STOCHASTIC ANALYSIS OF A COLD STANDBY COMPUTER SYSTEM WITH UP-GRADATION PRIORITY AND FAILURE OF SERVICE FACILITY

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Abstract

We describe the development of a stochastic model for a computer system with cold standby redundancy, priority and failure of service facility. A computer system (called a single unit) means the simultaneous working of its hardware and software components. The system has one more unit (called computer system) that can be used as and when required at the failure of any of the hardware/software components of the initially operative computer system. A single repair facility is made available to rectify the faults which occur due to the failure of hardware and software components. The failed hardware component undergoes for repair immediately while failed software is up-graded. The service facility is subjected to failure during hardware repair. The provision of perfect treatment has been made for the failed service facility. The components work as new after repair and up-gradation with the same life time distribution. The priority is given to the software up-gradation over the hardware repair. In steady state, the expressions for some important reliability measures have been derived using the well known semi-Markov process and regenerative point technique. The behavior of some useful reliability characteristics has been observed for particular values of the parameters related to failure times, repair and up-gradation times and treatment time which follow negative exponential distribution.

Keywords: Computer System, Unit Wise Redundancy, Priority, Failure of Service Facility and Stochastic Modelling

I. Introduction

Over the years an overwhelming transformation of the modern society into the digitalization World has been observed with the advent of advanced technology and frequent use of computer systems. As a result of which we are now in a position to complete the assigned jobs within time limits and perfectness. In the modern World of today the use of computer systems cannot be ignored completely or partially in order to survive in the competitive markets. On the other hand, the burden for the heavy use of computer systems grabs the attention of reliability engineers and scientists to identify all possible ways and means to improve the reliability and performance of these systems. The researchers in the field of reliability have succeeded somehow in identifying the reliability improvement techniques. The provisions of standby redundancy in both parallel and cold standby have been frequently being used by the system developers. The other means such as priority in repair disciplines and proper repair facility have also been suggested by the researchers while analyzing profit of repairable and non repairable systems. The reliability can also be improved by giving priority to repair activities. Many researchers including Goel et al. [3], Leung et al. [11] and Malik [13] explained the model with the help of priority concept. Kumar and Saini [6] three models are developed under different priority policies. Kumar and Yadav [5] described a computer system with priority given to software up-gradation over hardware repair. Kumar et al. [7] the reliability of single unit system is calculated subject to arrival time of server. Kumar et al. [8] assumed the single server to handle the repair activities of computer system. Subramanian and Anantharaman [19] described the reliability analysis of a complex redundant system where standby unit is in cold state for a certain amount of time before it is allowed to become warm.

In most of the research work authors have analyzed the system models of repairable systems under a common assumption that the service facility cannot fail while performing jobs. This assumption seems to be unrealistic in case system has some complex failures and the service facility is very careless. In that situation the treatment to the failed service facility may be given in order to resume the jobs with full efficiency and perfectness. Kuo and Ke [9] compared system availability among three configurations with unreliable server and switching failure. Meng et al. [14] described a two unit cold standby system with switch failure and equipment maintenance. Nandal and Malik [15] evaluated reliability of a single unit system subject to arrival time of the server. Singh [18] evaluated the expected profit by taking repair man appearance and disappearance for a two unit cold standby system. Sridharan and Mohanavadivu [17] analyzed the two unit cold standby redundant system, two types of repairmen (regular and expert). It is also proved that component wise redundancy is better than that of unit wise redundancy so far as reliability of the system is concerned. Friedman and Tran [2] used the combined hardware/software systems. Gupta et al. [4] gave an idea of single server to determine the profit of two unit standby system model in which priority unit is in operation and ordinary unit is in cold standby. Lai et al. [10] determined the system availability for distributed hardware/software system. Mahmoud and Moshref [12] had taken the human error failure with hardware failure for cold standby system. Bhardwaj and Singh [1] considered the failure of server in steady state behavior of cold standby system. Poonam and Malik [16] analyzed a stochastic parallel system with the assumption of failure of service facility. Yadav and Malik [20] analyzed the computer system with unit wise cold standby redundancy.

In view of the above facts and observations here we describe the stochastic modeling of a computer system with cold standby redundancy (unit wise), priority in repair discipline and failure of service facility. A computer system (called a single unit) means the simultaneous working of its hardware and software components. The system has one more unit (called computer system) that can be used as and when required at the failure of any of the hardware/software components of the initially operative computer system. A single repair facility is made available to rectify the faults which occur due to the failure of hardware and software components. The failed hardware component undergoes for repair immediately while failed software is up-graded. The service facility is subjected to failure during hardware repair. The provision of perfect treatment has been made for the failed service facility. The components work as new after repair and up-gradation with the same life time distribution. The priority is given to the software up-gradation over the hardware repair. In steady state, the expressions for some important reliability measures including MTCSF, availability and profit function have been derived using the well known semi-Markov process and regenerative point technique. The behavior of some useful reliability characteristics has been observed for particular values of the parameters related to failure times, repair and up-gradation times and treatment time which follow negative exponential distribution.

In section 2, notations and abbreviations are explained. In section 3, assumptions and state descriptions are described. In section 4, the reliability measures are calculated. Section 5 determines the profit analysis. The particular values are given in section 6. Section 7 describes the graphical behavior of reliability measures. The numerical example is illustrated in section 8. Section 9 comprises of conclusion of the present study. In final, the relevant references are incorporated.

II. Assumptions and State Descriptions

1. There is a computer system comprises hardware & software components which function independently.
2. The hardware and software components fail independently.
3. The system is a cold standby in which one unit (called computer system) is initially operative and the other unit (computer system) is kept as spare.
4. There is a single service facility that repairs the hardware and upgrades the software.
5. The service facility (server) can fail during hardware repair.
6. The h/w repairs, s/w up-gradation and treatments are perfect.
7. The h/w and s/w failures (s/w failure occurs when it fails to furnish the jobs as per the instructions) are assumed to be constant.
8. The distributions for repair, up-gradation and treatment rates are considered as arbitrary.
9. S_0 is an initial state in which one unit (computer system) is in operation and another unit (computer system) is in cold standby.
10. S_1 is the operative state in which one unit is in operation and second unit's failed h/w component is under repair.
11. S_2 is the failed state in which one unit's h/w component is continued under repair from state S_1 while second unit's h/w component is waiting for repair.
12. S_3 is the operative state in which one unit is in operation and second unit's failed s/w component is under up-gradation.
13. S_4 is the operative state in which the failed server is under treatment, one unit is in operation and second unit's h/w component is waiting for repair.
14. S_5 is the failed state in which the failed server is under treatment, one unit's h/w component is continued waiting for repair from state S_2 while second unit's h/w component is waiting for repair.
15. S_6 is the failed state in which the failed server is under continued treatment from state S_4 while one unit's h/w component is continued waiting for repair from state S4 and second unit's h/w component is waiting for repair.
16. S_7 is the failed state in which one unit's h/w component is continued waiting for repair from state S_5 and second unit's h/w component is under repair.
17. S_8 is the failed state in which the failed server is under continued treatment from state S_4 while one unit's h/w component is continued waiting for repair from state S_4 and second unit's s/w component is waiting for up-gradation.
18. S_9 is the failed state in which one unit's h/w component is waiting for repair from while second unit's s/w component is under up-gradation.
19. S_{10} is the failed state in which one unit's s/w component is under continued up-gradation from state S_3 while second unit's s/w component is waiting for up-gradation.
20. S_{11} is the failed state in which one unit's s/w component is under up-gradation while h/w

component of second unit is continued waiting for repair from state S_8.

21. S_{12} is the failed state in which one unit's h/w component is waiting for repair while second unit's s/w component is under continued up-gradation from state S_3.

The state transition diagram shown in the figure 1 as:

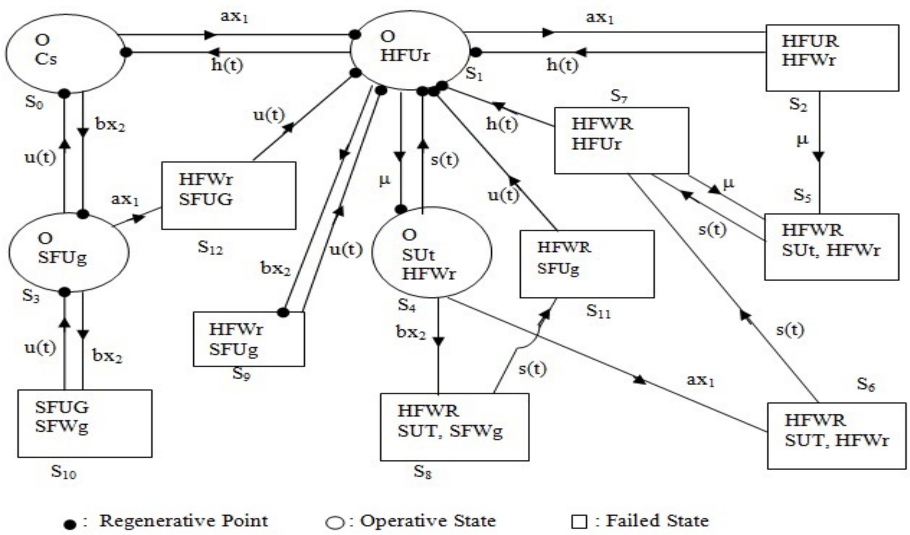

Figure 1: *State Transition Diagram*

a) Notations and Abbreviations

MTCSF	Mean Time to Computer System Failure
SMP	Semi-Markov Process
RPT	Regenerative Point Technique
MST	Mean Sojourn Time
O/Cs	The unit is operative/ in cold standby
a/b	Probability of hardware/software failure
x1/x2/ μ	Hardware/software/ server failure rates
HFUr/HFWr	The failed hardware is under/waiting for repair
HFUR/HFWR	The failed hardware is continuously under/waiting for repair from prior state
SFUg/SFWg	The failed software is under/waiting for up-gradation
SFUG/SFWG	The failed software is continuously under/waiting for up-gradation from prior state
SUt	The failed server (service facility) is under treatment
SUT	The failed server (service facility) is continuously under treatment from prior state
h(t)/H(t)	pdf/cdf of hardware repair time
u(t)/U(t)	pdf/cdf of software repair time
s(t)/S(t)	pdf/cdf of server treatment time
m(t)/M(t)	pdf/cdf of hardware preventive maintenance time
q_{ij}/Q_{ij}	pdf/cdf of first passage time
m_{ij}	Contribution to MST (μi) in state Si when system transits directly to state Sj
$M_i(t)$	Probability that the system up initially in regenerative state Si is up at time t without visiting any other regenerative state
$W_i^H(t)$	Probability that the server is busy in the state Si due to hardware failure up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states

$W_i^S(t)$	Probability that the server is busy in the state Si due to software up-gradation up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states
Ⓢ/©	Standard notation for Laplace-Stieltjes convolution/Laplace convolution
*/**	Symbol for Laplace Transform (LT)/Laplace Stieltjes Transform (LST)
P	Profit function by considering busy period cost of the server per unit time due to hardware repair/ software up-gradation and treatment cost of the server per unit time
Z_1	System revenue per unit up-time
Z_2/Z_3	Busy period cost of the server per unit time due to hardware repair/ software up gradation
Z_4	Treatment cost of the server per unit time

III. Reliability Measures of the System

a) Transition Probabilities

The differential transition probabilities for state S_0 are given by

$$dQ_{01}(t) = ax_1 e^{-(ax_1+bx_2)t}dt \text{ , } dQ_{02}(t) = bx_2 e^{-(ax_1+bx_2)t}dt$$

Taking LST of above equations and using the following results

$$p_{ij} = \lim_{s\to 0} \emptyset_{ij}^{**}(s) = \emptyset_{ij}^{**}(0) = \int_0^\infty dQ_{ij}(t) = \int_0^\infty q_{ij}(t)dt, \text{ we get}$$

$$p_{01} = \int_0^\infty ax_1 e^{-(ax_1+bx_2)t}\, dt = \frac{ax_1}{ax_1+bx_2},\ p_{02} = \int_0^\infty bx_2 e^{-(ax_1+bx_2)t}\, dt = \frac{bx_2}{ax_1+bx_2}$$

Similarly, the other transition probabilities for remaining states are given by

$$p_{10} = h^*(ax_1+bx_2+\mu), p_{12} = \frac{ax_1}{ax_1+bx_2+\mu}\{1-h^*(ax_1+bx_2+\mu)\},\ p_{21} = p_{71} = h^*(\mu)$$

$$p_{14} = \frac{\mu}{ax_1+bx_2+\mu}\{1-h^*(ax_1+bx_2+\mu)\}, p_{19} = \frac{bx_2}{ax_1+bx_2+\mu}\{1-h^*(ax_1+bx_2+\mu)\},$$

$$p_{21} = p_{71} = h^*(\mu),\ p_{25} = p_{75} = 1-h^*(\mu), p_{30} = u^*(ax_1+bx_2), p_{41} = s^*(ax_1+bx_2)$$

$$p_{3,10} = p_{33.10} = \frac{bx_2}{ax_1+bx_2}\{1-u^*(ax_1+bx_2)\}, p_{3,12} = p_{31.12} = \frac{ax_1}{ax_1+bx_2}\{1-u^*(ax_1+bx_2)\},\ p_{41} =$$

$$s^*(ax_1+bx_2), p_{46} = \frac{ax_1}{ax_1+bx_2}\{1-s^*(ax_1+bx_2)\}, p_{57} = p_{8,11} = p_{67} = s^*(0)$$

$$p_{48} = p_{41.8,11} = \frac{bx_2}{ax_1+bx_2}\{1-s^*(ax_1+bx_2)\},\ p_{91} = p_{10,3} = p_{11,1} = p_{12,1} = u^*(0),$$

$$p_{11.2} = p_{12}p_{21},\ p_{41.67} = p_{46}p_{71},\ p_{11.2(5,7)^n} = p_{12}p_{25},\ p_{41.67(5,7)^n} = p_{46}p_{75}$$

From the above transition probabilities, the following relations are obtained as follows:

$$p_{01}+p_{03} = p_{10}+p_{12}+p_{14}+p_{19} = p_{21}+p_{25} = p_{30}+p_{3,10}+p_{3,12} = p_{41}+p_{46}+p_{48} = 1, p_{71}+p_{75} =$$

$$p_{57} = p_{67} = p_{8,11} = p_{9,1} = p_{10,3} = p_{11,1} = p_{12,1} = p_{30}+p_{33.10}+p_{31.12} = 1,$$

$$p_{10}+p_{14}+p_{11.2}+p_{11.2(5,7)^n}+p_{19} = p_{41}+p_{41.67}+p_{41.67(5,7)^n}+p_{41.8,11} = 1$$

b) Mean Sojourn Times

The expected time taken by the system in a particular state before transiting to any other state is known as mean sojourn time or mean survival time in the state. If T_i be the sojourn time in the state i, then the mean sojourn time in the state i is

The MST (μ_i) in state S_i are calculated by the following relations

$$m_{ij} = \left|-\frac{d}{ds}Q_{ij}^{**}(s)\right|_{s=0} = -Q_{ij}^{**\prime}(0) \text{ and } \mu_i = \sum_j m_{ij} \text{ where } Q_{ij}^{**}(s) = \int_0^\infty e^{-st}dQ_{ij}(t).$$

Thus, we have

$$\mu_0 = m_{01}+m_{03},\ \mu_1 = m_{10}+m_{12}+m_{14}+m_{19}, \mu_3^{'} = m_{30}+m_{33.10}+m_{31.12},$$

$$\mu_3 = m_{30}+m_{3,10}+m_{3,12},\ \mu_4 = m_{41}+m_{46}+m_{48},\ \mu_9 = m_{91},$$

$$\mu_1' = m_{10} + m_{11.2} + m_{11.2(5,7)^n} + m_{14} + m_{19}, \mu_4' = m_{41} + m_{41.67} + m_{41.67(5,7)^n} + m_{41.8,11}$$

c) Reliability and MTCSF

Let $\emptyset_i(t)$ be the c.d.f. of first passage time from regenerative state S_i to a failed state. Regarding the failed state as absorbing state, we have following recursive relations for $\emptyset_i(t)$:

$$\emptyset_i(t) = \sum_j Q_{ij}(t) Ⓡ \emptyset_j(t) + \sum_k Q_{ik}(t) \tag{1}$$

where S_j is an un-failed regenerative state to which the given regenerative state S_i can transit and S_k is a failed state to which the state S_i can transit directly. Thus, the following equations are obtained by using (1) as:

$$\emptyset_0(t) = Q_{01}(t) Ⓢ \emptyset_1(t) + Q_{03}(t) Ⓢ \emptyset_3(t)$$
$$\emptyset_1(t) = Q_{10}(t) Ⓢ \emptyset_0(t) + Q_{12}(t) + Q_{14}(t) Ⓢ \emptyset_4(t) + Q_{19}(t)$$
$$\emptyset_3(t) = Q_{30}(t) Ⓢ \emptyset_0(t) + Q_{3,10}(t) + Q_{3,12}(t)$$
$$\emptyset_4(t) = Q_{41}(t) Ⓢ \emptyset_1(t) + Q_{46}(t) + Q_{48}(t)$$

Taking LST of above equations, we get

$$\emptyset_0^{**}(s) = Q_{01}^{**}(s)\emptyset_1^{**}(s) + Q_{03}^{**}(s)\emptyset_3^{**}(s)$$
$$\emptyset_1^{**}(s) = Q_{10}^{**}(s)\emptyset_0^{**}(s) + Q_{12}^{**}(s) + Q_{14}^{**}(s)\emptyset_4^{**}(s) + Q_{19}^{**}(s)$$
$$\emptyset_3^{**}(s) = Q_{30}^{**}(s)\emptyset_0^{**}(s) + Q_{3,10}^{**}(s) + Q_{3,12}^{**}(s)$$
$$\emptyset_4^{**}(s) = Q_{41}^{**}(s)\emptyset_1^{**}(s) + Q_{46}^{**}(s) + Q_{48}^{**}(s)$$

By using Cramer Rule, $\emptyset_0^{**}(s)$ is calculated as

$$\emptyset_0^{**}(s) = \frac{\Delta_1}{\Delta}$$

Where
$$\Delta_1 = \begin{vmatrix} 1 & -Q_{01}^{**}(s) & -Q_{03}^{**}(s) & 0 \\ -Q_{10}^{**}(s) & 1 & 0 & -Q_{14}^{**}(s) \\ -Q_{30}^{**}(s) & 0 & 1 & 0 \\ 0 & -Q_{41}^{**}(s) & 0 & 1 \end{vmatrix}$$

and
$$\Delta = \begin{vmatrix} 0 & -Q_{01}^{**}(s) & -Q_{03}^{**}(s) & 0 \\ Q_{12}^{**}(s) + Q_{19}^{**}(s) & 1 & 0 & -Q_{14}^{**}(s) \\ Q_{3,10}^{**}(s) + Q_{3,12}^{**}(s) & 0 & 1 & 0 \\ Q_{46}^{**}(s) + Q_{48}^{**}(s) & -Q_{41}^{**}(s) & 0 & 1 \end{vmatrix}$$

Now, we have $R^*(s) = \frac{1-\emptyset_0^{**}(s)}{s}$

The reliability of the computer system model can be obtained by

$$R(t) = L^{-1}[R^*(s)]$$

The MTCSF is given by

$MTCSF = \lim_{s\to 0} R^*(s) = R^*(0) = \frac{N_1}{D_1}$, where $N_1 = (1 - p_{14}p_{41})(p_{03}\mu_3 + \mu_0) + p_{01}(p_{14}\mu_4 + \mu_1)$ and $D_1 = (1 - p_{14}p_{41})(1 - p_{03}p_{30}) - p_{01}p_{10}$

d) Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at epoch 't' given that the computer system entered regenerative state S_i at $t = 0$. The recursive relations for $A_i(t)$ are given as

$$A_i(t) = M_i(t) + \sum_j q_{ij}^{(n)}(t) © A_j(t) \tag{2}$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Thus, the following equations are obtained by using (2) as:

$$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{03}(t) © A_3(t)$$
$$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + [q_{11.2}(t) + q_{11.2(5,7)^n}(t)] © A_1(t) + q_{14}(t) © A_4(t) + q_{19}(t) © A_9(t)$$
$$A_3(t) = M_3(t) + q_{30}(t) © A_0(t) + q_{31.12}(t) © A_1(t) + q_{33.10}(t) © A_3(t)$$
$$A_4(t) = M_4(t) + [q_{41}(t) + q_{41.67}(t) + q_{41.67(5,7)^n}(t) + q_{41.8,11}(t)] © A_1(t)$$
$$A_9(t) = q_{91}(t) © A_1(t)$$

Where,

$M_0(t) = e^{-(ax_1+bx_2)t}$, $M_1(t) = e^{-(ax_1+bx_2+\mu)t}\bar{H}(t)$, $M_3(t) = e^{-(ax_1+bx_2)t}\bar{U}(t)$, $M_4(t) = e^{-(ax_1+bx_2)t}\bar{S}(t)$

Taking LT of above equations and solving for $A_0^*(s)$, the steady state availability is calculated by

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$$

Where $N_2 = (p_{14}\mu_4 + \mu_1)(1 - p_{3,10} - p_{03}p_{30}) + p_{10}[\mu_0(1 - p_{3,10}) + \mu_3 p_{03}]$

$$D_2 = (p_{14}\mu_4' + \mu_1' + p_{19}\mu_9)(1 - p_{3,10} - p_{03}p_{30}) + p_{10}[\mu_0(1 - p_{3,10}) + \mu_3' p_{03}]$$

and

$$\mu_i = M_i^*(0), i = 1,2,3,4$$

e) Busy Period of the Repairman Due to Repairs

Let $B_i^R(t)$ be the probability that server is busy in repairing the unit at epoch 't' given that the system entered state S_i at $t = 0$. The recursive relations for $B_i^R(t)$ are given as:

$$B_i^R(t) = W_i^R(t) + \sum_j q_{ij}^{(n)}(t) © B_j^H(t) \qquad (3)$$

where S_j is any successive regenerative state to which the regenerative state S_i can transit through n transitions. Thus, the following equations are obtained by using (3) as:

i)Repair of Hardware

$$B_0^H(t) = q_{01}(t)©B_1^H(t) + q_{03}(t)©B_3^H(t)$$
$$B_1^H(t) = W_1^H(t) + q_{10}(t)©B_0^H(t) + [q_{11.2}(t) + q_{11.2(5,7)^n}(t)]©B_1^H(t) + q_{14}(t)©B_4^H(t) + q_{19}(t)©B_9^H(t)$$
$$B_3^H(t) = q_{30}(t)©B_0^H(t) + q_{31.12}(t)©B_1^H(t) + q_{33.10}(t)©B_3^H(t)$$
$$B_4^H(t) = [q_{41}(t) + q_{41.67}(t) + q_{41.67(5,7)^n}(t) + q_{41.8,11}(t)]©B_1^H(t)$$
$$B_9^H(t) = q_{91}(t)©B_1^H(t)$$

Where, $W_1^H(t) = [e^{-(ax_1+bx_2+\mu)t} + (ax_1 e^{-(ax_1+bx_2+\mu)t} © \mu e^{-\mu t} © s(t) © 1) + (ax_1 e^{-(ax_1+bx_2+\mu)t} © 1)]\bar{H}(t)$

Taking LT of above equations and solving for $B_0^{H^*}(s)$, then busy period of server due to h/w repair is given by

$$B_0^H(\infty) = \lim_{s\to 0} sB_0^{H^*}(s) = \frac{N_3}{D_2}, \text{ where } N_3 = (1 - p_{3,10} - p_{03}p_{30})W_1^{H^*}(0) \text{ and}$$
$$D_2 = (p_{14}\mu_4' + \mu_1' + p_{19}\mu_9)(1 - p_{3,10} - p_{03}p_{30}) + p_{10}[\mu_0(1 - p_{3,10}) + \mu_3' p_{03}]$$

ii)Software Up-gradation

$$B_0^S(t) = q_{01}(t)©B_1^S(t) + q_{03}(t)©B_3^S(t)$$
$$B_1^S(t) = q_{10}(t)©B_0^S(t) + [q_{11.2}(t) + q_{11.2(5,7)^n}(t)]©B_1^S(t) + q_{14}(t)©B_4^S(t) + q_{19}(t)©B_9^S(t)$$
$$B_3^S(t) = W_3^S(t) + q_{30}(t)©B_0^S(t) + q_{31.12}(t)©B_1^S(t) + q_{33.10}(t)©B_3^S(t)$$
$$B_4^S(t) = [q_{41}(t) + q_{41.67}(t) + q_{41.67(5,7)^n}(t) + q_{41.8,11}(t)]©B_1^S(t)$$
$$B_9^S(t) = W_9^S(t) + q_{91}(t)©B_1^S(t)$$

Where, $W_3^S(t) = [e^{-(ax_1+bx_2)t} + (ax_1 e^{-(ax_1+bx_2)t} © 1) + (bx_2 e^{-(ax_1+bx_2)t} © 1)]\bar{U}(t)$ and $W_9^S = \bar{U}(t)$

Taking LT of above equations and solving for $B_0^{S^*}(s)$ (same as 4.3), busy period of server due to s/w up-gradation is given by

$$B_0^S(\infty) = \lim_{s\to 0} sB_0^{S^*}(s) = \frac{N_4}{D_2}, \text{ where } N_4 = W_3^{S^*}(0)p_{03}p_{10} + W_9^{S^*}(0)p_{19}(1 - p_{3,10} - p_{03}p_{30}) \text{ and}$$
$$D_2 = (p_{14}\mu_4' + \mu_1' + p_{19}\mu_9)(1 - p_{3,10} - p_{03}p_{30}) + p_{10}[\mu_0(1 - p_{3,10}) + \mu_3' p_{03}]$$

f) Expected Number of Server Treatment

Let $T_i^R(t)$ be the expected number of repairs of the unit by the server in (0, t] such that the system entered regenerative state i at t = 0. The recursive relation for $T_i^R(t)$ are given as:

$$T_i^R(t) = \sum_j Q_{i,j}^{(n)}(t) Ⓢ [\delta_j + T_i^R(t)] \qquad (4)$$

Where j is any regenerative state to which the given regenerative state i transits and $\delta_j = 1$ if j is the regenerative state where the server does job afresh, otherwise, $\delta_j = 0$. Thus, the following equations are obtained by using (4) as:

$$T_0(t) = Q_{01}(t)ⓈT_1(t) + Q_{03}(t)ⓈT_3(t)$$
$$T_1(t) = Q_{10}(t)ⓈT_0(t) + \left[Q_{11.2}(t) + Q_{11.2(5,7)^n}(t)\right]ⓈT_1(t) + Q_{14}(t)ⓈT_4(t) + Q_{19}(t)ⓈT_9(t)$$
$$T_3(t) = Q_{30}(t)ⓈT_0(t) + Q_{31.12}(t)ⓈT_1(t) + Q_{33.10}(t)ⓈT_3(t)$$
$$T_4(t) = [Q_{41}(t) + Q_{41.67}(t) + Q_{41.67(5,7)^n}(t) + Q_{41.8,11}(t)]Ⓢ[1 + T_1(t)]$$
$$T_9(t) = Q_{91}(t)ⓈT_1(t)$$

Taking LST of above relation and solving for $T_0^{**}(s)$ (same as 4.3). The expected no. of the server treatments is given by

$$T_0(\infty) = \lim_{s\to 0} sT_0^{**}(s) = \frac{N_5}{D_2} \text{ where } N_5 = \left(1 - p_{3,10} - p_{03}p_{30}\right)(p_{12}p_{25} + p_{14}) \text{ and}$$
$$D_2 = (p_{14}\mu_4' + \mu_1' + p_{19}\mu_9)\left(1 - p_{3,10} - p_{03}p_{30}\right) + p_{10}\left[\mu_0\left(1 - p_{3,10}\right) + \mu_3' p_{03}\right]$$

IV. Profit Analysis

The profit function in time 't' of the computer system is given by
P (t) = Expected revenue in (0, t] – expected total cost in (0, t]
In steady state, the profit of the computer system model can be obtained by the following formula:

$$P = Z_1A_0(\infty) - Z_2B_0^H(\infty) - Z_3B_0^S(\infty) - Z_4T_0(\infty) \quad (5)$$

V. Particular Cases

Let us assume $h(t) = \alpha e^{-\alpha t}$, $u(t) = \beta e^{-\beta t}$ and $s(t) = \gamma e^{-\gamma t}$ then reliability measures are determined as follows:

$$p_{10} = \frac{\alpha}{ax_1+bx_2+\mu+\alpha}, p_{12} = \frac{ax_1}{ax_1+bx_2+\mu+\alpha}, p_{14} = \frac{\mu}{ax_1+bx_2+\mu+\alpha}, p_{19} = \frac{bx_2}{ax_1+bx_2+\mu+\alpha},$$
$$p_{25} = \frac{\mu}{\mu+\alpha}, p_{30} = \frac{\beta}{ax_1+bx_2+\beta}, p_{3,10} = \frac{bx_2}{ax_1+bx_2+\beta}, p_{41} = \frac{\gamma}{ax_1+bx_2+\gamma}, \mu_0 = \frac{1}{ax_1+bx_2},$$
$$\mu_1 = \frac{1}{ax_1+bx_2+\mu+\alpha}, \mu_3 = \frac{1}{ax_1+bx_2+\beta}, \mu_4 = \frac{1}{ax_1+bx_2+\gamma}, \mu_1' = \frac{\gamma\alpha+ax_1(\mu+\gamma)}{\gamma\alpha(ax_1+bx_2+\mu+\alpha)},$$
$$\mu_3' = \frac{1}{\beta} = W_3^{S^*}(0) = W_9^{S^*}(0), W_1^{H^*}(0) = \frac{(\mu+\alpha)(\alpha+\gamma)(\alpha+ax_1)+\mu\gamma ax_1}{\alpha(ax_1+bx_2+\mu+\alpha)(\mu+\alpha)(\alpha+\gamma)},$$
$$\mu_4' = \frac{\beta\alpha(ax_1 + bx_2 + \gamma) + \gamma\alpha + \beta ax_1(\mu + \gamma)}{\beta\gamma\alpha(ax_1 + bx_2 + \gamma)}$$
$$MTSF = \frac{N_1}{D_1}, A_0(\infty) = \frac{N_2}{D_2}, B_0^H(\infty) = \frac{N_3}{D_2}, B_0^S(\infty) = \frac{N_4}{D_2}, T_0(\infty) = \frac{N_5}{D_2}$$

where

$$N_1 = \frac{\{(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)-\gamma\mu\}\{ax_1+2bx_2+\beta\}+ax_1(ax_1+bx_2+\mu+\gamma)(ax_1+bx_2+\beta)}{(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)(ax_1+bx_2)(ax_1+bx_2+\beta)}$$
$$D_1 = \frac{\{(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)-\gamma\mu\}\{(ax_1+bx_2)(ax_1+bx_2+\beta)-bx_2\beta\} - ax_1\alpha(ax_1+bx_2+\gamma)(ax_1+bx_2+\beta)}{(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)(ax_1+bx_2)(ax_1+bx_2+\beta)}$$
$$N_2 = \frac{(ax_1+bx_2+\mu+\gamma)+\alpha(ax_1+bx_2+\gamma)}{(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)(ax_1+bx_2)}$$
$$D_2 = \frac{ax_1(ax_1+bx_2+\beta)[(ax_1+bx_2+\gamma)\{\alpha\beta\gamma+\beta ax_1(\mu+\gamma)+\alpha\beta\mu+\alpha\gamma bx_2\}+\alpha\gamma\mu+\beta\mu ax_1(\mu+\gamma)] + \alpha^2\gamma(ax_1+bx_2+\gamma)\{\beta(ax_1+bx_2+\beta)+bx_2(ax_1+bx_2)\}}{\alpha\beta\gamma(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2+\gamma)(ax_1+bx_2)(ax_1+bx_2+\beta)}$$
$$N_3 = \frac{ax_1\{(\mu+\alpha)(\gamma+\alpha)(\alpha+ax_1)+\gamma\mu ax_1\}}{\alpha(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2)(\mu+\alpha)(\gamma+\alpha)},$$
$$N_4 = \frac{bx_2(\alpha+ax_1)}{\beta(ax_1+bx_2+\mu+\alpha)(ax_1+bx_2)},$$
$$N_5 = \frac{\mu ax_1(\alpha + ax_1 + \mu)}{(ax_1 + bx_2 + \mu + \alpha)(ax_1 + bx_2)(\mu + \alpha)}$$

VI. Graphical Presentation

The graphical representation of MTCSF, availability and profit function has been shown in figures 2, 3 and 4 respectively to check their behavior with respect to the values of the parameters associated with failure and repair rates. From Figure 2, it is observed that the MTCSF of the system decreases when failure rate of hardware and software is increased from 0.01 to 0.1. Also, MTCSF increases with an increase in hardware repair rate, software up-gradation rate and treatment rate of the server.

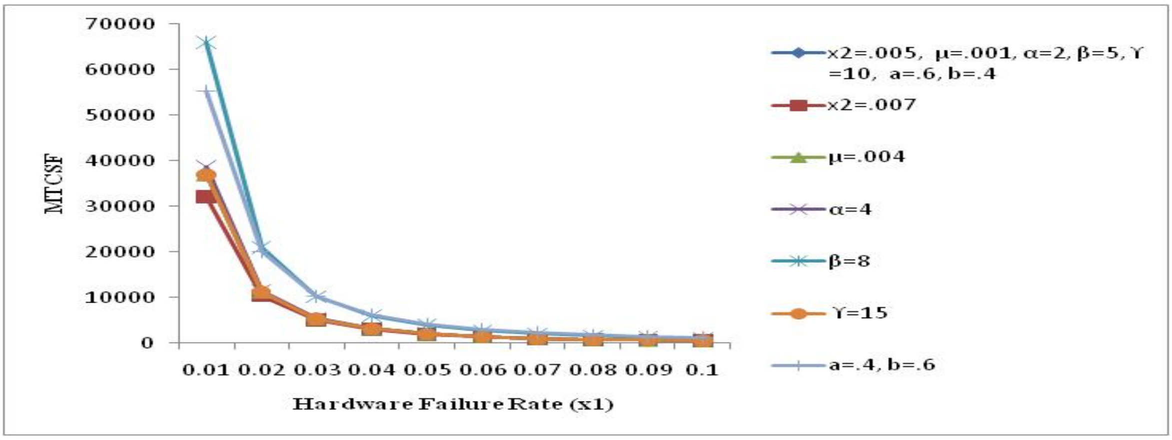

Figure 2: *MTCSF Vs Hardware Failure Rate (X_1)*

From Figure 3, it is clearly seen that the availability of the system decreases rapidly with increase of failure rate of hardware and software. Also, availability of the system increases with an increase of hardware repair, software up-gradation and treatment rate of the server.

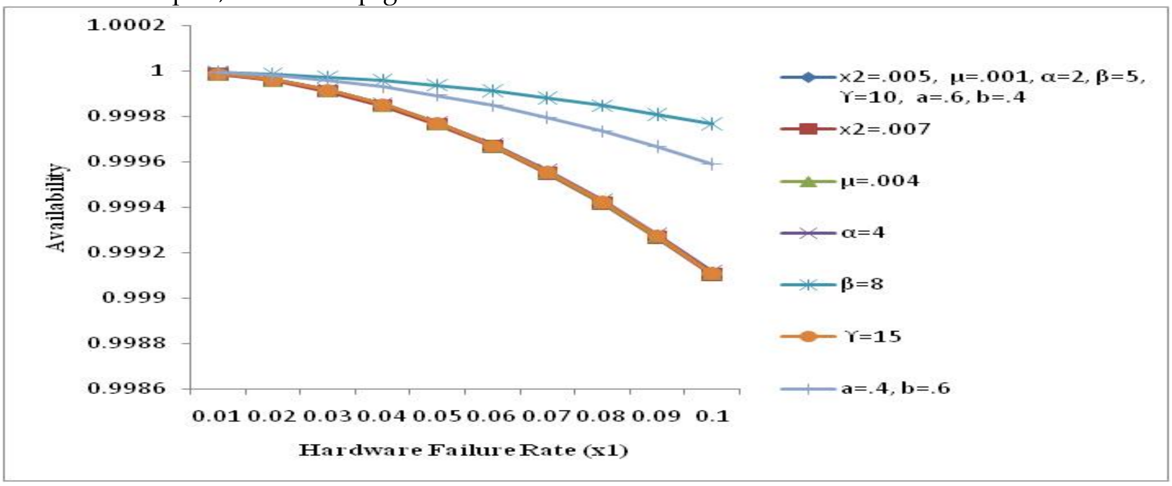

Figure 3: *Availability Vs Hardware Failure Rate (X_1)*

From Figure 4, it is observed that the profit decreases when failure rate of the hardware and software increases. Also, the profit of the system is increases with an increase of hardware repair rate, software up-gradation rate and treatment rate of the server.

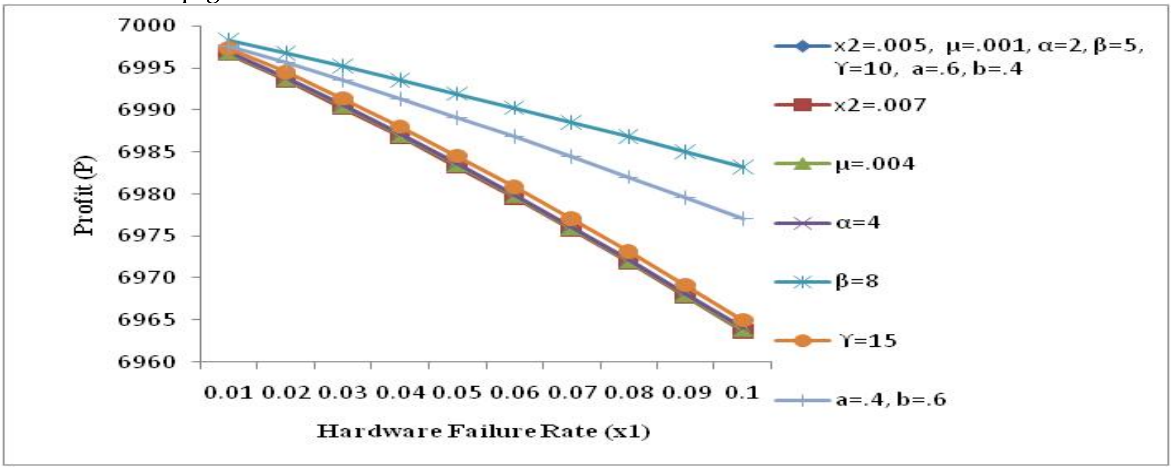

Figure 4: *Profit Vs Hardware Failure Rate (X_1)*

VII. Conclusion

The present study mainly focuses on MTCSF, availability and profit analysis of a computer system with unit wise redundancy and failure of service facility. The preference is given to the software up-gradation over hardware repair. The graphical behavior of some important measures such as MTCSF, availability and profit has been observed w.r.t. hardware failure rate (x_1) and for the fixed values of server failure rate, repair rates of components and server's treatment rate as shown in the respective figures (Fig.2, Fig.3 and Fig.4). From these figures, it is concluded that MTCSF (Fig.2), availability (Fig.3) and profit (Fig.4) decrease with increase in hardware failure rate (x_1) & software failure rate (x_2) and increase with increase of hardware repair rate (α) and software up-gradation rate (β) and treatment rate (Υ) of server. It is also examined that the provision of priority to software up-gradation of one unit over the hardware repair of other unit can only be helpful in increasing the profit of the system model provided the software up-gradation rate is increased.

VIII. Illustration

Suppose the department office has two computers for furnishing day to day assigned jobs. The official starts the jobs initially at a single computer (unit) and the other computer system is kept as spare in order to makes its use as and when required at any type of problems which occur in the initial operative computer system. The computer can have problems in both hardware and software like damage of RAM, defects in CPU and short-circuit in the monitor as the hardware problems while software can fail to follow the instructions due to malware in the system and failure of drivers. In that situation it becomes necessary to take the help of another computer system in order to complete the assigned jobs in time. In order to secure the data from any kind of malware attack the priority to up-grade the software is required instead of repair of any type of hardware faults. On the other hand, it is not necessary that the service facility can be made available immediately to rectify the faults and in that case we can consider the failure of the service facility. On the basis of the experience and practices the present study is illustrated on a computer system by considering the ideas of unit wise redundancy, priority to software up-gradation and failure of the service facility. The reliability characteristics such as MTCSF, availability and profit have been obtained by taking the hypothetical values for the parameters as:

$$\text{Here, } x_1 = 0.04, x_2 = 0.007, \mu = 0.001, \alpha = 2, \beta = 5, \Upsilon = 10, a = 0.6 \text{ and } b = 0.4,$$
$$Z_1 = 7000, Z_2 = 1000, Z_3 = 800, Z_4 = 500.$$

We have

$$\text{MTCSF} = 3046.581, \text{Availability} = 0.999848 \text{ and Profit} = 6986.86$$

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