{r}^{r\beta-1}}{\Gamma(r\beta)}\exp\left(-\gamma s_{r}\right) \tag{2}$$

Proof. From (1), the joint pdf is

$$f^{*}\left(\underline{y}_{(i)},i=1,2,\ldots n;\gamma,\beta,\mu\right)=n!\prod_{i=1}^{n}G'\left(y_{(i)}^{-1};\mu\right)\exp\left\{-\gamma\sum_{i=1}^{n}G\left(y_{(i)}^{-1};\mu\right)\right\} \tag{3}$$

When we integrate $y_{(r+1)}, y_{(r+2)}, \ldots\ldots\ldots, y_{(n)}$ throughout the region $y_{(r)} \leq y_{(r+1)} \leq \ldots \leq y_{(n)}$, we get the likelihood as

$$h\left(\underline{y}_{(i)}, (i = 1, 2, \ldots\ldots.r); \gamma, \mu\right) = n(n-1)\ldots\ldots\ldots\ldots(n-r+1)\gamma^r \prod_{i=1}^{r} G'\left(y_{(i)}^{-1}, \mu\right) \exp\left(-\gamma s_r\right) \tag{4}$$

S_r is sufficien by Fisher -Neyman factorization theorem [15]. In (1), put $A = G\left(y^{-1}; \mu\right)$, the pdf is

$$k(a; \gamma, \beta, \underline{\theta}) = \frac{\gamma^{\beta} a^{\beta-1}}{\Gamma(\beta)} \exp\left(-\gamma a\right); a > 0$$

S_r go by Johnson and Kotz [12] discovered the additive property of gamma distribution. Meanwhile, S_r is associated with the exponential distributions family, and it is still complete. ■

Theorem 1. The UMVUE of γ^{-p} is, for $p \in (-\infty, \infty)$,

$$\hat{\gamma}^{-p} = \begin{cases} \frac{\Gamma(r\beta)}{\Gamma(r\beta+p)} s_r^p, & n\beta + p > 0 \\ 0, & \text{other wise} \end{cases}$$

Proof. From Lemma 1,

$$E\left(s_r^p\right) = \frac{\gamma^{n\beta}}{\Gamma(n\beta)} \int_0^{\infty} s_r^{n\beta+p-1} \exp\left(-\gamma s_r\right) ds_r = \left\{\frac{\Gamma(n\beta+p)}{\Gamma(n\beta)}\right\} \gamma^{-p}$$

and the theorem observes from Lehmann-Scheffe theorem [15]. ■

Remark 1. We can write (1) as

$$f(y; \gamma, \beta, \mu) = \frac{G^{\beta-1}\left(y^{-1}; \mu\right) G'\left(y^{-1}; \mu\right)}{y^2 \Gamma(\beta)} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} G'\left(y^{-1}; \mu\right) \cdot \gamma^{i+\beta}$$

From Chaturvedi and Tomar [5] (Lemma 1) and theorem 1, for integer -yalued β, the UMVUE of $f(y; \alpha, \beta, \mu)$ for stipulated point y

$$\hat{f}(y; \gamma, \beta, \mu) = \frac{G^{\beta-1}\left(y^{-1}; \mu\right) G'\left(y^{-1}; \mu\right)}{y^2 \Gamma(\beta)} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} G'\left(y^{-1}; \mu\right) \hat{\gamma}^{i+\beta}$$
$$= \frac{G^{\beta-1}\left(y^{-1}; \mu\right) G^2\left(y^{-1}; \mu\right)}{y^2 \Gamma(\beta)} \sum_{i=0}^{\infty} \frac{(-1)^i}{i!} G^i\left(y^{-1}; \mu\right) \frac{\Gamma(r\beta)}{\Gamma(r\beta+i)} s_r^{i+\beta},$$

Theorem 2. The UMVUE of $f(y; \alpha, \beta, \mu)$ for a stipulated point y

$$\hat{f}_{II}(y, \gamma, \beta, \mu) = \left\{ \frac{G^{\beta-1}\left(y^{-1}; \mu\right) G^t\left(y^{-1}; \mu\right)}{y^2 S_r^{\beta} B((r-1)\beta, \beta)} \left[1 - \frac{G\left(y^{-1}; \mu\right)}{S_r}\right]^{r-1)\beta-1}, \quad G\left(y^{-1}; \mu\right) < S_r \right.$$

Proof. Using remark 1 and theorem 1, we acquire the required solution ■

Theorem 3. The UMVUE of $R(t)$
Where

$$I_z(s, q) = \frac{1}{\beta(s, q)} \int_0^z x^{s-1} (1-x)^{q-1} dx$$

The incomplete beta function

Proof. Now, let us suppose the expectation

$$\int_t^{\infty} \hat{f}_{II}(y; \gamma, \beta, \mu) dy$$

The integration to S_r

$$\begin{aligned}
&= \int_t^\infty \left\{ \int_t^\infty \hat{f}(y;\gamma,\beta,\mu)dy \right\} k\left(s_r;\gamma,\beta,\mu\right) ds_r \\
&= \int_t^\infty \left[E_{S_r}\{\hat{f}(y;\gamma,\beta,\mu)\} \right] dy \\
&= \int_t^\infty f_{II}(y;\gamma,\beta,\mu)dy \\
&= R_{II}(t)
\end{aligned}$$

■

Suppose two independent rv's X and Y follow the inverse distributions families $f_1\left(x_9,\gamma_1,\beta_1,\mu_1\right)$ and $f_2\left(y;\gamma_2,\beta_2,\mu_2\right)$, sequentially ,

$$f_{1II}\left(x;\gamma_1,\beta_1,\mu_1\right) = \frac{\gamma_1^{\beta} G^{\beta-1}\left(x^{-1};\mu_1\right) G'\left(x^{-1};\mu_1\right)}{x^2\Gamma\left(\beta_1\right)} \exp\left(-\gamma_1 G\left(x^{-1};\mu_1\right)\right)$$
$$x > 0, \gamma_1 > 0, \beta_1 > 0$$
$$f_{2II}\left(y;\gamma_2,\beta_2,\mu_2\right) = \frac{\gamma_2^{\beta_2} H^{\beta_2-1}\left(y^{-1};\mu_2\right) H'\left(y^{-1};\mu_2\right)}{y^2\Gamma\left(\beta_2\right)} \exp\left(-\gamma_2 H\left(y^{-1};\mu_2\right)\right)$$
$$y > 0, \gamma_2 > 0, \beta_2 > 0$$

Where β_1, β_2, μ_1 and μ_2 are well-known, however γ_1 and γ_2 are unknown. Suppose n objects arranged X and m objects arranged Y are subjected to a lifespan test, and that the expiry quantities for X and Y are r and r', separately . As well as notation by $S = \sum_{i=1}^{r} G\left(x_i^{-1};\mu_1\right)$ and $T = \sum_{i=1}^{r'} H\left(y_i^{-1};\mu_2\right)$

Theorem 4. The UMUVE of P is The summations range from 0 to $(r-1)\beta_1 - 1$ in case $(r-1)\beta_1$ is an integer .

Proof. From theorem 3

$$\hat{f}_{1II}\left(x;\gamma_1,\beta_1,\mu_1\right) = \begin{cases} \frac{G^{\beta_1-1}\left(x^{-1};\mu_1\right)G'\left(y^{-1};\mu_1\right)}{x^1 T^{\beta}\beta((r-1)\beta_1,\beta_1)} \left[1 - \frac{G\left(y^{-1};\mu_1\right)}{S} \right. & , \mathrm{G}\left(x^{-1};\mu_1\right) < S \\ 0, & \text{other wise} \end{cases} \Bigg]^{(r-1)\beta_{-}1} \quad (5)$$

$$\hat{f}_{2II}\left(y;\gamma_2,\beta_2,\mu_2\right) = \begin{cases} \frac{H^{\beta_2-1}\left(y^{-1};\mu_2\right)H'\left(y^{-1};\mu_2\right)}{y^2 T^{\beta_2}\beta((r'-1)\beta_2,\beta_2)} \left[1 - \frac{H\left(y^{-1};\mu_2\right)}{T} \right. & , \mathrm{H}\left(y^{-1};\mu_2\right) < T \\ 0, & \text{other wise} \end{cases} \Bigg]^{(r'-1)\beta_2-1} \quad (6)$$

The UMVUES of $f_1\left(x;\gamma_1,\beta_1,\mu_1\right)$ and $f_2\left(y;\gamma_2,\beta_2,\mu_2\right)$ for specifie pints $'x'$ and $'y'$ separately , similarly , from theorem 4, we get the UMVUE of P

$$\hat{P}_{II} = \int_{y=0}^{\infty}\int_{x=y}^{\infty} \hat{f}_{1I}\left(x;\gamma_1,\beta_1,\mu_1\right)\hat{f}_{2I}\left(y;\gamma_2,\beta_2,\mu_2\right) dxdy$$

Using (5) and (6) we get
$\hat{P}_{II} = \frac{1}{B((r-1)\beta_1,\beta_1)B((r'-1)\beta_2,\beta_2)S^{\beta_1}T^{\beta_2}}$

$$\int_{y=[H^*(T)]^{-1}}^{\infty}\int_{x=y}^{\infty} \left\{ \frac{G^{\beta-1}\left(x^{-1};\mu_1\right)G'\left(x^{-1};\mu_1\right)}{x^2} \right\} \left[1 - \frac{G\left(x^{-1};\mu_1\right)}{S}\right]^{(r-1)\beta_1-1}$$

$$\left\{ \frac{H^{\beta_2-1}\left(y^{-1};\mu_2\right)H'\left(y^{-1};\mu_2\right)}{y^2} \right\} \left[1 - \frac{H\left(y^{-1};\mu_2\right)}{T}\right]\Bigg]^{(r'-1)\beta_2-1} dxdy$$

■

Corollary 1. If $\mu_1 = \mu_2 = \mu$, and $G\left(x^{-1};\mu\right) = H\left(x^{-1};\mu\right)$

$$\hat{P}_{II} = \begin{cases} \frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\frac{S}{T}\right)^{\beta_2}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\begin{pmatrix}\{(n-1)\beta_1\}-1\\ i\end{pmatrix} \\ \sum_{j=0}^{\infty}\frac{(-1)^j}{(\beta_1+\beta_2+i+j)}\begin{pmatrix}(m-1)\beta_2-1)\left(\frac{S}{T}\right)^j,\\ j\end{pmatrix} \text{ if } S<T \\ \frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\left(\frac{T}{S}\right)^{\beta}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\begin{pmatrix}\{(n-1)\beta_1\}-1\\ i\end{pmatrix} \\ \left(\frac{T}{S}\right)^i B\left(\beta_1+\beta_2+i,(m-1)\beta_2\right), \quad \text{if } S>T \end{cases}$$

The summatio n over i, from 0 to $\{(n-1)^*\beta_1\}-1$, if $(n-1)\beta_1$ is an integer and the summation over j, from 0 to $(m-1)\beta_2$, if $(m-1)\beta_2$ is an integer .

Proof. we get From Theor em 4 for $S \leqslant T$,

$$\hat{P}_{II} = \left\{\frac{1}{B\left((n-1)\beta_1,\beta_1\right)B\left((m-1)\beta_2,\beta_2\right)}\right\}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)}\begin{pmatrix}\{n-1\}\beta_1-1\\ i\end{pmatrix} \cdot \int_0^{\frac{s}{T}} w^{\beta_2-1}(1-w)^{(m-1)\beta_2-1}\left(\frac{Tw}{S}\right)^{\beta+i}dw$$

and for $S > T$, from Theor em 2,

$$\hat{P}_{II} = \frac{1}{\beta\left((n-1)\beta_1,\beta_1\right)\beta\left((m-1)\beta_2,\beta_2\right)}\left(\frac{T}{S}\right)^{\beta}\sum_{i=0}^{\infty}\frac{(-1)^i}{(\beta_1+i)} \cdot \begin{pmatrix}\{n-1\}\beta_1-1\\ i\end{pmatrix}\left(\frac{T}{S}\right)^i\int_0^1 w^{\beta_1+\beta_2+i-1}(1-w)^{(m-1)\beta_2-1}dw$$

and the second contention pr oved. ■

Remark 2. (i) UMVUES of $R(t)$ and P are calculated independently using sampling pdf under type II censoring of UMVUES $R(t)$ and P, as pr oved in theor ems 3 and 4. As a result, we identifie two estimation concer ns that indicated inter dependence.
(ii) The UMVUES of P was achie ved using type-II censoring, wher eas X and Y follo wed a similar distribution, possibly with dissimilar parameters or possibly with similar parameters, also while X and Y follo wed distinct distributions under all thr ee conditions.
(iii) In theor em 4, if $n \to \infty$ then $\mathrm{Var}(\hat{\gamma}) \to 0$. We kno w that, $\hat{f}(y;\gamma,\beta,\mu)_2\hat{R}(t)$ and $\hat{P}$ are consistent estimators of $f(y;\gamma,\beta,\mu), R(t)$ and P, respectiv ely because these are continuous functions. So, $\hat{\gamma}$ is a consistent estimator of γ.

4. MLES of γ and Reliability Functions

Using the lemma 1

$$\tilde{\gamma}^{-p} = \left(\frac{r}{S_r}\right)^{-p} \tag{7}$$

Theorem 5. The MLE for a specifie point y

$$\tilde{f}_{II}(y;\gamma,\beta,\mu) = \frac{(\tilde{\gamma})^{\beta}G^{\beta-1}\left(y^{-1};\mu\right)G'\left(y^{-1};\mu\right)}{y^2\Gamma(\beta)}\exp\left(-\tilde{\gamma}G\left(y^{-1};\mu\right)\right)$$

Proof. We can obtain from (7) and use the MLE's one-to-one pr operty . ■

Theorem 6. $\tilde{R}(t)$ is the MLE of R(t)

$$\tilde{R}(t) = J_{\frac{r}{S_r}G\left(t^{-1};\mu\right)}(\beta), \text{ and } J_y(p) = \frac{1}{\Gamma(p)}\int_0^{\infty} x^{p-1}e^{-x}dx$$

This is an incomplete gamma function.

Proof. Using MLE's invariance property and theorem 5

$$\begin{aligned}\tilde{R}(t) &= \int_t^{\infty} \tilde{f}_{II}(y;\gamma,\beta,\mu)dy \\ &= \frac{\left(\frac{r}{S_r}\right)^{\beta}}{\Gamma(\beta)}\int_t^{\infty}\frac{G^{\beta-1}\left(y^{-1};\mu\right)G'\left(y^{-1};\mu\right)}{y^2}\exp\left(-\frac{r}{S_r}G\left(y^{-1};\mu\right)\right)dy \\ &= \frac{1}{\Gamma(\beta)}\int_{\frac{r}{S_r}G\left(t^{-1};\mu\right)}^{\infty} x^{\beta-1}e^{-x}dx\end{aligned}$$

■

Corollary 2. When $\beta = 1$,

$$\tilde{R}(t) = \exp\left(-\frac{r}{S_r}G\left(t^{-1};\mu\right)\right)$$

Theorem 7. $\tilde{P}$ is the MLE of P

$$\tilde{P} = \frac{(\tilde{\gamma}_2)^{\beta_2}}{\Gamma(\beta_1)\Gamma(\beta_2)}\int_{y=0}^{Y_{(v)}}\left[\int_{z-\left(\tilde{\gamma}g\left(\tilde{x}_{(\tau)}^{-1};\mu 4\right)\right)}^{\gamma_1 g\left(y^{-1};\mu_4\right)} e^{-z}z^{\beta-1}dz\right]$$

$$\frac{H^{\beta_2-1}\left(y^{-1};\mu_2\right)H'\left(y^{-1};\mu_2\right)}{y^2}\exp\left(-\tilde{\gamma}_2 H\left(y^{-1};\mu_2\right)\right)dy$$

Proof. Using the MLE's one-to-one condition and theorem 5,

$$= \frac{(\tilde{\gamma}_1)^{\beta_1}(\tilde{\gamma}_2)^{\beta_2}}{\Gamma(\beta_1)\Gamma(\beta_2)}\int_{y=0}^{y_{(m)}}\int_{x=y}^{X_{(n)}}\left\{\frac{G^{\tilde{\beta}-1}\left(x^{-1};\mu_1\right)G'\left(x^{-1};\mu_1\right)}{x^2}\right\}$$

$$\exp\left(-\tilde{\gamma}_1 G\left(x^{-1};\mu_1\right)\right)\left\{\frac{H^{\beta_2-1}\left(y^{-1};\mu_2\right)H'\left(y^{-1};\mu_2\right)}{y^2}\right\}\exp\left(-\tilde{\gamma}_2 H\left(y^{-1};\mu_2\right)\right)dxdy$$

$$= \frac{(\tilde{\gamma}_1)^{\beta_1}(\tilde{\gamma}_2)^{\beta_2}}{\Gamma(\beta_1)\Gamma(\beta_2)}\int_{y=0}^{y'_r}\frac{H^{\tilde{\beta}_2-1}\left(y^{-1};\mu_2\right)H'\left(y^{-1};\mu_2\right)}{y^2}\exp\left(-\tilde{\gamma}_2 H\left(y^{-1};\mu_2\right)\right)$$

$$\left\{\int_{z=\tilde{\gamma}_1 G\left(X_{(r)}^{-1};\mu_1\right)}^{\tilde{\gamma}_1 G\left(y^{-1};\mu_1\right)} e^{-z}\left(\frac{z}{\tilde{\gamma}_1}\right)^{\beta_1-1}\frac{dz}{\tilde{\gamma}_1}\right\}dy$$

■

Remark 3. (i) UMVUES are acceptable for the MLES under remarks 2.
(ii) There is no need to use reliability function expressions to obtain UMVUES and MLES.

5. Hypotheses Testing

Putting the hypothesis to the test $H_0 : \gamma = \gamma_0$ versus $H_1 : \gamma \neq \gamma_0$, from *eq.*(1), The likelihood function for γ

$$L(\gamma/\underline{y}) = n\cdot(n-1).---(n-r-1)\cdot\gamma^{r\beta}\prod_{i=1}^{r}\left\{\frac{G^{\beta-1}\left(y^{-1};\mu\right)G\left(y^{-1};\mu\right)}{x^2}\right\}\exp\left(-\gamma S_r\right) \quad (8)$$

For H_0

$\text{Sup } L(\gamma/\underline{y}) = n \cdot (n-1) \cdot - - - .(n-r-1) \cdot \gamma_0^{r\beta} \prod_{i=1}^{r} \left\{ \frac{G^{\beta-1}(y^{-1};\mu)G(y^{-1};\mu)}{x^2} \right\} \exp\left(-\gamma_0 S_r\right)$

$\Theta_0 = \{\gamma : \gamma = \gamma_0\}$

$SupL(\gamma/\underline{y}) = n.(n-1). - - - .(n-r-1). \left(\frac{r}{S_r}\right)^r \prod_{i=1}^{r} \left\{ \frac{G^{\beta-1}(y^{-1};\mu)G'(y^{-1};\mu)}{x^2} \right\} \exp\left(-r\right)$

$\Theta = \{\gamma : \gamma = \gamma_0\}$

Likelihood ratio

$$\Phi(\underline{x}) = \left\{ \frac{\text{Sup . L}(\gamma/\underline{x})}{\text{SupL }(\gamma/\underline{x})} \right\} = \left(\frac{\gamma_0^{\beta} S_r}{r} \right) \exp\left(-\left(r + \gamma_0 S_r\right)\right) \quad (9)$$

And if $\beta = 1$

$$\Phi(\underline{x}) = \left(\frac{\gamma_0 S_r}{r} \right) \exp\left(-\left(r + \gamma_0 S_r\right)\right) \quad (10)$$

We note from (10) we get $2\gamma_0 S_r \sim \chi^2_{2r}$, the rejection region is given by

$$\{0 < S_r < m_0\} \cup \{m_0' < S_r < \infty\}$$

Where m_0 and m' obtained from

$$P\left[\chi^2_{2r} < 2\gamma_0^{\beta} m_0 \text{ or } 2\gamma_0^{\beta} m_0' < \chi^2_{2r}\right] = \alpha$$

Thus

$$m_0 = 2\gamma_0^{\beta}\chi^2_{2r}\left(1 - \frac{\alpha}{2}\right) \text{ and } m_0' = 2\gamma_0^{\beta}\chi^2_{2r}\left(\frac{\alpha}{2}\right)$$

For $H_0 : \gamma \leq \gamma_0$ against $H_1 : \gamma > \gamma_0$, It follows from (8) that, for $\gamma_1 < \gamma_2$

$$\frac{k\left(y_{(1)}, y_{(2)}, \ldots\ldots\ldots\ldots\ldots y_{(r)}; \gamma_2, \mu\right)}{k\left(y_{(1)}, y_{(2)}, \ldots\ldots\ldots\ldots\ldots y_{(r)}; \gamma_1, \mu\right)} = \left(\frac{\gamma_2}{\gamma_1}\right)^r \exp\left(-\left(\gamma_2 - \gamma_2\right) S_r\right) \quad (11)$$

It follows that from (11) that $\left(y_{(1)}, y_{(2)}, \ldots\ldots\ldots\ldots\ldots\ldots y_{(r)}; \gamma_2, \mu\right)$ has a maximum likelihood ratio in S_r. Thus, the UMPCR for analysis $H_0 : \gamma \leq \gamma_0$ against $H_1 : \gamma > \gamma_0$ is given by

$$\Phi\left(y_{(1)}, y_{(2)}, \ldots\ldots\ldots\ldots\ldots\ldots y_{(r)}\right) = \left[\begin{array}{ll} 1, & s, \leq m_0' \\ 0, & \text{otherrise} \end{array} \right.$$

Where m' obtained from

$$P\left[\chi^2_{2r} < 2\gamma_0^{\beta} m_0\right] = \alpha$$

Therefore

$$m_0' = 2\gamma_0^{\beta}\chi^2_{2r}\left(1 - \frac{\alpha}{2}\right)$$

Now for $H_0 : P = P_0$ against $H_1 : P \neq P_0$ under type II censoring. Then H_0 is equivalent to $\gamma_1 = m\gamma_0$, under H_0

$$\hat{\gamma}_1 = \frac{m\left(r + r'\right)}{mS + T} \quad \hat{\gamma}_2 = \frac{\left(r + r'\right)}{mS + T}$$

For m the likelihood of γ_1 and γ_2 for, $\underline{x}_{(i)}; i = 1, 2, \ldots\ldots\ldots r$ and $\underline{y}_{(j)}; j = 1, 2, \ldots\ldots\ldots r$ is given by Then

$L\left(\gamma_1, \gamma_1/\underline{x}_{(i)}, \underline{y}_{(j)}\right) = m\gamma_1^r \gamma_2^{r'} \exp\left(-\left(\gamma_1 S - \gamma_2 T\right)\right)$

$$\text{Sup } L\left(\gamma_1, \gamma_1/\underline{x}_{(i)}, \underline{y}_{(j)}\right) = \frac{mm' \exp\left(-\left(r + r'\right)\right)}{(mS + T)^{r+r'}}$$

$$\text{Sup } L\left(\gamma_1, \gamma_1/\underline{x}_{(i)}, \underline{y}_{(j)}\right) = \frac{m \exp\left(-\left(r + r'\right)\right)}{S^r T^{r'}}$$

Then likelihood ratio

$$\lambda\left(\gamma_1, \gamma_1 / \underline{x}_{(i)}, \underline{y}_{(j)}\right) = m \frac{\left(\frac{mS}{T}\right)^{r}}{\left[1+\frac{mS}{T}\right]^{r+r'}}$$

The F-statistic and using the statistic that

$$\frac{S}{T} \sim \frac{r_1\gamma_1}{r'\gamma_2} F_{(2r,2r)}(.) \quad \left\{\frac{S}{T} < m_2 \text{ and } \frac{S}{T} > m_2\right\}$$

m_2 and m_2 obtained from the given condition

$$P\left[\frac{r'mS}{rT} < F_{2r,2r'} \cup \frac{r'mS}{rT} > F_{2r',2r'}\right] = \alpha$$

$$m_2 = \frac{r}{r'm} F_{(2r,2r)}\left(1-\frac{\alpha}{2}\right) \text{ and } m_2' = \frac{r}{r'm} F_{(2r,2r')}\left(\frac{\alpha}{2}\right)$$

6. Result

We can see in remarks 2(iii) where $\hat{\gamma}, f(x;\gamma,\beta,\mu)$, $R(t)$ and P are consistent estimators.

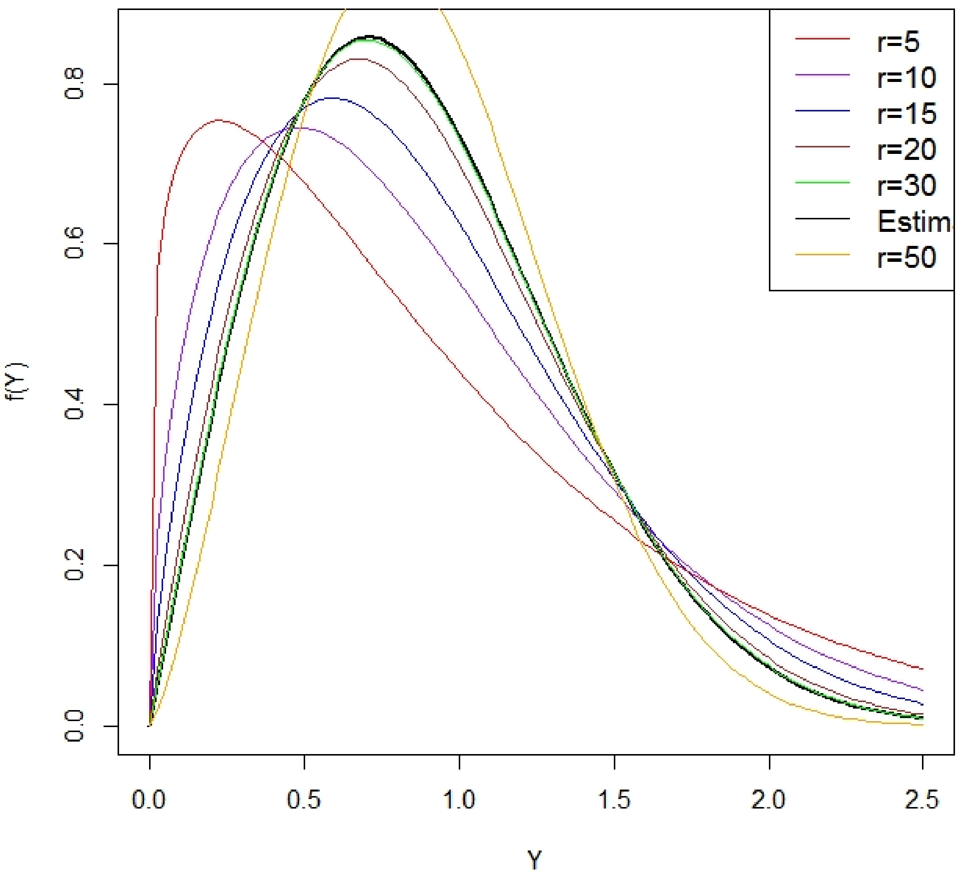

Figure 1: *Uniformly Minimum Variance Unbiased Estimator*

Table 1: *Estimate of R(t) Using Simulation Approach*

r		**10**		**15**		**50**	
t	$R(t)$	$\tilde{R}(t)$	$\hat{R}(t)$	$\tilde{R}(t)$	$\hat{R}(t)$	$\tilde{R}(t)$	$\hat{R}(t)$
15	0.988256	0.982199	0.987858	0.985162	0.989151	0.987757	0.988996
		- 0.006058	- 0.000399	- 0.003095	0.000894	- 0.000500	0.000740
		0.000342	0.000271	0.000167	0.000134	0.00004	0.000038
		0.050549	0.042173	0.039335	0.034016	0.020675	0.019854
		79.2216	73.58140	83.6338	80.0726	87.3715	86.9576
20	0.917915	0.910388	0.914626	0.915237	0.918637	0.918618	0.919682
		- 0.007528	- 0.003289	- 0.002678	0.000722	0.000703	0.001767
		0.003464	0.003984	0.002017	0.002234	0.000565	0.000585
		0.178809	0.188914	0.145604	0.152525	0.079166	0.080517
		86.77420	85.8389	89.1520	88.8500	89.8782	89.8563
25	0.798104	0.79897	0.793371	0.801307	0.797564	0.80119	0.799987
		0.000866	- 0.004733	0.003204	- 0.000540	0.003086	0.001883
		0.008834	0.010559	0.005178	0.005855	0.001404	0.001456
		0.295848	0.323471	0.237045	0.252153	0.125086	0.127418
		88.3448	88.3294	90.0189	90.0201	90.3765	90.3809
30	0.670807	0.680869	0.666155	0.679482	0.669284	0.675551	0.672371
		0.010061	- 0.004652	0.008675	- 0.001523	0.004743	0.001564
		0.012591	0.014486	0.007144	0.007823	0.001827	0.001874
		0.356619	0.38318	0.278715	0.291637	0.14255	0.144326
		88.0549	87.9343	89.7265	89.6547	90.4366	90.4364
45	0.389714	0.409555	0.387121	0.402778	0.387598	0.394959	0.390400
		0.019841	- 0.002593	0.013064	- 0.002116	0.005245	0.000686
		0.011188	0.011276	0.005701	0.005672	0.001285	0.001279
		0.331236	0.331191	0.245934	0.244903	0.118951	0.118657
		85.906	85.5138	88.3818	88.2268	90.2805	90.2736
50	0.32968	0.349176	0.32759	0.342179	0.327687	0.334531	0.330208
		0.019486	- 0.002090	0.012499	- 0.001993	0.004851	0.000528
		0.009403	0.009186	0.004661	0.004541	0.001024	0.001013
		0.301375	0.296388	0.221558	0.218317	0.106079	0.105504
		85.3279	84.9272	88.0684	87.9203	90.2329	90.2261
55	0.281492	0.300028	0.279791	0.293153	0.279655	0.285911	0.281906
		0.018536	- 0.001701	0.011661	- 0.001837	0.004419	0.000414
		0.007747	0.007379	0.003752	0.003595	0.000808	0.000795
		0.27164	0.263625	0.198166	0.193649	0.094143	0.093416
		84.8402	84.4478	87.8163	87.6789	90.1936	90.1871
60	0.242535	0.25984	0.241133	0.253268	0.24086	0.246532	0.242865
		0.017305	- 0.001402	0.010733	- 0.001675	0.003997	0.00033
		0.006327	0.005901	0.003004	0.00284	0.000636	0.000625
		0.243921	0.23421	0.176869	0.171694	0.083515	0.082718
		84.4303	84.0555	87.6127	87.4872	90.1612	90.1552
70	0.184604	0.199307	0.183617	0.193547	0.183228	0.187856	0.184825
		0.014703	- 0.000987	0.008943	- 0.001377	0.003252	0.000221
		0.004203	0.003797	0.001936	0.001794	0.000401	0.000391
		0.196726	0.185941	0.141401	0.135939	0.066187	0.065382
		83.7942	83.4651	87.3111	87.2079	90.1126	90.1076

Table 2: *Estimation of P Using Simulation Approach*

r, r'	**(10, 10)**		**(10, 15)**		**(15, 15)**		**(25, 25)**	
(m, n)	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$	$\tilde{P}$	$\hat{P}$
(5, 5)	0.66625	0.55623	0.79245	0.4739	0.85131	0.38238	0.88938	0.30929
	-0.00042	-0.11044	-0.00755	-0.32610	-0.00583	-0.47476	0.0005	-0.57960
	0.0131	0.00542	0.00825	0.0149	0.00521	0.01713	0.00226	0.01262
	0.37465	0.22246	0.29287	0.37771	0.21917	0.42422	0.15231	0.36142
	89.4513	80.3704	88.3766	85.7414	85.8522	89.4548	87.8912	88.9059
(5, 10)	0.66939	0.536	0.80224	0.46939	0.853	0.3872	0.88923	0.3223
	0.00272	-0.13067	0.00224	-0.33061	-0.00415	-0.46995	0.00034	-0.56659
	0.01343	0.00452	0.00617	0.01127	0.00525	0.01481	0.00244	0.0124
	0.38062	0.1954	0.25112	0.31962	0.23214	0.40524	0.15582	0.35937
	89.8872	78.3713	88.461	84.4431	87.5116	90.3086	87.722	89.3048
(10, 10)	0.66096	0.65675	0.79749	0.70795	0.84474	0.65375	0.88476	0.5824
	-0.00591	-0.00991	-0.00251	-0.09205	-0.01241	-0.20340	-0.00413	-0.30649
	0.00709	0.0055	0.0032	0.00192	0.00329	0.00603	0.00155	0.00975
	0.26423	0.21508	0.17838	0.119	0.18275	0.24016	0.12176	0.32224
	88.238	82.9781	87.8595	74.2657	86.8354	84.0915	87.1155	89.1193
(15, 15)	0.66441	0.66743	0.80245	0.77868	0.85492	0.76714	0.88795	0.71988
	-0.00226	0.00076	0.00245	-0.02132	-0.00223	-0.09000	-0.00094	-0.16901
	0.00604	0.00588	0.00166	0.0005	0.00127	0.00126	0.00092	0.00398
	0.26138	0.25845	0.13777	0.06267	0.11497	0.10138	0.09689	0.19464
	89.9864	89.6238	90.897	75.9468	88.7437	77.2489	88.1403	85.3064
(15, 25)	0.66978	0.66943	0.79561	0.76784	0.85674	0.76313	0.88892	0.72191
	0.00311	0.00277	-0.00439	-0.03216	-0.00040	-0.09401	0.00003	-0.16698
	0.00324	0.00312	0.00234	0.00088	0.00111	0.00131	0.00065	0.00287
	0.18845	0.18624	0.15194	0.08226	0.10749	0.10513	0.08171	0.16528
	90.1082	90.2303	87.77	75.2671	89.2306	75.7539	88.6398	85.857
(25, 25)	0.66432	0.66728	0.79942	0.79972	0.85627	0.84052	0.88749	0.84112
	-0.00234	0.00061	-0.00058	-0.00028	-0.00088	-0.01662	-0.00140	-0.04777
	0.00296	0.00304	0.00182	0.00149	0.00084	0.00032	0.00054	0.00027
	0.17797	0.18026	0.14352	0.12584	0.09115	0.04498	0.0799	0.05119
	89.8014	89.7715	90.4715	88.2366	88.2113	73.121	90.712	76.8041
(25, 30)	0.66545	0.66744	0.79985	0.7996	0.85393	0.83576	0.88492	0.84001
	-0.00122	0.00077	-0.00015	-0.00040	-0.00322	-0.02138	-0.00397	-0.04888
	0.00278	0.00285	0.00145	0.0012	0.0013	0.00054	0.00062	0.00031
	0.17136	0.1736	0.1295	0.11518	0.11215	0.05916	0.07973	0.04987
	89.5562	89.6143	90.8001	89.2747	87.3483	72.9983	88.8342	74.4235
(30, 30)	0.66466	0.67086	0.79943	0.80168	0.85183	0.84563	0.88852	0.86285
	-0.00201	0.00419	-0.00057	0.00168	-0.00531	-0.01151	-0.00037	-0.02604
	0.00223	0.0019	0.00127	0.00117	0.00115	0.00067	0.00048	0.00014
	0.16182	0.14227	0.11788	0.11381	0.10656	0.07456	0.0739	0.02872
	91.3111	89.3985	89.6646	89.5152	87.8413	80.5097	90.0725	71.648

Figure 1 is plotted $\hat{f}(y, \gamma, \beta, \mu)$ under type II censoring for various values of r $= 5(5), 10, 15, 20,$ 30 and 50 and concludes that the curves of $\hat{f}(y, \gamma, \beta, \mu)$ getting closer to the curve of $f(y; \gamma, \beta, \mu)$ as r increases.For $r = 30$, validates the consistency property of the estimators, because the curves overlap.We have presented a simulation study when γ is unknown with the bootstrap re-sampling procedure for r $= 10(5)15$ and 50 while other parameters are known. If $G\left(y^{-1}; \mu\right) = y^2, \beta = 1$, and $\gamma = 1$. Table 1 shows computation using 500 bootstrap replications with a 95% confidenc coefficien to obtain the estimated value of UMVUES and MLES for $R(t)$, bias, variance, and MSES, for different values of t.Also display simulation trials using the bootstrap re-sampling procedure for (n, m) $= (5, 10)$,$(10, 10), (15, 15), (15, 25), (25, 25), (25, 30), (30, 30))$ across different $(r', r') = (10, 10), (10, 15), (15, 15)$ and $(25, 25)$, while γ_1 and γ_2 are unidentifie but the other parameters are identifie to estimate P. The free sample is produced as of (1), if $G\left(x^{-1}; \mu\right) = \log(x)$, $\beta_1 = \beta_2 = 1$, $G\left(y^{-1}, \mu\right) = \log(y), \gamma_1 = 1$ and $\gamma_2 = \frac{1}{2}, \frac{1}{4}, \frac{1}{6}$ and $\frac{1}{8}$. Table 2 shows computations using 500 bootstrap replications with a 95% confidenc coefficien to obtain the estimated value of UMVUES and MLES P, bias, variance, and mean sum of squares (MSES).

7. Discussion

We established estimation algorithms for the inverse distributions family based on type-II censoring in this paper.The point estimates are taken into consideration. Hypotheses were generated for many parametric functions, and UMPCR was achieved. Simulation techniques are used to study the efficiency of the UMVUES and MLES of reliability functions, as well as other parameters. For type-II censorship, the UMVUE of $R(t)$ is superior to the MLE of $R(t)$ for different t. Furthermore, for all values of (r, r'), the MLE of P outperforms the UMVUE of P. On the other hand for large t, UMVUE comes to be more effective than MLE of $R(t)$. From the study of P it has been determined that for $m < n$, UMVUE is superior to MLE for P. Alternatively, for $n < m$, it is concluded that MLE is superior to UMVUE for P. As n and m rise both estimators yield equally effective. Using Figure 1, we validated the consistency property of the estimators under censoring approaches.

8. References

[1] Awad, A. M and Gharraf, M.K. (1986). Estimation of $P(Y < X)$ in the Burr case: comparative study *Commun.statist. B- Simul. Comp.*, 15(2), 389-403.

[2] Chaturvedi, A. ,Kang SB and Pathak T. (2016). Estimation and testing procedures for the reliability functions of generalized half logistic distribution *J. Korean Stat Soc.*, 45, 314-328.

[3] Chaturvedi, A. ,Kumari T. (2017). Estimation and testing procedures for the reliability functions of a general class of distributions *Commun. Stat-Theor. Meth.*, 46(22), 70-82.

[4] Chaturvedi, A. ,Kumari T. (2019). Robust Bayesian Analysis of a generalized inverted family of distributions *Comm. Stat. Simul. Comput.*, 48(8), 2244-2268.

[5] Chaturvedi, A. and Tomer, S.K. (2002). Classical and Bayesian reliability estimation of the negative binomial distribution *Jour. Applied Statist. Sci.*, 11 (1), 33-43.

[6] Chaudhary, A. and Chauhan, K. (2009). Estimation and testing procedures for the reliability function of Weibull distribution under type I and type II censoring, Journal of statistics sciences *Jour. Applied Statist. Sci.*, 1(2), 121-136.

[7] Chaturvedi, A. ,Pathak, A. and Kumar, N. (2019). Statistical inferences for the reliability functions in the proportional hazard rate models based on progressive type-II right censoring *Journal of Statistical Computation and Simulation*, 89 (1), 1-31.

[8] Chaturvedi, A. and Malhotra, A. (2018) Estimation of $P(X > Y)$ for the positive exponential family of distributions *Statistica*, 78(2), 149-167.

[9] Constantine, K. , Karson, M. and Tse, S. K. (1986). Estimators of P (Y<X) in the gamma case *Commun. Statist. - Simul. Comp.*, 15, 365-388.

[10] Enis, P. and Geisser, S. (1971). Estimation of the probability that $Y < X$ *Jour. Amer. Statist. Assoc.*, 66, 162-168.

[11] Erto, P. (1989). Genesis, properties, and identificatio of the inverse Weibull lifetime model. Statistica Applicata *Jour. Applied Statist. Sci.*, 1, 117-128.

[12] Johnson, N. L. and Kotz, S. (1970). Continuous Univariate Distributions-I *John Wiley and Sons, New York*.

[13] Lin, C.T. , Duran, B. S. and Lewis, T. O. (1989). Inverted gamma as a life distribution *Microelectron Reliab.*, 29, 613-626.

[14] Nigm, El. S.M. , Aboeleneen, Z. (2007). Estimation of the parameters of Inverse Weibull distribution with progressive censoring data *Journal of Applied Statistical Science*, 15,39-47.

[15] Rohatgi, V. K. , and Saleh, A. K. Md. E. (2012). An introduction to probability and statistics, . *John Wiley and Sons, New York*.

[16] Surles, J. G. and Padgett, W. J. (2001). Inference for reliability and stress-strength for a scaled Burr type X distribution. *Lifetime Data Analysis*, , 7, 187-200.

[17] Weerahandi, S. and Johnson, R. A. (1992). Testing reliability in a stress-strength model when x and y are normally distributed. *Technometrics*, 34(1), 83-91

[18] Yadav, A.S. , Singh, S.K. and Singh U. (2018). Estimation of stress-strength reliability for inverse Weibull distribution under progressive type-II censoring scheme. , *Journal of Industrial and Production Engineering*, 35(1), 48-55.