Solving Bi-objective Assignment Problem under Neutrosophic Environment

S. Sandhiya[1], D. Anuradha[2]

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[1, 2] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore, Tamil Nadu 632014, India.
[1]sandhiyanive98@gmail.com , [2]anuradhadhanapal1981@gmail.com
* Correspondence: anuradhadhanapal1981@gmail.com

Abstract

The assignment problem (AP) is a decision-making problem that is used in production planning, industrial organizations, the economy and so on. As the single objective AP is no longer sufficient to handle today's optimization problems, bi-objective AP (BOAP) is considered. This research article introduces BOAP in neutrosophic environment. The neutrosophic BOAP (NBOAP) is formulated by adding the elements of cost matrices with single-valued trapezoidal neutrosophic numbers (SVTrNNs). A new method namely, fixing point approach (FPA) is proposed in this paper. The aim of this study is not only to determine the set of efficient solutions but also to find the optimal compromise solution for NBOAP using FPA. The proposed approach is elucidated with a numerical example and its solutions are plotted in a graph using MATLAB, which demonstrates its efficiency and optimality in practical aspects. This approach is more profitable for decision makers (DMs) and more efficient than other existing approaches because it provides the best optimal compromise solution in a neutrosophic environment.

Keywords: Bi-objective neutrosophic assignment problem, Hungarian method (HM), Fixing point approach, Ideal solution, Efficient solution, Optimal compromise solution.

1. Introduction

AP is one of the most fundamental combinatorial optimization problems which is widely enforced in both mechanized and repair systems and it is one of the most anticipated optimization problems in administration discipline. Many researchers have employed a variety of ways such as HM, linear programming, neural networks, and evolutionary algorithms to solve the AP. As the name suggests, the BOAP consists of two objectives and in solving it, the individual is assigned with a single task in order to optimize the outcomes. Hamou and Mohamed [1] developed the method to construct the set of efficient solutions to MOAP. Sobana and Anuradha [2] primarily focused on determining the set of all solutions to bi-objective interval AP. Przybylski et al. [3] adopted the two-phase technique to solve the BOAP. Bufardi [4] investigated the efficiency of feasible solutions for multi-criteria AP. Adiche et al. [5] developed a hybrid strategy to develop an efficient solution to MOAP. Son et al. [6] developed a compromise programming approach to MOAP. Tilva and Dhodiya [7] modified the algorithm in exponential membership functions for solving MOAP. In some conventional approaches, the parameters are usually defined in an uncertain manner due to the inability of the

DMs to assign accurate values to parameters as they have no idea of the actual value of the parameters. Zadeh [8] introduced the fuzzy sets (FS) which is determined by its membership functions to handle the problems involving imprecise information. Kar et al. [9] developed three different approaches for solving fuzzy BOAP. Pramanik and Biswas [10] analyzed a MOAP with uncertain price, time and ineffectiveness using priority-based fuzzy goal programming technique. Vinoliah et al. [11] proposed a unique approach for the solution of generalised fuzzy AP. Raj et al. [12] investigated an approach involving modified best candidate for solving pentagonal fuzzy AP. In such cases, the results or decisions based on the given data do not seem to be satisfactory. Intuitionistic fuzzy sets (IFS), determined by their membership and non-membership functions and useful in dealing with situations involving uncertainty information were introduced by Atanassov [13]. In such cases, generalisation of the FS eventually failed to deal with difficulties involving imprecise or inconsistent data. To overcome this, Smarandache [14] introduced neutrosophic sets (NS) which is an extension of FS and IFS. The neutrosophic set is determined by the membership, the non-membership and the indeterminacy functions which are independent of one another. The BOAP is examined in a neutrosophical framework to address the truth, indeterminacy and falsity of the data which were caused by issues such as the uncertain magnitude of the problem, imprecise data and inefficient forecasting. Wang et al. [15] introduced the concept of single-valued neutrosophic sets (SVNS) in many real-life situations. A methodology for solving decision-making problems with SVNNs was presented by Deli and Subas [16]. Khalifa [17] proposed a method for solving the MOAP in a neutrosophic environment based on the weighting tchebycheff programme. Khalifa and Pavan kumar [18] developed a neutrosophic AP using the interval-valued trapezoidal neutrosophic number. Bera and Mahapatra [19] proposed a solution methodology for solving AP with neutrosophic costs. Harnpornchai and Wonggattaleekam [20] proposed a neutrosophic set-based relative AP. Risk-Allah et al. [21] developed the neutrosophic compromise programming approach to solve the multi-objective transportation model under neutrosophic environment.

In the literature, many researchers have proposed various methods to solve NBOAP where the solutions are in deterministic form. To fill this gap, we have proposed FPA to determine the set of all efficient solutions and optimal compromise solution for NBOAP in neutrosophic quantities. In traditional BOAP, the DM is supposed to know the precise values of the coefficients of the variables in the objective functions, resources and activities of the product. In real world situations, the precise knowledge of all the parameters of the BOAP may not be possible due to uncontrollable situations. Solution methods based on neutrosophic theory generally have the advantages of not requiring prior prediction of regularities or posterior frequency distributions, as well as that they can handle with unpredictable information based on the subjective judgment of the DM. In general, most of the existing techniques provide only deterministic solution to the optimization problems under neutrosophic environment. Practically, the DM may not have specific, reliable and detailed information regarding these solutions. This motivates us to solve NBOAP under neutrosophic environment. In this paper, the parameters of both objectives for NBOAP are considered as SVTrNNs. The neutrosophic number provides an ideal approach to a decision-making process dealing with the uncertainty of truth, falsity, and an indeterminant state of information. Without converting the given problem into deterministic form, the proposed approach provides the set of all possible solutions for NBOAP. The set of all efficient solutions and a neutrosophic optimal compromise solution can be chosen from the obtained possible solutions of NBOAP. This approach enables the DMs to choose a solution that suits their economic situations and satisfying their goals.

This research article is formulated as follows: In Section 2, basic concepts and preliminaries are presented. In Section 3, assumptions and notations of the proposed NBOAP models are listed. Section 4 proposes solution approach for obtaining neutrosophic optimal compromise solution. Mathematical illustration and graphical interpretations are shown in Section 5. Section 6 illustrates a comparison of the solution method with other existing methods. Section 7 summarizes the conclusions and directions for future research.

2. Preliminaries and Essential Definitions

Some basic definitions related to NS, SVNS and SVTrNNs applied throughout this paper are introduced briefly in this section.

Definition 2.1 Neutrosophic set [14]

Let X be a universe. A NS A over X is defined by $\bar{A}^N = \{\langle x, P_{\bar{A}^N}(x), Q_{\bar{A}^N}(x), R_{\bar{A}^N}(x)\rangle : x \in X\}$ where $P_{\bar{A}^N}, Q_{\bar{A}^N}, R_{\bar{A}^N} : X \to \left]0^-, 3^+\right[$ are called the truth, indeterminacy and falsity membership function of the element $x \in X$ to the set $\bar{A}^N$ with $0^- \le P_{\bar{A}^N}(x) + Q_{\bar{A}^N}(x) + R_{\bar{A}^N}(x) \le 3^+$

Definition 2.2 Single-valued neutrosophic sets [15]

A SVNS $\bar{A}^{SVN}$ of a non-empty set X is defined as follows:
$\bar{A}^{SVN} = \{\langle x, P_{\bar{A}^N}(x), Q_{\bar{A}^N}(x), R_{\bar{A}^N}(x)\rangle : x \in X\}$ where $P_{\bar{A}^N}(x), Q_{\bar{A}^N}(x)$ *and* $R_{\bar{A}^N}(x) \in [0,1]$ for each $x \in X$ and $0 \le P_{\bar{A}^N}(x) + Q_{\bar{A}^N}(x) + R_{\bar{A}^N}(x) \le 3$

Definition 2.3 Single-valued trapezoidal neutrosophic number [16]

Let $\sigma_{\tilde{d}}, \lambda_{\tilde{d}}, \tau_{\tilde{d}} \in [0,1]$ and $p,q,r,s \in {}^\sim$ such that $p \le q \le r \le s$ Then a SVTrNN, $\tilde{d}^N = \langle (p,q,r,s) : \sigma_{\tilde{d}}, \lambda_{\tilde{d}}, \tau_{\tilde{d}}\rangle$ is a special NS on $^\sim$, whose truth membership, indeterminacy membership and falsity membership functions are given below:

$$\mu_{\tilde{d}^N}(x) = \begin{cases} \sigma_{\tilde{d}^N}\left(\dfrac{x-p}{q-p}\right), & p \le x < q \\ \sigma_{\tilde{d}^N}, & q \le x \le r \\ \sigma_{\tilde{d}^N}\left(\dfrac{s-x}{s-r}\right), & r \le x \le s \\ 0, & otherwise \end{cases}$$

$$\delta_{\tilde{d}^N}(x) = \begin{cases} \dfrac{q-x+\lambda_{\tilde{d}^N}(x-p)}{l-k}, & p \le x < q \\ \lambda_{\tilde{d}^N}, & q \le x \le r \\ \dfrac{x-r+\lambda_{\tilde{d}^N}(s-x)}{s-r}, & r \le x \le s \\ 1, & otherwise \end{cases}$$

$$\rho_{\tilde{d}^N}(x)=\begin{cases}\dfrac{q-x+\tau_{\tilde{d}^N}(x-p)}{q-p}, & p\le x<q\\ \tau_{\tilde{d}^N}, & q\le x\le r\\ \dfrac{x-r+\tau_{\tilde{d}^N}(s-x)}{s-r}, & r\le x\le s\\ 1, & otherwise\end{cases}$$

where $\sigma_{\tilde{d}},\lambda_{\tilde{d}}$ *and* $\tau_{\tilde{d}}$ denote the maximum truth, minimum indeterminacy, and minimum falsity membership degrees respectively. A SVTrNN $\tilde{d}^N=\langle (p,q,r,s):\sigma_{\tilde{d}},\lambda_{\tilde{d}},\tau_{\tilde{d}}\rangle$ may be expressed as an ill-defined quantity of p, which is approximately equal to $[q,r]$.

Definition 2.4 Arithmetic operations on SVTrNNs [16]

Let $\tilde{d}^N=\langle (p,q,r,s):\sigma_{\tilde{d}^N},\lambda_{\tilde{d}^N},\tau_{\tilde{d}^N}\rangle$ and $\tilde{g}^N=\langle (p',q',r',s');\eta_{\tilde{g}^N},\mu_{\tilde{g}^N},\theta_{\tilde{g}^N}\rangle$ be two SVTrNNs. The arithmetic operations on $\tilde{d}^N$ and $\tilde{g}^N$ are

1. $\tilde{d}^N+\tilde{g}^N=\langle (p+p',q+q',r+r',s+s');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle$
2. $\tilde{d}^N-\tilde{g}^N=\langle (p-p',q-q',r-r',s-s');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle$
3. $\tilde{d}^N*\tilde{g}^N=\begin{cases}\langle (pp',qq',rr',ss');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s,s'>0\\ \langle (ps',qr',qr',ps');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s<0,s'>0\\ \langle (ss',qq',rr',pp');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s<0,s'<0\end{cases}$
4. $\tilde{p}^N/\tilde{q}^N=\begin{cases}\langle (p/s',q/r',r/q',s/p');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s,s'>0\\ \langle (s/s',r/r',q/q',p/p');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s<0,s'>0\\ \langle (s/p',r/q',q/r',p/s');\sigma_{\tilde{d}^N}\wedge\sigma_{\tilde{g}^N},\lambda_{\tilde{d}^N}\vee\lambda_{\tilde{g}^N},\tau_{\tilde{d}^N}\vee\tau_{\tilde{g}^N}\rangle, s<0,s'<0\end{cases}$
5. $d\tilde{g}^N=h(x)=\begin{cases}\langle (dp,dq,dr,ds);\sigma_{\tilde{g}^N},\lambda_{\tilde{g}^N},\tau_{\tilde{g}^N}\rangle, d>0\\ \langle (ds,dr,dq,dp);\sigma_{\tilde{g}^N},\lambda_{\tilde{g}^N},\tau_{\tilde{g}^N}\rangle, d<0\end{cases}$
6. $\tilde{g}^{N^{-1}}=\langle (1/s',1/r',1/q',1/p');\sigma_{\tilde{g}^N},\lambda_{\tilde{g}^N},\tau_{\tilde{g}^N}\rangle, \tilde{g}^N\neq 0$

Definition 2.5 Efficient solution

A feasible solution U° is said to be an efficient solution to the problem if there exists no other feasible X° such that $Z^{1(N)}(X^\circ)\le Z^{1(N)}(U^\circ)$ and $Z^{2(N)}(X^\circ)\le Z^{2(N)}(U^\circ)$ (or) $Z^{1(N)}(X^\circ)<Z^{1(N)}(U^\circ)$ and $Z^{2(N)}(X^\circ)<Z^{2(N)}(U^\circ)$. Otherwise, it is called non-efficient solution to the problem.

Definition 2.6 Optimal compromise solution

An optimal compromise solution $\left(Z^{1(N)}\left(U^{\circ}\right), Z^{2(N)}\left(V^{\circ}\right)\right)$ is an efficient solution which is closest to the ideal solution $\left(Z^{1(N)}\left(X^{\circ}\right), Z^{2(N)}\left(Y^{\circ}\right)\right)$ where $\tilde{Z}^{1(N)}\left(X^{\circ}\right)$ is an optimal solution to the first objective problem with all constraints and $\tilde{Z}^{2(N)}\left(Y^{\circ}\right)$ is an optimal solution of the second objective problem with all constraints.

3. Description and formulation of NBOAP

For defining a mathematical model, assumption, indices, formulation and related theorem are presented in this section.

3.1. Assumption

Let there be n activities to be completed by n resources whose costs are determined by their specific task. There must be only one to one relation between the activity and the resource.

3.2. Indices

i: Resources.
j: Activities.

3.3. Formulation

In real life, the goal of every DM is to achieve numerous targets at the same time when the products are assigned under neutrosophic environment. This has motivated the researchers to develop NBOAP. In NBOAP, the quantity $(\tilde{x}_{ij}{}^{N})$ is to be assigned from resources $i\ (i=1,2,...,n)$ to activities $j\ (j=1,2,...,n)$ with cost $(\tilde{c}_{ij}{}^{N})$ where $(\tilde{c}_{ij}{}^{N})$ can be shipping cost, shipping time, deterioration cost, consumption of energy or minimizing the risk while shipping goods, etc. The two objectives $\tilde{Z}^{(1)N}$ and $\tilde{Z}^{(2)N}$ are related to shipping cost and deterioration cost during shipping. The single-valued trapezoidal NBOAP (A) can be represented mathematically as follows:

$$\textit{Minimize } \tilde{Z}^{(1)N}\left(x\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}{}^{(1)N}\tilde{x}_{ij}{}^{N}$$

$$\textit{Minimize } \tilde{Z}^{(2)N}\left(x\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{c}_{ij}{}^{(2)N}\tilde{x}_{ij}{}^{N}$$

subject to

$$\sum_{j=1}^{n}\tilde{x}_{ij}{}^{N}=1^{N}, i=1,2,...,n \text{ (only i}^{th}\text{ resource would be assigned to the j}^{th}\text{ activity)} \tag{1}$$

$$\sum_{i=1}^{n}\tilde{x}_{ij}{}^{N}=1^{N}, j=1,2,...,n \text{ (only j}^{th}\text{ activity would be selected by the i}^{th}\text{ resource)} \tag{2}$$

$$\tilde{x}_{ij}^{N}=0^{N} \textit{ or } 1^{N} \textit{ for all } i \textit{ and } j \tag{3}$$

3.4. Parameter

$\tilde{c}_{ij}^{(1)N} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4, c_{ij}'^1, c_{ij}'^2, c_{ij}'^3\right)$ denotes the first objective single valued trapezoidal neutrosophic shipping cost associated with i^{th} resource to j^{th} activity.

$\tilde{c}_{ij}^{(2)N} = \left(c_{ij}^1, c_{ij}^2, c_{ij}^3, c_{ij}^4, c_{ij}'^1, c_{ij}'^2, c_{ij}'^3\right)$ denotes the second objective single valued trapezoidal neutrosophic deterioration cost associated with i^{th} resource to j^{th} activity.

$\tilde{x}_{ij}^{N} = \left(x_{ij}^{1}, x_{ij}^{2}, x_{ij}^{3}, x_{ij}^{4}; x_{ij}'^{1}, x_{ij}'^{2}, x_{ij}'^{3}\right)$ denotes the single valued trapezoidal neutrosophic variable assuming 0 or 1 depending upon the entire assignment of j^{th} activity fulfilled from i^{th} resource.

3.5. Theorem

Let $X^{\circ N} = \left\{x_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ be an optimal solution to (A_1) where

$$\left(A_1\right) \ Minimize \ \tilde{Z}^{(1)N}\left(x\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}^{(1)N} \tilde{x}_{ij}^{N}$$

Subject to (1), (2) and (3)

and $Y^{\circ N} = \left\{y_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ be an optimal solution to (A_2) where

$$\left(A_2\right) \ Minimize \ \tilde{Z}^{(2)N}\left(x\right) = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{c}_{ij}^{(2)N} \tilde{x}_{ij}^{N}$$

Subject to (1), (2) and (3)

Then $U^{\circ N} = \left\{u_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$, obtained from $X^{\circ N}$ (or) $Y^{\circ N}$ is an efficient solution to the problem (A).

Proof:

Let the problem (A_1) be a square matrix of order 'n'.

Since, $X^{\circ N} = \left\{x_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ is an optimal solution of (A_1), $X^{\circ N} = \left\{x_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ is a feasible solution of (A_2).

Clearly, $X^{\circ N} = \left\{x_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ is an efficient solution to the problem (A) which is trivial.

Let the allocated cell with maximum a_{ij} in (A_2) be chosen. Here a_{ij} is placed where the i^{th} row and the j^{th} column intersect.

Deleting the i^{th} row and j^{th} column of (A_2), we obtain a sub-matrix of order (n-1). Let $U^{\circ N} = \left\{u_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ be the solution to the sub-matrix obtained using the HM.

Repeat the procedure for the remaining allocated cells and obtain the solutions to the problem (A).

The procedure can be repeated for all the remaining cells in decreasing order of their magnitude to obtain all the efficient solutions to the problem (A).

By Definition 5, $U^{\circ N} = \left\{u_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ is an efficient solution to the problem (A).

In the same way, an efficient solution to the problem (A) from the optimal solution $Y^{\circ N} = \left\{y_{ij}^{\circ N}, i = 1,2,..,n; j = 1,2,..,n\right\}$ of (A_2) can be obtained.

Hence the theorem.

4. Solution approach

As to solve bi-objective problems under neutrosophic environment, it is necessary to find the optimal compromise solution. Here we have proposed an approach to find the efficient solutions which lead to optimal compromise solution. The following steps are given to proceed with the proposed approach:

Step 1 Consider the given problem (A) with A_1 as first objective neutrosophic AP (FNAP) and A_2 as second objective neutrosophic AP (SNAP).
Step 2 Determine an optimal solution of A_1 and A_2 by HM.
Step 3 Consider the optimal solution of A_1 as a feasible solution of A_2 which is an efficient solution to the problem (A).
Step 4 Select the allocated cell with the highest cost of problem (A_2) and delete its corresponding row and column. Determine the solution for the resultant sub-matrix using HM.
Step 5 Repeat Step 4 and obtain all the solutions for the remaining cells. The same process can be repeated to all the cells in decreasing order of their magnitude.
Step 6 Consider the optimal solution of A_2 as a feasible solution of A_1 which is an efficient solution to the problem (A).
Step 7 Steps 4 and 5 for A_1 are repeated.
Step 8 Combining all the solutions of A obtained using the optimal solutions of A_1 and A_2, the set of all efficient solutions and optimal compromise solution to the problem (A) can be worked out.

5. Application of MMA in NBOAP

Now we interpret the proposed approach to determine the application for the BOAP under neutrosophic environment. A numerical illustration predicts the shipping cost and deterioration cost of the cargoes in cargo ships. The following subsection discusses the procedure for obtaining the application with FPA.

5.1 Numerical illustration

Let three cargo ships be used for shipping goods from one port to another. Any ship can be chosen at random for each journey. Let us assume that there are two objectives to be considered: (i) the total shipping cost of the cargoes must be minimized; and (ii) the total deterioration cost of the cargoes must be minimized. The shipping cost and deterioration cost of the cargoes are represented as SVTrNNs as shown in Table 1.

Table 1

		Ports					
		P_1		P_2		P_3	
Ships	SH_1	$\tilde{c}_{11}^{(1)N}$	$\tilde{c}_{11}^{(2)N}$	$\tilde{c}_{12}^{(1)N}$	$\tilde{c}_{12}^{(2)N}$	$\tilde{c}_{13}^{(1)N}$	$\tilde{c}_{13}^{(2)N}$
	SH_2	$\tilde{c}_{21}^{(1)N}$	$\tilde{c}_{21}^{(2)N}$	$\tilde{c}_{22}^{(1)N}$	$\tilde{c}_{22}^{(2)N}$	$\tilde{c}_{23}^{(1)N}$	$\tilde{c}_{23}^{(2)N}$
	SH_3	$\tilde{c}_{31}^{(1)N}$	$\tilde{c}_{31}^{(2)N}$	$\tilde{c}_{32}^{(1)N}$	$\tilde{c}_{32}^{(2)N}$	$\tilde{c}_{33}^{(1)N}$	$\tilde{c}_{33}^{(2)N}$

where $\tilde{c}_{11}^{(1)N}$ = (14,17,21,28;0.8,0.2,0.6); $\tilde{c}_{11}^{(2)N}$ = (12,20,25,29;0.9,0.3,0.2);
$\tilde{c}_{12}^{(1)N}$ = (13,18,20,24;0.6,0.4,0.5); $\tilde{c}_{12}^{(2)N}$ = (22,25,30,34;0.8,0.2,0.4);
$\tilde{c}_{13}^{(1)N}$ = (20,25,30,35;0.8,0.4,0.2); $\tilde{c}_{13}^{(2)N}$ = (12,18,21,24;0.7,0.4,0.5);
$\tilde{c}_{21}^{(1)N}$ = (15,18,23,30;0.9,0.2,0.3); $\tilde{c}_{21}^{(2)N}$ = (15,17,19,24;0.7,0.2,0.3);
$\tilde{c}_{22}^{(1)N}$ = (11,16,25,28;0.8,0.3,0.2); $\tilde{c}_{22}^{(2)N}$ = (28,32,35,40;0.9,0.3,0.2);
$\tilde{c}_{23}^{(1)N}$ = (14,15,24,26;0.9,0.1,0.1); $\tilde{c}_{23}^{(2)N}$ = (17,18,22,26;0.8,0.2,0.3);
$\tilde{c}_{31}^{(1)N}$ = (11,17,22,25;0.6,0.5,0.4); $\tilde{c}_{31}^{(2)N}$ = (23,27,30,31;0.9,0.3,0.4);
$\tilde{c}_{32}^{(1)N}$ = (12,14,24,30;0.8,0.6,0.2); $\tilde{c}_{32}^{(2)N}$ = (12,19,24,25;0.8,0.5,0.4);
$\tilde{c}_{33}^{(1)N}$ = (14,16,21,23;0.7,0.5,0.3); $\tilde{c}_{33}^{(2)N}$ = (13,18,23,25;0.9,0.2,0.2)

Now, A_1 and A_2 of the problem A are shown in Table 2.

Table 2

	A_1			A_2		
	P_1	P_2	P_3	P_1	P_2	P_3
SH_1	(14,17,21,28; 0.8,0.2,0.6)	(13,18,20,24; 0.6,0.4,0.5)	(20,25,30,35; 0.8,0.4,0.2)	(12,20,25,29; 0.9,0.3,0.2)	(22,25,30,34; 0.8,0.2,0.4)	(12,18,21,24; 0.7,0.4,0.5)
SH_2	(15,18,23,30; 0.9,0.2,0.3)	(11,16,25,28; 0.8,0.3,0.2)	(14,15,24,26; 0.9,0.1,0.1)	(15,17,19,24; 0.7,0.2,0.3)	(28,32,35,40; 0.9,0.3,0.2)	(17,18,22,26; 0.8,0.2,0.3)
SH_3	(11,17,22,25; 0.6,0.5,0.4)	(12,14,24,30; 0.8,0.6,0.2)	(14,16,21,23; 0.7,0.5,0.3)	(23,27,30,31; 0.9,0.3,0.4)	(12,19,24,25; 0.8,0.5,0.4)	(13,18,23,25; 0.9,0.2,0.2)

Using HM, the optimal allotment of A_1 and A_2 are highlighted in Table 3.

Table 3

	A_1			A_2		
	P_1	P_2	P_3	P_1	P_2	P_3
SH_1	(14,17,21,28; 0.8,0.2,0.6)	**(13,18,20,24; 0.6,0.4,0.5)**	(20,25,30,35; 0.8,0.4,0.2)	(12,20,25,29; 0.9,0.3,0.2)	(22,25,30,34; 0.8,0.2,0.4)	**(12,18,21,24; 0.7,0.4,0.5)**
SH_2	(15,18,23,30; 0.9,0.2,0.3)	(11,16,25,28; 0.8,0.3,0.2)	**(14,15,24,26; 0.9,0.1,0.1)**	**(15,17,19,24; 0.7,0.2,0.3)**	(28,32,35,40; 0.9,0.3,0.2)	(17,18,22,26; 0.8,0.2,0.3)
SH_3	**(11,17,22,25; 0.6,0.5,0.4)**	(12,14,24,30; 0.8,0.6,0.2)	(14,16,21,23; 0.7,0.5,0.3)	(23,27,30,31; 0.9,0.3,0.4)	**(12,19,24,25; 0.8,0.5,0.4)**	(13,18,23,25; 0.9,0.2,0.2)

The optimal allotment and the optimal shipping cost of A_1 are $SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$ and $SH_3 \rightarrow P_1$ and (38,50,66,75;0.6,0.5,0.5) respectively. The optimal allotment and the optimal deterioration cost of A_2 are $SH_1 \rightarrow P_3$, $SH_2 \rightarrow P_1$ and $SH_3 \rightarrow P_2$ and (39,54,64,73;0.7,0.5,0.5) respectively.
Now as in Step 3, consider the optimal solution of A_1 as a feasible solution of A_2 as shown in Table 4.

Table 4

	P_1	P_2	P_3
SH_1	(12,20,25,29;0.9,0.3,0.2)	**(22,25,30,34;0.8,0.2,0.4)**	(12,18,21,24;0.7,0.4,0.5)
SH_2	(15,17,19,24;0.7,0.2,0.3)	(28,32,35,40;0.9,0.3,0.2)	**(17,18,22,26;0.8,0.2,0.3)**
SH_3	**(23,27,30,31;0.9,0.3,0.4)**	(12,19,24,25;0.8,0.5,0.4)	(13,18,23,25;0.9,0.2,0.2)

Thus ((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3.0.4)) is the bi-objective value of NBOAP for the feasible allotment $SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$ and $SH_3 \rightarrow P_1$

Using Step 4, the solution for the resultant sub-matrix obtained using HM is shown in Table 5.

Table 5

	P_2	P_3
SH_1	**(22,25,30,34;0.8,0.2,0.4)**	(12,18,21,24;0.7,0.4,0.5)
SH_2	(28,32,35,40;0.9,0.3,0.2)	**(17,18,22,26;0.8,0.2,0.3)**

Thus ((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3.0.4)) is the bi-objective value of NBOAP for the feasible allotment $SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$ and $SH_3 \rightarrow P_1$.

Since all the highest cost cells for A_1 to A_2 are fixed, we terminate the process. Therefore, the set of all possible solutions S_1 from A_1 to A_2 are given in Table 6.

Table 6

S.No	Optimal allotments	Possible solutions (S_1)
1.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_1$, $SH_3 \rightarrow P_3$	((42,52,64,77;0.6,0.5,0.5), (50,60,72,83;0.7,0.2,0.4))
2.	$SH_1 \rightarrow P_1$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_2$	((40,46,69,84;0.8,0.6,0.6), (41,57,71,80;0.8,0.5,0.4))
3.	$SH_1 \rightarrow P_3$, $SH_2 \rightarrow P_1$, $SH_3 \rightarrow P_2$	((47,57,77,95;0.8,0.6,0.3), (39,54,64,73;0.7,0.5,0.5))
4.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_1$	((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3.0.4))

Similarly, by using Steps 6 and 7, we obtain the set of all possible solutions S_2 from A_2 to A_1 as given in Table 7.

Table 7

S.No.	Optimal allotments	Possible solutions (S_2)
1.	$SH_1 \rightarrow P_3$, $SH_2 \rightarrow P_2$, $SH_3 \rightarrow P_1$	((42,58,77,88;0.6,0.5,0.4), (63,77,86,95;0.7,0.4,0.5))
2.	$SH_1 \rightarrow P_1$, $SH_2 \rightarrow P_2$, $SH_3 \rightarrow P_3$	((39,49,67,79;0.7,0.5,0.6), (53,70,83,94;0.9,0.3,0.2))
3.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_1$	((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3,0.4))
4.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_1$, $SH_3 \rightarrow P_3$	((42,52,64,77;0.6,0.5,0.5), (50,60,72,83;0.7,0.2,0.4))
5.	$SH_1 \rightarrow P_1$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_2$	((40,46,69,84;0.8,0.6,0.6), (41,57,71,80;0.8,0.5,0.4))

Now, using Step 8, combine the set of all possible solutions S to the problem (A) obtained from A_1 to A_2 and from A_2 to A_1 as given in Table 8.

Table 8

S.No.	Optimal allotments	Possible solutions ($S=S_1 \cup S_2$)
1.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_1$, $SH_3 \rightarrow P_3$	((42,52,64,77;0.6,0.5,0.5), (50,60,72,83;0.7,0.2,0.4))
2.	$SH_1 \rightarrow P_1$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_2$	((40,46,69,84;0.8,0.6,0.6), (41,57,71,80;0.8,0.5,0.4))
3.	$SH_1 \rightarrow P_3$, $SH_2 \rightarrow P_1$, $SH_3 \rightarrow P_2$	((47,57,77,95;0.8,0.6,0.3), (39,54,64,73;0.7,0.5,0.5))
4.	$SH_1 \rightarrow P_2$, $SH_2 \rightarrow P_3$, $SH_3 \rightarrow P_1$	((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3.0.4))
5.	$SH_1 \rightarrow P_3$, $SH_2 \rightarrow P_2$, $SH_3 \rightarrow P_1$	((42,58,77,88;0.6,0.5,0.4), (63,77,86,95;0.7,0.4,0.5))
6.	$SH_1 \rightarrow P_1$, $SH_2 \rightarrow P_2$, $SH_3 \rightarrow P_3$	((39,49,67,79;0.7,0.5,0.6), (53,70,83,94;0.9,0.3,0.2))

From Table 8, we obtain the set of all efficient solutions and the optimal compromise solution which is closest to the ideal solution. The obtained ideal, efficient and optimal compromise solutions are plotted in a graph using the MATLAB which are shown in Figure 1.

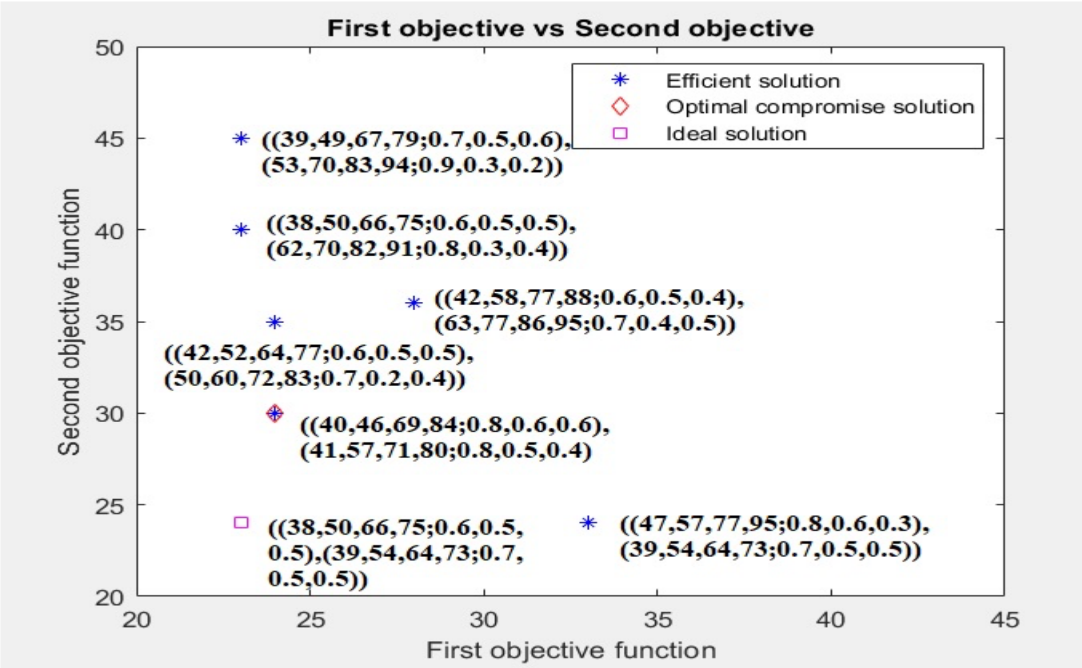

Figure 1: *Graphical representation of all solutions obtained by FPA*

6.Comparative study and discussions

We compare the above example with the existing methods from the literature to prove the efficiency of our approach. Using the method of Risk-Allah et al. [21] we obtain the ideal solution as ((38,50,66,75;0.6,0.5,0.5), (39,54,64,73;0.7,0.5,0.5)) and optimal compromise solution as ((38,50,66,75;0.6,0.5,0.5), (62,70,82,91;0.8,0.3.0.4)) and using the method of Khalifa [17], we obtain the optimal compromise solution as ((47,57,77,95;0.8,0.6,0.3), (39,54,64,73;0.7,0.5,0.5)). Using our proposed approach, we obtain the ideal solution as ((38,50,66,75;0.6,0.5,0.5), (39,54,64,73;0.7,0.5,0.5)), efficient solutions as ((42,52,64,77;0.6,0.5,0.5), (50,60,72,83;0.7,0.2,0.4)); ((40,46,69,84;0.8,0.6,0.6),(41,57,71,80;0.8,0.5,0.4));((47,57,77,95;0.8,0.6,0.3),(39,54,64,73;0.7,0.5,0.5));((38,50,66,75;0.6,0.5,0.5),(62,70,82,91;0.8,0.3.0.4));((42,58,77,88;0.6,0.5,0.4),(63,77,86,95;0.7,0.4,0.5));((39,49,67,79;0.7,0.5,0.6), (53,70,83,94;0.9,0.3,0.2)) and optimal compromise solution as ((40,46,69,84;0.8,0.6,0.6), (41,57,71,80;0.8,0.5,0.4)).

In this comparative study, we find out that our proposed approach provides the set of all efficient solutions and the best optimal compromise solution of the given problem as compared to the other two approaches which are clearly shown in Table 9 and Figure 2.

Table 9: *Comparisons between the proposed approach with other existing approaches*

Methods	Ideal solution	Efficient solutions	Optimal compromise solution
FPA	✓	✓	✓
Rizk-Allah et al. [21]	✓	-	✓
Khalifa [17]	-	-	✓

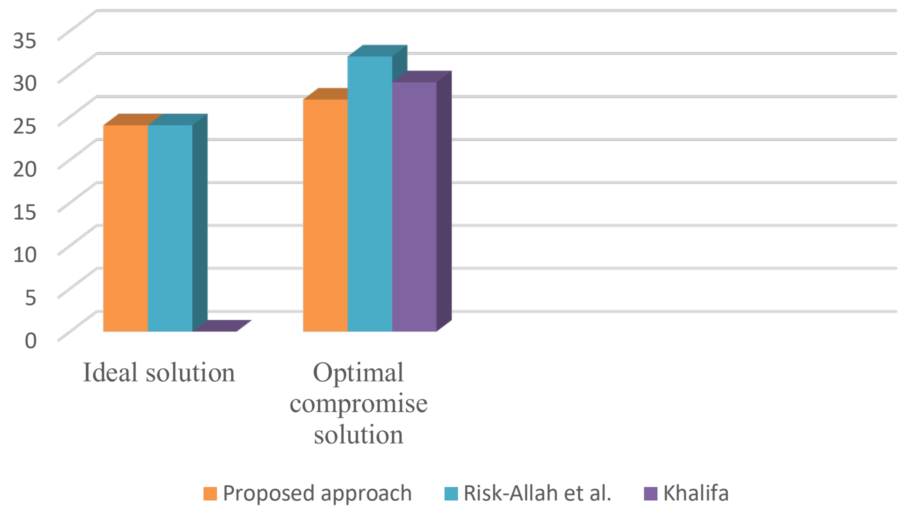

Figure 2: *Comparison between proposed and existing approaches*

7.Conclusions and future scopes

In this research article, the parameters of the model are expressed as SVTrNNs which improve the capacity of DM to make more realistic decisions. The main advantage of our approach is that the efficient solutions and the optimal compromise solution we obtain are neutrosophic quantities rather than deterministic values and they provide greater flexibility to the DM. Our proposed approach provides the best optimal compromise solution to the given problem as compared to the other two approaches which are shown in graph. When the DM deals with a range of logistical issues, the set of all efficient solutions obtained by our proposed approach can serve as a valuable tool. Though our approach analyses the solutions of NBOAP in the best way, there may be some limitations in predicting the solutions of qualitative and complex data due to the computational complexity in handling higher dimensional problems, they can be resolved using evolutionary algorithms. In the future research, one may incorporate this concept in neutrosophic bi-objective fractional assignment problem. The solution approach presented in this article can be aptly used by the DM when dealing with type-2 fuzzy parameters. Furthermore, in areas such as management science, finance, etc. wherever the assignment problems arise in neutrosophic environment, this solution approach will be a great resource.

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