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A NEW ALGORTHIM TO SOLVE FUZZY TRANSPORTATION MODELWITH L-R TYPE HEXAGONAL FUZZY NUMBERS USING RANKING FUNCTION

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Abstract

The transportation problems have much utilization in logistics and supply chains for minimizing costs. In real life circumstances, the limitations of transportation models may not be known absolutely because of unmanageable elements. In the several research papers the transportation costs, availability and demands of the commodity are shown as general fuzzy numbers and L-R flat fuzzy numbers for minimizing the transportation cost using different algorithms. But in this article, proposed the fuzzy costs, supply, and demands of the commodity at origins and destinations are taken as L-R type hexagonal fuzzy numbers for obtaining the optimal solution of unbalanced and balanced fuzzy transportation model by using ranking function to get minimum transportation cost. Here in, the numerical examples are also included. It is very simple to express and execute in real world transportation problem for decision maker.

Keywords: L-R HFN's, ranking function, Transportation problem, balanced transportation problem and unbalanced transportation problem.

1. Introduction

The transportation is the demand of proliferation. All societies are arriving closely not only by ability, tradition and custom but demand and inventory of goods and equipments are transferred from warehouses to retailers through identical vehicles. In actual life, the shipping problem and its optimal solution techniques are used to huge circumstances of practical fields, trades and manufacturing implementations but the constraints are ambiguous and imprecise. To accomplish the aim, the quantity of available supplies and the amount demanded must be known. The transportation models have vast utilizations in logistics and supply chain for minimize the cost. A fuzzy transportation problem contains fuzzy costs, supply and demand of the shipping algorithms. These are all characterized by L-R hexagonal fuzzy numbers. The main concept of the transportation problem is to obtain the optimal solution of the transportation model to minimize the transportation cost of a commodity for gratifying the demand at destinations using the supply at origins. Several literatures are discussed about the transportation problem to reduce the fully fuzzy transportation cost.

The modern process for obtaining fuzzy transportation problem in which the costs, availabilities and demands for the commodity are taken as non negative L-R flat fuzzy numbers by using standard transportation simplex algorithm in [1] and also the results are compare with other existing methods. A novel algorithm is introduced for finding fuzzy transportation problems in [2] by considered that the inventor is unpredictable about the accurate amount of the transportation costs, supply and demands of the commodity are taken as general fuzzy numbers using modern ranking function. A modern algorithm is introduced for finding the special cases of transportation problem in [3] by considering that the decision maker has doubt about the exact values of shipping cost. In the transportation problem the constraints are taken as general fuzzy numbers. The advanced method called as Mehar's method for finding fully fuzzy linear programming problems are presented in [4] in which the constraints are taken as L-R flat fuzzy numbers and the numerical example is also given. The permanent of both inter valued and triangular number fuzzy matrices are defined with examples and few properties, propositions to the permanent of inter valued and triangular number fuzzy matrices are proved in [5]. A new method for solving optimal solution of fully fuzzy linear programming problem is proposed in [6] by utilize the ranking technique with hexagonal fuzzy numbers. The permanent of square L-R hexagonal fuzzy matrix by using various techniques from partial derivatives and derived some properties and constant matrix of the permanent of non square L-R hexagonal fuzzy numbers are investigated in [7]. A fuzzy inventor model with allowable shortage with new L-R hexagonal fuzzy numbers are considered in [8] for obtaining the fuzzy optimal cost and optimal order amount in which the constraints are described by L-R hexagonal fuzzy numbers. The mathematical computation in novel arithmetic operations on α - cut of hexagonal fuzzy numbers, ranking function and their properties are proposed in [9]. Novel procedures of matrix inversion method with the hexagonal fuzzy number matrices for finding fuzzy linear system of equations are established in [10]. [11] has proposed the permanent of a square matrix from Ryser's formula or standard definition. It was calculated two formulas by using in several approaches. One is related to symmetric tensors another one is algebraic method. [12] have compared the optimal solution for reducing the minimum transportation cost of balanced and unbalanced fuzzy transportation problems which is solved by using ranking technique with hexagonal fuzzy numbers. The advanced ranking technique based on the hexagonal fuzzy numbers using centroid of the triangle and rectangle is proposed in [13]. In this method hexagonal fuzzy numbers transferred to crisp number. The constant type-2 triangular fuzzy matrices are proposed in [14]. It is extension of type-2 fuzzy sets whose membership function define in [0,1]. In this, the properties and the examples of constant type-2 triangular fuzzy matrices are verified with the help of type-2 fuzzy sets. Two identities for the estimation of permanents like the formulas of Binet and Minc and of Ryser are obtained in [15]. It is used to reduce in an easy approach. The notion of triangular fuzzy matrices are defined and their new properties, special cases like pure and fuzzy triangular, symmetric, skew-symmetric, singular, semi singular etc. using the elementary operations and main properties of triangular fuzzy matrices are given in [16]. [17] have proposed a new ranking technique for determining the fuzzy transportation model, in which the constraints are taken as trapezoidal fuzzy numbers. These numbers represents the costs, supply and demands for the product. A fuzzy linear programming with hexagonal fuzzy numbers by using simplex method is investigated in [18] for finding the optimal solution and compare with existing algorithms. The concept of the determinant of the permanent of a square matrix is introduced in [19]. In this, it is concentrated on graphs and theorem for the determinant. Also various auxiliary facts are proved.

In view of this article, a novel approach for obtaining the balanced and sun balanced fuzzy transportation problem using ranking system. The fuzzy transportation problem considering that the decision maker is undetermined about the accurate values of shipping cost only even so unpredictability about the supply and demands of the product are not there. In proposed method shipping costs, supply and demands are taken as L-R hexagonal fuzzy numbers. For illustration of proposed method a numerical example has been given.

The structure of this article organized as follows: 2.Preliminaries: A few fundamental definitions, arithmetic operators and Ranking function of L-R hexagonal fuzzy numbers are presented 3. Formulation of transportation model is discussed 4. Proposed method is explained 5. Illustration of the numerical examples are presented 6.Explained the conclusions.

2. Preliminaries

Definition 1: [8] A fuzzy number $\widetilde{P}_h = (a, b, c, d, e, f)$ is said to be hexagonal fuzzy numbers (HFN's). Which are belongs to real numbers and its membership function is as follows

$$\mu_{\widetilde{P}_h}(x) = \begin{cases} 0 & x < a \\ \frac{1}{2}\left(\frac{x-a}{b-a}\right) & a < x < b \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x-b}{c-b}\right) & b < x < c \\ 1 & c < x < d \\ 1 - \frac{1}{2}\left(\frac{x-d}{e-d}\right) & d < x < e \\ \frac{1}{2}\left(\frac{f-x}{f-e}\right) & e < x < f \\ 0 & x > f \end{cases}$$

Definition 2: [8] A fuzzy number $\widetilde{P}_{hLR} = (a, b, c_1, c_2, d_1, d_2)$ is said to be L-R hexagonal fuzzy number. Where $(a, b, c_1, c_2, d_1, d_2)$ are belongs to real numbers satisfying $a \leq b,\ c_1 \geq c_2\ and\ d_1 \geq d_2$ and its membership function is given by

$$\mu_{\widetilde{P}_{hLR}} = \begin{cases} 0 & x \leq a - (c_1 + c_2) \\ 1 - \left(\frac{a-x}{c_1 + c_2}\right) & a - (c_1 + c_2) \leq x \leq a - c_1 \\ 1 - \frac{1}{2}\left(\frac{a-x}{c_1}\right) & a - c_1 \leq x \leq a \\ 1 & a \leq x \leq b \\ 1 + \frac{1}{2}\left(\frac{b-x}{d_1}\right) & b \leq x \leq b + d_1 \\ 1 + \frac{2}{3}\left(\frac{b-x}{d_1 + d_2}\right) & b + d_1 \leq x \leq b + (d_1 + d_2) \\ 0 & x \geq b + (d_1 + d_2) \end{cases}$$

Here a and b are the points with membership value of 1 is known as the flat region of mean value and c_1, c_2, d_1, d_2 are the four different left and right shapes of $\widetilde{P}_{hLR}$ respectively.

Definition 3: [8] An L-R hexagonal fuzzy number is called as symmetric, if the addition of both its shapes are equal, i.e; if $c_1 + c_2 = d_1 + d_2$ and it is denoted as $\widetilde{P}_{hLR} = (a, b, c_1, c_2)_{LR}$

Definition 4: [7] Arithmetic operations on L-R hexagonal fuzzy numbers

Let $\widetilde{P}_{hLR} = (a, b, c_1, c_2, d_1, d_2)_{LR}$ and $Q_{hLR} = (p, q, r_1, r_2, s_1, s_2)_{LR}$ are two L-R hexagonal fuzzy numbers. Then

(i) Addition: $\widetilde{P}_{hLR} + \widetilde{Q}_{hLR} = (a+p, b+q, c_1+r_1, c_2+r_2, d_1+s_1, d_2+s_2)_{LR}$

(ii) Subtraction: $\widetilde{P}_{hLR} - \widetilde{Q}_{hLR} = (a-p, b-q, c_1+r_1, c_2+r_2, d_1+s_1, d_2+s_2)_{LR}$

(iii) Multiplication: $\widetilde{P}_{hLR}\ (\times)\ \widetilde{Q}_{hLR} = \left(\frac{a}{6}\sigma_v, \frac{b}{6}\sigma_v, \frac{c_1}{6}\sigma_v, \frac{c_2}{6}\sigma_v, \frac{d_1}{6}\sigma_v, \frac{d_2}{6}\sigma_v \right)$

Where $\sigma_v = (3p + 3q - r_1 - r_2 + s_1 + s_2)$

(iv) Division: $\widetilde{P}_{hLR}\ (\div)\ \widetilde{Q}_{hLR} = \left(\frac{6a}{\sigma_v}, \frac{6b}{\sigma_v}, \frac{6c_1}{\sigma_v}, \frac{6c_2}{\sigma_v}, \frac{6d_1}{\sigma_v}, \frac{6d_2}{\sigma_v} \right)_{LR}$

If $\sigma_v \neq 0$, Where $\sigma_v = (3p + 3q - r_1 - r_2 + s_1 + s_2)$

(v) Scalar Multiplication: If $k \neq 0$ is scalar, then $k\,\widetilde{P}_{hLR}$ is defined as

$$k\,\widetilde{P}_{hLR} = \begin{cases} (ka, kb, kc_1, kc_2, kd_1, kd_2) & if\ k \geq 0 \\ (kb, ka, -kc_1, -kc_2, -kd_1, -kd_2) & if\ k < 0 \end{cases}$$

Definition 5: [7] If $\widetilde{R}: F(R) \to R$ maps every numbers to real line $F(R)$ represented the set of all hexagonal fuzzy numbers. If R be any linear ranking function, then we write the ranking function is given below

$$\widetilde{R}(\widetilde{P}_{hLR}) = \left(\frac{3a + 3b - c_1 - c_2 + d_1 + d_2}{6} \right)$$

Definition 6: [7] An L-R hexagonal fuzzy number $\widetilde{P}_{hLR}$ is called Zero L-R hexagonal fuzzy number if $\widetilde{P}_{hLR} = (0,0,0,0,0,0,)$. It is denoted as 0_{LR}.

Definition 7: [7] If $\widetilde{R}(\widetilde{P}_{hLR}) = 0$ then $\widetilde{P}_{hLR}$ is called Zero equivalent L-R hexagonal fuzzy number and is denoted as $\widetilde{0}_{LR}$.

Definition 8: [7] An L-R hexagonal fuzzy number $\widetilde{P}_{hLR}$ is called unit L-R hexagonal fuzzy number if $\widetilde{P}_{hLR} = (1,1,0,0,0,0,)$. It is denoted as 1_{LR}.

Definition 9: [7] If $\widetilde{R}(\widetilde{P}_{hLR}) = 1$ then $\widetilde{P}_{hLR}$ is called unit- equivalent L-R hexagonal fuzzy number and it is denoted as $\widetilde{1}_{LR}$.

3. Formulation of Fuzzy Transportation Model

In traditional transportation problem, it is considered that the decision maker is confident about the correct data of shipping cost, availability and demand of the production. In real life utilizations, few constraints in the shipping algorithms may not be known exactly because uncertain elements. For instance, in real world problems are the following positions may appear: Consider a product is to be shipped first time at destination and skilled have no idea about the shipping cost then there exist unpredictability about the shipping cost. For finding transportation algorithms the costs, supply and demands of the commodity are taken as L-R hexagonal fuzzy numbers. The shipping problem, in which a decision maker has a doubt about the exact values of shipping cost from i[th] source to j[th] destination, even so the decision maker confident about the supply and demand of the commodity can be mathematically given as below

Minimize Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \otimes t_{ij}$

$$\text{Subject to} \quad \sum_{j=1}^{n} t_{ij} \le p_i, \qquad i = 1,2,3,\ldots,m$$
$$\sum_{i=1}^{m} t_{ij} \le q_j, \qquad j = 1,2,3,\ldots,n$$
$$t_{ij} \ge 0 \qquad \forall\, i, j$$

where p_i is the total availability of the commodity at i[th] origin, q_j is the total demand of the commodity at j[th] destination, $c_{ij}$is an approximate cost for shipping one unit amount of the commodity from i[th] origin to j[th] destination and $t_{ij}$is the number of units of the commodity that must be shipped from the i[th] origin to j[th] destination or decision variables.

If $\sum_{i=1}^{m} p_i = \sum_{j=1}^{n} q_j$ called balance fuzzy transportation problem and if it is not equal then it is unbalanced fuzzy transportation problem.

4. Proposed methodology

There are so many procedures in the several research papers [1, 2, 4, 3, 17] for obtaining initial basic feasible solution[IBFS] and fuzzy optimal solution of balanced fuzzy transportation problem by using various algorithms or different ranking functions with general fuzzy numbers or L-R flat fuzzy numbers. But in the few articles [6, 12, 18] are finding only optimal solution by using several methods and ranking function with general hexagonal fuzzy numbers.

In this article, proposed a new algorithm for solving IBFS and fuzzy optimal solution of balanced and unbalanced transportation problem by using ranking function, in which the transportation costs, supply and demands are represented by L-R hexagonal fuzzy numbers in place of general hexagonal fuzzy numbers. The procedure and the numerical examples are presented below.

The process is given below for obtaining initial basic feasible solution and optimal solution.

Step: 1 General Hexagonal fuzzy numbers transformed to L-R type hexagonal fuzzy numbers.

Step: 2 Check the transportation table is balanced or unbalanced.

Step: 3 If the table is balanced transportation table then continue the following steps. If the table is unbalanced transportation table then it is converted to balanced transportation table after that continue the below steps.

Step: 4 Construct a transportation table using ranking technique with L-R type hexagonal fuzzy numbers.

Step: 5 Obtain initial basic feasible solution using various following algorithms.

4.1 Generalized fuzzy north-west corner method:

The steps to finding initial basic feasible solution using GFNWCM.

Case 1: Start the allotment from the left hand side of the top most corner (North West corner) wing in the transportation matrix and construct a allotment based on availability and demand.

Case 2: After attain the availability or requirements for that row or column respectively, then delete that row or column and prepare a next table.

Case 3: Continue this way until the all allotments of North West corner is completed.

Case 4: Write all allotments of each cell and compute the IBFS.

4.2 Generalized fuzzy Least cost method:

The process is obtaining for IBFS using GFLCM.

Case 1: Select a least cost of the complete transportation table and allot the smallest supply and demand.

Case 2: Delete that row or column whose supply and demands are completed and construct another table.

Case 3: Repeat this process until all allotments are fulfilled.
Case 4: After all allocations are fulfilled then write the allocations and calculate IBFS.
4.3 Generalized fuzzy Vogel's approximation method:
The steps are finding IBFS using GVAM.
Case 1: Check out the each row and column variance of the fuzzy transportation table.
Case 2: Choose the row or column with largest variation in the fuzzy transportation table and allocate a penalty in second smallest cost wing.
Case 3: Delete that row or column whose supply and demands are allocated i.e. all allotment cells with least cost connected with specified largest row or column variance. Next prepare a new transportation table.
Case 4: Maintain this process until all penalties are over in the entire table then take all the allotments in the matrix.
Case5: Obtain the IBFS or least transportation cost.
Step: 4 find the optimal solution of the transportation problem using modified distribution method. This method gives the lowest cost to the fuzzy transportation problem.
4.4 Generalized fuzzy modified distribution method:
The way to find the optimal solution using GFMODI.
Case 1: Determine the IBFS using 4.1, 4.2 or 4.3 methods.
Case 2: Determine the values u_i *and* v_j of dual variables using $c_{ij} = u_i + v_j$.
Case 3: Find the penalty costs using $\Delta_{ij} = c_{ij} - (u_i + v_j)$.
Case 4: Examine the sign of every penalty. If the penalties of all the vacant cells are either positive or zero then the optimum solution of the given problem is obtained. If the penalty has negative then the optimum solution is not gained. So go to further process for shipping costs are possible.
Case 5: Choose the vacant cell with the lowest negative penalty as the cell to be together with immediate solution.
Case 6: Draw a closed path for the vacant cell pick out in the preceding step. Mark that the right angle rotate in this path is allowed only at settled cells at the actual vacant wing.
Case 7: Mark another plus and minus sign at the vacant cells on the corner points of the closed path with a plus sign at the cell being analyzed.
Case 8: Solve the large number of units that must be transported to the vacant cell. The least point with a negative sign on the closed path denoted as the number of units that can be transported to the existing wing.
Case 9: Add this amount to all the cells on the corner points of the closed loop is noted with positive signs and subtract it from those cells marked with negative signs. In this way, a vacancy cell changed to a settled cell.
Case 10: Repeat this way until fuzzy optimum solution is determined for reducing the least fuzzy transportation cost.

UNBALANCED FUZZY TRANSPORTATION PROBLEM CONVERT IN TO BALANCED FUZZY TRANSPORTATION PROBLEM AS GIVEN:

An Unbalanced fuzzy transportation problem transformed in to Basic fuzzy transportation problem by established a temporary source or a temporary destination which will gives for the hugely availability or the prerequisite cost of shipping a unit from this temporary source or destination to any other area is represented by zero. After transforming the unbalanced fuzzy transportation problem to balanced fuzzy transportation problem, take up the regular process for finding the balanced fuzzy transportation problem. A numerical example for the unbalanced fuzzy transportation problem is presented.

5. Numerical Example

In this segment, two examples are given using proposed method for solving fuzzy transportation problem with L-R hexagonal fuzzy numbers using ranking system.

Example 1: The table 1 occupies general hexagonal fuzzy numbers of transportation costs of the product from several origins to several destinations.

Table 1: *Hexagonal fuzzy numbers of fuzzy transportation costs*

	D_1	D_2	D_3	D_4	Supply
O_1	1 4,16,18, 12,16,20	0,1,2, -1,1,3	7,8,9, 6,8,9	11,13,15, 10,13,16	2,4,6, 1,4,7
O_2	8,11,14, 7,11,15	3,4,5, 2,4,6	5,7,9, 4,7,10	8,10,12, 6,10,14	5,6,7, 4,6,8
O_3	6,8,10, 5,8,11	13,15,17, 12,15,18	7,9,11, 6,9,12	1,2,3, 0,2,4	7,8,9, 5,8,11
Demand	3,4,5, 2,4,6	3,5,7, 1,5,9	10,12,14, 8,12,16	6,7,8 5,7,9	

L-R type hexagonal fuzzy numbers for solving fuzzy transportation costs are presented in table-2.

Table 2: *L-R hexagonal fuzzy numbers of transportation costs*

	D_1	D_2	D_3	D_4	Supply
O_1	18,12,2, 2,4,4	2,-1,1, 1,2,2	9,6,1, 1,2,2	15,10,2, 2,3,3	6,1,2, 2,3,3
O_2	14,7,3, 3,4,4	5,2,1, 1,2,2	9,4,2, 2,3,3	12,6,2, 2,4,4	7,4,1, 1,2,2
O_3	10,5,2, 2,3,3	17,12,2, 2,3,3	11,6,2, 2,3,3	3,0,1 1,2,2	9,5,1, 1,3,3
Demand	5,2,1, 1,2,2	7,1,2, 2,4,4	14,8,2, 2,4,4	8,5,1, 1,2,2	

Table 2 represents unbalanced transportation table. So it is changed to balanced transportation problem introducing dummy origin represented in Table – 3.

Table 3: *The balanced Fuzzy Transportation costs with L-R hexagonal fuzzy numbers*

	D_1	D_2	D_3	D_4	Supply
O_1	18,12,2, 2,4,4	2,-1,1, 1,2,2	9,6,1, 1,2,2	15,10,2, 2,3,3	6,1,2, 2,3,3
O_2	14,7,3, 3,4,4	5,2,1, 1,2,2	9,4,2, 2,3,3	12,6,2, 2,4,4	7,4,1, 1,2,2
O_3	10,5,2, 2,3,3	17,12,2, 2,3,3	11,6,2, 2,3,3	3,0,1 1,2,2	9,5,1, 1,3,3
O_4	0,0,0 0,0,0	0,0,0, 0,0,0	0,0,0 0,0,0	0,0,0 0,0,0	12,6,2, 2,4,4
Demand	5,2,1, 1,2,2	7,1,2, 2,4,4	14,8,2, 2,4,4	8,5,1, 1,2,2	

$\Re(\widetilde{P}_{hLR})$is determined for the fuzzy costs in table - 3using the formula

$$\widetilde{R}(\widetilde{P}_{hLR}) = \left(\frac{3a + 3b - c_1 - c_2 + d_1 + d_2}{6} \right)$$

After implementing the ranking function, the L-R hexagonal Fuzzy transportation problem is shown in Table 4.

6.

Table 4: *After Fuzzy ranking Fuzzy transportation table*

	D_1	D_2	D_3	D_4	Supply
O_1	15.7	0.8	7.8	12.8	3.8
O_2	10.8	3.8	6.8	9.2	5.8
O_3	7.8	14.8	8.8	1.8	7.7
O_4	0	0	0	0	9.7
Demand	3.8	4.7	11.7	6.8	

The various methods used for IBFS and the optimal solution of the fuzzy minimum transportation cost are presented in Table 5.

Table 5: *The optimal solution of the transportation problem*

Methods used for IBFS	IBFS for minimum transportation cost	Number of iterations of fuzzy MODI method for finding the fuzzy optimal solution by using finding IBFS	The total Fuzzy optimal cost
GFNWCM	159	5	61.4
GFLCM	93.4	4	61.4
GFVAM	62.4	2	61.4

Example 2: The table 6 contains general hexagonal fuzzy numbers of transportation costs of the product from various origins to various destinations.

Table 6: *Hexagonal fuzzy numbers of fuzzy transportation costs*

	D_1	D_2	D_3	D_4	Supply
O_1	3,7,11; 15,19,24	13,18,23; 28,33,40	6,13,20; 28,36,45	15,20,25; 31,38,45	7,9,11; 13,16,20
O_2	16,19,24; 29,34,39	3,5,7; 9,10,12	5,7,10; 13,17,21	20,23,26; 30,35,40	6,8,11; 14,19,25
O_3	11,14,17; 21,25,30	7,9,11; 14,18,22	2,3,4; 6,7,9	5,7,8; 11,14,17	9,11,13; 15,18,20
Demand	3,4,5; 6,8,10	3,5,7; 9,12,15	6,7,9; 11,13,16	10,12,14; 16,20,24	

From table 6, the fuzzy transportation costs are changed to LR- type hexagonal fuzzy numbers of fuzzy transportation costs are given in Table 7.

Table 7: *L-R hexagonal fuzzy numbers of transportation costs*

	D_1	D_2	D_3	D_4	Supply
O_1	11,15,4; 4,4,5	23,28,5; 5,5,7	20,28,7; 7,8,9	25,31,5; 5,7,7	11,13,2; 2,3,4
O_2	24,29,5; 3,5,5	7,9,2; 2,1,2	10,13,3; 2,4,4	26,30,3; 3,5,5	11,14,3; 2,5,6
O_3	17,21,3; 3,4,5	11,14,2; 2,4,4	4,6,1; 1,1,2	8,11,1; 2,3,3	13,15,2; 2,3,2

Demand	5,6,1; 1,2,2	7,9,2; 2,3,3	9,11,2; 1,2,3	14,16,2; 2,4,4	

Table 7 represents balanced transportation table. So using the ranking function ($\Re(\widetilde{P}_{hLR})$) is determined for the fuzzy costs in table 7.

$$\Re(\widetilde{P}_{hLR}) = \left(\frac{3a + 3b - c_1 - c_2 + d_1 + d_2}{6} \right)$$

After applying the ranking function, the L-R hexagonal Fuzzy transportation problem is shown in Table 8.

Table 8: *After Fuzzy ranking Fuzzy transportation table*

	D_1	D_2	D_3	D_4	Supply
O_1	13.17	25.83	24.5	28.67	12.5
O_2	26.83	7.83	12	28.67	13.5
O_3	19.5	13.17	5.17	10	14.17
Demand	5.83	8.33	10.33	15.67	

The several methods are used for IBFS and the optimal solution of the fuzzy minimum transportation cost is presented in Table 9.

Table 9: *The optimal solution of the transportation problem*

Methods used for IBFS	IBFS for minimum transportation cost	Number of iterations of fuzzy MODI method for finding the fuzzy optimal solution by using finding IBFS	The total Fuzzy optimal cost
GFNWCM	6 03.67	4	506.36
GFLCM	577.40	2	506.36
GFVAM	509	2	506.36

6. Conclusion

In view of this work, a new method is introduced to gain IBFS and optimum solution of balanced and unbalanced fuzzy transportation model in which the constraints like transportation costs, supply and demand of the product are taken as L-R hexagonal fuzzy numbers. In this, the comparison of UBFTP and BFTP give the optimal transportation cost on LR- hexagonal fuzzy numbers are working to get the fuzzy optimal solutions. We observed that UBFTP get less transportation cost than BFTP. A numerical example shows that the proposed work produce the quality results which are genuine and general in fuzzy environment. It can be also used for other algorithms appearing in real world circumstances.

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