CONSTRUCTION AND SELECTION OF SKIP LOT SAMPLING PLAN OF TYPE SKSP-V FOR LIFE TESTS BASED ON PERCENTILES OF EXPONENTIATED RAYLEIGH DISTRIBUTION

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Abstract

This study uses percentiles under the exponentiated Rayleigh distribution to build a skip lot sampling plan of the SkSP-V type for a life test. A truncated life test may be carried out to determine the minimum sample size to guarantee a specific percentage life time of products. In particular, this paper highlights the construction of the Skip lot Sampling Plan of the type SkSP-V by considering the Singe Sampling Plan as reference plans for life tests based on percentiles of Exponentiated Rayleigh Distribution. Calculations are made for various quality levels to determine the minimum sample size, prescribed ratio, and operational characteristic values. The proposed sampling plan, which is appropriate for the manufacturing industries for the selection of samples, is also analyzed in terms of its parameters and metrics. The curve is produced after tabulating the operating characteristic data of the plan. Illustrations are provided to help you comprehend the plan. In addition, it addresses the feasibility of the new strategy.

Keywords: Exponentiated Rayleigh Distribution, Percentiles, Life tests, Single Sampling Plan, Double Sampling Plan, SkSP –V.

I. Introduction

The word used for statistical quality control (SQC) describes the collection of qualitative statistical methods used by Professionals in manufacturing. One of the main industries of Acceptance sampling is the regulation of statistical consistency. A sampling of acceptance is a methodology that addresses Procedures under which an approval or rejection decision is focused on sample inspection. Acceptance sampling plans in statistical quality control are concerned with accepting or rejecting a submitted lot of large-size products based on the quality of products inspected through the sample taken from the lot, whereas reliability is the "probability of performing without failure a specified function under the given condition for a specified period." Therefore, reliability testing usually involves the simulation of conditions under which the item will be used during its lifespan.

Dodge [5] proposed skip-lot sampling plans based on the principle of continuous sampling plan of type CSP-1 for a series of lots or consignments of material. Balamurali and Chi-Hyuck Jun [4] derived the new skip-lot sampling plan, which is designated as SkSP-V. It is based on the principles of a continuous sampling plan of type CSP-V and derived the cost model for the SkSP-V plan. Huge authors like Epstein [6], Sobel and Tischendr [14], Goode and Kao [7], Gupta and Groll [8], Kantam et al. [9,10], Baklizi [2], Tsai and Wu [16], Balakrishnan et al. [3], Aslam and Shahbaz [1], Rao et.al.[13], Pradeepa Veerakumari and Ponneeswari [11], Pradeepa Veerakumari et.al [12], Suganya and Pradeepa Veerakumari [15] have fascinated the methodology of time-truncated acceptance sampling plans.

The paper focuses on building an SkSP-V life-test plan with single sampling plan as a percentile-based comparison plan with Exponential Rayleigh Distribution. Rayleigh Exponential Distribution is Important distribution of life testing and analysis of reliability. It has some of the essential structural properties and exhibits great mathematical consistency. Most features of the Exponentiated Rayleigh distribution are close to those of gamma, Weibull, and exponential distribution. ERD's functions for distribution and density are in similar forms. As a result, this is quickly extended to the truncated plans. The cumulative function of the ERD distribution is given by,

$$F(t;\tau,\theta) = \left[1 - e^{-1/2(t/\tau)^2}\right]^{\theta},\ t>0,\ 1/\tau>0,\ \theta>0 \quad (1)$$

Where, τ and θ are the scale and shape parameters respectively. The first derivative of any cumulative distribution function is its probability density function. Hence the probability density function of ERD can be written as,

$$f(t;\tau,\theta) = \theta\left[1 - e^{-1/2(t/\tau)^2}\right]^{\theta-1}\left[\frac{t}{\tau^2}e^{-1/2(t/\tau)^2}\right] \quad (2)$$

Pradeepa Veerakumari and Ponneeswari [11] proposed SSP and DSP for life testing based on the percentiles of ERD. Subsequently, Pradeepa Veerakumari et.al [12] developed Skip-lot sampling plans for life testing based on the percentiles of ERD.

II. Operating procedure for Skip-lot sampling plan of type SkSP-V

The operating procedure for skip-lot sampling plan of type SkSP-V is given by

- The procedure begins with a normal inspection of samples using a suitable reference sampling plan procedure.
- Under the normal inspection, if i number of successive lots are accepted discontinue normal inspection and switch on to skipping inspection.
- Under skipping inspection, fraction f of lots are randomly selected and inspected based on the conditions of the assigned reference plan. Continue the skipping inspection until a sampled and inspected lot is nonconforming.
- Again under skipping inspection if fraction f of the lots are rejected before k consecutively sampled lots are accepted, go to the normal inspection step (1) as above.

- When k consecutive lots are accepted under skipping inspection then go to normal inspection with reduced clearance number x as per step (6) given below.
- Under normal inspection with clearance number x, lots are inspected one by one in the order of being submitted to inspection.
- When a lot is rejected, immediately return to normal inspection with clearance number i as per (1) given above.
- When x lots are accepted on normal inspection mode, immediately stop the normal inspection and switch to skipping inspection as per (3) above.
- When a lot is rejected, perform screening inspection and substitute all the non-conforming units found with conforming units in the rejected lots in the case of non-destructive testing.

III. Operating procedure of SkSP-V with SSP as a reference plan based on percentiles of ERD

A random sample of size n is drawn and put Draw a random sample of size and place on test for time t_0.

- The numbers of defectives d are counted and a comparison is made with the acceptance number c.
- If d>c , then reject the lot.
- If d≤c, then accept the lot.
- If d>c, is obtained before the specified time t_0, terminate the test, and reject the lot.

IV. Operating characteristic function for SkSP-V using Single Sampling Plan

OC function is the most applied technique to measure the efficiency of the sampling plan and from where the probability of acceptance is derived. It provides the probability that the lot can be accepted. The OC function of SSP for life tests based on the percentiles of ERD is as follows,

$$L(p) = \sum_{i=0}^{c} \binom{n}{i} p^i (1-p)^{n-i} \tag{3}$$

Where $P = F(t, \delta_0)$ represents the failure probability at time t given a determined 100q[th] percentile of the lifetime t_q^0 and p depends only on $\delta_0 = t/t_q^0$. The OC values are tabulated in Table 3 of Pradeepa Veerakumari [11].

The OC function of SkSP-V for the lot quality p is given by,

$$P_a(p) = \frac{fP+(1-f)P^i+fP^{k+1}(P^i-P^x)}{(1+P^{i+k}-P^{2k})+(1-f)P^i} \tag{4}$$

Then, the Average Sample number is

$$ASN\ (p) = ASN\ (R)F \tag{5}$$

Where, R- represents the Average Sample number of the reference plan, P represents the probability of acceptance of the reference plan.

4.1. Illustration

Presume that the lifetime of the electric goods follows ERD. Skip lot sampling plan of type SkSP-V with SSP as a reference plan based on the 10[th] percentile is applied for testing. The parameters for the life testing is as follows: θ=2,t=40hrs, $t_{0.1}$=20hrs, c=0, α=0.05 and β=0.05 then η = 0.871929 from the equation and the ratio is found to be $t/t_{0.1}$ =2.00 by applying the minimum sample size according to the requirements is n=3 and the corresponding OC values L(p) for the Single Sampling plan for the life tests based on percentiles of ERD (n,c, $t/t_{0.1}$ = 3,0,0.7921) with $P^* = 0.95$. $L(p)$ is the P-value for SkSP-V with SSP as a reference plan for life tests based on the percentiles of ERD. For i=1,k=2, and f=1/3; the probability of acceptance $L(p)$ values of SkSP-V with SSP for life tests grounded on percentiles of ERD is derived from Eqn. 4 as,

$t/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$L(p)$	0.0285	0.5350	0.8802	0.9678	0.9909	0.9973	0.9991	0.9997	0.9999

From the illustrations, it is indicated that the actual 10th percentile is almost equal to the required 10th percentile ($t/t_{0.1}$ = 1.00) the producer's risk is approximately 0.9715 (1-0.0285). Also, the producer's risk is nearly equal to 0.05 or less and the actual producer risk is large or nearly equal to 1.5 times the required percentile. The OC curve is provided for the illustration as fig.1.

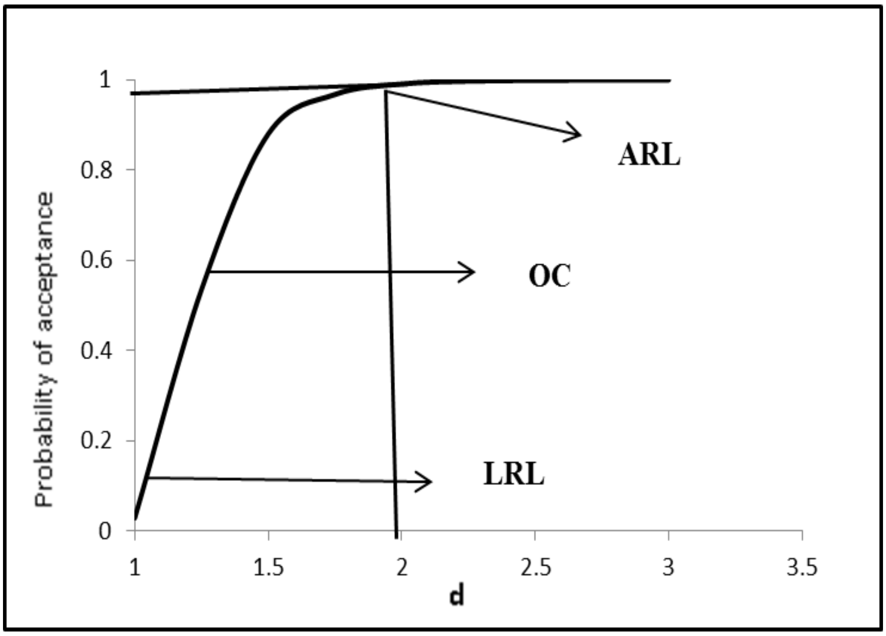

Figure 1. *OC Curve for i=1,k=2, f=1/3, P*=0.95, d=$d_{0.1}$ and θ=2*

Figure 1 clearly says that the plan attains ARL when the actual lifetime percentile is in close proximity to 2 times greater than the specified 10th percentile and attains LRL when the actual lifetime percentile is roughly equal to the specified lifetime percentile. For the purpose of convenience OC values of the table are constructed and tabulated with parameters *i=1,k=2, f=1/3,* and *c=0* in Table 1

Table 1: *Gives the OC values for sampling plan (n,c = 2, $t/t_{0.1}$) for a given P^* under ERD when θ=2*

P^*	$t/t_{0.1}^0$	$t_{0.1}/t_{0.1}^0$								
		0.7	**0.9**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**
0.75	0.7	0.5025	0.9070	0.9814	0.9960	0.9990	0.9997	0.9999	0.9999	0.9999
0.75	0.9	0.5016	0.9006	0.9791	0.9953	0.9988	0.9997	0.9999	0.9999	0.9999
0.75	1	0.4870	0.8932	0.9768	0.9946	0.9986	0.9996	0.9999	0.9999	0.9999
0.75	1.5	0.4907	0.8757	0.9690	0.9919	0.9978	0.9993	0.9998	0.9999	0.9999
0.75	2	0.3738	0.8070	0.9440	0.9830	0.9947	0.9983	0.9994	0.9998	0.9999
0.75	2.5	0.3420	0.7623	0.9220	0.9734	0.9908	0.9967	0.9988	0.9995	0.9998

0.9	0.7	0.2489	0.8205	0.9614	0.9910	0.9977	0.9994	0.9998	0.9999	0.9999
0.9	0.9	0.2413	0.8051	0.9559	0.9893	0.9972	0.9992	0.9998	0.9999	0.9999
0.9	1	0.2434	0.8001	0.9536	0.9885	0.9970	0.9991	0.9997	0.9999	0.9999
0.9	1.5	0.2135	0.7469	0.9326	0.9810	0.9944	0.9983	0.9994	0.9998	0.9999
0.9	2	0.2214	0.7167	0.9149	0.9733	0.9914	0.9971	0.9990	0.9996	0.9998
0.9	2.5	0.1160	0.5668	0.8473	0.9456	0.9801	0.9926	0.9971	0.9989	0.9995
0.95	0.7	0.1360	0.7467	0.9434	0.9863	0.9965	0.9990	0.9997	0.9999	0.9999
0.95	0.9	0.1330	0.7284	0.9363	0.9839	0.9957	0.9988	0.9996	0.9999	0.9999
0.95	1	0.1342	0.7214	0.9331	0.9827	0.9953	0.9986	0.9996	0.9998	0.9999
0.95	1.5	0.1275	0.6700	0.9094	0.9738	0.9921	0.9975	0.9992	0.9997	0.9999
0.95	2	0.1204	0.6165	0.8801	0.9613	0.9871	0.9956	0.9984	0.9994	0.9998
0.95	2.5	0.1160	0.5668	0.8473	0.9456	0.9801	0.9926	0.9971	0.9989	0.9995
0.99	0.7	0.0294	0.5609	0.8928	0.9723	0.9924	0.9978	0.9993	0.9998	0.9999
0.99	0.9	0.0285	0.5350	0.8802	0.9678	0.9909	0.9973	0.9991	0.9997	0.9999
0.99	1	0.0288	0.5248	0.8743	0.9655	0.9901	0.9970	0.9990	0.9997	0.9999
0.99	1.5	0.0216	0.4274	0.8202	0.9449	0.9824	0.9942	0.9980	0.9993	0.9997
0.99	2	0.0143	0.3240	0.7441	0.9127	0.9690	0.9887	0.9958	0.9984	0.9993
0.99	2.5	0.0081	0.2214	0.6406	0.8631	0.9467	0.9787	0.9914	0.9964	0.9985

V. Operating procedure of SkSP-V with DSP as a reference plan based on percentiles of ERD

The modus operandi of SkSP-V with DSP as a reference plan based on percentiles of ERD are as follows:
Step 1: A random sample of size n_1 is drawn and put on a life test.
Step 2: The number of defectives $d_1$is counted and a comparison is made with the acceptance number c.
i. If $d_1>c_1$, then reject the lot.
ii. If $d_1\leq c_1$, then accept the lot.
Step 3: If $d_1<c_2$, is obtained before the specified time t_0, terminate the test, and reject the lot.
Step 4: If $c_1< d_1 \leq r_1$, take a second sample of size n_2 from the remaining lot and put them on test for time $t_0$and count the number of non-conformities (d_2).
Step 5:
If $d_1+d_2 \leq r_1$, accept the lot.
If $d_1+d_2 > r_1$, reject the lot.

VI. Operating characteristic function for SkSP-V using Double Sampling Plan

OC function is the most applied technique to measure the efficiency of the sampling plan and from where the probability of acceptance is derived. It provides the probability that the lot can be accepted. The OC function of DSP for life tests based on the percentiles of ERD is as follows,

$$L(p) = \sum_{d_1}^{c_1} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} . \sum_{d_1+c_{1+1}}^{c_2} \binom{n_1}{d_1} p^{d_1}(1-p)^{n_1-d_1} . \sum_{d_2=0}^{c_2-d_2} \binom{n_2}{d_2} p^{d_2}(1-p)^{n_2-d_2} \quad (6)$$

Where $P = F(t, \delta_0)$ represents the failure probability at time t given a determined 100qth percentile of the lifetime t_q^0 and p depends only on $\delta_0 = t/t_q^0$. The ASN Value of DSP is calculated from the equation,

$$ASN = n_1 p_1 + (n_1 + n_2)(1 - p_1) = n_1 + n_2(1 - p_1) \quad (7)$$

The OC function of SkSP-V for the lot quality p is given by,

$$P_a(p) = \frac{fP + (1-f)P^i + fP^{k+1}(P^i - P^x)}{(1 + P^{i+k} - P^{2k}) + (1-f)P^i} \quad (8)$$

Then, the Average Sample number is

$$ASN\ (p) = ASN\ (R)F \quad (9)$$

Where ASN (R) represents the Average Sample number of the reference plan; P represents the probability of acceptance of the reference plan.

6.1 Illustration

Presume that the lifetime of the electric goods follows ERD. Skip lot sampling plan of type SkSP-V with DSP as a reference plan based on the 10th percentile is applied for testing. The parameters for the life testing is as follows θ=2,t=40hrs, $t_{0.1}$=20hrs, c=0, α=0.05 and β=0.05 then η = 0.871929 from the equation and the ratio is found to be $t/t_{0.1}$ =2.00 by applying the minimum sample size according to the requirements is $n_1=9, n_2=11$ and the corresponding OC values L(p) for the Double Sampling plan for the life tests based on percentiles of ERD $n_1, n_2, c_1, c_2, {}^{t}/_{t_{0.1}}$ = (9,11,0,3,0.9379) with P* = 0.99. *L (p)* is the *P*-value for SkSP-V with DSP for life tests as a reference plan defined on the percentiles of ERD. For *i=1,k=3,* and *f=1/5;* the probability that SkSP-V with DSP will consider *L(p)* values for life tests based on percentiles of ERD is found from Eqn; 8 For,

$t/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
L(p)	0.0434	0.3454	0.7235	0.9158	0.9678	0.9957	0.9991	0.9998	0.9999

From the illustrations, it is indicated that the actual 10th percentile is almost equal to the required 10th percentile $t/t_{0.1}^0$ the producer's risk is approximately 0.9566 (1-0.0434). Moreover, the producer's risk is closely equal to 0.05 or less and the actual producer risk is large or nearly equal to 2 times the required percentile. The OC curve is provided for the illustration as fig 2.

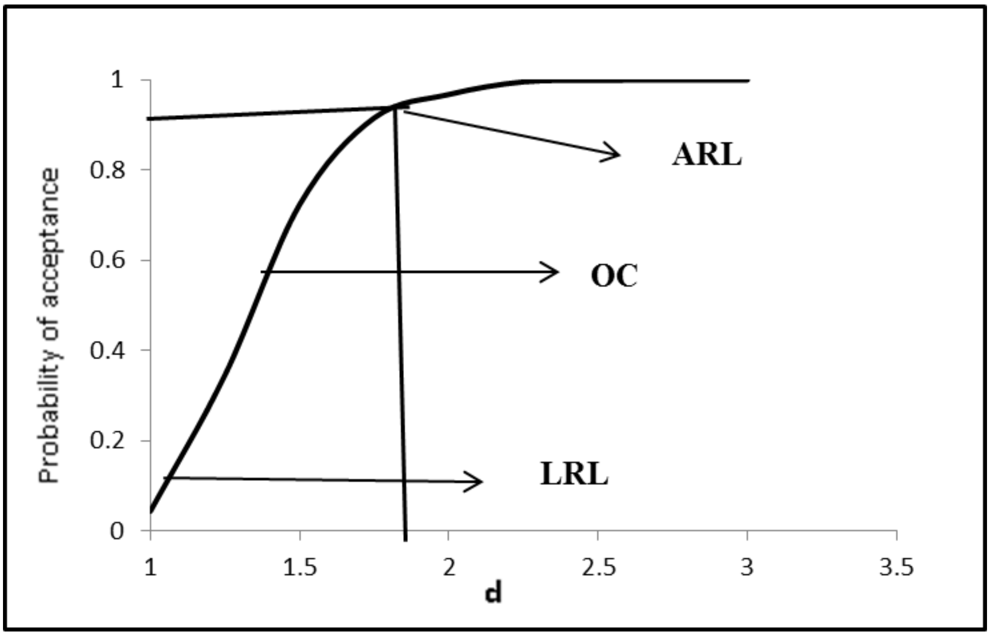

Figure 2. *OC Curve for i=1,k=3, f=1/5, P*=0.99, d=d0.1 and θ=2*

Figure 2.clearly says that the plan attains ARL when the actual lifetime percentile is in close proximity to 1.85 times greater than the specified 10th percentile and attains LRL when the actual lifetime percentile is approximately equal to the specified lifetime percentile. For the purpose of convenience OC values of the table are constructed and tabulated with parameters *i=1,k=3, f=1/5* and $c_1=0, c_2=3$ in Table 2.

Table 2 : *Gives the OC values for Sampling Plan (n, $c_1=0$, $c_2=1$, $t_{0.1}/t^0_{0.1}$) for a given P* under ERD when θ=2*

P^*	$t/t^0_{0.1}$	$t_{0.1}/t^0_{0.1}$								
		1	**1.25**	**1.5**	**1.75**	**2**	**2.25**	**2.5**	**2.75**	**3**
0.75	0.7	0.5047	0.8428	0.9569	0.9880	0.9964	0.9988	0.9995	0.9998	0.9999
0.75	0.9	0.4943	0.8408	0.9559	0.9874	0.9961	0.9986	0.9995	0.9998	0.9999
0.75	1	0.4928	0.8411	0.9557	0.9871	0.9959	0.9986	0.9994	0.9998	0.9999
0.75	1.5	0.4787	0.8325	0.9499	0.9843	0.9947	0.9980	0.9992	0.9996	0.9998
0.75	2	0.4777	0.8322	0.9468	0.9820	0.9935	0.9975	0.9989	0.9995	0.9998
0.75	2.5	0.3021	0.7179	0.8951	0.9578	0.9819	0.9918	0.9961	0.9980	0.9989
0.9	0.7	0.2495	0.6922	0.9070	0.9718	0.9909	0.9968	0.9988	0.9995	0.9998
0.9	0.9	0.2478	0.6895	0.9051	0.9706	0.9903	0.9966	0.9986	0.9994	0.9997
0.9	1	0.2493	0.6741	0.8954	0.9667	0.9889	0.9960	0.9985	0.9993	0.9997
0.9	1.5	0.2379	0.6615	0.8870	0.9617	0.9864	0.9948	0.9979	0.9991	0.9996
0.9	2	0.2095	0.6615	0.8841	0.9580	0.9838	0.9933	0.9971	0.9986	0.9993
0.9	2.5	0.1505	0.5815	0.8422	0.9385	0.9746	0.9890	0.9950	0.9976	0.9987

0.95	0.7	0.1352	0.5971	0.8770	0.9620	0.9874	0.9954	0.9982	0.9992	0.9996
0.95	0.9	0.1327	0.6009	0.8781	0.9615	0.9868	0.9951	0.9980	0.9991	0.9996
0.95	1	0.1330	0.5902	0.8714	0.9587	0.9857	0.9946	0.9978	0.9990	0.9995
0.95	1.5	0.1206	0.5561	0.8487	0.9476	0.9806	0.9923	0.9967	0.9985	0.9993
0.95	2	0.0966	0.5297	0.8286	0.9355	0.9738	0.9887	0.9948	0.9975	0.9987
0.95	2.5	0.1034	0.4777	0.7873	0.9165	0.9657	0.9854	0.9935	0.9969	0.9985
0.99	0.7	0.0297	0.4122	0.8088	0.9387	0.9783	0.9916	0.9965	0.9984	0.9992
0.99	0.9	0.0291	0.3965	0.7954	0.9326	0.9756	0.9904	0.9959	0.9981	0.9991
0.99	1	0.0297	0.3823	0.7832	0.9275	0.9735	0.9895	0.9955	0.9979	0.9990
0.99	1.5	0.0259	0.3365	0.7393	0.9059	0.9633	0.9847	0.9932	0.9968	0.9984
0.99	2	0.0265	0.2432	0.6105	0.8397	0.9340	0.9718	0.9874	0.9941	0.9971
0.99	2.5	0.0210	0.2620	0.6464	0.8533	0.9366	0.9711	0.9862	0.9931	0.9964

VII. Conclusion

In this study, life testing plans based on percentiles of ERD for Skip-lot Sampling plan of type-V with SSP and DSP as reference plan are developed. Skip-lot Sampling plan of type-V with SSP and DSP as reference plan requires minimum sample size and also has better-operating characteristics values. Thus, results in a reduction of inspection cost and better efficiency. The proposed plan can be further extended to other sampling plans for instance SkSP and other probability distribution.

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