BAYESIAN INTERVAL ESTIMATION FOR THE PARAMETERS OF POISSON TYPE RAYLEIGH CLASS MODEL

Rajesh Singh
•
Department of Statistics, S. G. B. Amravati University, Amravati, India
rsinghamt@hotmail.com

Preeti A. Badge
•
Department of Statistics, S. G. B. Amravati University, Amravati, India
preetibadge10@gmail.com

Pritee Singh
•
Department of Statistics, Institute of Science, Nagpur, India.
priteesingh25@gmail.com

Abstract

In this article, two sided Bayesian interval is proposed for the parameters of Poisson type Rayleigh class software reliability growth model. In this work, the failure intensity function, mean time to failure function and likelihood function of this model have been derived by considering parameter total number of failures i.e. γ_0 and scale parameter γ_1. The mathematical expressions of Bayesian interval for the parameters have been obtained by considering non informative priors. The performance of proposed Bayesian interval is studied on the basis of average length and coverage probability. Average length and coverage probability is obtained by using Monte Carlo simulation technique after generating 1000 random samples. From the obtained results, it is concluded that Bayesian interval of parameters perform better for appropriate choice of execution time and certain values of parameters.

Keywords: Rayleigh distribution, Non informative prior, Software reliability growth model, Bayesian interval, average length, coverage probability.

1. Introduction

Software reliability is the quality characteristic of operation system which can measure, predict and estimate quality of software system. In last several decades various model have been proposed to assess software reliability. Most of them are probabilistic models. Software modeling techniques can be divided into two categories: Prediction and estimation models. Estimation models determines the current software reliability by applying statistical inference techniques to failure data while the prediction models determines future software reliability based upon available software metrics and measures. The parameters included in software reliability models can be estimated by using some basic procedures like maximum likelihood, least square estimation and Bayesian point estimation, etc.

The research in this area of software reliability has been started since 1950's and a large number of researchers have done work in this field. Most of the past research work in software reliability modeling has concentrated on the point estimation of the parameters. The uncertainty of the estimates by using interval estimation has not been fully discussed. The most commonly applied interval estimation technique is based on the central limit theorem assuming large sample size. However, in real world testing the number of software failures observed is usually not large enough. Whereas Bayesian approaches produce interval estimates even in the case of small sample size, by utilizing prior knowledge.

This paper considers Poisson type Rayleigh model as per classification scheme of Musa and Okumoto [10] Rayleigh distribution has wide application in life time data especially in reliability theory and survival analysis. A specific case of Weibull distribution exhibiting aging effect with an integer valued shape parameter is known as "Rayleigh distribution." Dey and Dey [3] presented Bayes estimators for the parameter of Rayleigh model on the basis of loss function. Also provided Highest Posterior Density (HPD) for the unknown parameter of Rayleigh model. Lee et al [8] proposed software reliability growth model (SRGM) and obtained confidence interval using Obha's inflection S shaped model that can assess software developers optimal release time of software testing tasks. Rabie and Li [12] has studied Bayesian and E-Bayesian approaches under squared error loss function LINEX loss functions and constructed confidence interval for maximum likelihood and credible interval. Xie et al [18] estimated software reliability using Goel-Okumoto model and obtained confidence interval for failure intensity. Wu et al [17] obtained Bayes estimates and credible interval for Rayleigh distribution for the parameters and reliability function. Shrestha and Kumar [13] have obtained Bayes parameter estimates such as reliability function, hazard function under loss function for lomax distribution. Also provides Bayesian credible interval and Highest Posterior Density (HPD) interval for the corresponding parameters. Ogura and Yanagimoto [11] proposed a novel credible interval of the binomial proportion by improving the Highest Posterior Density (HPD) interval using logit transformation. The 100(1-α)% confidence interval through MLE is compared with corresponding level of credible interval. The reason for this is that MLE is preferred by researchers and Bayesian inference is effective for small sample size. Cunha and Rao [2] estimated credible interval and confidence interval through MLE for lognormal distribution also compared average length and coverage probability of the calculated interval. Fang and Yeh [4] proposed a software reliability estimation process that uses stochastic differential equations (SDEs) with fault detection function to construct confidence interval of mean value function $m(t)$ of SRGMs. Song et al [16] proposed a new NHHP software reliability model and estimated confidence interval.

The association of the paper is such that section 2 presents derivation of failure intensity and expected number of failures using Rayleigh distribution. Section 3 presents selection of priors and posterior distribution of model. Section 4 presents two sided Bayes interval for the parameters γ_0 and γ_1. Results are discussed in the section 5 while concluding remarks are provided in section 6.

2. Model Formulation

Considering that software failure time of a system following Rayleigh distribution with scale parameter γ_1 and software failures occurred in Poisson manner. Let t be the positive random variable having Rayleigh distribution then its probability density function is given by

$$f(t) = \{t\,\gamma_1^{-2} e^{-\frac{1}{2}\left[\frac{t}{\gamma_1}\right]^2} \quad , \ t > 0, \gamma_1 > 0 \tag{1}$$

Assuming that the total number of failures remaining in the program at time t = 0 is a Poisson random variable with mean γ_0 then the failure intensity λ (t) = γ_0 f (t) can be obtained as follows (cf Musa et al [9])

$$\lambda(t) = \gamma_0 \gamma_1^{-2} t e^{-\frac{1}{2}\left[\frac{t}{\gamma_1}\right]^2} \quad , \ t > 0, \gamma_1 > 0, \gamma_0 > 0 \tag{2}$$

The mean time to failure function i.e. expected number of failures at time t_e using equation (2) comes to be

$$\mu(t_e) = \ \gamma_0 \gamma_1^{-2} \int_0^{t_e} x e^{-\frac{1}{2}\left[\frac{x}{\gamma_1}\right]^2} dx$$

On simplification it can be written as

$$\mu(t_e) = \ \eta_0 [1 - e^{-\frac{1}{2}\left(\frac{t_e}{\eta_1}\right)^2}] \tag{3}$$

Now assuming that m_e software failures for a system are experienced at times t_i, i = 1,2,……,m_e up to execution time t_e ($\geq t_{me}$) and the likelihood function of parameters γ_0 and γ_1 can be obtained as $L(\Theta, \underline{t}) = \left[\prod_{i=1}^{m_e} \lambda(t_i)\right] exp\, exp\, [-\mu(t_e)]$ (cf. Singh et al [14]). Using failure intensity function given in (2) and mean time to failure function given in (3) the likelihood function is obtained as

$$L(\Upsilon_0, \Upsilon_1) = \Upsilon_0^{m_e} \Upsilon_1^{-2m_e} \left[\prod_{i=1}^{m_e} t_i\right] e^{-\frac{1}{2} T \Upsilon_1^{-2}} e^{-\Upsilon_0} exp \left\{ \Upsilon_0 \, e^{-\frac{1}{2}\left(\frac{t_e}{\Upsilon_1}\right)^2} \right\} \tag{4}$$

3. Choice of priors and Posterior distribution

In Bayesian estimation appropriate choice of the prior(s) for the parameter is necessary. However Bayesian analyst pointed out that there is no perfect technique from which one can conclude that one prior is better than the other. Very often, priors are chosen according to one's subjective knowledge and beliefs. However, in case of adequate information about the parameter is available one can use informative prior(s) otherwise it is preferable to use non informative prior(s). Bayesian estimation is a method that combines prior information with information obtained from sample data. While testing the software, the experimenter have very little knowledge relative to the total number of failures present in the software i. e. γ_0 and γ_1. Here insufficient prior information is available about parameters γ_0 and γ_1, hence non-informative priors are considered.

Jeffrey's [6] has suggested the use of non-informative priors. Jeffrey's prior is widely used because it is proper under slight conditions. It requires likelihood function from which the prior is then derived using Jeffrey's rule. More discussion properties of Jeffrey's prior has been studied by Chen et al [1].The following non informative prior distributions g (γ_0) and g (γ_1) are considered for parameters γ_0 and γ_1 which are as follows:

$$g(\gamma_0) \propto \{\gamma_0^{-1} \quad ,\gamma_0 \epsilon\, [0,\infty) \tag{5}$$

and

$$g(\gamma_1) \propto \{\gamma_1^{-1} \quad ,\gamma_1 \epsilon\, [0,\infty) \tag{6}$$

The joint posterior of Υ_0 and Υ_1 given $\underline{t}$ (= t_i, i=1, 2,……….m_e) is obtained by using equations (4),(5) and (6) is as follows:

$$\pi(\underline{t}) = D^{-1}\gamma_0^{m_e-1}\gamma_1^{-2m_e-1}e^{-\frac{1}{2}T\gamma_1^{-2}}e^{-\gamma_0}exp\left\{\gamma_0\, e^{-\frac{1}{2}\left(\frac{t_e}{\gamma_1}\right)^2}\right\} \qquad m_e < \gamma_0 < \infty, 0 < \gamma_1 < \infty \tag{7}$$

Where D is normalizing constant

$$D = \sum_{j=1}^{\infty} \frac{\Gamma(m_e + j, m_e)\Gamma(2m_e)}{j!} (2/S)^{(2m_e)}$$

Where, $S = (T + jt_e^2)$, $T = \sum_{i=1}^{m_e} t_i^2$

The marginal posterior distribution of γ_0 given $\underline{t}$ is obtained by integrating equation (8) over the whole range of γ_1 i.e.

$$\pi(\underline{t}) = D^{-1}\sum_{j=0}^{\infty}\left[\frac{(2/S)^{(2m_e)}\Gamma(2m_e)}{j!}\right]\left[\gamma_0^{m_e+j-1}e^{-\gamma_0}\right], \qquad m_e < \gamma_0 < \infty \tag{8}$$

Similarly, the marginal posterior distribution of γ_1 given $\underline{t}$ is as

$$\pi(\underline{t}) = D^{-1}\sum_{j=0}^{\infty}\left[\frac{\Gamma(m_e+j,m_e)}{j!}\right]\left[\gamma_1^{-2m_e-1}e^{-\frac{1}{2}S\gamma_1^{-2}}\right], 0 < \gamma_1 < \infty \tag{9}$$

4. Bayesian interval estimation of parameters γ_0 and γ_1

The equal tailed 100(1- α)% Bayes probability interval is given as:

$\int_{-\infty}^{\gamma_*} \pi(\underline{t})dt = \alpha/2$ and $\int_{\gamma^*}^{\infty} \pi(\underline{t})dt = \alpha/2$

Where $(\underline{t})$ is the marginal posterior distribution and γ_* lower limit and γ^* upper limit of the Bayesian interval respectively. For details see Martz and Waller [9], B.K. Kale [7], S. K. Sinha [15].

Now by integrating equations (8) and (9) w.r.t. γ_0 and γ_1 respectively 100(1- α)% two sided Bayesian interval for the parameter γ_0 and γ_1 can be obtained as follows:

$$\widetilde{\gamma_{0l}} = D^{-1}\sum_{j=1}^{\infty}\frac{(2/S)^{(2m_e)}\Gamma(2m_e)}{j!}\,\Gamma(\,m_e+j,\gamma_{0*}\,)$$

$$\widetilde{\gamma_{0u}} = D^{-1}\sum_{j=1}^{\infty}\frac{(2/S)^{(2m_e)}\Gamma(2m_e)}{j!}\,\Gamma(\,m_e+j,\gamma_0^*\,)$$

$$\widetilde{\gamma_{1l}} = D^{-1}\sum_{j=1}^{\infty}\frac{\Gamma(\,m_e+j,me\,)}{j!}\left(\frac{2}{S}\right)^{(2m_e)}\Gamma(2m_e,S/2\,\gamma_{1*})$$

$$\widetilde{\gamma_{1u}} = D^{-1}\sum_{j=1}^{\infty}\frac{\Gamma(\,m_e+j,me\,)}{j!}\left(\frac{2}{S}\right)^{(2m_e)}\Gamma(2m_e,S/2\,\gamma_1^*)$$

Where, $\widetilde{\gamma_{0l}}$ and $\widetilde{\gamma_{0u}}$ is the Bayes lower limit and upper limit of parameter γ_0 i.e. total number of failures, $\widetilde{\gamma_{1l}}$ and $\widetilde{\gamma_{1u}}$ is the Bayes lower limit and upper limit of parameter γ_1. And $\Gamma(\,m_e+j,\gamma_0^*\,)$ and $\Gamma(2m_e,S/2\,\gamma_1^*)$ are incomplete gamma functions.

The details about the incomplete gamma function can be seen from Gradshteyn and Ryzhik [5].

5. Discussion

Table (1) to (8) represents average length and coverage probability of the Bayesian two sided interval is obtained for the parameter γ_0 i.e. total number of failures and parameterγ_1. The Bayesian interval depends upon the values of execution time i.e. t_e and m_e failures experienced at times t_i, i = 1,2………..,m_e. Bayesian interval is studied by calculating the average length and coverage probability of the simulated interval. To study the performance, a sample size was generated from the Rayleigh distribution and it is repeated 1000 times. Average length and coverage probability is calculated for Bayes two sided interval for different execution time t_e for different values of parameters. Monte Carlo simulation is used to study the performance of Bayesian interval. Average length and coverage probability have been calculated by assuming parameter γ_0 (=1(1)5) and $\gamma_1(=0.25(0.25)1.25)$ using 1000simulations.

From tables (1) to (4) it is observed that Bayesian interval's average length decreases as $\gamma_0$increases and it is increases as γ_1 increases for different execution time i.e. t_e. It can be seen that as execution time increases average length also increases. Here assumes that Bayesian interval maintains the credible level if the estimated coverage probability is in between the range of 0.940 to 0.960 i.e. (1-α)$\pm$0.01 where, α = 0.05. It was found that the coverage probabilities of the interval decreases as $\gamma_0$increases and coverage probability increases as γ_1 increases.

Here, table (5) to (8) represents average length and coverage probability for Bayesian two sided interval for the parameter γ_1.It can be seen that average length computed for Bayesian interval is increases as total number of failures i.e. γ_0 increases. And as $\gamma_1$increases average length also increases. It also observes that average length decreases as execution time i.e. t_e increases. From tables it can be seen that coverage probability increases as $\gamma_0$and γ_1 increases. When average length will be shorter coverage probability will decreases. As execution time t_e increases coverage probability decreases.

Table 1: *Average length and coverage probability of Bayesian interval $\widetilde{\gamma_0}$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 5$*

γ_1 \ γ_0	1	2	3	4	5
0.25	2.14713 (0.996)	2.023915 (0.996)	1.574669 (0.995)	1.02334 (0.995)	0.295229 (0.994)
0.50	2.14714 (0.996)	2.058673 (0.996)	1.604102 (0.995)	1.04157 (0.995)	0.375861 (0.994)
0.75	2.14715 (0.996)	2.066402 (0.996)	1.621196 (0.995)	1.30856 (0.995)	0.624627 (0.994)
1	2.14723 (0.996)	2.078305 (0.996)	1.683277 (0.995)	1.40131 (0.995)	0.674976 (0.994)
1.25	2.14768 (0.997)	2.081441 (0.996)	1.751818 (0.995)	1.41164 (0.995)	0.718004 (0.994)

*The values in the parenthesis are coverage probability.

Table 2: *Average length and coverage probability of Bayesian interval $\widetilde{\gamma_0}$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 10$*

γ_1 \ γ_0	1	2	3	4	5
0.25	2. 42186 (0.998)	2.14698 (0.996)	2.13883 (0.996)	2.051196 (0.995)	1.664166 (0.994)
0.50	2.46526 (0.998)	2.22720 (0.996)	2.147932 (0.996)	2.06754 (0.995)	1.673916 (0.994)
0.75	2.48655 (0.998)	2.247208 (0.996)	2.147905 (0.996)	2.07153 (0.995)	1.67429 (0.994)
1	2.48658 (0.998)	2.327472 (0.997)	2.14932 (0.997)	2.07210 (0.996)	1.67455 (0.994)
1.25	2.49144 (0.999)	2.348076 (0.998)	2.15390 (0.997)	2.074465 (0.996)	1.68610 (0.994)

Table 3: *Average Length and Coverage Probability of Bayesian interval $\widetilde{\gamma_0}$calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 15$*

γ_1 \ γ_0	1	2	3	4	5
0.25	2.43761 (0.998)	2.24616 (0.998)	2.18678 (0.997)	2.151995 (0.997)	2.08162 (0.996)
0.50	2.43786 (0.998)	2.26656 (0.998)	2.18701 (0.997)	2.152415 (0.997)	2.082083 (0.996)
0.75	2.43791 (0.998)	2.271643 (0.998)	2.20720 (0.997)	2.15321 (0.997)	2.083737 (0.996)
1	2.44656 (0.998)	2.33112 (0.998)	2.21472 (0.998)	2.153597 (0.997)	2.084265 (0.996)
1.25	2.48656 (0.999)	2.35643 (0.998)	2.22476 (0.998)	2.15383 (0.998)	2.102458 (0.997)

*The values in the parenthesis are coverage probability.

Table 4: *Average Length and Coverage Probability of Bayesian interval $\widetilde{\gamma_0}$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 20$*

γ_1 \ γ_0	1	2	3	4	5
0.25	2.46155 (0.998)	2.25882 (0.998)	2.14701 (0.996)	2.14309 (0.995)	2.10662 (0.994)
0.50	2.46237 (0.998)	2.26956 (0.998)	2.14701 (0.996)	2.14382 (0.995)	2.10883 (0.994)
0.75	2.46256 (0.998)	2.27436 (0.998)	2.14720 (0.996)	2.14445 (0.995)	2.11537 (0.995)
1	2.46506 (0.998)	2.34452 (0.998)	2.147682 (0.996)	2.14894 (0.995)	2.12780 (0.995)
1.25	2.49164 (0.999)	2.36436 (0.998)	2.14807 (0.996)	2.14979 (0.995)	2.12858 (0.995)

*The values in the parenthesis are coverage probability.

Table 5: *Average length and coverage probability of Bayesian interval $\tilde{\gamma}_1$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 5$*

γ_1 \ γ_0	1	2	3	4	5
0.25	0.001098 (0.995)	0.003211 (0.996)	0.006615 (0.997)	0.009148 (0.998)	0.011176 (0.998)
0.50	0.001114 (0.996)	0.003322 (0.996)	0.007081 (0.997)	0.009530 (0.998)	0.012966 (0.998)
0.75	0.001116 (0.996)	0.003406 (0.996)	0.007276 (0.997)	0.010573 (0.998)	0.013338 (0.998)
1	0.001116 (0.996)	0.003485 (0.996)	0.007954 (0.998)	0.013072 (0.998)	0.014761 (0.998)
1.25	0.001125 (0.996)	0.003821 (0.997)	0.008098 (0.998)	0.013388 (0.998)	0.015302 (0.998)

*The values in the parenthesis are coverage probability.

Table 6: *Average length and coverage probability of Bayesian interval $\tilde{\gamma}_1$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 10$*

γ_1 \ γ_0	1	2	3	4	5
0.25	0.001071 (0.995)	0.001090 (0.995)	0.001723 (0.996)	0.002980 (0.996)	0.002112 (0.997)
0.50	0.001074 (0.995)	0.001102 (0.995)	0.002021 (0.996)	0.00302 (0.996)	0.002406 (0.997)
0.75	0.001074 (0.995)	0.001102 (0.995)	0.002023 (0.996)	0.003133 (0.996)	0.002988 (0.997)
1	0.001078 (0.995)	0.001103 (0.996)	0.002036 (0.996)	0.003106 (0.996)	0.003079 (0.997)
1.25	0.001079 (0.995)	0.001103 (0.996)	0.002221 (0.996)	0.003374 (0.996)	0.003684 (0.997)

*The values in the parenthesis are coverage probability.

Table 7: *Average Length and Coverage Probability of Bayesian interval $\tilde{\gamma}_1$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 15$*

γ_1 \ γ_0	**1**	**2**	**3**	**4**	**5**
0.25	0.001069 (0.995)	0.001082 (0.995)	0.001091 (0.996)	0.001114 (0.996)	0.001116 (0.997)
0.50	0.001071 (0.995)	0.001085 (0.995)	0.001113 (0.996)	0.001115 (0.996)	0.001119 (0.997)
0.75	0.001072 (0.995)	0.001086 (0.995)	0.001115 (0.996)	0.001115 (0.996)	0.001120 (0.997)
1	0.001075 (0.995)	0.001086 (0.995)	0.001116 (0.996)	0.001116 (0.996)	0.001122 (0.997)
1.25	0.001077 (0.995)	0.001088 (0.995)	0.001119 (0.996)	0.001117 (0.997)	0.001122 (0.997)

*The values in the parenthesis are coverage probability.

Table 8: *Average Length and Coverage Probability of Bayesian interval $\tilde{\gamma}_1$ calculated for different values of parameters γ_0 and γ_1 when execution time $t_e = 20$*

γ_1 \ γ_0	**1**	**2**	**3**	**4**	**5**
0.25	0.001068 (0.994)	0.001079 (0.995)	0.001082 (0.995)	0.001085 (0.995)	0.001090 (0.995)
0.50	0.001068 (0.994)	0.001080 (0.995)	0.001084 (0.995)	0.001088 (0.995)	0.001092 (0.995)
0.75	0.001067 (0.995)	0.001081 (0.995)	0.001085 (0.995)	0.001089 (0.995)	0.001093 (0.996)
1	0.001066 (0.995)	0.001083 (0.995)	0.001085 (0.995)	0.001092 (0.995)	0.001094 (0.996)
1.25	0.001064 (0.995)	0.001086 (0.995)	0.001088 (0.995)	0.001093 (0.996)	0.001095 (0.996)

*The values in the parenthesis are coverage probability.

6. Conclusion

In this research paper, Bayesian interval has been proposed considering Poisson type Rayleigh class software reliability growth model as function with parameters total number of failures i.e. γ_0 and scale parameter γ_1. Bayesian analysis is carried out by considering non informative prior. The performance of two sided Bayesian interval is studied using Monte Carlo simulation technique. Average length and coverage probability of Bayesian interval is calculated for both the parameters γ_0 and γ_1 for different execution time t_e. From study it is concluded that proposed Bayesian interval has shorter average length for both parameters. Bayesian interval maintained coverage probability for both the parameters γ_0 and γ_1 for different execution time for different values of parameters. In future, confidence interval will be obtained for proposed model and will be compared with Bayesian interval on the basis of average length and coverage probability.

References:

[1] Chen, M. H., Ibrahim, J.G. and Kim, S. (2008). Properties and Implementation of Jeffrey's Prior in Binomial Regression Models. *Journal of the American Statistical Association*, 103:1959-1664.

[2] D'Cunha, J. G. and. Rao, K. A. (2016). Frequentist Comparison of the Bayesian Credible and Maximum Likelihood Confidence interval for the Median of the Lognormal Distribution for the Censored Data. *International Journal of Scientific and Research Publications*, 6:61-66.

[3] Dey, S. and Dey, T. (2012). Bayesian estimation and prediction intervals for Rayleigh distribution under a conjugate prior. *Journal of Statistical Computation and Simulation*, 82:1651-1660.

[4] Fang C.C. and Yeh C.W. (2016). Effective confidence interval estimation of fault detection process of software reliability growth model. *International Journal of Systems science*, 47: 2878-2892. http://dx.doi.org/10.1080/00207721.2015.1036474

[5] Gradshteyn, I. S and Ryzhik, I. M. Table of Integrals, Series, and Products, Alan Jeffrey (editor) 5th Ed., Academic Press, New York, (1994).

[6] Jeffreys, H. Theory of Probability, Third Edition, Clarendon Press, Oxford(1961).

[7] Kale, B. K. A First Course on Parametric Inference (Second Edition) Narosa, (1999).

[8] Lee, T. Q. Fang, C. C. and Yeh C.W. (2013). Confidence Interval Estimation of Software reliability growth Models based on Obha's Inflection S shaped Model. *Journal of Industrial and Intelligent Information*, 1:196-200.

[9] Martz H.F. and Waller R.A. Bayesian Reliability Analysis, New York, Wiley (1982).

[10] Musa, J. D. and Okumoto, K. (1984), "A logarithmic Poisson execution time model for software reliability measurement", Proceedings of Seventh International conference on software engineering Orlando, pp 230-238.

[11] Ogura, T. and Yanagimoto, T. (2018). Improvement of Bayesian Credible interval for a small Binomial Proportion using Logit Transformation, *Current Research in Biostatistics*, 8.

[12] Rabie A. and Li J. (2019). E-Bayesian Estimation based on Burr-X Generalized Type II Hybrid censored data, *Symmetry*, 11: 626-640.

[13] Shrestha S.K. and Kumar V. (2014). Bayesian analysis of Extended Lomax distribution, *International Journal of Mathematics Trends and Technology*, 7: 33-41.

[14] Singh, R. Singh, P. and Kale, K. (2018). Bayes Estimators for the parameters of Binomial type Rayleigh class software reliability growth model using non- informative priors. *Journal of Reliability and Statistical Studies*, 11: 01-07.

[15] Sinha, S. K. Reliability and life testing, Wiley, New Delhi (1985).

[16] Song, K. Y. Chang I. H. and Pham H. (2017). A software Reliability Model with a Weibull Fault Detection rate Function Subject to Operating Environments. *Journals of Applied Sciences*, 7: 983-999.

[17] Wu, S.J. Chen, D.H. and Chen S.T. (2006). Bayesian inference for Rayleigh distribution under progressive censored Sample, *Applied Stochastic models in business and industry*, 22:269-279.

[18] Xie, M. Hong G. Y. and Wohlin C. (1997). A practical method for the Estimation of Software Reliability Growth in the Early Stage of Testing, *Proc. IEEE 7th International Symposium on software Reliability Engineering*, Albuquerque, 116-123.