M/M/∞ Queue with Catastrophes and Repairable Servers

Gulab Singh Bura

•

Department of Mathematics and Statistics, Banasthali Vidyapith, Rajasthan, INDIA
gulabsingh@banasthali.in

Abstract

An infinite server Markovian queueing system with randomly occurring breakdowns and non zero exponentially distributed repair time is proposed. Upon arrival, a catastrophes deactivate all the servers and system is under catastrophic failure. Immediately, a repair process is started and after successful repair the system is ready to serve the newly arrived customers. Continued fraction techniques have been used to obtain the time dependent probabilities of the studied model. The stationary probability distribution for the number of customers in the system is also derived. Some important stationary as well as transient moments are also determined. Further, The availability and reliability of the system under consideration are investigated. Finally, some graphical results are presented to visualize the model practically.

Keywords: M/M/∞ Queue, Server Breakdown, Transient Analysis,Steady State Solution, Confluent Hypergeometric Function, Reliability and Availability.

1. Introduction

Here, we consider a classical M/M/∞ queueing model subjected to randomly occurring breakdowns (catastrophes). Upon arrival, a catastrophes deactivate all the servers and system is under disasters breakdown. Immediately a repair process is started and after successful repair the system is again restart their functioning and provide service to a newly arrived customer. We analyze this model and provide steady state and time dependent solution.

During the last four decades the interest in catastrophic queueing model has been increased by a rapid phase. Therefore queueing models in the presence of catastrophes has been analyzed by many researchers.(see e.g., [1],[2],[3], [12],[17],[19],[21][22]). Occurrence of catastrophes destroys all present customers and also breakdown the servers. Some authors assumes that whenever catastrophes occurs, it flush out all present customers and immediately the server is ready for service for a newly arrived customer(see e.g. [1], [3],[8], [9]). And some assumes that the server or system may take a non zero repair time for their re-functioning whenever it is affected by a catastrophic failure(see e.g. [4], [18], [21], [25]).

Infinite servers queueing models are also analyzed by many researchers with the possibility of catastrophes. Gursoy *et* al. [15] analyzed an infinite server queue with randomly occurring interruption and provide steady state solution . Giorno *et* al. [26] have discussed the various properties of a bilateral birth-death process, affected by randomly occurring catastrophes. Linton and Purdue[6] have obtained the stationary and transient distribution of the probabilities for an M/G/∞ queue with catastrophes. Yechiali [25] considered an M/M/∞ queues with catastrophes and studied the impatient behavior of customers when server is down. The transient solution of an infinite servers Markovian queue subjected to catastrophes has been obtained by Krishna Kumar *et* al. [5] and Gulab Singh Bura [8].

In this work, we present an M/M/∞ queuing system with catastrophes, Server breakdown and

non-zero repair time. Although,the operating model has been already analyzed by Sophia and Murali [23]. Then, objective of this paper is to illustrate a different approach and provide some additional important measures of the system under consideration.
The M/M/∞ queueing system with repairable servers finds its application in telecommunication field. Our system under consideration is a University campus which provide free Wi-Fi service to their students. Within the campus, each mobile is considered as one queueing server. Whenever a breakdown occur i.e. (connectivity loss or signal failure), all the servers gets deactivated and none of them works until that breakdown is repaired. So, an M/M/∞ queue with system failure and repair is a suitable approximation.

The paper is arranged in the following way. Next section describe the formulation of the model. The transient solution of the model have been obtained in section 3. Under section 4, we have obtained some moments of the model in transient form. Section 5, gives the time independent solution of the model. In Section 6, we have discussed about the availability and reliability of the system. Section 7 presents some graphical illustrations to observe the system performance with the effect of various parameters. Conclusion is given in the last Section.

2. MATHEMATICAL MODEL

An infinite servers Markovian queueing system with server breakdown and repair is in operation. Arrivals occur one by one in a Poisson stream with mean rate λ. Service times are exponentially distributed with parameter θ. The system may fails due the disastrous breakdown occurs at a Poisson rate γ. Whenever a catastrophes occur all the servers are deactivated and the system is under disasters breakdown. Immediately a repair process is started and the repair time distribution is exponential with rate η. After successful repair the system again restart their functioning and provide service to a newly arrived customer. Also, it is assumed that, no customer is allowed to enter in to the system during the repair process of failed servers. Let the random variable $C(t)$ represents the number of customers present in the system at time t and $P_n(t)$ denotes its probability.

3. TRANSIENT ANALYSIS UNDER MARKOVIAN SETUP

This section provides the probability mass function of the random variable $C(t)$. For this, the differential-difference equations are given as:

$$F'(t) = \gamma(1 - F(t) - P_0(t)) - \eta F(t) \tag{1}$$

$$P_0'(t) = \theta P_1(t) + \eta F(t) - \lambda P_0(t) \tag{2}$$

$$P_n'(t) = (n+1)\theta P_{n+1}(t) + \lambda P_{n-1}(t) - (\lambda + n\theta + \gamma)\, P_n(t), \;\; n = 1, 2, 3, ... \tag{3}$$

Initially, at $t = 0$,

$$P_n(0) = \begin{cases} 1 & \text{if } n = 0; \\ 0 & \text{if } n \neq 0. \end{cases} \tag{4}$$

Taking Laplace transform of Eq.(1),Eq.(2),Eq.(3) and by the use of Eq.(4), we have

$$(s + \gamma + \eta)\, F^*(s) = \gamma(\frac{1}{s} - P_0^*(s)) \tag{5}$$

$$(s + \lambda)\, P_0^*(s) = 1 + \theta P_1^*(s) + \eta F^*(s) \tag{6}$$

$$(s + \lambda + n\theta + \gamma)\, P_n^*(s) = (n+1)\theta P_{n+1}^*(s) + \lambda P_{n-1}^*(s), \tag{7}$$

After some manipulation, Eq.(7), gives an expression

$$\frac{P_n^*(s)}{P_{n-1}^*(s)} = \frac{\frac{\lambda}{\theta}}{\frac{s+\lambda+\gamma}{\theta} + n) - (n+1)\frac{P_{n+1}^*(s)}{P_n^*(s)}}$$

$$\frac{\lambda}{\theta}\frac{P^*_{n-1}(s)}{P^*_n(s)} = \left(\frac{s+\lambda+\gamma}{\theta}+n\right) - \cfrac{(n+1)\frac{\lambda}{\theta}}{\left(\frac{s+\lambda+\gamma}{\theta}+n+1\right) - \cfrac{(n+2)\frac{\lambda}{\theta}}{\left(\frac{s+\lambda+\gamma}{\theta}+n+2\right)-\cdots}} \tag{8}$$

Now using the identity given by Lorentzen and Waadeland [13]

$$\frac{{}_1F_1(q+1;r+1;z)}{{}_1F_1(q;r;z)} = \frac{r}{r-z+}\frac{(q+1)z}{r-z+1+}\frac{(q+2)z}{r-z+2+}\cdots$$

rewritten as

$$\frac{{}_1F_1(q;r;z)}{{}_1F_1(q+1;r+1;z)} = \frac{r-z}{r+}\frac{(q+1)z}{r-z+1+}\frac{(q+2)z}{r-z+2+}\cdots, \tag{9}$$

by using Eq.(9) in Eq.(8), we have

$$\frac{P^*_n(s)}{P^*_{n-1}(s)} = \frac{\lambda}{\theta}\frac{{}_1F_1(q+1;r+1,z)}{\left(\frac{s+\gamma}{\theta}+n\right)\ {}_1F_1(q;r;z)}, \tag{10}$$

therefore for $n \geq 1$,we have

$$P^*_n(s) = \left(\frac{\lambda}{\theta}\right)^n \frac{{}_1F_1(n+1;\frac{s+\gamma}{\theta}+n+1;-\frac{\lambda}{\theta})}{\prod_{j=1}^n\left(\frac{s+\gamma}{\theta}+j\right)\ {}_1F_1(1;\frac{s+\gamma}{\theta}+1;-\frac{\lambda}{\theta})}P^*_0(s), \tag{11}$$

$$P^*_n(s) = \zeta^*_n(s)P^*_0(s), \tag{12}$$

where

$$\zeta^*_n(s) = \left(\frac{\lambda}{\theta}\right)^n \frac{{}_1F_1(n+1;\frac{s+\gamma}{\theta}+n+1;-\frac{\lambda}{\theta})}{\prod_{j=1}^n\left(\frac{s+\gamma}{\theta}+j\right)\ {}_1F_1(1;\frac{s+\gamma}{\theta}+1;-\frac{\lambda}{\theta})}, \tag{13}$$

It is well known that

$$F^*(s) + \sum_{n=0}^{\infty} P^*_n(s) = \frac{1}{s}, \tag{14}$$

by the use of Eq.(12) and Eq.(5), we get

$$P^*_0(s) = (1+\frac{\eta}{s})\left[(s+\lambda+\eta) - \theta\zeta^*_1(s) + \eta\sum_{n=1}^{\infty}\zeta^*_n(s)\right]^{-1} \tag{15}$$

after simplification Eq.(15) reduces to

$$P^*_0(s) = (1+\frac{\eta}{s})\sum_{j=o}^{\infty}\frac{(-1)^j}{(s+\lambda+\eta)^{j+1}}\left[\sum_{k=1}^{\infty}(\eta-\delta_k\theta)\zeta^*_k(s)\right]^j \tag{16}$$

on inversion, we get

$$\begin{aligned} P_0(t) &= \sum_{j=0}^{\infty}(-1)^j\int_0^t e^{-(\lambda+\eta)(t-u)}(t-u)^j\left[\sum_{k=1}^{\infty}(\eta-\delta_k\theta)\zeta_k(u)\right]^{*J} du \\ &\quad +\eta\sum_{j=0}^{\infty}(-1)^j\int_0^t e^{-(\lambda+\eta)x}\frac{x^j}{j!}\left[\sum_{k=1}^{\infty}(\eta-\delta_k\theta)\zeta_k(x)\right]^{*J} dx \end{aligned} \tag{17}$$

Now for $P_n(t)$, consider Eq.(12), which on inversion, gives

$$P_n(t) = \zeta_n(t) * P_0(t), \tag{18}$$

where the symbol * denotes the convolution and $P_0(t)$ given in Eq.(17).
Next we derive the expression for $\zeta_n(t)$, where $\zeta_n(t)$ represents the inverse Laplace transform of

$\zeta_n^*(s)$.
From Eq.(13)

$$\zeta_n^*(s) = \left(\frac{\lambda}{\theta}\right)^n \frac{{}_1F_1(n+1; \frac{s+\gamma}{\theta}+n+1; \frac{-\lambda}{\theta})}{\prod_{j=1}^{n}\left(\frac{s+\gamma}{\theta}+j\right) {}_1F_1(1; \frac{s+\gamma}{\theta}+1; \frac{-\lambda}{\theta})}.$$

We known that

$${}_1F_1(n+1; \frac{s+\gamma}{\theta}+n+1; \frac{-\lambda}{\theta}) = \sum_{k=0}^{\infty} \frac{(n+1)_k \left(\frac{-\lambda}{\theta}\right)^k}{(\frac{s+\gamma}{\theta}+n+1)_k k!}$$

where $(b)_k$ represents the Pochhammor symbol, i.e.

$$(b)_k = \begin{cases} 1 & \text{if } k=0; \\ b(b+1)(b+2)...(b+k+1) & \text{if } k=1,2,3,.... \end{cases}$$

Therefore

$$\frac{{}_1F_1(n+1; \frac{s+\gamma}{\theta}+n+1; \frac{-\lambda}{\theta})}{\prod_{j=1}^{n}\left(\frac{s+\gamma}{\theta}+j\right)} = \sum_{k=0}^{\infty} \frac{\binom{n+k}{k}\left(-\frac{\lambda}{\theta}\right)^k}{\prod_{j=1}^{n+k}\left(\frac{s+\gamma}{\theta}+j\right)}$$

Applying partial fraction expansion, the above equation can be written as

$$\begin{aligned} \frac{{}_1F_1(n+1; \frac{s+\gamma}{\theta}+n+1; \frac{-\lambda}{\theta})}{\prod_{j=1}^{n}\left(\frac{s+\gamma}{\theta}+j\right)} =& \theta\sum_{k=0}^{\infty}\binom{n+k}{k}\left(-\frac{\lambda}{\theta}\right)^k \\ & \sum_{j=1}^{n+k}\frac{(-1)^{j-1}}{(j-1)!\,(n+k-j)!\,(s+\gamma+j\theta)}. \end{aligned} \tag{19}$$

Also

$${}_1F_1\left(1; \frac{s+\gamma}{\theta}+1; \frac{-\lambda}{\theta}\right) = \sum_{k=0}^{\infty}(-\lambda)^k\, d_k^*(s),$$

where

$$d_k^*(s) = \frac{1}{\prod_{j=1}^{k}(s+\gamma+j\theta)} \quad and \quad d_0^*(s)=1.$$

$$\frac{1}{{}_1F_1(1; \frac{s+\gamma}{\theta}+1; \frac{-\lambda}{\theta})} = \sum_{k=0}^{\infty}(\lambda)^k e_k^*(s), \tag{20}$$

where $e_0^*(s)=1$, and for k=1,2,3,...

$$e_k^*(s) = \begin{vmatrix} d_1^*(s) & 1 & & . & . & . & & \\ d_2^*(s) & d_1^*(s) & 1 & . & . & . & & \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ d_{k-1}^*(s) & d_{k-2}^*(s) & d_{k-3}^*(s) & . & . & . & d_1^*(s) & 1 \\ d_k^*(s) & d_{k-1}^*(s) & d_{k-2}^*(s) & . & . & . & d_2^*(s) & d_1^*(s) \end{vmatrix}$$

$$= \sum_{l=1}^{k}(-1)^{l-1}e_{k-l}^*(s)d_l^*(s).$$

By substituting Eq.(19) and Eq.(20) in Eq.(13), we get

$$\zeta_n^*(s) = (\lambda)^n\sum_{i=0}^{\infty}(-\lambda)^i\binom{n+i}{i}d_{n+i}^*(s)\sum_{k=0}^{\infty}(\lambda)^k e_k^*(s).$$

On inversion, we obtain

$$\zeta_n(t) = (\lambda)^n \sum_{i=0}^{\infty}(-\lambda)^i \binom{n+i}{i} d_{n+i}(t) \sum_{k=0}^{\infty}(\lambda)^k e_k(t), \tag{21}$$

where

$$d_k(t) = \frac{1}{(\theta)^{k-1}} \sum_{j=1}^{k} \frac{(-1)^{j-1}}{(k-j)!\,(j-1)!} e^{(-j\theta+\gamma)t}, k = 1, 2, 3, ...,$$

$$e_k(t) = \sum_{j=1}^{k}(-1)^{j-1} d_j(t) * e_{k-j}(t), \quad k = 2, 3, 4, ...; \quad e_1(t) = d_1(t)$$

Now from Eq(5) , we have

$$F^*(s) = \frac{\gamma}{s+\gamma+\eta}\left(\frac{1}{s} - P_0^*(s)\right)$$

On inversion,

$$F(t) = \gamma \int_0^t (1 - P_0(z))e^{-(\gamma+\eta)(t-z)}dz \tag{22}$$

4. TIME DEPENDENT MOMENTS

4.1. MEAN

Let A(t) denote the mean value of the random variable C(t),therefore

$$A(t) = E(C(t)) = \sum_{n=1}^{\infty} nP_n(t) \tag{23}$$

Initially, at t=0, Eq(23) gives

$$A(0) = 0,$$

which implies

$$A'(t) = \sum_{n=1}^{\infty} nP'_n(t), \tag{24}$$

where $A'(t)$ denotes the derivative of $A(t)$. Application of Eq.(3) in Eq.(24),after some calculation gives

$$A'(t) = -(\theta + \gamma)A(t) + \lambda \tag{25}$$

which is a linear differential equation in $A(t)$, whose solution gives

$$A(t) = \frac{\lambda}{\theta+\gamma}[1 - e^{-(\theta+\gamma)t}] \tag{26}$$

4.2. VARIANCE

An average is not sufficient to understand completely the distribution of the random variable $C(t)$).Hence, variance is also needed for better understanding. Let $Var(C(t))$ represents the variance of the random variable $C(t)$, then

$$Var(C(t)) = E[C(t) - E(C(t))]^2$$

Which may be written as

$$Var(C(t)) = c(t) - [A(t)]^2, \tag{27}$$

and

$$c(t) = E(C^2(t)) = \sum_{n=1}^{\infty} n^2 P_n(t),$$

with

$$c(0) = 0,$$

and

$$c'(t) = \sum_{n=1}^{\infty} n^2 P_n'(t) \tag{28}$$

Substitution of $P_n'(t)$ in Eq.(28), after some calculation results in the form of a linear differential equation in $c(t)$ i.e.

$$c'(t) = -(2\theta + \eta)c(t) + (2\lambda + \theta)A(t) + \lambda \tag{29}$$

which after integration gives

$$\begin{aligned} c(t) = & \frac{(2\lambda + \theta)\lambda(\theta - e^{-(2\theta+\gamma)t}(3\theta + \gamma) + e^{-(\theta+\gamma)t}(2\theta + \gamma))}{(2\theta + \gamma)\theta(\theta + \gamma)} \\ & + \frac{\lambda}{(2\theta + \gamma)}[1 - e^{-(2\theta+\gamma)t}]. \end{aligned} \tag{30}$$

subsitutation of Eq.(30) in Eq.(27), gives the expression of $Var(C(t))$.

5. STEADY STATE SOLUTION

Here, we derive an expression for the stationary probabilities of the operating model

Theorem 5.1. Stationary probabilities of the system under consideration are given as

$$F = \frac{\gamma}{\gamma + \eta}(1 - \eta\rho_1)$$
$$P_n = \eta\rho_n\rho_1$$
$$P_0 = \eta\rho_1$$

where

$$\rho_n = \left(\frac{\lambda}{\theta}\right)^n \frac{{}_1F_1(n+1; \frac{\gamma}{\theta} + n + 1; \frac{-\lambda}{\theta})}{\prod_{j=1}^{n}\left(\frac{\gamma}{\theta} + j\right)\ {}_1F_1(1; \frac{\gamma}{\theta} + 1; \frac{-\lambda}{\theta})}.$$

and

$$\rho_1 = \sum_{j=o}^{\infty} \frac{(-1)^j}{(\lambda + \eta)^{j+1}} \left[\sum_{n=1}^{\infty} (\eta - \delta_n\theta)\rho_n\right]^j$$

Proof. Multiplying by s on both side of Eq.(16) and taking limit as $s \to 0$, and using $\lim_{s\to 0} sP_0^*(s) = P_0$, we get

$$P_0 = \eta\rho_1 \tag{31}$$

where

$$\rho_1 = \sum_{j=o}^{\infty} \frac{(-1)^j}{(\lambda + \eta)^{j+1}} \left[\sum_{n=1}^{\infty} (\eta - \delta_n\theta)\rho_n\right]^j$$

For $n = 1, 2, ...,$

Multiplying by s on both side of Eq.(12) and taking limit as $s \to 0$, and using $\lim_{s\to 0} sP_n^*(s) = P_n$, we get

$$P_n = \eta\rho_n\rho_1, \tag{32}$$

where

$$\rho_n = \left(\frac{\lambda}{\theta}\right)^n \frac{{}_1F_1(n+1; \frac{\gamma}{\theta} + n + 1; \frac{-\lambda}{\theta})}{\prod_{j=1}^{n}\left(\frac{\gamma}{\theta} + j\right)\ {}_1F_1(1; \frac{\gamma}{\theta} + 1; \frac{-\lambda}{\theta})}.$$

The failure distribution is obtained by multiplying s on both sides of Eq(5) and using Tauberian theorem after taking the limit as $s \to 0$, we get

$$F = \frac{\gamma}{\gamma + \eta}(1 - \eta\rho_1) \tag{33}$$

■ It is observed that the stationary solution exist only if $\rho_1 < 1$.

5.1. Mean and Variance

Taking limit as $t \to \infty$ in Eq.(26) and in Eq.(27) after putting the values of $c(t) and A(t)$, we get directly an expression for steady state mean and variance i.e.

$$A = \frac{\lambda}{(\theta + \gamma)} \tag{34}$$

$$Var(C) = \frac{1}{2\theta + \gamma}\left[(2\lambda + \theta)A + \lambda - (2\theta + \gamma)A^2\right] \tag{35}$$

6. RELIABILITY AND AVAILABILITY ANALYSIS

The probability that a system perform well without any failure for a given period of time is known as its reliability. In this section, we derive an expression for availability and reliability of the system. Let $Av(t)$ be the probability that a repairable system is available at a given point of time t. Therefore, from Eq(22), the availability of the system is obtained as

$$Av(t) = 1 - F(t)$$

$$= \frac{1}{(\eta + \gamma)}(\gamma + \eta e^{-(\gamma+\eta)t}) + \gamma \int_0^t P_0(x) e^{-(\eta+\gamma)(t-x)} dx, \tag{36}$$

where $P_0(t)$ is given by Eq(17).
Next, we obtain an expression for the average availability of the system i.e.

$$Av(t)^* = \frac{1}{t}\int_0^t Av(y)dy$$

$$= \frac{1}{(\eta + \gamma)}\left(\gamma + \frac{\eta}{(\eta + \gamma)t}[1 - e^{-(\eta+\gamma)t}]\right) + \frac{\gamma}{(\eta + \gamma)t}\int_0^t P_0(y)[1 - e^{-(\eta+\gamma)(t-y)}]dy, \tag{37}$$

If $\eta = 0$, then we get from Eq(22)

$$F(t) = 1 - e^{-\gamma t} - \gamma \int_0^t P_0(x) e^{-\gamma(t-x)} dx$$

Therefore $R(t)$, the system reliability is obtained as

$$R(t) = 1 - F(t)$$

$$= e^{-\gamma t}\left(1 + \gamma \int_0^t e^{\gamma x} P_0(x) dx\right) \tag{38}$$

7. NUMERICAL ANALYSIS

Here, some graphical results are presented to study the behavior of the probability P_0 and $E(C)$ with various parameters i.e. arrival rate λ, catastrophic rate γ and service rate θ.

In fig.(1 to 2) we have plotted the probability P_0 as a function of (λ, γ) and (θ, γ) respectively. We observe that the value of P_0 is decreasing with increasing value of λ and increasing with increasing value of θ. Also, in both the figures P_0 increases with increasing γ i.e. the probability of an empty system increases with the increase in catastrophic rate. Fig.(3 and 4), illustrates that the expected number of customers decreases with the increasing service and catastrophic rates and increases with the corresponding increase in arrival rate.

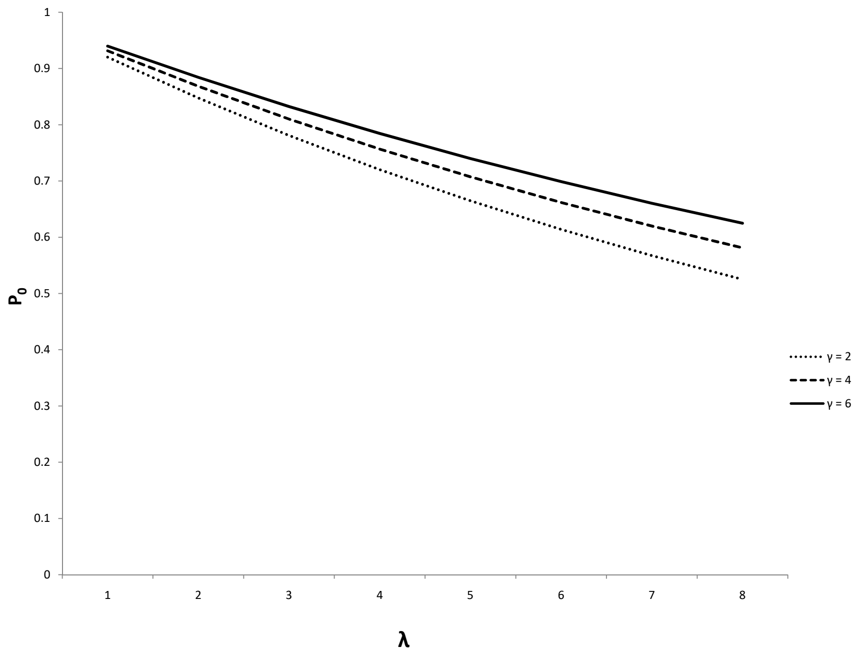

Figure 1: P_0 *as a function of* λ *for* $\theta = 10$

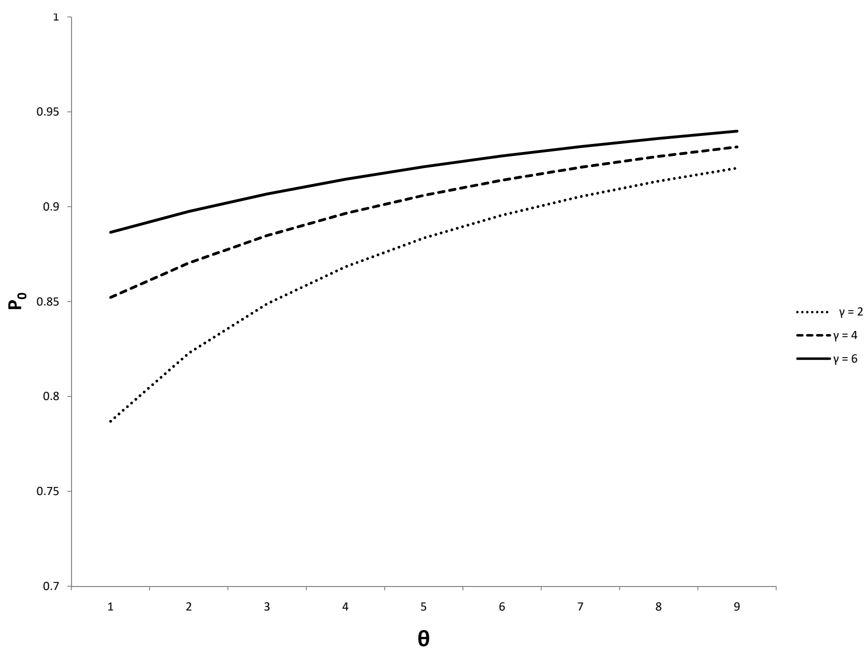

Figure 2: P_0 *as a function of* θ *for* $\lambda = 1$

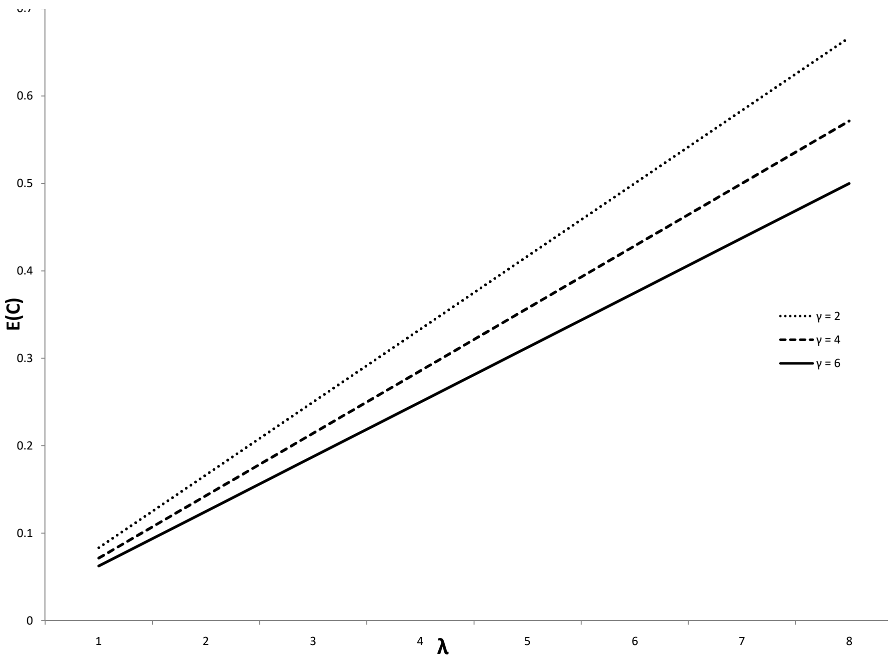

Figure 3: $E(C)$ *as a function of* λ *for* $\theta = 10$

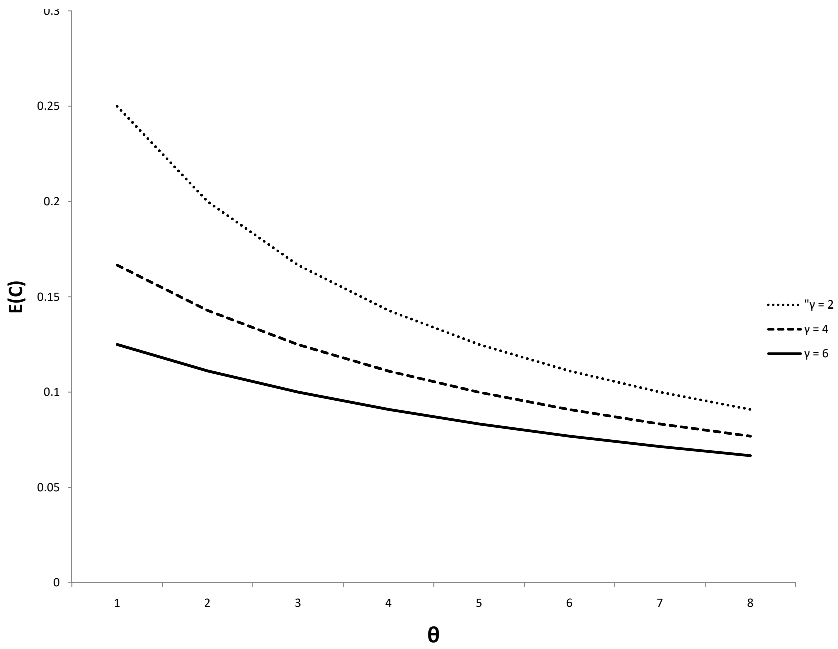

Figure 4: $E(C)$ *as a function of* θ *for* $\lambda = 1$

8. CONCLUSION

In this paper, we have considered an infinite servers Markovian queueing system with catastrophes and repairable servers. The transient and stationary probabilities are obtained analytically. The system availability and reliability are two important characteristics for those queueing system which are failed and repaired. Therefore, these two measures are also investigated for the system. Some graphical results are also added to visualize the model in practical situations.

References

[1] Crescenzo A. Di., Giorno V.,Nobile AG. and Ricciardi LM.(2003). On the M/M/1 queue with catastrophes and its continuous approximation. *Queueing Syst.*, 43: 329–347.

[2] Crescenzo A. Di.,Giorno V.,Nobile AG. and Ricciardi LM.(2008). A note on birth-death processes with catastrophes. *Statistics and Probability Letter*, 78: 2248–2257.

[3] Krishna Kumar B. and Arivudainambi D. (2000). Transient solution of an M/M/1 queue with catastrophes. *Comput. Math.Appl*,40: 1233–1240.

[4] Krishna Kumar B., Krishnamoorthy A., Pavai Madheswari S. and Sadiq Basha S. (2007). Transient analysis of a single server queue with catastrophes, failure and repairs. *Queueing Syst.*, 56: 133–141.

[5] Krishna Kumar B., Vijaykumar A. and Sophia S.(2008) Transient Analysis for State-Dependent Queues with Catastrophes. *Stochastic Analysis and Applications*, 26: 1201–1217.

[6] Linton D. and Purdue P. (1982). An M/G/∞ queue with catastrophes. *Opsearch*, 19(3): 183–186.

[7] Gross D. and Harris C. M. Fundamental of queueing theory, Wiley, Singapore 2003.

[8] Bura Gulab Singh (2019). Transient solution of an $M/M/\infty$ queue with catastrophes. *Communication in Statistics-Theory and Methods*, 48(14): 3439–3450.

[9] Bura Gulab Singh and Gupta Shilpi (2019). Time Dependent Analysis of an $M/M/2/N$ Queue With Catastrophes. *Reliability theory and Applications*, 14:79–86.

[10] Srivastava H. M. and Kashyap B. R. K. Special functions in queuing theory, Academic Press, New York 1982.

[11] Abate J. and Whitt W. (1999). Computing Laplace transforms for numerical inversion via continued fraction. *INFORMS J.Comput*,11:394–405.

[12] Kaliappan K., Gopinath S., Gnanaraj J. and Ramnath K.(2012). Time dependent analysis of an M/M/1/N queue with catastrophes and a repairable server. *OPSEARCH*, 49(1): 39–61.

[13] Lorentzen L. and Waadeland H. Continued fraction with application. Studies in computional mathematics, Elesvier, Amsterdam 1992.

[14] Andrews L. W. Special function of Mathematics for engineers, McGraw Hill., Singapore 1992.

[15] Baykal-Gursoy M. and Xiao W.(2004). Stochastic decomposition in M/M/∞ queues with Markov modulated services rates. *Queueing Syst.*, 48: 75–88.

[16] Mederer M. (2003). Transient solutions of Markov processes and generalized continued fractions. *IMA Journal of Applied Mathematics*, 68: 99–118.

[17] Paz Noam and Yechiali U.(2014). An M/M/1 queue in random environment with disasters. *Asia-Pacific journal of operational research*, 31: 1450016(12 pages).

[18] Jain N. K. and Bura Gulab Singh (2011). M/M/2/N queue subject to modified Binomially distributed catastrophic intensity with restoration time. *Journal of the Indian Statistical Association*, 49(2): 135–147.

[19] Pollett P., Zhang H. and Cairns BJ. (2007). A note on extinction times for the general birth, death and catastrophe process. *Journal of Applied Probability* 44: 566–569.

[20] Wartenhorst P. (1995). N parallel queueing system with server breakdown and repair. *European Journal of Operations Research*, 82: 302–322.

[21] Sudhesh R. (2010). Transient analysis of a queue with system disaster and customer impatience. *Queueing Syst.*, 66: 95–105.

[22] Dimou S. and Economou A.(2013). The single server queue with catastrophes and geometric reneging. *Methodology and Computing in Applied Probability*, 15: 595–621.

[23] Sophia S. and Murali T. S. (2018). Transient Analysis of an Infinite Server Queue with Catastrophes and Server Failures. *International Journal of Pure and Applied Mathematics*, 119(11): 253–261 .

[24] Satty T. L. Elements of queueing theory with applications, McGraw Hill.,New York 1961.

[25] Yechiali U.(2007). Queues with system disaster and impatient customers when system is down. *Queueing Syst.*, 56: 195–202.

[26] Giorno V., Nobile AG. and Spina S. (2014). On some time non-homogeneous queueing systems with catastrophes. *Applied Mathematics and Computation*, 245: 220–234.

[27] Giorno V., Nobile AG. (2013). On a bilateral birth and death process with catastrophes. *In Computer Aided Systems Theory - EUROCAST 2013 (Moreno Diaz R., Pichler F. and Quesada Arencibia A., eds.) Lecture Notes in Computer Science, Springer-Verlag, Berlin,*. 8111: 28–35.