About the Teacher. Alexander D. Soloviev[1]

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Abstract

The article talks about a remarkable man and an outstanding scientist – Alexander Dmitrievich Soloviev. He was Doctor of Physics and Mathematics, Professor, Laureate of the State Prize of the USSR, Professor of the Probability Theory Department of the Faculty of Mechanics and Mathematics at Lomonosov Moscow State University. Alexander Dmitrievich lived an amazing creative life that can serve as an example for modern researchers. Victor Kashtanov, his student, shares his recollections and reflections on some episodes in the life of his teacher and friend.

Key words: memories, Soloviev, mathematicians, reliability assessment, redundancy
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Such events happen in life that you only realize their significance many years later. One of such events in my life I consider my meeting with Alexander Dmitrievich Soloviev (Doctor of Physical and Mathematical Sciences, Professor, Laureate of the State Prize of the USSR, Professor of the Department of Probability Theory of the Faculty of Mechanics and Mathematics, Moscow State University named after M.V. Lomonosov, Professor, Department of Probability Theory, Faculty of Mechanics and Mathematics, Moscow State University, M.V. Lomonosov). Already, being a young specialist, working for a year after graduating from the Faculty of Mechanics and Mathematics of Moscow State University, in 1958 I went to work at Scientific Research Institute № 17 (NII-17) in the Mathematical Laboratory. At that time, A. D. Soloviev, a thirty-year-old associate professor at the Department of Mathematical Analysis of the Faculty of Mechanics and Mathematics, Moscow State University, was "moonlighting" in this laboratory (this was the former name of my part-time job). It was there that our first meeting took place. Later it turned out that we could have met earlier. I was told that he led classes in mathematical analysis in some groups of our course, but in our student group he was not. Later, my classmates talked about him as a good teacher - knowledgeable, fair, calm.

In the mathematical laboratory, Alexander Dmitrievich solved a wide variety of problems. Naturally, practical problems were solved. Consequently, solutions had to be brought to numbers, complex cumbersome formulas had to be simplified, and the accuracy of the obtained approximated results had to be evaluated. This is where his highest mathematical qualification as an analyst became apparent. In 1955 Alexander Dmitriyevich defended his PhD thesis "The problem of moments for integer analytic functions", the thesis supervisor was the corresponding member of the USSR Academy of Sciences, Professor A. O. Gelfond. It is also necessary to note fruitful cooperation of Alexander Dmitrievich with Prof. M.A. Evgrafov (see joint works published in the Doklady of the

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USSR). (see joint papers published in Reports of the Academy of Sciences of the USSR: "On one general criterion of basis", vol. 113, no. 3; "Determination of convergence class of interpolation series for some problems", no. 113, no. 5; "On one class of reversible operators in the ring of analytic functions", no. 114, no. 6) - an important expert on asymptotic methods (M.A. Evgrafov "Asymptotic estimates and integer functions", Moscow, 1962). Alexander Dmitrievich had a perfect command of subtle analytical methods for constructing asymptotic expansions and asymptotic evaluations, which he successfully used in the study of practical problems.

Mathematicians were also faced with the problem of studying stochastic models. At creation of radio equipment, the tasks of random processes processing, construction of estimations of correlation functions and spectral densities were solved. Tasks of evaluating reliability of developed equipment also arose. I have in front of me report "Mathematical Problems of Reliability of Radio-Electronic Equipment", signed by Aleksandr Dmitrievich (original signature) and approved by the management on March 19, 1958. The content of the report is surprising. Firstly, it is felt that there is still no unified terminology accepted in this science. Therefore, the reliability of an element is understood as the probability of failure-free operation, there is no concept of failure rate, this function is called a reliability characteristic. On the other hand, the concept of an aging element is used, estimates are constructed from below of the probability of no-failure operation of an aging element, which depend on the numerical characteristics determined by statistical tests. It is characteristic for all works of Alexander Dmitrievich – to bring mathematical formulas to practical use because the numerical characteristics can be obtained by the results of statistical tests, the estimates from below give the guaranteed value of the indicator. Reliability of systems with arbitrary structure is investigated, reliability of elements of which depends on a condition of others.

The content of a simple ordinary technical report shows how far Alexander Dmitrievich has advanced in formulating, solving, and understanding reliability problems.

In the late 50's - early 60's of the last centuries there were quantitative accumulations of results in the mathematical reliability theory, separate mathematical models under different, sometimes very significant, limitations were investigated, in a certain sense the terminology was formed, and specialists began to speak the same language and to understand each other better. Alexander Dmitrievich took an active part in forming the principles of constructing the mathematical theory of reliability. Suffice it to point out the work "Mathematical justification of the reliability theory", published in 1958 (Radioelectronic Industry, No.4).

Much later, a joint work "Mathematics and Reliability Theory" by B.V. Gnedenko and A.D. Soloviev was written (Izdatel'nye Znanie. New in Life, Science and Technology. Series "Mathematics, Cybernetics". №10. 1982), in which the authors outlined the history and their participation in this process.

In 1960, the leadership of the laboratory (Yuri Alexandrovich Arkhangelsky, later Doctor of Physical and Mathematical Sciences, Professor of the Department of Theoretical Mechanics at the Faculty of Mechanics and Mathematics, Moscow State University) offered us young specialists, who had worked in the laboratory for three years, to go to graduate school. To my indescribable joy, Alexander Dmitrievich agreed to be my supervisor. I hope I did not let my teacher down in the future, since I was his first student, who defended both his master's and doctoral dissertations. So, our scientific cooperation and collaboration began.

Since the early 60's Alexander Dmitrievich has been working with postgraduate students. But from the beginning of the 70's after his return from Cuba (Alexander Dmitriyevich spent several years there, being engaged in teaching and scientific work) the work with graduate students acquired a mass character. It should be noted that more than 30 postgraduate students under the guidance of Alexander Dmitrievich defended their doctoral theses. Some of them later became Doctor of Science. Let me mention the names of Doctor of Physical and Mathematical Sciences, Professor O. P. Vinogradov, Doctor of Physical and Mathematical Sciences, Professor A. M. Zubkov, Doctor of Physical and Mathematical Sciences, Professor O. Sakhobov. Alexander Dmitrievich never left his graduate students to the mercy of fate. His help was concrete and substantial. He spent a lot of time talking with a graduate student, showing ways to solve the problem, correcting mistakes.

Criticism was always benevolent. He was generous in imparting knowledge and new results. Suffice it to say that I had no joint work with my teacher until 1983.

If we adhere to chronology, then the beginning and the middle of the 1960s contain the formation of the remarkable scientific team of B.V. Gnedenko, Yu.K. Belyaev, A.D. Soloviev, organization of the seminar on mathematical reliability theory at the Mechanical and Mathematical Faculty of MSU, organization of the Reliability Study and cycles of lectures at the Polytechnic Museum on reliability and progressive methods of quality control of products. Of special note is the writing of the monograph "Mathematical Methods in the Reliability Theory" in 1965 (Moscow. Nauka.1965).

Writing a monograph summarizing the results of the development of the mathematical theory of reliability was an overdue necessity. Now, evaluating the appearance of this book, we can say that it has shaped domestic mathematical reliability theory and determined further ways of its development. It is a milestone in development of the domestic mathematical theory of reliability.

Boris Vladimirovich Gnedenko defined in this monograph the subject of the mathematical theory of reliability, highlighting the life cycles of complex technical systems, defining the theoretical basis of the theory and the ultimate practical tasks facing it.

"A general scientific discipline that studies general methods and techniques to be followed in designing, manufacturing, accepting, transporting and operating products to ensure maximum efficiency in the process of their use, as well as developing general methods of calculating the quality of devices according to the known qualities of their components" - this is the definition by B. V. Gnedenko.

The book, published in 1965, on the one hand, summed up the development of the domestic mathematical theory of reliability, on the other hand, determined the further directions of development of this theory, defined the relationship of the mathematical theory of reliability with the classical probability theory, the theory of random processes with the theory of mass service, mathematical statistics.

On this basis, the following areas were intensively developed at the time of writing:

- Problems of predicting reliability and durability, studies of distributions of positive random variables and their properties (aging and aging distributions),
- reliability characteristics of various structures (systems) under given distributions of no-failure times of their individual parts (study of distributions of functions from a set of random quantities) for restorable and non-restorable systems,
- evaluation of reliability characteristics based on test results, construction of various test plans.

At the stage of theory formation these very directions were considered by the authors to be the main ones. Therefore, basing on the conception of formation of mathematical reliability theory proposed by B. V. Gnedenko, the authors set forth in the monograph modern (at that time) results on estimation of reliability characteristics by test results (Y. K. Belyaev) and on research of reliability characteristics of different structures with elements of operation - restoration of failed elements (A. D. Soloviev).

In the sections of the monograph written by Alexander Dmitrievich the strengths of his mathematical and analytical qualifications were evident. Yu. K. Belyaev wrote about it in "Reliability" magazine (№4, 2006). He meant the time when their joint work had not yet begun. Let us cite this quote: "At that time, I learned from V.A. Kashtanov that very similar problems interested Alexander Dmitrievich Soloviev, whom Victor Alexeevich considered (and it was in fact) an unsurpassed virtuoso of asymptotic methods of mathematical analysis.

To the period of the early 1960s we start negotiations of Alexander Dmitrievich with Andrey Nikolayevich Kolmogorov about his transfer to the "Probability Theory" department. When this transfer took place, a group of authors was formed, which created a classical work called "Mathematical Methods in Reliability Theory".

Much later, reviewing his scientific work, Alexander Dmitrievich wrote in a letter to Igor Nikolayevich Kovalenko about the stages of his scientific activity (we will cite this excerpt in full):

"My scientific work went through several stages:

1. 1960s-70s. The construction of a mathematical theory of reliability;
2. 1970-80s. Creation of asymptotic theory, which allows to estimate the reliability of restored systems under small load. The main thing here is the proof of *the limit theorems of the uniform type*, in which in the limit transition all parameters and functions defining the system change, and the limit transition itself is defined by some small functional;
3. 1980s-90s. The transition from limit theorems to asymptotically exact bilateral inequalities from which, in particular, the limit theorems themselves follow;
4. I've had several themes in recent years:
   - The study of restorable systems with arbitrary service disciplines. Finding asymptotically optimal disciplines;
   - Reliability assessment of restorable systems with high redundancy and finite load."

As can be seen from the above quote, these periods have no clear boundaries, they overlap. For us it will be important to highlight the main ideas and achievements of Alexander Dmitrievich in these years. In the same letter Alexander Dmitrievich indicates his monograph "Mathematical Methods in the Reliability Theory" as the main work of the first period, which was republished many times in different countries. There are 7 editions of this monograph in Moscow, Berlin, Bucharest, New York, Budapest, and Japan during 1965-1972.

In addition, more than 40 articles were written during this period. If we evaluate in general the places of publication and the nature of publications, we can trace a deep interest of practitioners (industry representatives) in the theoretical research of mathematicians. The main publications that published Alexander Dmitrievich's works are departmental journals: "Radioelectronic Industry," "Problems of Radioelectronics," and "Automation and Computer Engineering. Three articles were published in the collected articles "Cybernetics to the service of Communism" (1964), which collected papers delivered at three scientific seminars: on reliability theory (Dependability Section of the Scientific Council on Cybernetics under the Presidium of the USSR Academy of Sciences); on mass service theory (the Mechanical and Mathematical Department of the Lomonosov Moscow State University, the Moscow State University, the Moscow Engineering Physics Institute, the Central Research Institute of the USSR Academy of Sciences); and on mass service theory (the Department of Physics and Technology of the Moscow State University). A seminar on the theory of reliability was held jointly by the Department of Mechanics and Mathematics, Lomonosov Moscow State University, and the Popov Radio Engineering and Telecommunications Scientific Research Institute. A.S. Popov). The academic journals "Proceedings of the USSR Academy of Sciences, Technical Cybernetics" and "Proceedings of the USSR Academy of Sciences, OTN, Power Engineering and Automation" also published Alexander Dmitrievich's works of that period.

For the second and third periods of AD's creative activity we refer to the monograph "Questions of the Mathematical Theory of Reliability", Moscow, Radio and Communications, 1983. In the preface to this monograph B.V. Gnedenko, describing the sections belonging to Alexander Dmitrievich, wrote: "The author managed to find an elegant manner of exposition, which allowed him to put an extensive material in a comparatively small volume. It is also noteworthy that the author does not make assumptions about the exponential distribution of the duration of no-failure operation or recovery time, and he managed to obtain general results under very broad assumptions. It is also important that in the limit theorems he obtained very accurate bilateral inequalities, which can be successfully used in practical situations.

Note that in addition to the "traditional" sections and research directions written by A. D. Soloviev and Yu. K. Belyaev, other sections appeared in the 1983 book. I. N. Kovalenko complemented the material with the subsection "Methods of statistical modeling" (the chapter "Multidimensional Markov Processes that describe complex systems and their statistical modeling" and the chapter "Analytical-statistical method of calculation of characteristics of high-reliability systems"). There also appeared the section "Problems of Optimization of Reliability and Efficiency of Functioning" written by E.Yu. Barzilovich, V.A. Kashtanov and I.A. Ushakov. If in the initial works the quality (efficiency) was estimated by distribution of the time of no-failure operation or by mathematical expectation of this time, then in the later works other indicators were investigated, which were defined as functionals built on trajectories of random processes describing evolution of a technical system. The solution of the problem was completed by the optimization of these functionalities (indicators).

From the mid-1970s to the mid-80s, Alexander Dmitrievich published about 30 works, many of them written jointly with his graduate students.

Note that Alexander Dmitrievich publishes his results during this period in the "Znanie" publishing house. This is since at the end of the 1960s the reliability cabinet begins to work on the premises of the Polytechnical Museum in Moscow, where the seminar on reliability and progressive methods of quality control of products, where mathematicians consult for industry representatives on practical problems of reliability arising during development of various apparatuses, is held. In the large auditorium of the Polytechnic Museum cycles of lectures on reliability for engineers are organized. The materials of the lectures are published in separate brochures. These were reviews of mathematical methods in reliability theory, the volume of materials was 40-50 pages. Let us point out three issues under the title "Fundamentals of the mathematical theory of reliability" (Moscow, Znanie, 1975). Of course, the mathematical results are adapted to the audience, but this gives the theory a practical orientation. Communication with the engineering audience allowed us to feel the applied problems, to describe new models, to formulate new problems. (Note that Alexander Dmitrievich had a great experience of delivering lectures at the Faculty of Mechanics and Mathematics of Moscow State University for specialists with higher engineering education and wishing to improve their mathematical qualification, for the so called "engineering stream"). Let's give the name of one of Alexander Dmitrievich's works, published after his lecture course for engineers: "Heuristic derivation of reliability characteristics of standby systems with fast restoration" (Moscow, Znanie, 1968). Such approach of Alexander Dmitrievich to the presentation of the material testifies to his desire to give practitioners a tool that could be easily used when solving practical problems.

The practical orientation of Alexander Dmitrievich's mathematical works is constantly traced. For him, the main task was not only to get some dependence (formula, equation, limit theorem), but also to show how this result can be used, to bring the research to number and to develop practical recommendations. He sought to formulate simple sufficient conditions, the verification of which allowed this mathematical result to be used in practice.

As noted above, Alexander Dmitrievich's research in the 80-90s dealt with subtle issues of asymptotic analysis of mass service and reliability models. He solves more complicated problems of constructing asymptotically accurate bilateral estimators, which allow not only to obtain the limiting values of the characteristics under study, but also to determine the convergence rates. The solution of these problems is related to the problem of summing up a random number of random terms. In his book "Boris Gnedenko in Memoirs of Students and Associates" (URSS. Moscow, 2006) I. N. Kovalenko wrote: "Boris Gnedenko's great merit was introducing into the mathematical theory of reliability the methods of summation theory of independent random variables. This stimulated the creation of a new direction - the limit theorems of the theory of redundant systems in the "triangular" scheme. The greatest contribution to the development of this direction was made by A.D. Solov'ev and his students...".

The works devoted to the construction of asymptotically exact bilateral estimates of the characteristics under study were mentioned above. In a certain sense, the chapters written by Alexander Dmitrievich in his 1983 monograph are a milestone work. These materials summarized the results of the studies that began with the publications of 1976-1977.

There are numerous examples in this book in which these two-way estimates can be used:

- Loaded duplication with recovery;
- Lightweight duplication with prophylactics;

- Temporary Reservation;
- Loaded duplication with recovery.

Let's mention works written together with O. Sahobov "Two-sided estimations of reliability of restored systems" (Izvestiya AS UzSSR, series Physics-Mat, ¹5, 1977), "Two-sided estimations of reliability in general redundancy model with one repair unit" (Izvestiya AS USSR. Technical Cybernetics, ¹4, 1977), "Two-sided estimations for system failure probability on one period of regeneration" (Izvestiya AS UzSSR, series Physics-Mat, ¹2, 1977).

In the 90's Alexander Dmitrievich published several works on reliability estimation of different systems. In collaboration with D.G. Konstantinidis he wrote the paper "Uniform reliability estimation of a complex restorable system with unlimited number of repair units" (Vestnik (Herald) of MSU, Series Mathematics, Mechanics. No.3, 1991), "Reliability Assessment of a Complex Reconstruction System with Unlimited Number of Repair Units" (Probability Theory and its Applications, vol. 37, issue 1, 1992), "Reliability Assessment of a Cold Reserving with Restoration Model in Case of Unlimited Number of Repair Units" (jointly with A. P. Polyakov, Vestnik (Herald) of MSU, Math. No.5, 1992), "An Estimation of the Average Lifetime of Reconstructed Systems" (jointly with N.G. Karaseva, Vestnik (Herald) of MSU, Math. №5, 1998).

All of Alexander Dmitrievich's co-authors mentioned in the latter papers were his graduate students, so it is obvious that all the mathematical ideas presented in these papers belong to his supervisor.

Researchers have long seen the connection between reliability models and mass maintenance models. Alexander Dmitrievich, investigating reliability models of restorable systems, devoted several his works to the analysis of various maintenance and restoration disciplines.

These works solve the problem of finding optimal recovery (maintenance) disciplines, which is fundamentally important from a practical point of view. One of the first works in this direction was the article "Optimal maintenance of restoring systems" (together with V.V. Kozlov. Izvestia of the Academy of Sciences of the USSR. Technical Cybernetics, Nos.3,4, 1977). This was followed by the paper "On a System with Maintenance Discipline of the First Demand with Minimum Remaining Length" (jointly with A. V. Pechinkin and S. F. Yashkov. Izvestia of the Academy of Sciences of the USSR. Technical Cybernetics, No.5, 1979). Let us also note the paper "Analysis of M/G/1/∞ system for different service disciplines" (The Theory of Mass Service. Proceedings of the All-Union School-Seminar. M. VNIISI, 1881), in which a wide range of service disciplines is investigated, a review of the results is given, and a list of characteristics obtained in closed form for various disciplines is given.

Besides his works devoted to the analysis of mass-service and reliability models, Alexander Dmitrievich wrote some historical and mathematical works. Let us mention the works related to the history of asymptotic methods of analysis, devoted to the problems very close to his scientific interests. These include the work "On the History of the Creation of the Passage Method" (published jointly with S. S. Petrova. SPb. These include: "On the History of the Creation of the Passage Method" (together with S. S. Petrova, SPb), "Historical and Mathematical Studies, Vol. 35, 1994", "P. A. Nekrasov and the Central Limit theorem of Probability Theory" (M. Historical and Mathematical Studies, Second Series, Issue 2(37), 1997), and "Asymptotic Methods of Laplace" (M. Historical and Mathematical Studies, Second Series, Issue 4(39), 1999).

Next, let us proceed to characterize the content of works on the mathematical theory of reliability and the ideas embedded in them.

The first works concerned the construction of probabilistic characteristics of reliability of systems by the characteristics of its individual parts. In other words, it was about the study of functions from random variables. However, when studying the process of functioning, when the model includes restoration of failed subsystems, it becomes necessary to consider the evolution of the system in time. Therefore, random processes are used to describe the model. In his first works, Alexander Dmitrievich used Markov processes. Regarding one of them (1964) it was written: "A. D. Soloviev's large work "On Reserving without Recovery" is a serious study, where the theory of reserving is systematically outlined, and many questions are far advanced based on random processes of death and reproduction" (see material from the editors in the article "Cybernetics - to the Service of Communism").

Alexander Dmitrievich sought to set and solve problems as much as possible under general assumptions with respect to the initial assumptions. Where it was possible to abandon some special assumptions (for example, exponentiality of some initial distributions), the problem was solved in general assumptions. With this approach, it is very rare to get a closed-form result. Then Alexander Dmitrievich's high erudition as a specialist in asymptotic methods worked. In his works, the terms asymptotic distribution, rare event, fast recovery, and the like appear. And all this in the study of reliability models. Here are the titles of some of these works: "Asymptotic Distribution of the Lifetime of a Duplicated Item" (Proceedings of the Academy of Sciences of the USSR, Technical Cybernetics, No. 5, 1964), "One Combinatorial Identity and its Application to the Problem of the First Occurrence of a Rare Event" (Probability Theory and its Applications, vol. XI, vol. 2, 1966), "Reserving with Fast Recovery" (Izvestia AS USSR, Technical Cybernetics, No.1, 1970), "Asymptotic Behavior of the Moment of the First Occurrence of a Rare Event in a Regenerating Process" (Izvestia AS USSR, Technical Cybernetics, No.6, 1971), "Asymptotic Analysis of Post-Failure Reliability Characteristics" (Proceedings of III All-Union School Meeting on Mass Service Theory, vol. 1, MSU, 1976).

Several works on asymptotic analysis dealt with mass service models. The relationship between reliability models and mass service models was mentioned above.

In 1972 Alexander Dmitrievich successfully defended his doctoral thesis "Systems of mass service with fast service" in the council of the Faculty of Mechanics and Mathematics of Moscow State University. Here is a citation from the abstract of this thesis, which fully demonstrates the characteristic features of Alexander Dmitrievich as a mathematician. Here is what he wrote in the thesis abstract.

Let us note two characteristic features of the work:

1. Almost everywhere the limit theorems have a uniform form, in other words, all the initial distributions and parameters change in the limit transition, and the topology of the limit transition is given by some small functional on distributions and parameters;
2. Each limit theorem looked for the most effective conditions, that is, conditions expressed explicitly and quite simply through the initial characteristics.

The study of numerous specific models of mass service and reliability eventually made it possible to develop a general basic mathematical model of a random process describing the evolution of the system under study, which can be used to judge the efficiency of its functioning.

It turns out to be a *regenerating random process* for which some event may occur at some point during the regeneration period. In specific models this event can be treated as the first loss of demand, system failure, etc. adverse events. Already in the 1983 monograph we find paragraphs and sections "Limit theorems for regenerating processes, exact distribution of the moment of the first event occurrence, regenerating processes of special type, estimation of event occurrence probability".

To demonstrate Alexander Dmitrievich's profound ideas, let us analyze the article "One General Model of Redundancy with Restoration," written jointly with D.B. Gnedenko (Izvestia of the Academy of Sciences of the USSR. Technical Cybernetics, No. 6, 1974).

In this paper we investigate a system consisting of *n+1th* element. During failures the elements are restored. There are r repair crews for repair, the duration of repair are independent random variables with an arbitrary distribution *G(x)*. In addition, with respect to the structure of the system, it is assumed that there are *n-r* places to wait for repairs. If at time *t* there are *k* failed elements in the system, $\xi(t) = k$, then the next failure appears after a random time distributed by the exponential law with the parameter λ_k . The failure of the system occurs at the moment of failure *of n+1th* element, $\xi(t) = n + 1$.

It is easy to see that the described reliability model coincides completely with a mass service system having r serving devices, *n-r* queue places, and for which the intensity of the input flow depends only on the number of demands in the system. An adverse event is the first loss of a demand.

Since service times are distributed arbitrarily, and several demands can be served simultaneously, the random process $\xi(t)$ - number of demands in the system at time t does not have good properties (such as the Markov property), which allows to involve known mathematical methods. The only property that can be used is the regeneration property. Moments of regeneration are moments of release of the system from requirements. Regeneration periods have two components – a free (random) period with an exponential distribution and an occupancy period when there are requirements in the system.

For such a regenerating process the theorem is proved

$$\lim_{\lambda_0 T_1 \to 0} P\{\lambda_0 q\tau > x\} = e^{-x},$$

where q *is the* probability of claim loss at one regeneration period, T_1 is the mathematical expectation of the employment period,τ is the moment of the first claim loss.

However, it is difficult to use the theorem in the presented form because it is necessary to express through the initial characteristics the mathematical expectation of the employment period and the probability of claim loss at one regeneration period.

Therefore, simpler sufficient conditions for convergence to the exponential distribution are formulated:

$$\text{If } T = \int_0^\infty x dG(x) \to 0 \text{ then } P\{\lambda_0 q\tau > x\} \to e^{-x}$$

Note the original method of proving this theorem - the construction of a *majority process*. We construct a random process $\bar{\xi}(t)$ - number of demands at time t for a single-channel mass service system with an infinite queue, which receives a Poisson flow of demands with parameter $\lambda = max_{0\le k\le n} \lambda_k$. It is argued that the process $\bar{\xi}(t)$ majorizes the process$\xi(t)$ in the sense that any realization of the process $\xi(t,\omega)$ is not superior to the *corresponding* realization of the process $\bar{\xi}(t,\bar{\omega})$,$\xi(t,\omega) \le \bar{\xi}(t,\bar{\omega})$. Here we need to clarify what the correspondence of the realizations of the two processes means. The realizations of random processes $\xi(t,\omega)$ *u* $\bar{\xi}(t,\bar{\omega})$ are defined by the intervals between neighboring moments of arrival of demands $\vec{t} = \{t_k, \ \ k \ge 1\}$ and the service times of each demand $\vec{\tau} = \{\tau_k, \ \ k \ge 1\}$. If these sequences are the same, then the above inequality is fulfilled by in the second model there is one servicing device. The same inequality holds for the occupancy periods $\nu_1(\vec{t},\vec{\tau}) \le \nu_2(\vec{t},\vec{\tau})$ of the first and second models. Obviously, the occupancy period is a nonincreasing function of the intervals t_k. Therefore, similar inequalities are true for the mathematical expectations of the occupancy periods, provided that the distributions of the intervals between the moments of arrival of demands have exponential distributions with parameters $\lambda \ge \lambda_k$. This proves the theorem under simply testable conditions.

However, the question remains open about determining the probability q of the loss of the claim on the regeneration period. And here an original solution is proposed. It is proved that under certain conditions the probability q is equivalent to the probability q_0 of loss of a claim along a monotone trajectory when no claim has been served during the regeneration period (monotone trajectory method). We end up with a very nice result

$$lim_{\frac{m_{n+1}}{m_1^n} \to 0} P\,\{\lambda_0 I\tau > x\} = e^{-x},$$

where $I = \lambda_1\lambda_2 \ldots \lambda_n \int_0^\infty \frac{\{\int_x^\infty [1-G(y)]dy\}^{r-1} x^{n-r}}{(r-1)!(n-r)!} [1-G(x)]dx$, $m_k = \int_0^\infty x^r dG(x)$ and easily verifiable conditions for its fulfillment. Some special cases are also given at r=n the equality $P\{\lambda_0 q\tau > x\} \to e^{-x}$ is valid, at r=1 the equality$I = \frac{\lambda_1\lambda_2 \ldots \lambda_n}{n!} m_n$ is valid.

This detailed analysis shows the depth of the ideas proposed by A. D. Soloviev, and the use of these ideas in the asymptotic analysis of other models testifies to their effectiveness.

Concluding these notes, I would like to say that in life Alexander Dmitriyevich Soloviev was a cheerful and benevolent man, treating any interlocutor with respect. He played the guitar, knew many stories and anecdotes, in the company was the soul of society. He liked to joke around. In 1967, at a Central Asian market in Tashkent, he would offer everyone a taste of bitter green pepper and immediately offer some fruit to eat to anyone who fell for his joke.

Time flies inexorably forward. It's been twenty-one years since Alexander Dmitrievich left us. Let's keep the memory of this wonderful man and be grateful to him for everything he did for us.

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